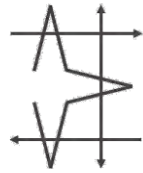


Neutrosophic Sets and Systems

<A> <neutA> <antiA>



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"Neutrosophic Sets and Systems" has been created for publications on advanced studies in neutrosophy, neutrosophic set, neutrosophic logic, neutrosophic probability, neutrosophic statistics that started in 1995 and their applications in any field, such as the neutrosophic structures developed in algebra, geometry, topology, etc.

The submitted papers should be professional, in good English, containing a brief review of a problem and obtained results.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea $\langle A \rangle$ together with its opposite or negation $\langle \text{anti}A \rangle$ and with their spectrum of neutralities $\langle \text{neut}A \rangle$ in between them (i.e. notions or ideas supporting neither $\langle A \rangle$ nor $\langle \text{anti}A \rangle$). The $\langle \text{neut}A \rangle$ and $\langle \text{anti}A \rangle$ ideas together are referred to as $\langle \text{non}A \rangle$.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on $\langle A \rangle$ and $\langle \text{anti}A \rangle$ only).

According to this theory every idea $\langle A \rangle$ tends to be neutralized and balanced by $\langle \text{anti}A \rangle$ and $\langle \text{non}A \rangle$ ideas - as a state of equilibrium.

In a classical way $\langle A \rangle$, $\langle \text{neut}A \rangle$, $\langle \text{anti}A \rangle$ are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that $\langle A \rangle$, $\langle \text{neut}A \rangle$, $\langle \text{anti}A \rangle$ (and $\langle \text{non}A \rangle$ of course) have common parts two by two, or even all three of them as well.

Neutrosophic Set and *Neutrosophic Logic* are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F), where T, I, F are standard or non-standard subsets of $]0, 1[$.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.

Neutrosophic Statistics is a generalization of the classical statistics.

What distinguishes the neutrosophics from other fields is the $\langle \text{neut}A \rangle$, which means neither $\langle A \rangle$ nor $\langle \text{anti}A \rangle$.

$\langle \text{neut}A \rangle$, which of course depends on $\langle A \rangle$, can be indeterminacy, neutrality, tie game, unknown, contradiction, ignorance, imprecision, etc.

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Neutrosophy applied in Physics: Extension from Half-Ice Half-Fire Material State and Phase to Partial-Ice Partial-Fire Material State and Phase

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Abstract: The recent groundbreaking discovery of a ‘Half-Ice, Half-Fire’ phase of matter at Brookhaven National Laboratory, by W. Yin & A. M. Tsvelik [3], has unveiled an intriguing new dimension in condensed matter physics. This novel state, characterized by a unique coexistence of ordered ‘cold’ spins and disordered ‘hot’ spins within a ferrimagnetic material, challenges our conventional understanding of phase transitions. This short note explores the implications of this discovery through the lens of neutrosophy and neutrosophic physics, and extends the concept to a more nuanced ‘Partial-Ice, Partial-Fire’ state, demonstrating the potential for richer, more complex interpretations of physical phenomena.

Keywords: Neutrosophy; Neutrosophic Physics; Neutrosophic Logic; Neutrosophic Sets; Indeterminacy; Thermodynamics; Condensed Matter Physics; Spintronics; Quantum Computing; Materials Science; Half-Ice, Half-Fire; Partial-Ice, Partial-Fire; Ferrimagnet; Electron Spins; Phase Transition.

1. Introduction

Neutrosophy [1] posits that any idea or entity (A) coexists with its opposite (antiA), and a neutral (neutA) component that is neither A nor antiA. Neutrosophic logic proposes this triadic structure, accounting for degrees of truth, falsehood, and indeterminacy. Applied to physics, this yields neutrosophic physics [2] — a paradigm capable of describing phenomena imbued with uncertainty, ambiguity, or dual nature.

The ‘Half-Ice, Half-Fire’ [3] state exemplifies this paradigm. In this ferrimagnetic state, ‘ice’ denotes spin order and low entropy, while ‘fire’ signifies spin disorder and high entropy. These coexisting states mirror the neutrosophic framework: A (‘ice’), antiA (‘fire’), and neutA (indeterminate co-presence of ‘ice’ and ‘fire’).

1.1. Partial-Ice State

This state can be thought of as a system where water exists in a mixed state of solid (ice) and liquid (water). The properties of this state can vary, exhibiting characteristics of both phases depending on the temperature and pressure conditions.

1.2. Partial-Fire State

Similarly, this could represent a state where a material exhibits both combustion (fire) and non-combustion properties. This might occur in certain chemical reactions or materials that are on the verge of ignition.

2. Implications

2.1. Thermodynamic Properties

In a neutrosophic context, these states could be characterized by non-binary properties, such as temperature not being a single value but a range that reflects the coexistence of phases. For instance, the latent heat of fusion could be represented as a neutrosophic quantity, where the exact state is not fully defined.

2.2. Physical Behavior

Materials in these states might display behaviors that are not easily categorized. For example, a partial-ice state might have a varying density and thermal conductivity that changes with the proportion of ice to water.

2.3. Applications

Understanding these mixed states could have implications in materials science, particularly in the development of new materials that can exist in transitional phases, such as phase-change materials used in thermal management systems.

3. From 'Half-Ice, Half-Fire' to 'Partial-Ice, Partial-Fire'

The 'Half-Ice, Half-Fire' state, discovered at Brookhaven National Laboratory, represents a groundbreaking phenomenon in condensed matter physics, wherein two subsystems of atomic spins within a ferrimagnetic material exhibit radically different behaviors simultaneously under the same temperature. One group of spins becomes highly ordered, behaving as if in a frozen, low-energy state ('ice'), while the other remains fully disordered, fluctuating chaotically like in a high-temperature, thermally agitated state ('fire').

This coexistence of magnetic order and disorder within a single material and thermal condition defies traditional thermodynamic expectations, which typically predict uniform responses to temperature changes. The phenomenon arises from a quantum-level interaction between the spin subsystems: one resists thermal agitation and locks into an ordered configuration, while the other becomes dynamically unstable.

This paradoxical behavior not only introduces a novel state of matter but also calls for new interpretative frameworks — such as neutrosophic logic — that can account for the simultaneous presence of opposites and indeterminacy within physical systems.

As such, the 'Half-Ice, Half-Fire' state opens exciting possibilities for material science, especially in fields that require precise control over quantum or magnetic states, such as spintronics and quantum computing.

The "half-ice, half-fire" phase of matter provides a physical example of this:

- "Ice" can be seen as representing a state of order (a form of "truth" or certainty).
- "Fire" can be seen as a state of disorder (a form of "falsehood" or uncertainty).
- The fact that they coexist demonstrates a "neutral" or "indeterminate" state, where both opposing characteristics are present.

Therefore, this physical phenomenon, provides a real world example of the theoretical neutrosophic concepts. It shows that in physics, like in philosophy, that there are states of being that contain opposing characteristics.

4. Bridge between Philosophy and Science

4.1. Coexistence of Opposites

- *Half-ice, half-fire*: Physical manifestation of two opposing phases.
- *Neutrosophy*: Intellectual framework for understanding how opposites and neutralities coexist.

4.2. Indeterminacy

- The physical system is not completely one thing or the other — there is a *superposition* or a *dynamic balance*.
- Neutrosophy embraces this *indeterminacy* (I) as a fundamental component of reality and truth.

4.3. Beyond Binary

- Classical physics wants systems to be one phase or another.

Neutrosophy, like quantum mechanics, accepts *contradiction* and *non-binariness* — and gives it a structured language.

5. Neutrosophic Interpretation

We propose an extension from the discrete notion of ‘half’ to a continuous range, i.e. ‘partial’: ‘partial ice, partial fire’. This allows the modeling of hybrid states via degrees of membership in ice-like or fire-like behavior, see Figure 1.

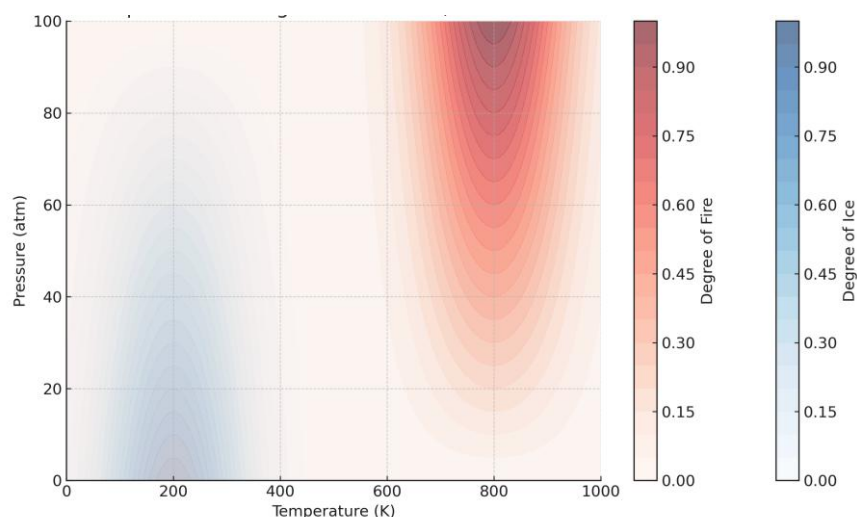


Figure 1. Neutrosophic phase diagram showing degrees of ice (blue) and fire (red) as functions of temperature and pressure.

Such degrees are not binary but exist on a spectrum, best captured by neutrosophic sets I_x , F_x , and N_x , denoting the degrees of ice, fire, and indeterminacy respectively, see Figure 2.

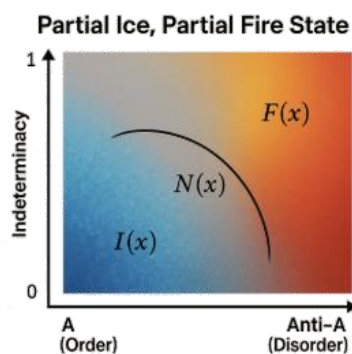


Figure 2. Spectrum of degrees captured by neutrosophic sets: $I(x)$, $F(x)$, and $N(x)$.

Therefore, using neutrosophic sets and logic, we define:

I_x : Degree of membership in the 'ice' state,

F_x : Degree of membership in the 'fire' state,

N_x : Degree of indeterminacy,

where $0 \leq I_x + F_x + N_x \leq 3$.

6. State of Matter vs. Phase of Matter

The terms 'state of matter' and 'phase of matter' are often used interchangeably in everyday language, but in physics and material science, they refer to distinct concepts.

A state of matter refers to the broad categories that describe the general physical form of a substance, based on the arrangement and energy of its constituent particles. The classical states are solid, liquid, gas, and plasma, and more recently, exotic states such as Bose–Einstein condensates, quark-gluon plasma, and half-ice, half-fire states have also been recognized. Each state is characterized by a particular degree of particle freedom, density, compressibility, and energy distribution.

A phase, on the other hand, refers to a distinct and homogeneous region within a material that has uniform physical and chemical properties, such as density, magnetization, or crystal structure. A single state of matter can include multiple phases.

For example, within the solid state, ice and dry ice (solid CO_2) are distinct phases due to differences in molecular structure and composition. Similarly, within a liquid state, oil and water form two immiscible phases because they do not mix, despite both being liquids.

7. Examples

A common and intuitive example is *water*: In a sealed container at the melting point, water can simultaneously exist as solid (ice) and liquid (water). Both are in the same state (condensed matter), but represent two different phases with distinct structures and energy levels.

Likewise, in *alloys* or *magnetic materials*, different phases can emerge within the same state due to variations in microscopic ordering or composition, such as in ferrimagnetic systems where sublattices may exhibit contrasting spin behaviors.

Understanding this distinction is crucial when analyzing complex systems, especially those that defy classical categorizations, such as the newly observed 'half-ice, half fire' matter state, which exhibits multiple phases with contradictory characteristics within a singular state.

8. Hybrid Material States and Phases: α -Ice and β -Fire

The conceptual models of α -Ice and β -Fire are proposed to describe intermediate or hybrid material states and phases that defy classical categorizations. These terms build on the neutrosophic logic framework, allowing for continuous degrees of order and disorder, rather than discrete, mutually exclusive states.

Let α and β be real numbers in the open interval $(0, 1)$, representing the degree to which a material exhibits ice-like (ordered) or fire-like (disordered) characteristics, respectively..

8.1. α -Ice State

An α -Ice state refers to a partially ordered configuration within a material. It may represent a mixture of crystalline and amorphous regions, or a spatial gradient between solid and liquid properties. Physically, this could correspond to conditions near a phase boundary—such as a supercooled liquid beginning to crystallize, or slush (a mixture of ice and water) in a metastable equilibrium. The value of α quantifies how "close" the system is to full solidification.

8.2. β -Fire State

A β -Fire state captures partial thermal excitation or combustion behavior. In physical systems, this might describe materials undergoing incipient ignition, chemical pre-reactions, or localized

hotspots within an otherwise stable medium. In magnetism, it could reflect the partial disruption of spin order due to thermal fluctuations, as observed in the 'half-ice, half-fire' state. A β approaching 1 implies high levels of energetic disorder; conversely, a β close to 0 indicates predominantly stable or non-excited behavior.

These hybrid states are particularly valuable in modeling non-equilibrium systems, mesophases, or complex adaptive materials that display coexisting or competing behaviors across different regions or scales.

9. Practical Relevance

In *materials science*, understanding and controlling α -Ice states is crucial for phase-change materials used in memory storage and thermal regulation.

In *energy systems*, β -Fire models help describe combustion efficiency, ignition control, or partial reactions in plasma and catalytic environments.

In *spintronics*, where the control of spin order and disorder is key to information processing, this hybrid model provides a richer vocabulary for describing partially coherent quantum states.

By quantifying α and β , researchers can construct *material phase maps* that describe gradients of behavior rather than sharp boundaries, aligning well with both experimental observations and the neutrosophic logic of partial, contradictory, and uncertain realities..

10. Conclusion

Analyzing the 'Half-Ice, Half-Fire' discovery through neutrosophic principles reveals new dimensions in state and phase theories. The "half-ice, half-fire" state and phase of matter embodies the neutrosophic idea of coexisting opposites and indeterminacy. It provides a tangible example of how seemingly contradictory properties can exist simultaneously within a system. AI was partially used in this paper. Extending this interpretation to partial states and phases enhances our capacity to model hybrid phenomena, supporting advancements in materials science, thermodynamics, and information systems. Neutrosophy enables a more nuanced understanding of physical reality.

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Triangular OverConorm with MCDM Methodology for Association Between Speaking Proficiency and Self-Assurance in English Language and Literature

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Abstract: A multifaceted assessment methodology for English language and literature courses in postsecondary educational institutions is presented in this study. The model incorporates qualitative and quantitative indicators across six fundamental criteria—Curriculum Design, Instructional Quality, Student Engagement, Research Output, Cultural Integration, and Language Proficiency Development—in recognition of the variety of educational techniques. These criteria are used to evaluate the efficacy of 18 options that reflect different departments, instructional approaches, or institutions. The multi-criteria decision-making (MCDM) methodology is used to deal with different criteria and alternatives. This study uses the COPRAS methodology to rank the alternatives. The triangular OverConorm is used in this study to deal with interval values of each criterion and alternatives. The sensitivity analysis is conducted to show the ranks of alternatives under different cases. We proposed six cases in weights criteria. The results contribute to the continuous improvement of the quality of humanities education by illuminating important program strengths and areas for improvement.

Keywords: Triangular OverConorm; MCDM Methodology; Self-Assurance; English Language and Literature.

1. Introduction

Speaking is a crucial supplementary tool in communication and one of the four fundamental language skills, the others being listening, writing, reading, and speaking—that is of paramount importance in day-to-day living. Speaking has numerous obstacles as a practical outcome, including psychological, physiological, physical, systematic, and attitudinal ones. While some are

external, such as physiological, physical, or systemic barriers, others are internal, such as psychological and attitude barriers. Lack of confidence can be categorized as psychological or attitude hurdles that make it difficult for learners to communicate in their own tongue or even in a foreign language[1], [2].

McIntyre proposed that a learner's propensity to speak in a foreign language is strongly influenced by their level of self-confidence. High levels of self-confidence are necessary for speaking, which entails being certain of one's actions and avoiding shyness and aggression in social settings. Therefore, self-confidence is one of the keys that unlock some significant communication hurdles to being a native-like speaker of a foreign language[3], [4].

Studies on language proficiency and instruction are overrepresented in the body of research on English Language Teaching (ELT). Even while speaking necessitates physiological competence and a physically adequate setting, it would be more reasonable to classify the studies under the headings of speaking skill and self-confidence[5], [6].

Self-confidence is characterized by a person's acceptance of his own skills, love for himself, and emotional intelligence. The researcher breaks down self-confidence into two subcategories, extrinsic and intrinsic, and goes on to explain these in the following manner: Intrinsic self-confidence refers to the feelings and ideas that people have about being happy or at peace with themselves. Positive thinking, articulating specific goals, self-love, self-knowledge, and self-esteem are the components of intrinsic self-confidence. Behavior and attitude toward other people are also examples of extrinsic self-confidence. Emotional regulation and communication are building blocks of extrinsic self-confidence. Extrinsic and intrinsic self-confidence were identified as two factors in a study to create a self-confidence measure[7], [8].

There are two categories of self-confidence: inner confidence and outer confidence. Love for oneself, self-awareness, goal-setting, and optimistic thinking are the components that make up inner confidence. An individual's thoughts and emotions that demonstrate how happy and content they are with themselves are known as inner confidence. Communication and the ability to regulate emotions are the components that make up exterior confidence. People with high levels of inner confidence are content with who they are and have a high sense of self-worths[9], [10].

Speaking, a generally overlooked aspect of language instruction, must be one of the primary priorities in an EFL context as communication and travel opportunities increase and national borders are eliminated. In communication, particularly speaking, self-confidence is becoming more and more important as a facilitator of accurate and fluent speech in both the mother tongue and the target language.

Considering all the previously mentioned factors, the current study intends to retest whether a relationship between speaking ability and self-confidence exists. Some measures to lower or

increase self-confidence in accordance with the research findings could be implemented after examining the correlation.

Then, using user-defined importance weights or other objective approaches applied to the criteria, multi-criteria decision-making (MCDM) algorithms choose the best option from the decision matrix. Several MCDM strategies have been used in numerous studies to determine parameter configurations for different machining performance goals[11], [12].

To ascertain the ranking agreement of various MCDM methodologies, comparative studies employ methods such as Spearman's rank correlation coefficient. These strategies support the validity of these techniques in determining the best machining parameters for a range of materials and performance requirements[13], [14]. As a result, MCDM approaches have shown success; they often use experimental data and methods like as ANOVA to evaluate how cutting parameters affect performance criteria. These methods are capable of efficiently determining appropriate machining parameters[15], [16].

This study suggests a novel framework for determining the Association between speaking proficiency and self-assurance in English language and literature by utilizing COPRAS MCDM approaches. The performance of MCDM parts made using the determined ideal criteria will be compared to a baseline set of parameters to assess the framework. This study's scope includes the following: Experiments and data collection are the first steps in the process. The average method is used to show the best creation. MCDM methods are used to achieve the appropriate optimal solution, considering COPRAS techniques[17], [18]. The triangular OverConorm is used to integrate the different decision matrix between the criteria and alternatives.

The suggested approach offers students substantial advantages by determining the ideal alternative and criterion. This method balances several performance goals to improve Association between speaking proficiency and self-assurance in English language and literature. The results also offer useful information for future studies on English language and literature.

2. Triangular OverConorm

We show the definitions of the triangular OverConorm in this section[19].

We can define the triangular OverConorm on the interval $[0, \theta]$, $\theta > 1$

$$T_{overconorm}: [0, \theta] \times [0, \theta] \rightarrow [0, \theta] \text{ where } \theta > 1 \quad (1)$$

$$\text{And for any } (G, H) \in [0, \theta] \times [0, \theta] \quad (2)$$

$$T_{overconorm}(G, H) = T_{overconorm}(H, G), \text{ commutativity} \quad (3)$$

$$T_{overconorm}(G, T_{overconorm}(H, I)) = T_{overconorm}(T_{overconorm}(G, H), I) \text{ associativity} \quad (4)$$

$$\text{if } H \leq I, \text{ then } T_{overconorm}(G, H) \leq T_{overconorm}(G, I) \text{ monotonicity} \quad (5)$$

$$T_{overconorm}(G, 0) = G, \text{ or the neutral elements is zero} \quad (6)$$

$$T_{overconorm}(G, H) = \max(G, H) \quad (7)$$

$$C(x) \circ^{\wedge} c(y) = \min\{c(x), c(y)\}$$

$$C(x) \circ^{\vee} c(y) = \max\{c(x), c(y)\}$$

The algebraic product T-norm and T-conorm

$$T - norm(x, y) = x \cdot y$$

$$T - conorm(x, y) = x + y - x \cdot y$$

The bounded $T - norm(x, y) = x \cdot y$

$$T - norm(x, y) = \max\{0, x + y - 1\}$$

$$T - conorm(x, y) = \min\{1, x + y\}$$

$$c(x) \circ^{\wedge} c(y) = \max\{\sigma, c(x) + c(y) - \rho\}$$

$$c(x) \circ^{\vee} c(y) = \min\{\rho, c(x) + c(y)\}$$

The neutrosophic offcomplement of T,I,F can be determined as:

$$< F, \sigma_I + \rho_I - I, T >$$

$$< \sigma_T + \rho_T - T, I, \sigma_F + \rho_F - F >$$

$$< \sigma_T + \rho_T - T, \sigma_I + \rho_I - I, \sigma_F + \rho_F - F >$$

The fuzzy complement

$$c(T) = 1 - T$$

$$c_0(T) = \sigma_T + \rho_T - T$$

$$c_0(I) = \sigma_I + \rho_I - I$$

$$c_0(F) = \sigma_F + \rho_F - F$$

We can define the neutrosophic N-offnorm and neutrosophic N-offconorm by three functions such as:

$$T_o \subseteq [\sigma_T, \rho_T]$$

$$I_o \subseteq [\sigma_I, \rho_I]$$

$$F_o \subseteq [\sigma_F, \rho_F]$$

$$\sigma_T, \sigma_I, \sigma_F, \leq 0$$

$$\rho_T, \rho_I, \rho_F \geq 1$$

Then we show the steps of the COPRAS method to rank the alternatives.

Create the decision matrix.

The decision matrix is created using the interval between 0 and greater than 1. Then we use the $T_{overconorm}$ to combine the decision matrix.

Compute the criteria weights by the average method.

Compute the normalized decision matrix

$$H_{ij} = \frac{x_{ij}}{\sum_{i=1}^n x_{ij}} \quad (8)$$

Compute the weighted decision matrix.

$$K_{ij} = H_{ij}w_j \quad (9)$$

The mini and max numbers are computed for positive and cost criteria.

$$Q_{+i} = \sum_{j=1}^g K_{ij} \quad (10)$$

$$Q_{-i} = \sum_{j=g+1}^n K_{ij} \quad (11)$$

Compute the relative numbers

$$R_o = Q_{+i} + \frac{\sum_{i=1}^m Q_{-i}}{Q_{-i} \sum_{i=1}^m \frac{1}{Q_{-i}}} \quad (12)$$

Rank the alternatives.

3. Case Study

This study proposed a case study to show the validation of the proposed approach. We use six criteria and 18 alternatives such as:

- Curriculum Design (C1):
- Assesses the comprehensiveness, coherence, and modern relevance of offerings.
- Instructional Quality (C2):
- Evaluates the pedagogical effectiveness and innovation of faculty teaching methods.
- Student Engagement (C3):
- Measures classroom participation, discussion quality, and learner motivation.
- Research Output (C4):
- Considers faculty and student involvement in academic research, publications, and presentations.
- Cultural Integration (C5):
- Assesses how well the program incorporates global and local cultural texts, contexts, and perspectives.
- Language Proficiency Development (C6):
- Focuses on student progress in critical language skills: reading, writing, speaking, and listening.
- ✓ A1: Program A from University 1

- ✓ A2: Program B from University 2
- ✓ A3: Project-based Learning Model
- ✓ A4: Literature Focused Curriculum
- ✓ A5: Communicative Language Teaching Approach
- ✓ A6: Hybrid Online-English Literature Program
- ✓ A7: Multicultural Literature Emphasis Program
- ✓ A8: Performance and Drama Integration Curriculum
- ✓ A9: British Literature Specialization
- ✓ A10: American Literature Specialization
- ✓ A11: Postcolonial Studies Emphasis
- ✓ A12: Linguistics-Integrated Literature Program
- ✓ A13: Critical Theory-Driven Instruction
- ✓ A14: Undergraduate Honors Program
- ✓ A15: Experiential Fieldwork and Internships Focus
- ✓ A16: Research-Intensive Graduate Program
- ✓ A17: Interdisciplinary English Studies Track
- ✓ A18: International Exchange-Based English Program

Three experts create the decision matrix between the criteria and alternatives as shown in Table 1. These decision matrices are combined using the $T_{overconorm}$. Then we compute the criteria weights.

C1 – (Weight: 0.1682)

This criterion holds significant weight, indicating that the structure, coherence, and relevance of the curriculum are crucial in evaluating program effectiveness. Emphasis is placed on whether the curriculum covers contemporary topics, classical literature, and skills development.

C2 – (Weight: 0.1649)

Slightly lower than Curriculum Design, this reflects the importance of teaching methods, faculty expertise, and classroom dynamics. Programs are evaluated based on how well instructors engage students and deliver content.

C3 – (Weight: 0.1632)

This weight shows a balanced yet slightly lesser emphasis on how actively students participate in learning. It includes involvement in discussions, collaborative projects, and literary activities.

C4 – (Weight: 0.1651)

Close in value to Instructional Quality, this criterion assesses both faculty and student contributions to scholarly publications, conference presentations, and literary research projects.

C5 – (Weight: 0.1678)

This weight underscores the value of incorporating global, local, and diverse cultural texts and perspectives. It assesses how well the program connects literature with real-world cultural dynamics.

C6 – (Weight: 0.1709)

The highest among all, this criterion reflects the top priority: enhancing students' skills in reading, writing, listening, and speaking. It gauges how effectively programs help students achieve fluency and academic command of the English language.

These weights suggest a well-balanced evaluation, with a slight preference for language proficiency and curriculum structure, which are foundational in language and literature education.

Table 1. The assessment values.

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
A ₁	[0,1.2]	[0,1.3]	[0,1.5]	[0,1.6]	[0,1.8]	[0,1.9]
A ₂	[0,1.9]	[0,1.1]	[0,1.2]	[0,1.3]	[0,1.5]	[0,1.6]
A ₃	[0,1.8]	[0,1.1]	[0,1.5]	[0,1.6]	[0,1.8]	[0,1.8]
A ₄	[0,1.6]	[0,1.2]	[0,1.3]	[0,1.2]	[0,1.1]	[0,1.2]
A ₅	[0,1.5]	[0,1.1]	[0,1.6]	[0,1.8]	[0,1.9]	[0,1.8]
A ₆	[0,1.3]	[0,1.6]	[0,1.5]	[0,1.2]	[0,1.9]	[0,1.2]
A ₇	[0,1.9]	[0,1.5]	[0,1.6]	[0,1.1]	[0,1.3]	[0,1.3]
A ₈	[0,1.1]	[0,1.3]	[0,1.1]	[0,1.6]	[0,1.5]	[0,1.5]
A ₉	[0,1.2]	[0,1.9]	[0,1.9]	[0,1.9]	[0,1.6]	[0,1.6]
A ₁₀	[0,1.8]	[0,1.6]	[0,1.5]	[0,1.3]	[0,1.9]	[0,1.8]
A ₁₁	[0,1.8]	[0,1.6]	[0,1.5]	[0,1.3]	[0,1.9]	[0,1.8]
A ₁₂	[0,1.8]	[0,1.2]	[0,1.1]	[0,1.6]	[0,1.5]	[0,1.3]
A ₁₃	[0,1.2]	[0,1.3]	[0,1.5]	[0,1.6]	[0,1.1]	[0,1.2]
A ₁₄	[0,1.8]	[0,1.6]	[0,1.5]	[0,1.3]	[0,1.9]	[0,1.8]
A ₁₅	[0,1.9]	[0,1.3]	[0,1.5]	[0,1.6]	[0,1.1]	[0,1.1]
A ₁₆	[0,1.9]	[0,1.1]	[0,1.2]	[0,1.8]	[0,1.2]	[0,1.9]
A ₁₇	[0,1.1]	[0,1.2]	[0,1.3]	[0,1.5]	[0,1.3]	[0,1.8]
A ₁₈	[0,1.6]	[0,1.8]	[0,1.5]	[0,1.1]	[0,1.5]	[0,1.6]
	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
A ₁	[0,1.1]	[0,1.3]	[0,1.5]	[0,1.6]	[0,1.8]	[0,1.2]
A ₂	[0,1.2]	[0,1.8]	[0,1.2]	[0,1.3]	[0,1.5]	[0,1.6]
A ₃	[0,1.3]	[0,1.8]	[0,1.5]	[0,1.6]	[0,1.8]	[0,1.1]
A ₄	[0,1.5]	[0,1.2]	[0,1.3]	[0,1.9]	[0,1.1]	[0,1.9]
A ₅	[0,1.6]	[0,1.8]	[0,1.6]	[0,1.1]	[0,1.2]	[0,1.3]
A ₆	[0,1.3]	[0,1.9]	[0,1.1]	[0,1.2]	[0,1.3]	[0,1.5]
A ₇	[0,1.9]	[0,1.3]	[0,1.2]	[0,1.3]	[0,1.5]	[0,1.6]
A ₈	[0,1.1]	[0,1.5]	[0,1.3]	[0,1.5]	[0,1.6]	[0,1.5]
A ₉	[0,1.2]	[0,1.6]	[0,1.5]	[0,1.6]	[0,1.6]	[0,1.6]
A ₁₀	[0,1.8]	[0,1.6]	[0,1.6]	[0,1.3]	[0,1.2]	[0,1.8]
A ₁₁	[0,1.8]	[0,1.6]	[0,1.6]	[0,1.3]	[0,1.9]	[0,1.8]
A ₁₂	[0,1.1]	[0,1.1]	[0,1.6]	[0,1.5]	[0,1.3]	[0,1.2]
A ₁₃	[0,1.9]	[0,1.3]	[0,1.5]	[0,1.6]	[0,1.1]	[0,1.2]
A ₁₄	[0,1.8]	[0,1.6]	[0,1.6]	[0,1.3]	[0,1.2]	[0,1.8]
A ₁₅	[0,1.3]	[0,1.5]	[0,1.6]	[0,1.8]	[0,1.3]	[0,1.2]
A ₁₆	[0,1.1]	[0,1.2]	[0,1.8]	[0,1.6]	[0,1.5]	[0,1.1]
A ₁₇	[0,1.1]	[0,1.2]	[0,1.3]	[0,1.5]	[0,1.6]	[0,1.9]
A ₁₈	[0,1.8]	[0,1.6]	[0,1.5]	[0,1.2]	[0,1.8]	[0,1.9]
	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
A ₁	[0,1.9]	[0,1.3]	[0,1.5]	[0,1.6]	[0,1.6]	[0,1.2]
A ₂	[0,1.9]	[0,1.6]	[0,1.2]	[0,1.3]	[0,1.5]	[0,1.6]
A ₃	[0,1.1]	[0,1.5]	[0,1.5]	[0,1.6]	[0,1.3]	[0,1.5]
A ₄	[0,1.6]	[0,1.3]	[0,1.6]	[0,1.2]	[0,1.2]	[0,1.3]
A ₅	[0,1.5]	[0,1.9]	[0,1.5]	[0,1.6]	[0,1.6]	[0,1.2]
A ₆	[0,1.3]	[0,1.6]	[0,1.3]	[0,1.5]	[0,1.5]	[0,1.6]
A ₇	[0,1.2]	[0,1.5]	[0,1.2]	[0,1.3]	[0,1.3]	[0,1.5]
A ₈	[0,1.8]	[0,1.3]	[0,1.8]	[0,1.9]	[0,1.9]	[0,1.3]
A ₉	[0,1.2]	[0,1.2]	[0,1.2]	[0,1.2]	[0,1.6]	[0,1.2]
A ₁₀	[0,1.1]	[0,1.6]	[0,1.5]	[0,1.3]	[0,1.2]	[0,1.8]
A ₁₁	[0,1.8]	[0,1.6]	[0,1.5]	[0,1.3]	[0,1.2]	[0,1.8]
A ₁₂	[0,1.8]	[0,1.2]	[0,1.8]	[0,1.6]	[0,1.5]	[0,1.3]
A ₁₃	[0,1.2]	[0,1.3]	[0,1.5]	[0,1.6]	[0,1.8]	[0,1.8]
A ₁₄	[0,1.1]	[0,1.6]	[0,1.5]	[0,1.3]	[0,1.2]	[0,1.8]

A_{15}	[0,1.2]	[0,1.3]	[0,1.5]	[0,1.6]	[0,1.1]	[0,1.2]
A_{16}	[0,1.2]	[0,1.8]	[0,1.2]	[0,1.8]	[0,1.1]	[0,1.3]
A_{17}	[0,1.8]	[0,1.9]	[0,1.3]	[0,1.5]	[0,1.9]	[0,1.5]
A_{18}	[0,1.6]	[0,1.8]	[0,1.5]	[0,1.3]	[0,1.8]	[0,1.6]

Eq. (8) is used to compute the normalized decision matrix as shown in Fig 1.

Eq. (9) is used to compute the weighted decision matrix as shown in Fig 2.

The mini and max numbers are computed for positive and cost criteria using eq. (10 and 11).

Eq. (12) is used to compute the relative numbers as shown in Fig 3.

Rank the alternatives as shown in Fig 4.

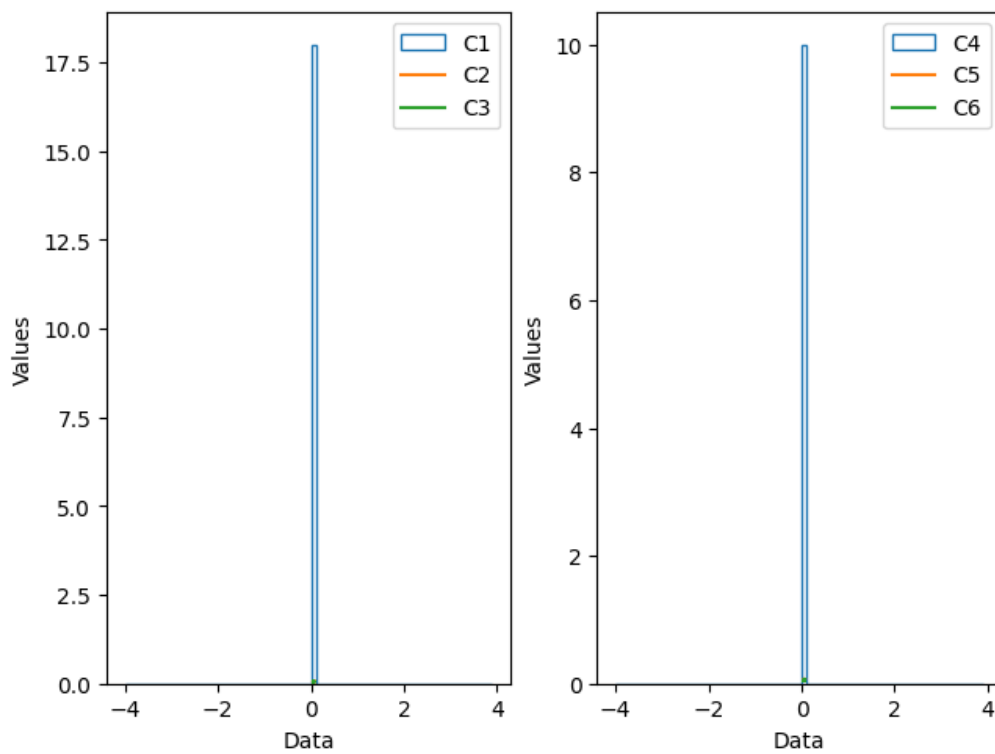


Fig 1. The normalized values.

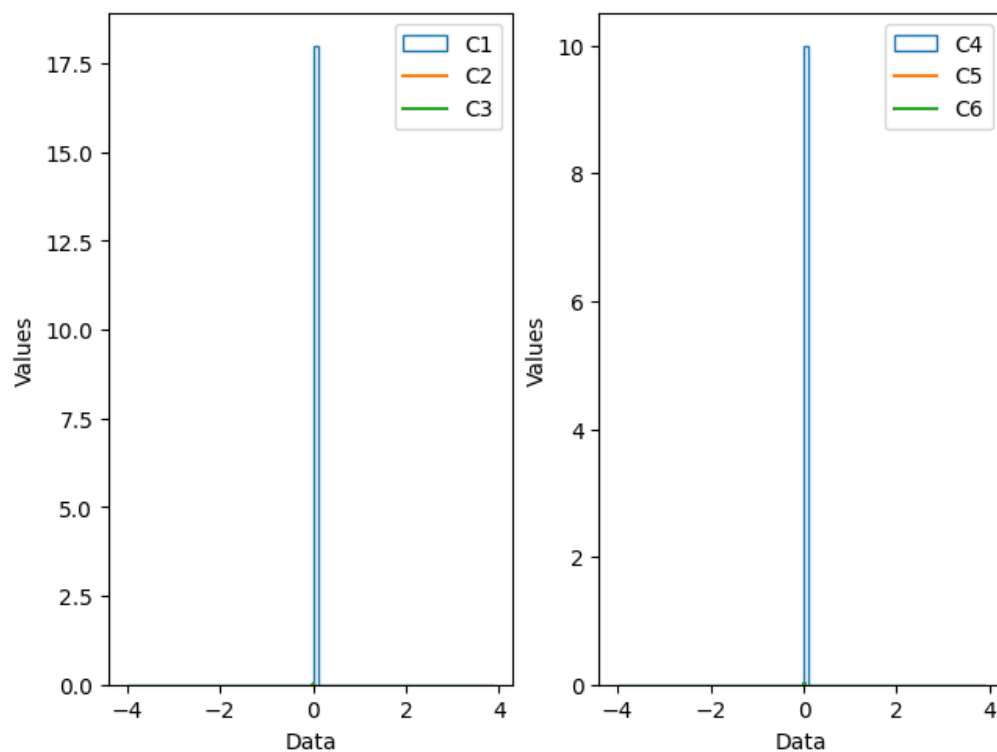


Fig 2. The weighted matrix.

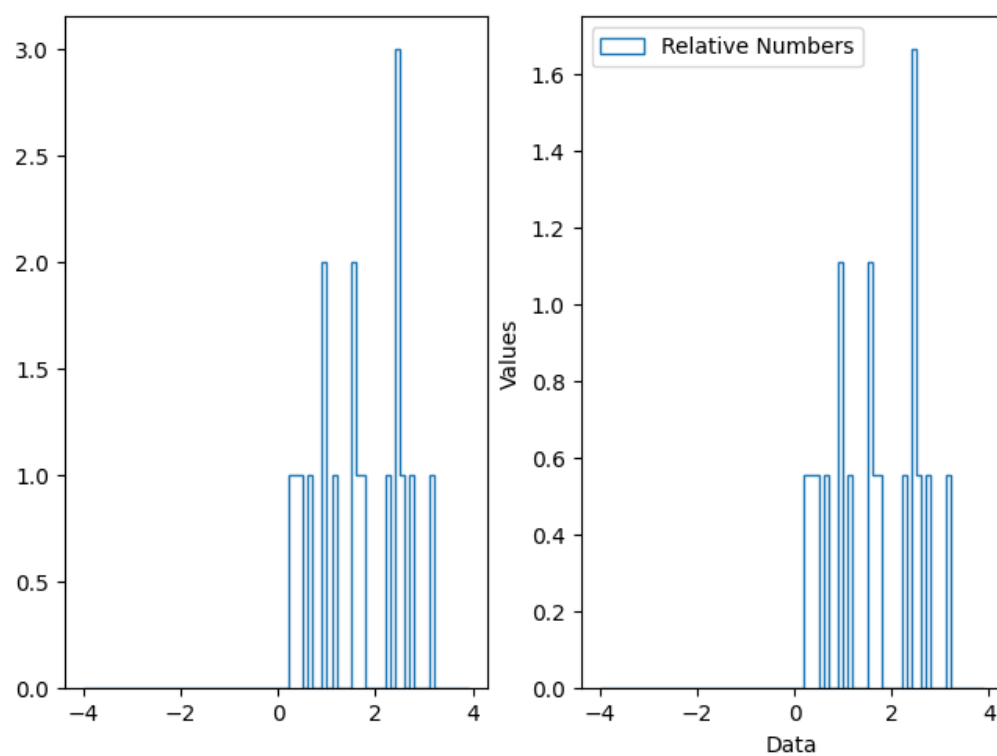


Fig 3. Relative numbers.

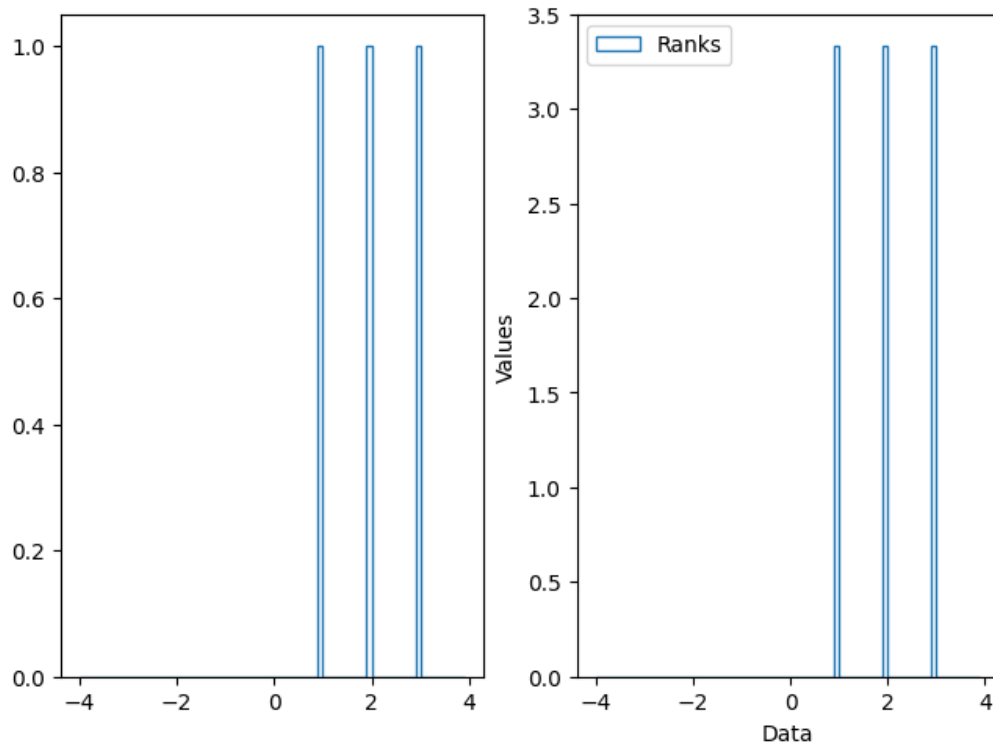


Fig 4. The ranks of alternatives.

4. Analysis

We show the sensitivity analysis between the ranks of alternatives. We change the criteria weights by six cases as shown in Fig 5. Then we apply the steps of the COPRAS method to show the ranks of alternatives. The relative numbers of six cases are shown in Fig 6. The ranks of alternatives are shown in Fig 7.

Criterion C1: Highly weighted in Case 1 (≈ 0.198): Minimal emphasis in other cases: This case likely focuses on the precision, clarity, and structural soundness of language or text.

Criterion C2: Highlighted in Case 2 (≈ 0.194)

Represents alignment with educational goals or the richness of thematic content.

Weight drops significantly in Case 2 for other criteria, indicating a targeted evaluation.

Criterion C3: Emphasis in Case 3 (≈ 0.192)

Assesses originality in language use, teaching methods, or literary interpretation.

Shows a marked drop in Case 3, spotlighting this aspect alone.

Criterion C4: Primary focus in Case 4 (≈ 0.194)

In this case, weight is placed on how well the alternative incorporates cultural literacy or relevance to context.

Criterion C5: Given the highest weight in Case 5 (≈ 0.198)

Indicates evaluation centered on student involvement and responsiveness in learning activities.

Criterion C6: Most emphasized in Case 6 (≈ 0.201): Focuses on the robustness of evaluation methods or success in achieving educational outcomes.

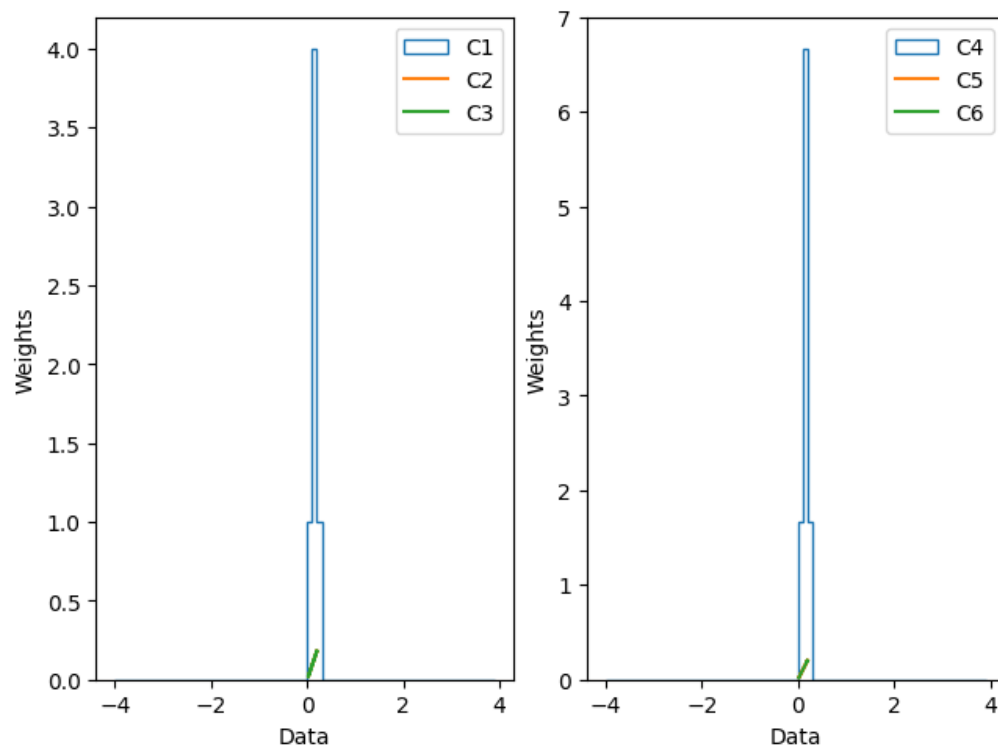


Fig 5. The six cases of criteria weights.

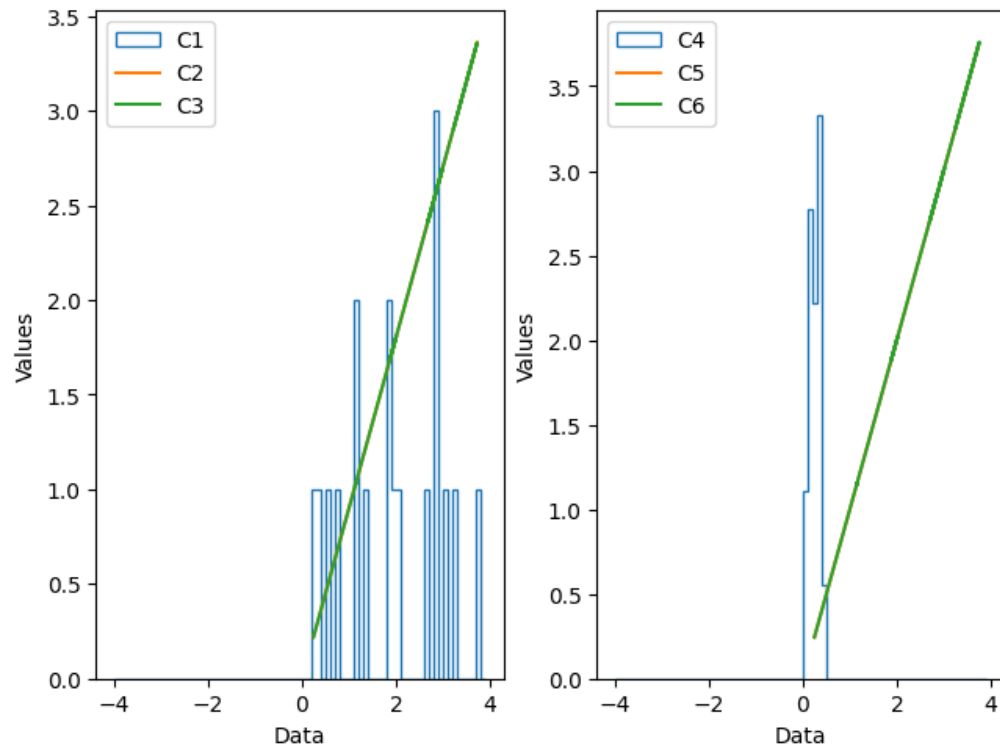


Fig 6. The six cases of relative numbers.

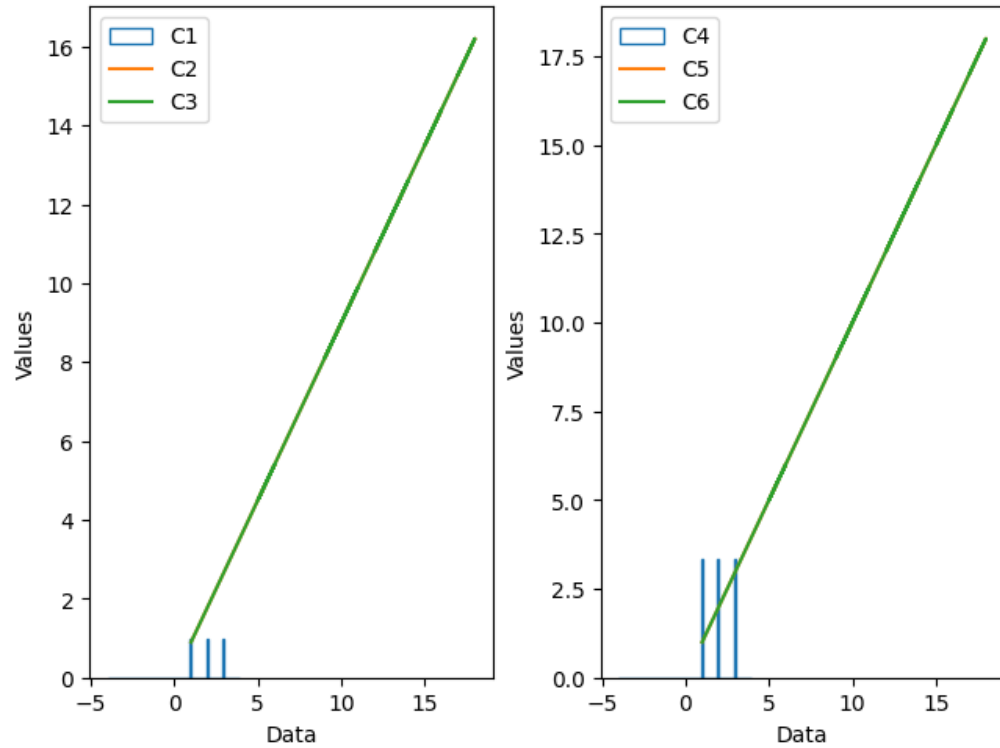


Fig 7. The six cases of ranks of alternatives.

A18 to A15 (Ranks 18–15): These are the top-performing alternatives across all cases: A18 (Rank 18) consistently stands out as the best among all alternatives, showing exceptional performance in all six cases.

A17 (Rank 17), A16 (Rank 16), and A15 (Rank 15) also maintain strong positions, suggesting high effectiveness in areas such as instructional strategy, student engagement, assessment methods, and integration of literature and language skills.

A14 to A12 (Ranks 14–12): These alternatives perform very well, just below the top tier: Likely demonstrate strong teaching approaches, innovative practices, and successful learning outcomes, though they may have slight room for improvement compared to the top 4.

A11 to A8 (Ranks 11–8): These are above-average performers: A11 (Ranks 8–11) and A8 (Rank 10) are steady, indicating reliability in meeting core educational objectives.

They likely reflect balanced programs with consistent delivery but may need targeted improvements to reach the top tier.

A7 to A5 (Ranks 7–5): These alternatives represent mid-level effectiveness: Programs here may have inconsistent performance, excelling in some evaluation criteria (e.g., content delivery) but underperforming in others (e.g., cultural integration or innovation).

A4 to A1 (Ranks 4–1): These alternatives show low effectiveness: A4 (Rank 16), A3 (Rank 15), A2 (Rank 18), and A1 (Rank 17) are the lowest ranked, consistently receiving the worst scores.

These may reflect outdated teaching methods, weak outcomes, poor student engagement, or insufficient integration of English literature and language components.

This rank distribution shows clear performance stratification, with A18–A15 leading as exemplary alternatives and A1–A4 lagging significantly. These insights are valuable for curriculum planning, strategy optimization, and targeted interventions in English language and literature instruction.

5. Conclusions

The evaluation model proposed in this study offers a comprehensive and adaptable approach to assessing English Language and Literature programs. By integrating six well-rounded criteria and evaluating a broad set of alternatives, the framework recognizes the diversity of educational approaches within the discipline. The findings highlight the importance of balancing curriculum innovation, pedagogical quality, and cultural inclusivity to improve academic outcomes. Future research could expand on this model by incorporating longitudinal student success data and alumni feedback to strengthen its application. We used the MCDM approach in this study. We used the COPRAS method to rank the alternatives. The average method is used to compute the criteria weights. Case study with six criteria and 18 alternatives is proposed. The sensitive

analysis is applied by changing the criteria weights by six cases. The results show the ranks of alternatives are stable under six different cases.

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SuperHyperSoft Set for Selection Strategies for Achieving Human Rights in Artificial Intelligence through Civil Public Interest Litigation

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Abstract: Making sure AI technologies respect and uphold human rights is crucial as AI gets more and more integrated into social structures. AI systems that are unethical or discriminatory may be challenged in court through civil public interest litigation (CPIL). To assess and choose strategic options for realizing human rights through CPIL in the AI domain, this study creates a SuperHyperSoft Set model with multicriteria decision making (MCDM) approach. We evaluate eight workable options and identify 14 criteria that represent ethical, legal, technological, and social aspects. The SuperHyperSoft Set is used to deal with different criteria values. ORESTE is used to rank the alternatives. The suggested model assists decision-makers in choosing the best advocacy and litigation tactics that are in line with AI deployment's human rights safeguards.

Keywords: SuperHyperSoft Set; MCDM; Human Rights; Artificial Intelligence; Civil Public Interest Litigation.

1. Introduction

Healthcare, finance, education, and governance are just a few of the industries that have been profoundly touched by the exponential expansion of artificial intelligence (AI). Although efficiency and creativity are promised by these developments, they also bring up important human rights issues like responsibility, nondiscrimination, privacy, and due process[1], [2]. Civil public interest litigation, or CPIL, has become a potent tool for addressing systemic problems that impact the general welfare. When it comes to AI, CPIL can fight against algorithmic prejudice, excessive surveillance, and rights abuses brought on by automated decision-making systems. Choosing the right legal and strategic frameworks is still difficult, even with CPIL's expanding involvement. These frameworks need to strike a balance between social justice goals, legal accuracy, and technological knowledge, especially in a sector as complicated and developing as artificial intelligence[3], [4].

When dealing with multi-attribute decision-making situations where parameter subsets differ in granularity, SuperHyperSoft set theory offers a solid mathematical framework. It is especially well-suited to the complex analyses needed to evaluate litigation tactics that take social, legal, and ethical factors into account[5], [6].

To assess selection strategies for realizing human rights in AI through CPIL, this research suggests a SuperHyperSoft set-based decision-making model. It presents 14 standards that cover the societal, technical, and legal effects of possible strategies[7], [8].

A variety of legal and strategic actions, ranging from algorithmic audits to constitutional litigation, are represented by the eight choices that have been outlined. To ascertain each alternative's efficacy and applicability in various situations, it is compared to the criteria.

The assessment process helps human rights advocates, legal experts, and politicians find the best approaches for addressing certain human rights issues in AI applications. The suggested approach improves the decision-making process for protecting human rights through strategic litigation and lobbying in the AI ecosystem by integrating SuperHyperSoft set theory, providing a sophisticated, multifaceted analysis[9], [10].

A significant interdisciplinary technique used in many different sectors to support intricate decision-making procedures incorporating numerous factors is multi-criteria decision-making (MCDM)[11], [12]. In contrast to conventional decision-making, which usually considers a single criterion, MCDM assesses options according to a variety of frequently incompatible criteria to identify the best course of action. MCDM helps decision-makers prioritize options and attain optimal results by evaluating influential aspects (criteria) and their relative relevance (weight)[13], [14].

Since its inception in the late 1950s, MCDM has developed into a strong field that facilitates efficient navigation of complex decision spaces¹. The exact definition of the problem and the determination of the decision criteria are the first two crucial elements in the MCDM process. Then, using techniques like pairwise comparisons or rating systems, each criterion is given weight according to its relative worth. After that, a thorough list of workable options is created. Each of these options is given a score determined on how well it performs when compared to the choice criteria. Following evaluation, each alternative's overall score is calculated by adding the values for each criterion[15], [16].

For this, a variety of aggregation techniques can be used. Sensitivity analysis is used to examine the effects of altering the weights of the criteria and the evaluation scores of the alternatives to guarantee the decision-making process's robustness. Ultimately, the alternative with the greatest total score is chosen after a decision is made based on the total scores. A balanced compromise or ideal solution that successfully meets the various criteria involved in the decision-making process is made possible by this methodical approach[17], [18].

The HyperSoft Set (2018), IndetermSoft Set (2022), IndetermHyperSoft Set (2022), SuperHyperSoft Set, TreeSoft Set (2022), and ForestSoft Set (2024) are the six new varieties of Soft Sets that Smarandache announced[19], [20], [21].

2. SuperHyperSoft Set

By mapping power set combinations of several values to subsets of a universal set, SuperHyperSoft Sets further generalize the idea and increase the usefulness of HyperSoft Sets. This higher-order method is an effective tool for modeling complex systems because it enables multidimensional decision-making and captures complicated interrelationships among characteristics[21], [22].

Let U be a universal set and $P(U)$ presents the power set of U . Let set attributes such as $a_1, a_2, \dots, a_n; n \geq 1$. Every criterion has different values A_i

$$A_i \cap A_j = \emptyset \text{ and } i \neq j \quad (1)$$

$p(A_i)$ is the power set of A_i . The cartesian product of the power set of criteria values is:

$$C = p(A_1) \times p(A_2) \times \dots p(A_n) \quad (2)$$

The SuperHyperSoft Set can be defined as[23], [24]:

$$F: C \rightarrow p(U) \quad (3)$$

$$(F, C) = \{(x, F(x)) | x \in C, F(x) \in P(U)\} \quad (4)$$

We show the steps of the ORESTE method under the SuperHyperSoft set to rank the alternatives based on a set of criteria.

Create the decision matrix.

Based on the opinions of experts, the decision matrix is built such as:

$$X = \begin{pmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{pmatrix} \quad (5)$$

Combine the decision matrix into a single matrix. The criteria weights are computed using the average method.

Compute the position matrix by ranking the alternatives.

Create the block distance.

$$S(0, A_{ij}) = \gamma x_{ij}(\gamma) + (1 - \gamma)x_j \quad (6)$$

Where $0 < \gamma < 1$

Order the alternatives based on the total rank of alternatives such as:

$$R(A_i) = \sum_{j=1}^n R(A_{ij}) \quad (7)$$

3. Results

This study discusses the results of the SuperHyperSoft Set model for ranking the alternatives based on the MCDM method. This study uses 14 criteria and eight alternatives.

The criteria with values are:

- I. Transparency of AI Systems {High, Medium}
- II. Legal Enforceability {Strong, Weak}
- III. Stakeholder Inclusivity {Government, NGOs, Technologists, Civil Society}
- IV. Ethical Accountability {Present, Absent}
- V. Impact on Marginalized Communities {High Impact, Medium Impact, Low Impact}
- VI. Technological Feasibility {Feasible, Partially Feasible}
- VII. Cost-effectiveness {Low Cost, Medium Cost, High Cost}
- VIII. Scalability of Litigation Effort {Scalable, Limited, Non-scalable}
- IX. Precedent-setting Potential {Strong Potential, Some Potential, No Potential}
- X. Public Awareness and Support {Widespread, Moderate, Minimal}
- XI. Policy Influence {Direct Influence, Indirect Influence, None}
- XII. Risk Mitigation for Plaintiffs {High, Medium, Low}
- XIII. Expert Collaboration Opportunities {High Collaboration, Moderate, None}
- XIV. Interdisciplinary Approach Strength {Strong, Moderate, Weak}

The alternatives are:

- I. Constitutional Litigation against Biased Algorithms
- II. Class-Action Lawsuits on Data Privacy Violations
- III. Judicial Review of AI in Public Sector Decision-Making
- IV. Strategic Partnerships for Algorithmic Audits
- V. Public Interest Petitions on Predictive Policing
- VI. Legislative Advocacy through CPIL
- VII. Filing Cases for Algorithmic Discrimination in Employment
- VIII. AI Ethical Standard Implementation via Legal Channels

Eq. (5) is used to create the decision matrix between the criteria and alternatives. The decision matrix is combined to obtain the criteria weights by the average method such as:

C1 has 0.0682– Represents how open and explainable AI models are to courts and the public, crucial for accountability.

C2 0.0721– Reflects the strength of the legal framework supporting AI-related human rights cases in court.

- C3 0.0779– Emphasizes the participation of civil society, tech experts, and marginalized groups in litigation planning and execution.
- C4 0.0718– Captures the extent to which ethical AI practices are recognized and enforced within legal boundaries.
- C5 0.0672– Indicates how effectively strategies protect vulnerable populations affected by AI deployment.
- C6 0.0644– Assesses whether the proposed litigation strategies can technically address the AI systems in question.
- C7 0.0595– Measures how financially viable and resource-efficient each litigation strategy is for sustained implementation.
- C8 0.0699– Describes the potential to apply the legal strategy across multiple similar cases or jurisdictions.
- C9 0.0747– Indicates the likelihood that a case could create significant legal precedents for future AI-related human rights claims.
- C10 0.0820– Reflects the level of social mobilization and public backing for the cause, which can influence litigation outcomes.
- C11 0.0714– Represents the ability of the legal strategy to trigger or influence AI policy reforms at governmental or institutional levels.
- C12 0.0772– Weighs how well the strategy protects litigants from legal, financial, and personal risks.
- C13 0.0720– Assesses the involvement of legal, technical, and ethical experts in building strong multidisciplinary cases.
- C14 0.0717– Captures how comprehensively the strategy blends law, technology, and ethics in tackling AI-related injustices.

We apply the steps of the ORESTE methodology based on the SuperHyperSoft set. We have indeterminacy in the first two criteria so, we divide the ORESTE method into four sets such as:

- I. Set 1: {High}, {Strong}, {Civil Society}, {Present}, {High Impact}, {Feasible}, {Medium Cost}, {Scalable}, {Strong Potential}, {Moderate}, {Direct Influence}, {High}, {High Collaboration}, {Strong}.
- II. Set 2: {High}, {Weak}, {Civil Society}, {Present}, {High Impact}, {Feasible}, {Medium Cost}, {Scalable}, {Strong Potential}, {Moderate}, {Direct Influence}, {High}, {High Collaboration}, {Strong}.

- III. Set 3: {Medium}, {Strong}, {Civil Society}, {Present}, {High Impact}, {Feasible}, {Medium Cost}, {Scalable}, {Strong Potential}, {Moderate}, {Direct Influence}, {High}, {High Collaboration}, {Strong}.
- IV. Set 4: {Medium}, {Weak}, {Civil Society}, {Present}, {High Impact}, {Feasible}, {Medium Cost}, {Scalable}, {Strong Potential}, {Moderate}, {Direct Influence}, {High}, {High Collaboration}, {Strong}.

In Set 1.

We compute the position matrix by ranking the alternatives. We create the block distance using Eq. (6) as shown in Fig 1. We order the alternatives based on the total rank of alternatives using eq. (7) as shown on Fig 2. The ranks of alternatives are shown in Fig 3.

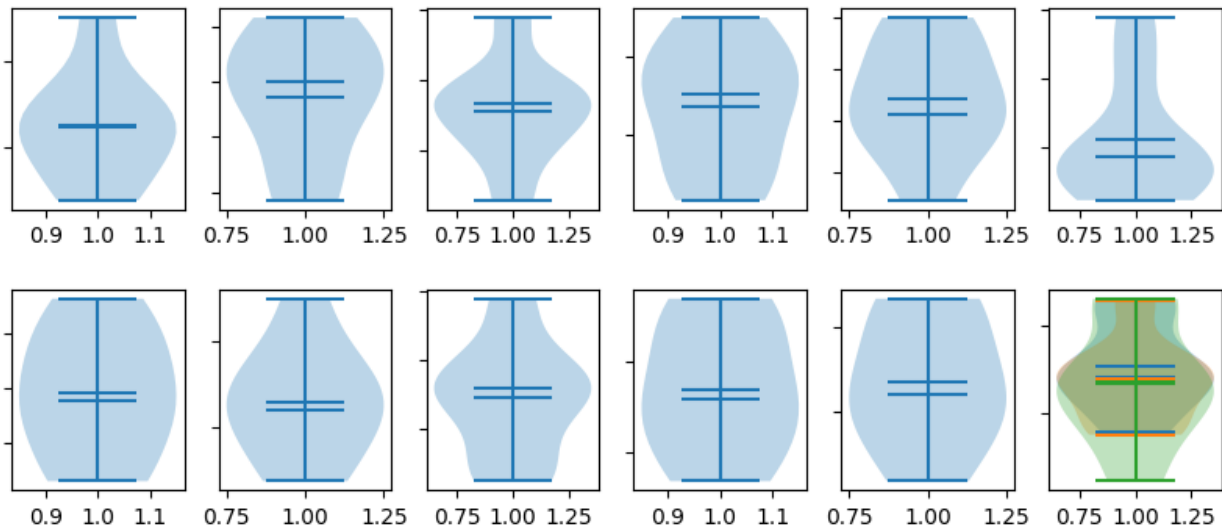


Fig 1. The block distance.

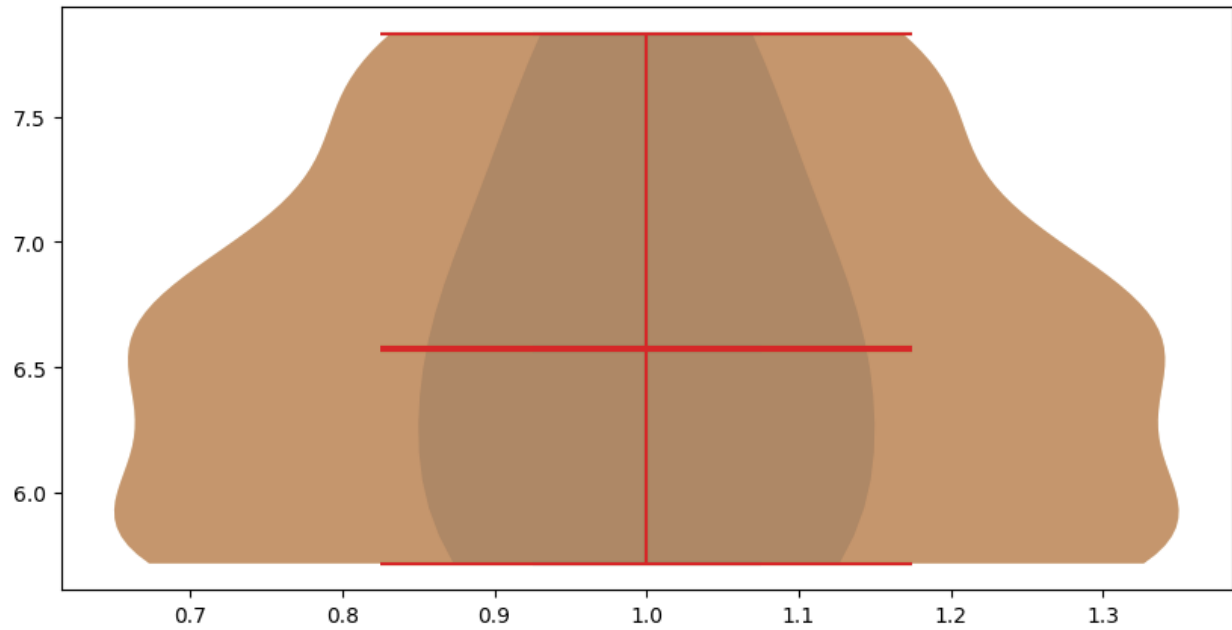


Fig 2. The total distance.

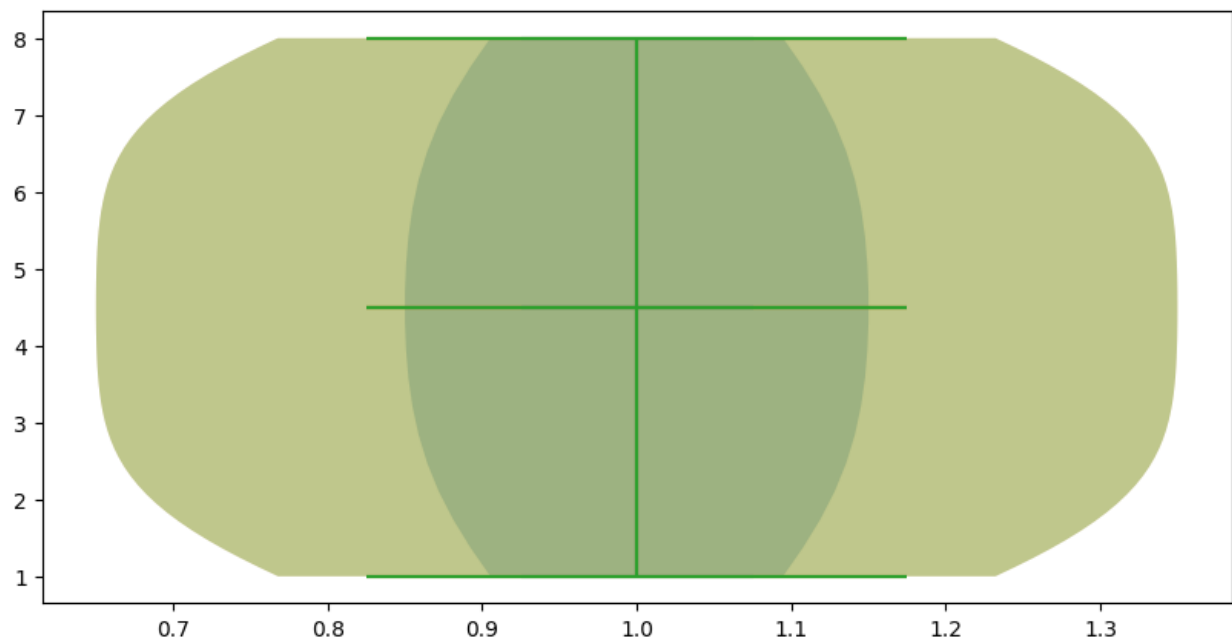


Fig 3. The ranks of alternatives.

In Set 2.

We compute the position matrix by ranking the alternatives. We create the block distance using Eq. (6) as shown in Fig 4. We order the alternatives based on the total rank of alternatives using eq. (7) as shown on Fig 5. The ranks of alternatives are shown in Fig 6.

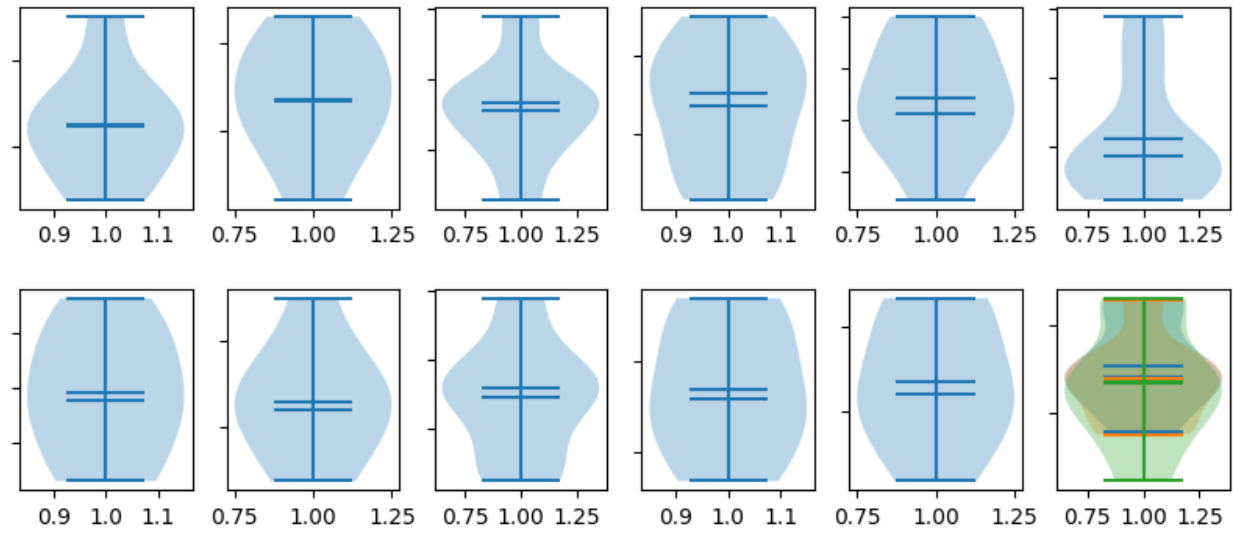


Fig 4. The block distance.

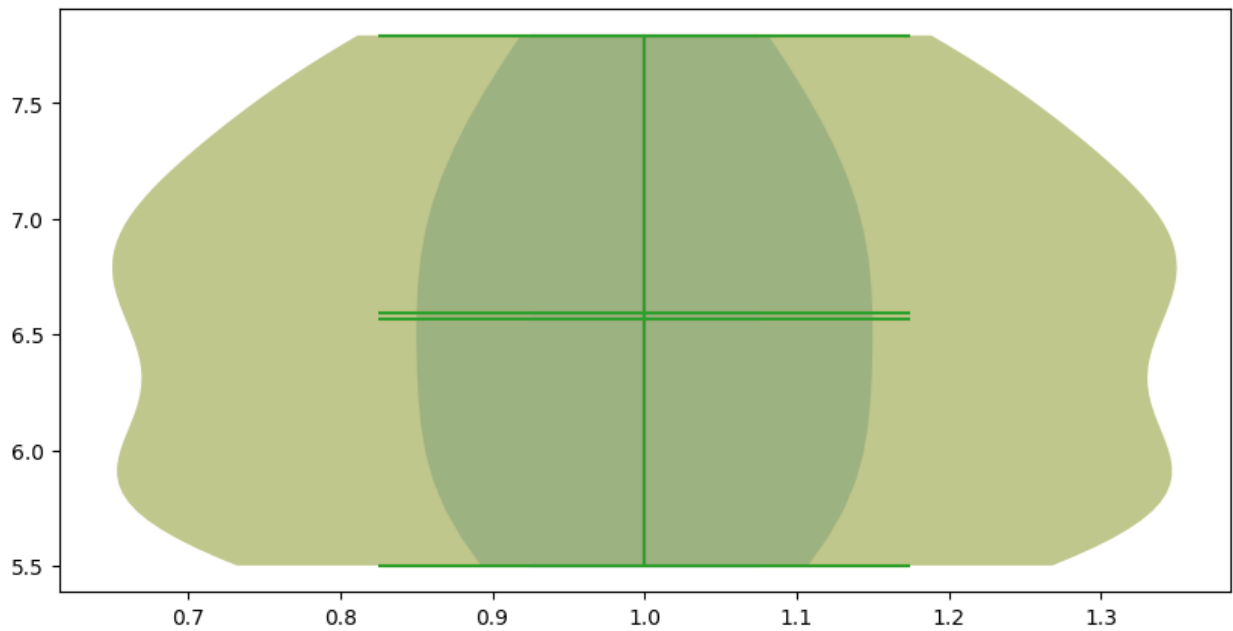


Fig 5. The total distance.

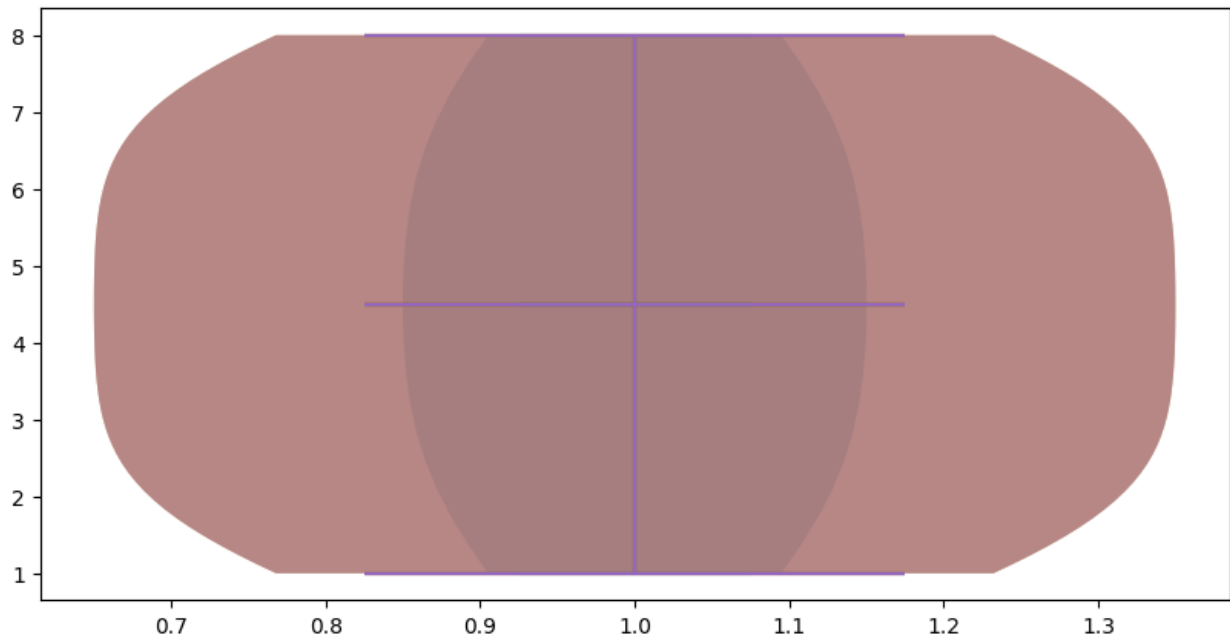


Fig 6. The ranks of alternatives.

In Set 3.

We compute the position matrix by ranking the alternatives. We create the block distance using Eq. (6) as shown in Fig 7. We order the alternatives based on the total rank of alternatives using eq. (7) as shown on Fig 8. The ranks of alternatives are shown in Fig 9.

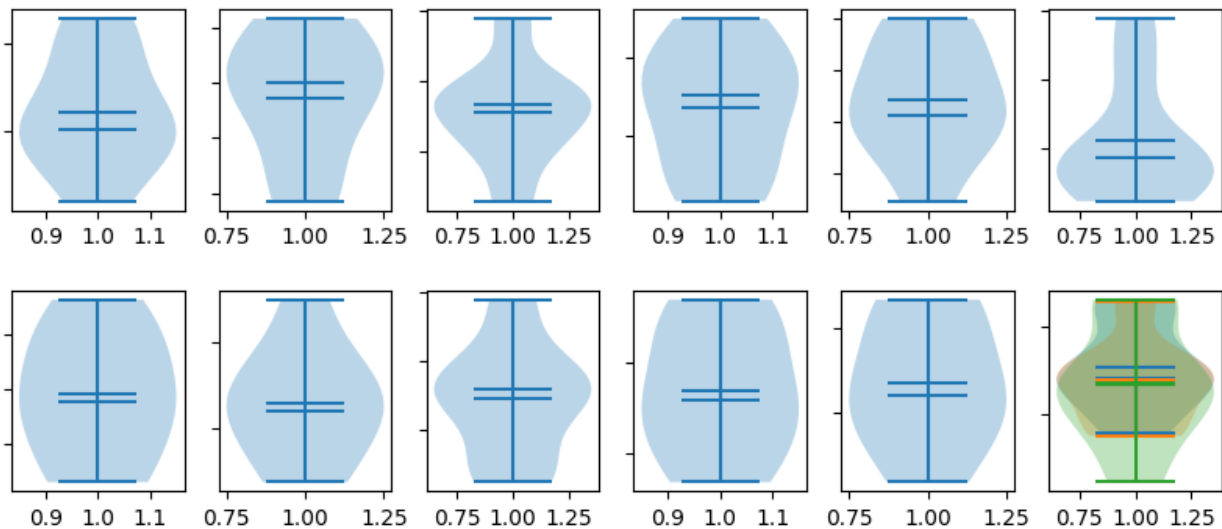


Fig 7. The block distance.

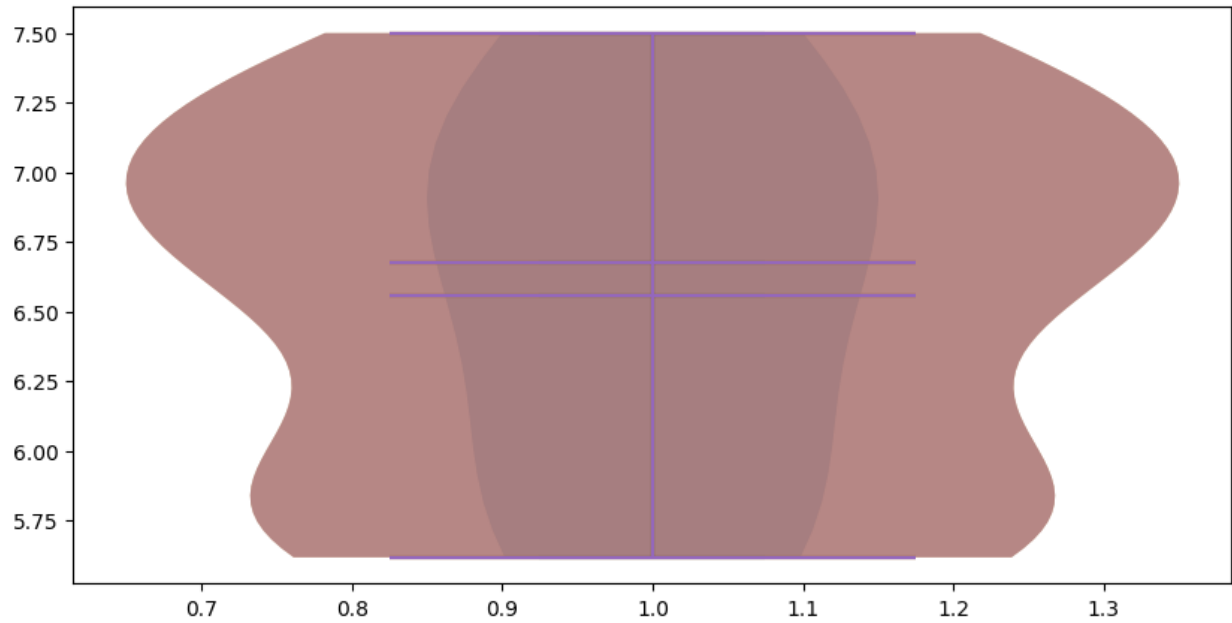


Fig 8. The total distance.

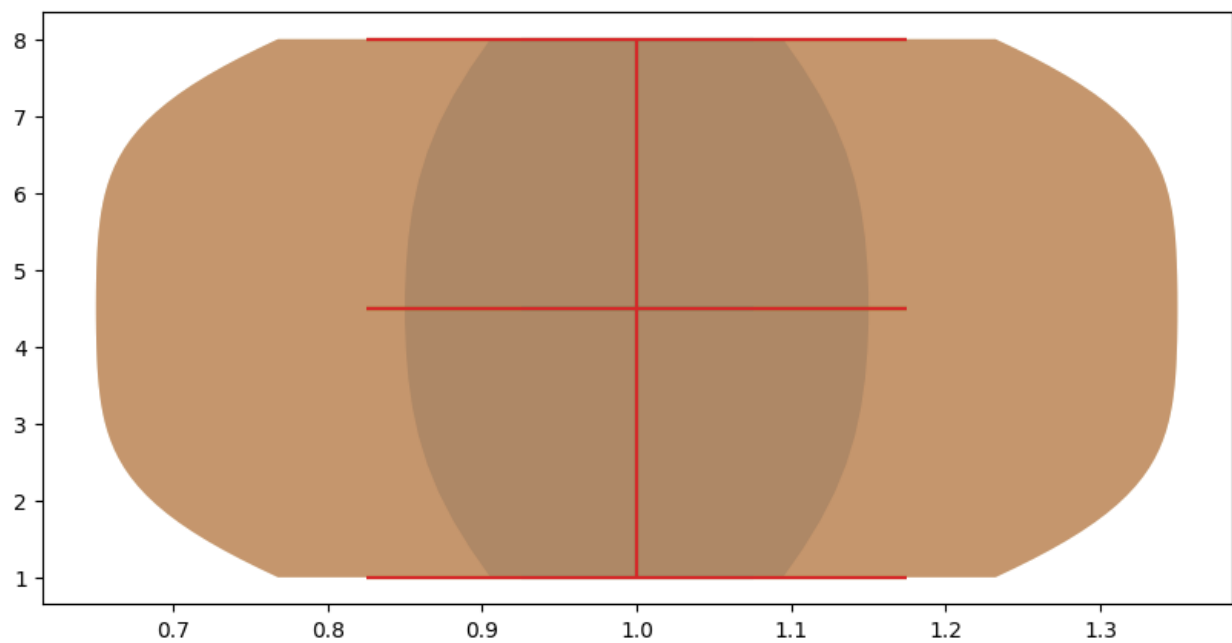


Fig 9. The ranks of alternatives.

In Set 4.

We compute the position matrix by ranking the alternatives. We create the block distance using Eq. (6) as shown in Fig 10. We order the alternatives based on the total rank of alternatives using eq. (7) as shown on Fig 11. The ranks of alternatives are shown in Fig 12.

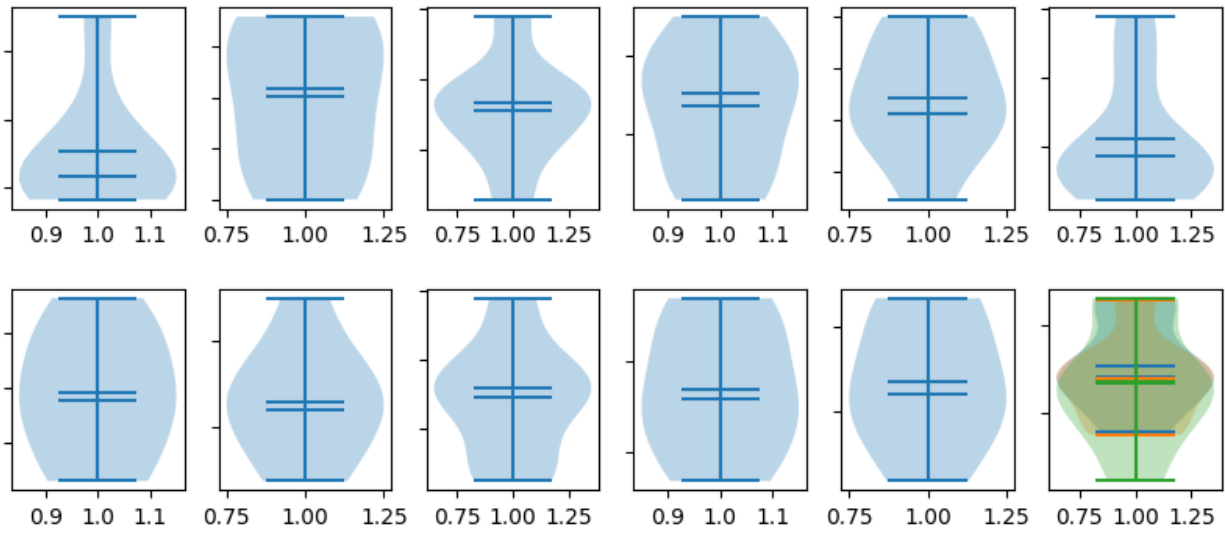


Fig 10. The block distance.

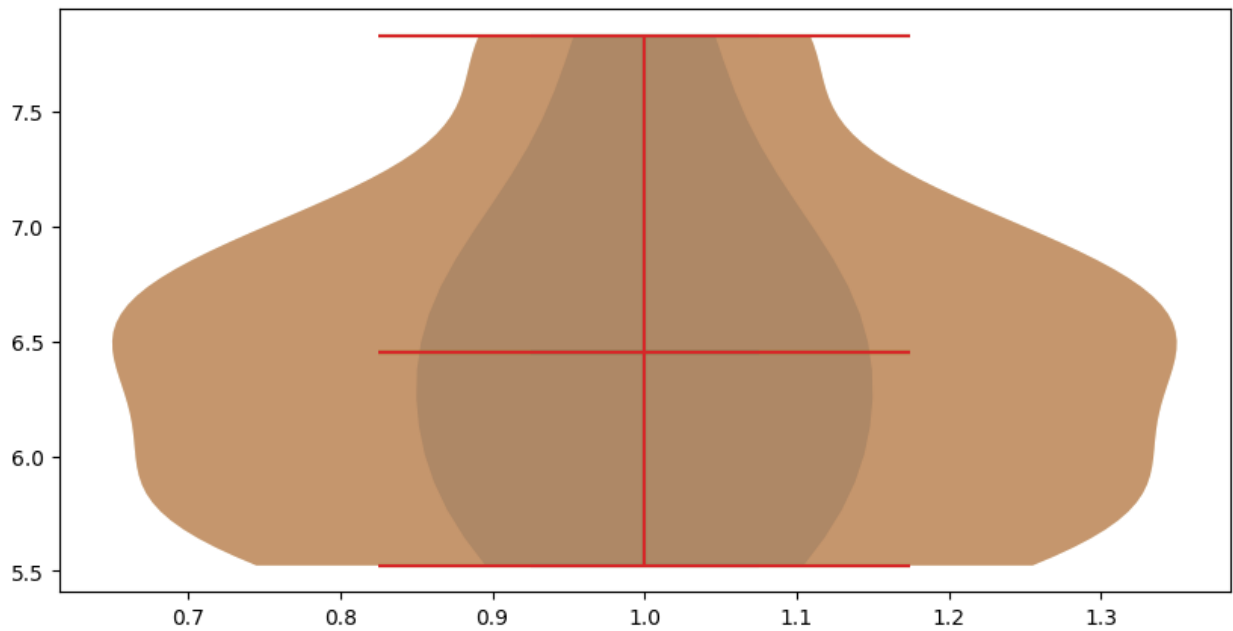


Fig 11. The total distance.

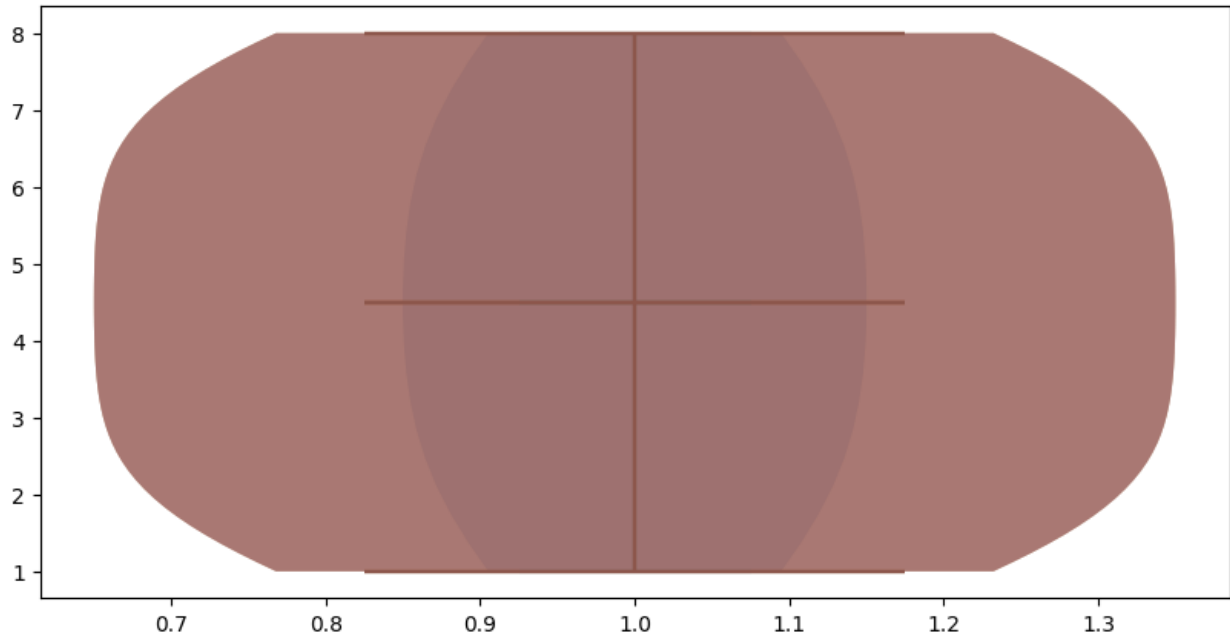


Fig 12. The ranks of alternatives.

- A1 and A4 dominate, frequently scoring 1st or 2nd. These represent strong strategic options in most evaluation dimensions.
- A5 is highly consistent in the top 3, showing strong reliability.
- A2 and A3 stay between 4th and 5th ranks, showing moderate effectiveness but not outperforming consistently.
- A6, A7, and A8 maintain static and low ranks (6th, 7th, and 8th respectively) across all sets, indicating limited utility or poor strategic alignment.

Alternative	Lowest Rank	Highest Rank	Repetitive Rank	Ranks
A1	1 (Set 2, Set 4)	2 (Set 1, Set 3)	2 (2 times)	Consistently high-performing and reliable.
A2	4 (Set 2, Set 3)	5 (Set 1, Set 4)	4 (2 times), 5 (2 times)	Mid-range performers with moderate consistency.
A3	4 (Set 1, Set 4)	5 (Set 2, Set 3)	5 (2 times), 4 (2 times)	Stable but not a top performer, stays mid-rank.
A4	1 (Set 1, Set 3)	3 (Set 2, Set 4)	1 (2 times), 3 (2 times)	Alternates between top and slightly lower, strong overall.
A5	2 (Set 2, Set 4)	3 (Set 1, Set 3)	3 (2 times), 2 (2 times)	Very stable, always ranked 2nd or 3rd.
A6	6 (All Sets)	6 (All Sets)	6 (4 times)	Perfectly consistent, but consistently low.
A7	7 (All Sets)	7 (All Sets)	7 (4 times)	Another low and repetitive performer.
A8	8 (All Sets)	8 (All Sets)	8 (4 times)	Lowest performer in every scenario

5. Conclusions

Protecting human rights in the rapidly developing field of artificial intelligence requires proactive, strategic involvement through civil public interest lawsuits. This study shows how selection strategies based on multiple, frequently competing criteria can be efficiently analyzed and prioritized using SuperHyperSoft set theory. A thorough assessment process that complies with human rights principles and practical viability is ensured by the integration of legal, ethical, technological, and social considerations. We used the MCDM methodology to deal with different criteria and alternatives. The ORESTE method is an MCDM method used to rank alternatives. The SuperHyperSoft set is used to deal with different criteria values. The suggested framework gives interested parties a methodical, flexible paradigm for making wise choices in the ever-evolving field of AI law.

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Optimizing of Universal DEA Model with Multi-Level Integration under Neutrosophic Environment

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Abstract. This research introduces a comprehensive Universal Data Envelopment Analysis (DEA) model to handle real-world problems fraught with uncertainty of every operational facet. Apart from all this, this framework has a feature of embracing many level variations: deterministic, stochastic, fuzzy, and neutrosophic, in both input and output variables, unlike the traditional DEA models, which are limited to deterministic. Besides accommodating different orientation types detected in models such as input-oriented and output-oriented, it also incorporates variable and constant return types of scale. The main objective of this study is to develop a flexible DEA model that can measure and rank, under uncertain conditions, the performance efficiency of organizations having comparable input-output structures. The proposed model seeks to identify inefficient organizations so as to improve specifically on those areas to have a much better outcome in overall efficiency. This provides decision-makers with very strong and versatile options for efficient measures, even in the context of heterogeneous and imprecise data. The paradigm for this study lies in bringing together several ways of handling uncertainty under one umbrella. This study is much better than the conventional DEA models. Possible restrictions, however, on applicability in very large data sets and computation time complexities would need further probing.

Keywords: Data Envelopment Analysis; Neutrosophic Analysis; Efficiency Assessment; Uncertainty Integration; Multi-Level Uncertainty; Healthcare Facility; Triangular neutrosophic number

1. Introduction

In these days, managers are now required not only to make collaborative decisions, but also interpret abundant data related to their business or organization. The biggest difficulty is finding valuable insight from this flood of data, and this insight can be used to improve organizational performance. Efficiency measurement represents a vital domain of interest within the complex nexus of dimensions that characterize organizational performance. One of the most rising challenges across organizations is questioning them on whether they believe themselves to be more efficient than one or more industry competitors. Performance measurement has evolved into necessary means of answering questions about how to measure productivity, including efficiency, effectiveness and accountability. Traditional performance measurement systems tend to present an incomplete picture of organizational performance leading to the risk of managers missing some important areas for development. In this sense efficiency is measured against established goals, whether it relates to the level of output produced, profit achieved, or cost avoided [1].

Data Envelopment Analysis (DEA) is a nonparametric method in operations research that is used to evaluate the relative efficiency and performance of diverse entities, such as firms, organizations, or functionally similar but independent operational units, often referred to as Decision-Making Units (DMUs) [2, 3]. These DMUs are usually specified by a vector of outputs and inputs. The main focus of a DEA investigation is to determine the relative efficiency of each DMU with respect to its peers. Based on this assessment, all entities are grouped in "efficient" or "inefficient" and it determines how much input/output slack is needed to convert an inefficient DMU into an efficient one. Radial distance from inefficient (DMU) to efficiency frontier indicates how much more efficient DMUs must be. An interesting property of the DEA algorithm is the fact that it can be applied without having an explicit specification (including a functional form) for the production function. Overall, the DEA model has the advantage of the ability to treat multiple inputs and outputs [4], which can improve the flexibility of this approach. Nevertheless, one important shortcoming of classical DEA models is that they do not deal with the uncertainty variations on input and output variables. Such a constraint makes the ordinary DEA models exceedingly exposed to *prima facie* uncertainty variables such as statistical error, data source scarcity, and usual randomness/vagueness/neutrosophicness of all economic phenomena. Thus, the use of classical DEA approaches to assess efficiency is very sensitive to variations in uncertainty, since such differences might cause DMU's changes between efficient and inefficient groups.

Recent research dealt with randomness of data within the DEA framework, and important steps have been taken. Among the output-oriented stochastic DEA models, two streams of approach are maintained. First, it is assumed that all variables are uniformly random [4, 5, 6], while in the second, only the output variables are random with inputs considered as deterministic [7, 8, 9]. Similarly, the input-oriented stochastic DEA models have also developed in two ways: one with outputs being random and inputs deterministic [10]. These models share common assumptions: they assume independence between the same random variables across different DMUs and identical distribution patterns for all stochastic variables. Expanding on these works, a few researchers have advanced novel robust stochastic DEA methodologies, which enhances the capability of the field to address uncertainty further. Researchers have recently studied randomness of data in DEA context, and development of the field further into important promises. Under the output-oriented stochastic DEA modeling, however, two streams of approach have been held. In the first, all variables are considered random [4, 5, 6]; in the second, the random ones are only considered output variables, while inputs are deterministic [7, 8, 9]. Along the same lines have come up these input-oriented stochastic DEA-models, one treating output random and inputs as deterministic [10]. These families of models share common assumptions: they assume the independence of the same random variables across different DMUs and by symmetry for all stochastic variables, the models are assumed to share the same distribution.

A few researchers built upon these works to develop novel robust stochastic DEA methodologies that further empower the field to address uncertainty. El-Demerdash et al. [11, 12] proposed a Stochastic Input-Oriented DEA model for evaluating the efficiency of decision-making units under uncertainty in input variables and/or output variables to provide realistic and reliable assessments of efficiency in environments where input values are subject to variability. Montazeri [13] furthered the construction of fuzzy stochastic DEA model to portray the synergistic relevance of joining fuzziness and randomness better to counter uncertainty in efficiency assessment. Sihotang et al. [14] modified DEA model for stochastic decision-making in production and supply chain planning under uncertain operational conditions. The reliability of efficiency measuring was improved by Syhotong et al. [15] providing for stochastic DEA considerations which in turn would stabilize DEA results when operating in unstable data conditions. In this respect, DEA models have been developed utilizing fuzzy logical approaches to capture the inherent vagueness of the variables. The development work follows two major avenues: First the classical fuzzy DEA models. Saati, et al., [16] developed a fuzzy DEA model to assess the efficiency and ranking of DMUs under uncertainty and emphasized handling imprecise inputs and outputs, offering a more flexible and realistic evaluation framework. Liu, et al., [17] applied fuzzy DEA to evaluate product design schemes, considering the vagueness inherent in design parameters and enable more accurate and practical assessments in the early stages of product development. Chiang and Che [18] proposed a fuzzy robust evaluation model combining Bayesian Belief Networks and weight-restricted DEA for ranking New Product Development project to manage uncertainty and enhance decision making in project selection. Khoshfetrat and Daneshvar [19] introduced fuzzy DEA models by refining the frontier structure to enhance discriminatory power and provide more precise efficiency analysis under fuzzy data conditions.

The input-output-oriented fuzzy DEA methods were presented as the second route. Kakao and Liu [20] were the pioneers in presenting the mathematical programming approach for ranking the DMUs within the framework of fuzzy DEA. They emphasized effective discrimination between DMUs and had a mechanism for handling imprecise input-output data very well. Liu [21] put forward the fuzzy DEA/Analytic Hierarchy Approach for determining

flexible manufacturing systems in uncertain conditions and situations. It aimed at supporting decision-making by combining the analysis of efficiency with preference-based ranking. Azadeh et al. [22] discussed hybrid methodologies combining neural networks through fuzzy DEA for optimal location for solar plants, all under uncertainty and complexity, while handling multiple uncertainties concerning the technical, economic, and environmental parameters involved. Zerafat Angiz, et al. [23] presented a discrete fuzzy DEA model for evaluating the efficiency of DMUs under imprecise data while retaining robustness in the efficiency assessment process. Kaleibar, et al., [24] modified a centralized resource allocation model under a variable return to scale DEA model using fuzzy data to better accommodate a more flexible and fair resource allocation among DMUs in uncertain environments, with routes establishing that all variables, inputs and/or outputs are considered fuzzy. These models are distinguished further by their assumption that all fuzzy variables have some standard, common individual membership function. This includes the most outstanding contributions, one being that of Hatami-Marbini et al. [25] which provided an adaptable but otherwise flexible cross-efficiency evaluation methodology through a fuzzy output-oriented DEA model conceived specifically for supplier performance assessment. Or Tharwat et al. [26, 27] developed the models about input-output orientation. However, Mohanta & Sharanappa [28] achieved this application to fuzzy DEA: the spherical fuzzy DEA, which can represent uncertainty of decision makers regarding assessment of data. Stanojević, B., & Stanojević, M. [29] presented an empirical Monte Carlo simulation to visualize fuzzy efficiencies within the wholly fuzzy DEA models.

The recent development on DEA methodology to deal with neutrosophic uncertainty of the novel approach in the efficiency analysis. Edalatpanah [30] was the first one who introduced neutrosophic components into inputs as well as outputs. Furthermore, Abdelfattah [31] worked by incorporating triangular neutrosophic variables to derive a more developed model capturing degrees of truth, indeterminacy, and falsity within values of data. The field progressed even further with many critical contributions in the field. Kahraman et al. [32] integrated methods of DEA to Neutrosophic Analytic Hierarchy Process to improve performance assessment. Yang et al. [33] proceeded further by considering all variables in the form of single-valued neutrosophic triangular number representation and Mao et al. [34] developed an efficient neutrosophic DEA model that has been uniquely considered on undesirable outputs based on an aggregation operator approach. Other fairly recent works are the input-oriented neutrosophic DEA model by El-Demerdash et al. [35], and the approach to model uncertainty of both input and output data by Farnam et al. [36]. A particularly comprehensive development was that of Almutairi et al. [37], who presented a model that could do the following: determining whether variables should be treated as deterministic or neutrosophic, accommodating both input and output orientations, and handling multiple types of returns to scale (constant and variable).

These advancements in DEA methodologies, incorporating stochastic, fuzzy or neutrosophic elements, represent a significant shift in the way decision-making units are evaluated, offering more nuanced and comprehensive approaches to handle uncertainty in real-world data. Even though the above literature survey in DEA has yielded new models, these models still exhibit certain limitations. Firstly, the DEA models that have been developed lack the generality needed to effectively handle both deterministic variables and variables characterized by various forms of uncertainty. Secondly, the existing DEA models that do address uncertainty in variables typically categorize all variables, whether they are inputs or outputs, into a single category of uncertainty, be either in their randomness, vagueness, or neutrosophic nature. Thirdly, the DEA models; that are introduced to handle different orientation types, whether input or output, and to manage distinct return-to-scale types, such as constant or variable; remain relatively scarce. Lastly, a number of these newly developed models are specifically tailored to address certain applications, limiting their versatility and adaptability across different domains.

So, the main objective of this study is to create a comprehensive DEA model capable of accommodating a wide spectrum of variable types, including deterministic, stochastic, fuzzy, and neutrosophic variations, applied to both input and output parameters. This model will also address various orientation types, whether output-oriented or input-oriented, and consider different return-to-scale scenarios, encompassing both constant return to scale (CRS) and variable return to scale (VRS). The ultimate goal is to utilize this model to assess and compare the relative efficiency of a collection of organizations that share common input-output characteristics. Furthermore, the model aims to provide actionable recommendations to organizations that are found to be inefficient, enabling them to enhance their overall performance.

This paper is structured as follows: In the second section, we outline the basic mathematical framework of DEA. In the third section, we introduce our novel contribution: a generalized DEA model that incorporates multi-level uncertainty integration. In the fourth section, we show the practical applicability of our proposed model by

implementing it in a case study. Section fifth introduces the comparison of the proposed approach with the existing method and the sensitivity analysis. Finally, in the sixth section, we conclude the paper with a comprehensive summary of our findings and discuss promising avenues for future research.

2. Basic Data Envelopment Analysis Mathematical Models

Arguably, there are two DEA models. The CRS model [2] that a given amount of input will yield a proportional increase in output. The second category is known as VRS model which is a refinement by Banker et al. [38] where one would not expect a constant relationship between changes in inputs and outputs. The VRS is advanced as a refinement of the CRS model in a way that all the efficient DMUs are covered with a convex curve representing efficient frontiers. Further, there are also output oriented and input-oriented DEA model's classification. Input-oriented DEA model's frontier is determined by reducing the scale of an input in proportion to its amount that is used to produce a given amount of output for a DMU. In a similar manner to input oriented models, output-oriented DEA models seek the maximum possible increase of output under an unchanged amount of input. So, we presented two basic distinct models for this study, an output-oriented CRS model presented in Model (M-1), and an input-oriented CRS model presented in Model (M-2).

A basic CRS Output-Oriented DEA model

$$\begin{aligned}
 & \text{Max } \emptyset \\
 & \text{s. t.} \\
 & \sum_{i=1}^n \lambda_i y_{ik} \geq \emptyset y_{pk}, \forall k = 1 \dots w \\
 & \sum_{i=1}^n \lambda_i x_{ij} \leq x_{pj}, \forall j = 1 \dots v \\
 & \lambda_i \geq 0, (i = 1, 2, \dots, n)
 \end{aligned} \tag{M-1}$$

A basic CRS Input-Oriented DEA model

$$\begin{aligned}
 & \text{Min } \theta \\
 & \text{s. t.} \\
 & \sum_{i=1}^n \lambda_i x_{ij} \leq \theta x_{pj}, \forall j = 1 \dots v \\
 & \sum_{i=1}^n \lambda_i y_{ik} \geq y_{pk}, \forall k = 1 \dots w \\
 & \lambda_i \geq 0, (i = 1, 2, \dots, n)
 \end{aligned} \tag{M-2}$$

where k : number of outputs (1 to 'w'); j : number of inputs (1 to 'v'); i : number of DMUs (1 to 'n'); y_{ik} is the amount of output k for DMU i , x_{ij} is the amount of input j for DMU i , and λ_i is the weight given to DMU i .

The models can be written as VRS models instead of CRS ones by adding a new constraint ($\sum_{i=1}^n \lambda_i = 1$) to both (M-1) and (M-2) models. This constraint is formed as a combination convex of DMUs with positive λ 's in the optimal solution.

3. Developing A Generalized Data Envelopment Analysis Model

A major development in DEA methodology is the formulation of more general and flexible models. This section introduces a new method that outperforms the traditional DEA models: the Generalized Mathematical Model for DEA. This model permits the incorporation of stochastic, fuzzy, and neutrosophic measures to assess performance efficiency under uncertainty. Great adaptability is one of the major strengths of this model. It moves effortlessly between input-oriented and output-oriented analyses, offering the means of relative efficiency measurement. Besides that, the model enables both constant return to scale and variable returns to scale assumptions, hence it can cope with efficiency analysis for any kind of operating scale.

This section will aim at an analysis of the changes in basic rules of this new DEA model regarding the patterns graph transformation using parametric techniques and thus increasing the robustness of the models applied even in the case of complex, uncertain and multi-dimensional situations. In this context, whereby managers are becoming more and more concerned with the framework design of organizational decision support systems, this model appears to be well suited for the practical applications for real life situational performance measurement and decision making. The subsequent remarks and propositions aim to establish a comprehensive DEA model for assessing the relative efficiency of each DMU while accommodating variables of distinct natures (deterministic, fuzzy, stochastic, and neutrosophic) independently. In addition, it accommodates the orientation types and allows for different kinds of return to scale.

Remark 1: Consider a versatile DEA model capable of addressing both input- and output-oriented problems, accommodating both CRS and VRS. The following model (M-3) represents a generalized adaptation of the traditional DEA model

$$\begin{aligned}
 & \text{Min } \xi_M \theta - (1 - \xi_M) \theta \\
 & \text{s. t.} \\
 & \sum_{i=1}^n \lambda_i x_{ij} \leq \xi_M x_{pj} + (1 - \xi_M) \theta x_{pj} \quad , \forall j = 1 \dots v \\
 & \sum_{i=1}^n \lambda_i y_{ik} \geq \xi_M \theta y_{pk} + (1 - \xi_M) y_{pk} \quad , \forall k = 1 \dots w \\
 & \xi_R \left[\sum_{i=1}^n \lambda_i - 1 \right] = 0 \\
 & \lambda_i \geq 0, (i = 1, 2, \dots, n)
 \end{aligned} \tag{M-3}$$

where ξ_M is the model type variable for orientation, which is defined as:

$$\xi_M = \begin{cases} 1 & \text{if the model is Output oriented} \\ 0 & \text{if the model is Input oriented} \end{cases}$$

and ξ_R is the model type variable for return to scale, which is defined as:

$$\xi_R = \begin{cases} 1 & \text{if the model is variable return to scale} \\ 0 & \text{if the model is constant return to scale} \end{cases}$$

As a result, we can identify three distinct cases:

- $\xi_M + \xi_R = 2$, represents a output-oriented VRS DEA model.
- $\xi_M + \xi_R = 1 \begin{cases} \text{If } \xi_M = 1, \text{ represents an output - oriented CRS DEA model.} \\ \text{If } \xi_R = 1, \text{ represents a input - oriented VRS DEA model.} \end{cases}$
- $\xi_M + \xi_R = 0$, represents an input-oriented CRS DEA model.

a- First stage - handle fuzzy variation: In this part, we have developed fuzzy generalized DEA models, which are unable to handle certain input or output variables. All other variables are considered deterministic in nature.

Remark 2: If either output and/or input are considered as fuzzy variables, then the fuzzy equivalent generalized DEA model to evaluate the p^{th} DMU efficiency level presented in Model (M-3) can be formulated as follows:

$$\begin{aligned}
 & \text{Min } \xi_M \theta - (1 - \xi_M) \theta \\
 & \text{s. t.} \\
 & \sum_{i=1}^n \lambda_i x_{ij} \leq \xi_M x_{pj} + (1 - \xi_M) \theta x_{pj} \quad , \forall j \in J_D \\
 & \sum_{i=1}^n \lambda_i \tilde{x}_{ij} \leq \xi_M \tilde{x}_{pj} + (1 - \xi_M) \theta \tilde{x}_{pj} \quad , \forall j \in J_F
 \end{aligned}$$

$$\begin{aligned}
\sum_{i=1}^n \lambda_i y_{ik} &\geq \xi_M \emptyset y_{pk} + (1 - \xi_M) y_{pk} & , \forall k \in K_D \\
\sum_{i=1}^n \lambda_i \tilde{y}_{ik} &\geq \xi_M \emptyset \tilde{y}_{pk} + (1 - \xi_M) \tilde{y}_{pk} & , \forall k \in K_F \\
\xi_R \left[\sum_{i=1}^n \lambda_i - 1 \right] &= 0 \\
\lambda_i &\geq 0, (i = 1, 2, \dots, n), & (M-4)
\end{aligned}$$

where $\tilde{x}_{ij}$ is the fuzzy input j for DMU i , $\tilde{y}_{ik}$ is the fuzzy output k for DMU i , J_D is the deterministic inputs set, J_F is the fuzzy inputs set, J are all the inputs set, K_D is the deterministic outputs set, K_F is the fuzzy outputs set, and K are all the outputs set. Taking into consideration, $J_D \cup J_F = J$ and $K_D \cup K_F = K$.

We can note that, when input, output, or both observations are treated as fuzzy variables, the deterministic inequalities are transformed into corresponding fuzzy inequalities. This transformation utilizes principles from fuzzy set theory as outlined in [39].

Proposition 1: considering fuzzy input variables ($\tilde{y}_{ik} \in K_F$) have triangular membership functions, then the crisp equivalent linear model for fuzzy generalized DEA model in the context of Model (M - 4) can be expressed as follows:

$$\begin{aligned}
&\text{Min } \xi_M \emptyset - (1 - \xi_M) \theta \\
&s. t. \\
&\sum_{i=1}^n \lambda_i x_{ij} \leq \xi_M x_{pj} + (1 - \xi_M) \theta x_{pj} & , \forall j \in J_D \\
&\sum_{i=1}^n \lambda_i \tilde{x}_{ij} \leq \xi_M \tilde{x}_{pj} + (1 - \xi_M) \theta \tilde{x}_{pj} & , \forall j \in J_F \\
&\sum_{i=1}^n \lambda_i y_{ik} \geq \xi_M \emptyset y_{pk} + (1 - \xi_M) y_{pk} & , \forall k \in K_D \\
&\sum_{i=1}^n \lambda_i \tilde{y}_{ik} \geq \xi_M \emptyset \tilde{y}_{pk} + (1 - \xi_M) \tilde{y}_{pk} & , \forall k \in K_F \\
&\alpha y_{ik}^M + (1 - \alpha) y_{ik}^L \leq \tilde{y}_{ik} \leq \alpha y_{ik}^M + (1 - \alpha) y_{ik}^U & , \forall k \in K_F \\
&\xi_R \left[\sum_{i=1}^n \lambda_i - 1 \right] = 0 \\
&\lambda_i \geq 0, (i = 1, 2, \dots, n), & (M-5)
\end{aligned}$$

where y_{ik}^L is the fuzzy output variable k for DMU i lower value, y_{ik}^M is the fuzzy output variable k for DMU i median value, y_{ik}^U is the fuzzy output variable k for DMU i upper value, and α is the fuzzy variables α -cut level.

Proof: Consider an input fuzzy numbers triangular membership function that are used for explaining input fuzzy for (M-5) as follows:

$$\mu_{\tilde{y}_{ik}} = \begin{cases} 0, & y_{ik} \leq y_{ik}^L \\ \frac{y_{ik} - y_{ik}^L}{y_{ik}^M - y_{ik}^L}, & y_{ik}^L \leq y_{ik} \leq y_{ik}^M \\ \frac{y_{ik}^U - y_{ik}}{y_{ik}^U - y_{ik}^M}, & y_{ik}^M \leq y_{ik} \leq y_{ik}^U \\ 0, & y_{ik} \geq y_{ik}^U \end{cases} \quad (1)$$

$$\tilde{y}_{ik} = (y_{ik}^L, y_{ik}^M, y_{ik}^U), \quad 0 \leq y_{ik}^L \leq y_{ik}^M \leq y_{ik}^U \rightarrow \tilde{y}_{ik} \in [y_{ik}^L, y_{ik}^U] \quad (2)$$

$\mu_{\tilde{y}_{ik}}$ as express in eq. (1) defines the α -cuts of outputs $\tilde{y}_{ik}$ for triangular fuzzy numbers of arithmetic operations. This approach yields intervals representing lower and upper bounds at different α -levels. The following outlines the application of α -cut interval operations to fuzzy inputs.

$$\mu_{\tilde{y}_{ik}} \geq \alpha \frac{y_{ik}^U - y_{ik}^L}{y_{ik}^M - y_{ik}^L} \geq \alpha \frac{y_{ik}^U - y_{ik}^L}{y_{ik}^U - y_{ik}^M} \geq \alpha \quad (3)$$

$$\tilde{y}_{ik} \in [\alpha y_{ik}^M + (1 - \alpha)y_{ik}^L, \alpha y_{ik}^M + (1 - \alpha)y_{ik}^U] \quad (4)$$

Proposition 2: considering fuzzy input variables ($\tilde{x}_{ij} \in J_F$) have triangular membership functions, then the crisp equivalent linear model for fuzzy generalized DEA model in the context of Model (M - 5) can be expressed as follows:

$$\text{Min } \xi_M \emptyset - (1 - \xi_M)\theta$$

$$\begin{aligned} & s. t. \\ & \sum_{i=1}^n \lambda_i x_{ij} \leq \xi_M x_{pj} + (1 - \xi_M)\theta x_{pj} \quad , \forall j \in J_D \\ & \sum_{i=1}^n \lambda_i \tilde{x}_{ij} \leq \xi_M \tilde{x}_{pj} + (1 - \xi_M)\theta \tilde{x}_{pj} \quad , \forall j \in J_F \\ & \alpha x_{ij}^M + (1 - \alpha)x_{ij}^L \leq \tilde{x}_{ij} \leq \alpha x_{ij}^M + (1 - \alpha)x_{ij}^U \quad , \forall j \in J_F \\ & \sum_{i=1}^n \lambda_i y_{ik} \geq \xi_M \emptyset y_{pk} + (1 - \xi_M) y_{pk} \quad , \forall k \in K_D \\ & \sum_{i=1}^n \lambda_i \tilde{y}_{ik} \geq \xi_M \emptyset \tilde{y}_{pk} + (1 - \xi_M) \tilde{y}_{pk} \quad , \forall k \in K_F \\ & \alpha y_{ik}^M + (1 - \alpha)y_{ik}^L \leq \tilde{y}_{ik} \leq \alpha y_{ik}^M + (1 - \alpha)y_{ik}^U \quad , \forall k \in K_F \\ & \xi_R \left[\sum_{i=1}^n \lambda_i - 1 \right] = 0 \\ & \lambda_i \geq 0, (i = 1, 2, \dots, n) \end{aligned} \quad (M - 6)$$

where x_{ij}^L is the input fuzzy variable j for DMU i lower value, x_{ij}^M is the input fuzzy variable j for DMU i median value, x_{ij}^U is the input fuzzy variable j for DMU i upper value, and α is the level of α -cut for fuzzy variables j

Proof: Consider an input fuzzy numbers triangular membership function that are used for explaining input fuzzy for (M-6) as follows:

$$\mu_{\tilde{x}_{ij}} = \begin{cases} 0, & x_{ij} \leq x_{ij}^L \\ \frac{x_{ij} - x_{ij}^L}{x_{ij}^M - x_{ij}^L}, & x_{ij}^L \leq x_{ij} \leq x_{ij}^M \\ \frac{x_{ij}^U - x_{ij}}{x_{ij}^U - x_{ij}^M}, & x_{ij}^M \leq x_{ij} \leq x_{ij}^U \\ 0, & x_{ij} \geq x_{ij}^U \end{cases} \quad (5)$$

$$\tilde{x}_{ij} = (x_{ij}^L, x_{ij}^M, x_{ij}^U), \quad 0 \leq x_{ij}^L \leq x_{ij}^M \leq x_{ij}^U \rightarrow \tilde{x}_{ij} \in [x_{ij}^L, x_{ij}^U] \quad (6)$$

$\mu_{\tilde{x}_{ij}}$ as express in eq. (5) defines the α -cuts of inputs $\tilde{x}_{ij}$ for triangular fuzzy numbers of arithmetic operations. This approach yields intervals representing lower and upper bounds at different α -levels. The following outlines the application of α -cut interval operations to fuzzy inputs.

$$\mu_{\tilde{x}_{ij}} \geq \alpha \frac{x_{ij}^L - x_{ij}^L}{x_{ij}^M - x_{ij}^L} \geq \alpha \frac{x_{ij}^U - x_{ij}^M}{x_{ij}^U - x_{ij}^M} \geq \alpha \quad (7)$$

$$\tilde{x}_{ij} \in [\alpha x_{ij}^M + (1 - \alpha) x_{ij}^L, \alpha x_{ij}^M + (1 - \alpha) x_{ij}^U] \quad (8)$$

- b- **Second Stage - handle stochastic variations:** In this part, we have constructed stochastic generalized DEA model, which integrates stochastic elements for any certain either output and/or input variable. However, the remaining variables are treated to be deterministic.

Proposition 4: assuming that either output and/or input are random variables, then the equivalent generalized chance-constraint DEA model to evaluate the efficiency level for DMU p within Model (M - 3) can be formulated as follows:

$$\text{Min } \xi_M \theta - (1 - \xi_M) \theta$$

$$\begin{aligned} & s. t. \\ & \sum_{i=1}^n \lambda_i x_{ij} \leq \xi_M x_{pj} + (1 - \xi_M) \theta x_{pj} \quad , \forall j \in J_D \\ & pr \left\{ \sum_{i=1}^n \lambda_i x_{ij} \leq \xi_M x_{pj} + (1 - \xi_M) \theta x_{pj} \right\} \geq (1 - \eta_j) \quad , \forall j \in J_S \\ & \sum_{i=1}^n \lambda_i y_{ik} \geq \xi_M \theta y_{pk} + (1 - \xi_M) y_{pk} \quad , \forall k \in K_D \\ & pr \left\{ \sum_{i=1}^n \lambda_i y_{ik} \geq \xi_M \theta y_{pk} + (1 - \xi_M) y_{pk} \right\} \geq (1 - \eta_k) \quad , \forall k \in K_S \\ & \xi_R \left[\sum_{i=1}^n \lambda_i - 1 \right] = 0 \\ & \lambda_i \geq 0, (i = 1, 2, \dots, n), \end{aligned} \quad (M - 7)$$

where η_j is the level of significance for stochastic input variable j , η_k is the level of significance for stochastic output variable k , J_S is the stochastic inputs set, J : all inputs sets, $J_D \cup J_S = J$, K_S is the stochastic outputs set, and K is the all outputs set, $K_D \cup K_S = K$.

Detailed proof is presented at [40].

Proposition 5: Assume the random input variable ($x_{ij} \in J_S$) is normally distributed, then the deterministic nonlinear equivalent model, for generalized stochastic DEA model presented in (M-7) for p^{th} DMU, is as follows:

$$\text{Min } \xi_M \theta - (1 - \xi_M) \theta$$

$$\begin{aligned} & s. t. \\ & \sum_{i=1}^n \lambda_i x_{ij} \leq \xi_M x_{pj} + (1 - \xi_M) \theta x_{pj} \quad , \forall j \in J_D \\ & \sum_{i=1}^n \lambda_i \mu_{ij} - \xi_M \mu_{pj} - (1 - \xi_M) \theta \mu_{pj} \leq e_j \sqrt{(\lambda_p - (1 - \xi_M) \theta)^2 \sigma_{pj}^2 + \sum_{i=1, i \neq p}^n \lambda_i^2 \sigma_{ij}^2} \quad , \forall j \in J_S \\ & \sum_{i=1}^n \lambda_i y_{ik} \geq \xi_M \theta y_{pk} + (1 - \xi_M) y_{pk} \quad , \forall k \in K_D \end{aligned}$$

$$\begin{aligned} & pr \left\{ \sum_{i=1}^n \lambda_i y_{ik} \geq \xi_M \emptyset y_{pk} + (1 - \xi_M) y_{pk} \right\} \geq (1 - \eta_k) \quad , \forall k \in K_S \\ & \xi_R \left[\sum_{i=1}^n \lambda_i - 1 \right] = 0 \\ & \lambda_i \geq 0, (i = 1, 2, \dots, n) , \end{aligned} \quad (M-8)$$

Proof: assume that each input $x_{ij} \in J_S$ is normally distributed with a mean μ_{ij} and variance σ_{ij}^2 . Now, define the random variable for all input $j \in J_S$

$$u_j = \sum_{i=1}^n \lambda_i x_{ij} - \xi_M x_{pj} - (1 - \xi_M) \theta x_{pj} \quad (9)$$

with mean and variance:

$$E(u_j) = \sum_{i=1}^n \lambda_i \mu_{ij} - \xi_M \mu_{pj} - (1 - \xi_M) \theta \mu_{pj} \equiv \mu_{u_j} \quad , \quad (10)$$

$$var(u_j) = (\lambda_p - (1 - \xi_M) \theta)^2 \sigma_{pj}^2 + \sum_{i=1, i \neq p}^n \lambda_i^2 \sigma_{ij}^2 \equiv \sigma_{u_j}^2 \quad . \quad (11)$$

Since the x_{ij} 's are normally distributed, $x_{ij} \sim N(\mu_{ij}, \sigma_{ij}^2)$, z_j denote the equivalent standardized normal value for the variable u_j , as follows,

$$z_j = \frac{u_j - \mu_{u_j}}{\sigma_{u_j}} \quad . \quad (12)$$

Hence,

$$pr \left\{ \sum_{i=1}^m \lambda_i x_{ij} \leq \xi_M x_{pj} + (1 - \xi_M) \theta x_{pj} \right\} = pr\{u_j \leq 0\} = pr \left\{ z_j \leq \frac{-\mu_{u_j}}{\sigma_{u_j}} \right\} \quad . \quad (13)$$

Using the normal distribution symmetry property,

$$pr \left\{ z_j \leq \frac{-\mu_{u_j}}{\sigma_{u_j}} \right\} = pr \left\{ z_j \geq \frac{\mu_{u_j}}{\sigma_{u_j}} \right\} = 1 - \varphi \left(\frac{\mu_{u_j}}{\sigma_{u_j}} \right) \quad , \quad (14)$$

where $\varphi(\cdot)$ denotes the distribution function for cumulative standard. The equivalent restriction for the chance-constrained DEA problem inequality as follows:

$$1 - \varphi \left(\frac{\mu_{u_j}}{\sigma_{u_j}} \right) \geq (1 - \alpha_j) \quad , \quad (15)$$

$$\varphi \left(\frac{\mu_{u_j}}{\sigma_{u_j}} \right) \leq \varphi(e_j) \quad . \quad (16)$$

$\varphi(e)$ is determined by referencing the standard normal distribution table. Therefore, the final equation can be written as

$$\mu_{u_j} \leq e_j \sigma_{u_j} \quad . \quad (17)$$

So, the next step is to substitute Eq. (10) and (11) in Eq. (17), i.e.,

$$\sum_{i=1}^n \lambda_i \mu_{ij} - \xi_M \mu_{pj} - (1 - \xi_M) \theta \mu_{pj} \leq e_j \sqrt{(\lambda_p - (1 - \xi_M) \theta)^2 \sigma_{pj}^2 + \sum_{i=1, i \neq p}^n \lambda_i^2 \sigma_{ij}^2} \quad (18)$$

Proposition 6: Assume both the random output and input variables ($y_{ik} \in K_s, x_{ij} \in J_s$) are normally distributed, then the deterministic nonlinear equivalent model, for generalized stochastic DEA model presented in (M-8) for the p^{th} DMU, is as follows:

$$\begin{aligned} & \text{Min } \xi_M \emptyset - (1 - \xi_M) \theta \\ & \text{s. t.} \\ & \sum_{i=1}^n \lambda_i x_{ij} \leq \xi_M x_{pj} + (1 - \xi_M) \theta x_{pj} \quad , \forall j \in J_D \\ & \sum_{i=1}^n \lambda_i \mu_{ij} - \xi_M \mu_{pj} - (1 - \xi_M) \theta \mu_{pj} \leq e_j \sqrt{(\lambda_p - (1 - \xi_M) \theta)^2 \sigma_{pj}^2 + \sum_{i=1, i \neq p}^n \lambda_i^2 \sigma_{ij}^2} \quad , \forall j \in J_S \\ & \sum_{i=1}^n \lambda_i y_{ik} \geq \xi_M \emptyset y_{pk} + (1 - \xi_M) y_{pk} \quad , \forall k \in K_D \\ & \sum_{i=1}^n \lambda_i \mu_{ik} - \xi_M \emptyset \mu_{pk} - (1 - \xi_M) \mu_{pk} \geq e_k \sqrt{(\lambda_p - \xi_M \emptyset)^2 \sigma_{pk}^2 + \sum_{i=1, i \neq p}^n \lambda_i^2 \sigma_{ik}^2} \quad , \forall k \in K_S \\ & \xi_R \left[\sum_{i=1}^n \lambda_i - 1 \right] = 0 \\ & \lambda_i \geq 0, (i = 1, 2, \dots, n), \end{aligned} \quad (M-9)$$

Proof: consider each output $y_{ik} \in K_s$ to be normally distributed, $y_{ik} \sim N(\mu_{ik}, \sigma_{ik}^2)$. so, we can define u_k as a random variable for all output variables taking into consideration $k \in K_s$

$$u_k = \sum_{i=1}^n \lambda_i y_{ik} - \xi_M \emptyset y_{pk} - (1 - \xi_M) y_{pk} \quad (19)$$

with mean and variance:

$$E(u_k) = \sum_{i=1}^n \lambda_i \mu_{ik} - \xi_M \emptyset \mu_{pk} - (1 - \xi_M) \mu_{pk} \equiv \mu_{uk} \quad (20)$$

$$\text{var}(u_k) = (\lambda_p - \xi_M \emptyset)^2 \sigma_{pk}^2 + \sum_{i=1, i \neq p}^n \lambda_i^2 \sigma_{ik}^2 \equiv \sigma_{uk}^2 \quad (21)$$

Since y_{ik} 's are normally distributed, $y_{ik} \sim N(\mu_{uk}, \sigma_{uk}^2)$, so let z_k denote the equivalent standardized normal value for the variable u_k , as follows,

$$z_k = \frac{u_k - \mu_{uk}}{\sigma_{uk}} \quad (22)$$

Hence,

$$\text{pr} \left\{ \sum_{i=1}^n \lambda_i y_{ik} \geq \xi_M \emptyset y_{pk} + (1 - \xi_M) y_{pk} \right\} = \text{pr} \{u_k \geq 0\} = \text{pr} \left\{ z_k \geq \frac{-\mu_{uk}}{\sigma_{uk}} \right\} \quad (23)$$

Using the normal distribution symmetry property,

$$pr \left\{ z_k \geq \frac{-\mu_{u_k}}{\sigma_{u_k}} \right\} = pr \left\{ z_k \leq \frac{\mu_{u_k}}{\sigma_{u_k}} \right\} = \varphi \left(\frac{\mu_{u_k}}{\sigma_{u_k}} \right) \quad (24)$$

where $\varphi(\cdot)$ denotes the distribution function for cumulative standard. The equivalent restriction for the chance-constrained DEA problem inequality [40] as follows:

$$\varphi \left(\frac{\mu_{u_k}}{\sigma_{u_k}} \right) \geq (1 - \alpha_k) \quad (25)$$

$$\varphi \left(\frac{\mu_{u_k}}{\sigma_{u_k}} \right) \geq \varphi(e_k) \quad (26)$$

The value of $\varphi(e)$ is determined by referencing the standard normal distribution table. Therefore, the final equation can be written as

$$\mu_{u_k} \geq e_k \sigma_{u_k} \quad (27)$$

Substitution equations (20) and (21) in equation (27) leads to

$$\sum_{i=1}^n \lambda_i \mu_{ik} - \xi_M \emptyset \mu_{pk} - (1 - \xi_M) \mu_{pk} \geq e_k \sqrt{(\lambda_p - \xi_M \emptyset)^2 \sigma_{pk}^2 + \sum_{i=1, i \neq p}^n \lambda_i^2 \sigma_{ik}^2}, \forall k \in K_s \quad (28)$$

c- **Third stage - handle neutrosophic variation:** In this part, we make the attempt to construct neutrosophic generalized DEA model with two assumptions; some specific variables either input and/or output are neutrosophic in nature while the remaining are deterministic under the first assumption.

- **Definition 1**[37]: Let U be the field of all objects and let u be an arbitrary object element in U , $u \in U$. A neutrosophic set of $(\tilde{W}^N)$ is expressed as $\tilde{W}^N = \{ \langle u: T_{\tilde{W}^N}(u), I_{\tilde{W}^N}(u), F_{\tilde{W}^N}(u) \rangle, u \in U, T_{\tilde{W}^N}(u), I_{\tilde{W}^N}(u), F_{\tilde{W}^N}(u) \in]0^-, 1^+[\}$, where $T_{\tilde{W}^N}(u)$, $I_{\tilde{W}^N}(u)$, and $F_{\tilde{W}^N}(u)$ are membership functions representing truth, indeterminacy, and falsity, respectively.

- **Definition 2** [37]: the general formulation of the neutrosophic set single value $(\tilde{W}^{SVN})$ concerning to non-empty set u is $\tilde{W}^{SVN} = \{ \langle u, T_{\tilde{W}^N}(u), I_{\tilde{W}^N}(u), F_{\tilde{W}^N}(u) \rangle, u \in U \}$. In this representation, $T_{\tilde{W}^N}(u), I_{\tilde{W}^N}(u), F_{\tilde{W}^N}(u)$ all take the values $0^- \leq T_{\tilde{W}^N}(u) + I_{\tilde{W}^N}(u) + F_{\tilde{W}^N}(u) \leq 3^+$, such that u is an element of the set U .

- **Definition 3** [37]: let $\tilde{v}^{TN} = \langle (a, b, c); L_{\tilde{v}^{TN}}, \delta_{\tilde{v}^{TN}}, F_{\tilde{v}^{TN}} \rangle$ be a triangular neutrosophic single valued (SVTFN), Then SVTFN as Score function

$$SF(\tilde{v}^{TN}) = \left(\frac{1}{4}(a + 2b + c) \right) \left(\frac{1}{3}(2 + L_{\tilde{v}^{TN}} - \delta_{\tilde{v}^{TN}} - F_{\tilde{v}^{TN}}) \right) \quad (29)$$

Remark 3: If certain variable either output and/or input are considered as neutrosophic variables, then the equivalent neutrosophic generalized DEA model to evaluate the p^{th} DMU relative efficiency level in model $(M - 3)$ can be formulated as follows:

$$\text{Min } \xi_M \emptyset - (1 - \xi_M) \theta$$

s. t.

$$\sum_{i=1}^n \lambda_i x_{ij} \leq \xi_M x_{pj} + (1 - \xi_M) \theta x_{pj}, \quad \forall j \in J_D$$

$$\sum_{i=1}^n \lambda_i \tilde{x}_{ij}^{TN} \leq \xi_M \tilde{x}_{pj}^{TN} + (1 - \xi_M) \theta \tilde{x}_{pj}^{TN}, \quad \forall j \in J_N$$

$$\sum_{i=1}^n \lambda_i y_{ik} \geq \xi_M \emptyset y_{pk} + (1 - \xi_M) y_{pk}, \quad \forall k \in K_D$$

$$\begin{aligned}
& \sum_{i=1}^n \lambda_i \tilde{y}_{ik}^{TN} \geq \xi_M \emptyset \tilde{y}_{pk}^{TN} + (1 - \xi_M) \tilde{y}_{pk}^{TN}, \forall k \in K_N \\
& \xi_R \left[\sum_{i=1}^n \lambda_i - 1 \right] = 0 \\
& \lambda_i \geq 0, (i = 1, 2, \dots, n),
\end{aligned} \tag{M-10}$$

where J_N is the neutrosophic inputs set, J is all the inputs set, $J_D \cup J_N = J$, and K_N is the neutrosophic outputs set, and K is all the outputs set, where $K_D \cup K_N = K$.

Remark 4: Assume that each input $\tilde{x}_{ij}^{TN} \in J_N$ and $\tilde{y}_{ik}^{TN} \in K_N$ are triangular neutrosophic variables, then the deterministic equivalent linear model explained in the neutrosophic generalized DEA model (M-10) for p^{th} DMU is given below based on a single valued triangular neutrosophic set definition (as shown in definition 3):

$$\begin{aligned}
& \text{Min } \xi_M \emptyset - (1 - \xi_M) \theta \\
& \text{s. t.} \\
& \sum_{i=1}^n \lambda_i x_{ij} \leq \xi_M x_{pj} + (1 - \xi_M) \theta x_{pj}, \forall j \in J_D \\
& \sum_{i=1}^n \lambda_i SF(\tilde{x}_{ij}^{TN}) \leq \xi_M SF(\tilde{x}_{pj}^{TN}) + (1 - \xi_M) \theta SF(\tilde{x}_{pj}^{TN}), \forall j \in J_N \\
& \sum_{i=1}^n \lambda_i y_{ik} \geq \xi_M \emptyset y_{pk} + (1 - \xi_M) y_{pk}, \forall k \in K_D \\
& \sum_{i=1}^n \lambda_i SF(\tilde{y}_{ik}^{TN}) \geq \xi_M \emptyset SF(\tilde{y}_{pk}^{TN}) + (1 - \xi_M) SF(\tilde{y}_{pk}^{TN}), \forall k \in K_N \\
& \xi_R \left[\sum_{i=1}^n \lambda_i - 1 \right] = 0 \\
& \lambda_i \geq 0, (i = 1, 2, \dots, n),
\end{aligned} \tag{M-11}$$

Finally, the proposed generalized DEA model, designed to accommodate various types of variable variations (deterministic, fuzzy, stochastic, and neutrosophic), is demonstrated in model (M-12). This model integrates the formulations from Models (M-3), (M-6), (M-9), and (M-11) into a unified framework represented by model (M-12).

$$\begin{aligned}
& \text{Min } \xi_M \emptyset - (1 - \xi_M) \theta \\
& \text{s. t.} \\
& \sum_{i=1}^n \lambda_i x_{ij} \leq \xi_M x_{pj} + (1 - \xi_M) \theta x_{pj}, \forall j \in J_D \\
& \sum_{i=1}^n \lambda_i \tilde{x}_{ij} \leq \xi_M \tilde{x}_{pj} + (1 - \xi_M) \theta \tilde{x}_{pj}, \forall j \in J_F \\
& \alpha x_{ij}^M + (1 - \alpha) x_{ij}^L \leq \tilde{x}_{ij} \leq \alpha x_{ij}^M + (1 - \alpha) x_{ij}^U, \forall j \in J_F \\
& \sum_{i=1}^n \lambda_i \mu_{ij} - \xi_M \mu_{pj} - (1 - \xi_M) \theta \mu_{pj} \leq e_j \sqrt{(\lambda_p - (1 - \xi_M) \theta)^2 \sigma_{pj}^2 + \sum_{i=1, i \neq p}^n \lambda_i^2 \sigma_{ij}^2}, \forall j \in J_S
\end{aligned}$$

$$\begin{aligned}
& \sum_{i=1}^n \lambda_i SF(\tilde{x}_{ij}^{TN}) \leq \xi_M SF(\tilde{x}_{pj}^{TN}) + (1 - \xi_M)\theta SF(\tilde{x}_{pj}^{TN}) \quad , \forall j \in J_N \\
& \sum_{i=1}^n \lambda_i y_{ik} \geq \xi_M \emptyset y_{pk} + (1 - \xi_M) y_{pk} \quad , \forall k \in K_D \\
& \sum_{i=1}^n \lambda_i \tilde{y}_{ik} \geq \xi_M \emptyset \tilde{y}_{pk} + (1 - \xi_M) \tilde{y}_{pk} \quad , \forall k \in K_F \\
& \alpha y_{ik}^M + (1 - \alpha) y_{ik}^L \leq \tilde{y}_{ik} \leq \alpha y_{ik}^M + (1 - \alpha) y_{ik}^U \quad , \forall k \in K_F \\
& \sum_{i=1}^n \lambda_i \mu_{ik} - \xi_M \emptyset \mu_{pk} - (1 - \xi_M) \mu_{pk} \geq e_k \sqrt{(\lambda_p - \xi_M \emptyset)^2 \sigma_{pk}^2 + \sum_{i=1, i \neq p}^n \lambda_i^2 \sigma_{ik}^2} \quad , \forall k \in K_s \\
& \sum_{i=1}^n \lambda_i SF(\tilde{y}_{ik}^{TN}) \geq \xi_M \emptyset SF(\tilde{y}_{pk}^{TN}) + (1 - \xi_M) SF(\tilde{y}_{pk}^{TN}) \quad , \forall k \in K_N \\
& \xi_R \left[\sum_{i=1}^n \lambda_i - 1 \right] = 0 \\
& \lambda_i \geq 0, (i = 1, 2, \dots, n) ,
\end{aligned} \tag{M-12}$$

4. Case Study

In this case study, we apply the proposed robust and versatile DEA framework in the assessment of relative efficiency performance for hospitals. Hospitals are quite complex organizations; therefore, the level of uncertainty surrounding some or all the inputs and/or outputs can provide suitable grounds for illustrating the versatility of the DEA model. Their operational inputs and outputs are usually driven by factors that can be deterministic, stochastic, fuzzy, or neutrosophic; therefore, the efficiency assessment approach should be flexible and robust. The case selected involves a comparison of five comparable hospitals, namely H₁, H₂, H₃, H₄, and H₅, operating within similar contexts and providing comparable healthcare services. Various inputs form parts of the analysis, such as the number of doctors, number of nurses, annual budget, and medical equipment available. In addition, outputs to be measured will include the number of patients treated, patient satisfaction rate, and treatment success rate. These reflect both certainty and uncertainty, classified as follows: deterministic variables include the Number of doctors, number of nurses; stochastic variables include Patient satisfaction rate, treatment success rate; fuzzy variables are medical equipment; neutrosophic variables are Annual budget, number of patients treated. It, therefore, gives an indication that this model can handle a wide range of data sets. These input and output variables data are presented in the following Table 1 and Table 2, respectively.

Table 1 Data for the hospital's input variables

Hospital	Number of Doctors	Number of nurses	Annual Budget (millions)	Medical Equipment
H ₁	20	50	$\langle (4.8, 5, 5.2); 0.9, 0.4, 0.1 \rangle$	(0.4, 0.6, 0.8)
H ₂	15	40	$\langle (3.8, 4, 5.5); 0.9, 0.7, 0.1 \rangle$	(0.2, 0.3, 0.4)
H ₃	30	70	$\langle (5.5, 6, 6.3); 0.9, 0.4, 0.1 \rangle$	(0.55, 0.6, 0.85)
H ₄	18	45	$\langle (4.5, 5, 5.5); 0.8, 0.5, 0.1 \rangle$	(0.2, 0.5, 0.7)
H ₅	25	60	$\langle (4.2, 4.7, 5.2); 0.9, 0.4, 0.1 \rangle$	(0.15, 0.4, 0.7)

Table 2 Data for the hospital's output variables

Hospital	Number of Patients Treated (thousands)	Patient Satisfaction Rate % (μ, σ^2)	Treatment Success Rate % (μ, σ^2)
H ₁	$\langle (1.8, 2, 2.1); 1.0, 0.0, 0.0 \rangle$	(85, 5)	(90, 3)
H ₂	$\langle (1.3, 1.4, 1.5); 1.0, 0.0, 0.0 \rangle$	(80, 6)	(85, 4)
H ₃	$\langle (2.8, 3, 3.2); 1.0, 0.0, 0.0 \rangle$	(90, 4)	(95, 2)
H ₄	$\langle (2.3, 2.45, 2.6); 1.0, 0.0, 0.0 \rangle$	(82, 4)	(88, 3)
H ₅	$\langle (1.9, 2, 2.2); 1.0, 0.0, 0.0 \rangle$	(87, 4)	(92, 2)

A proposed algorithm to address the problem under investigation can be implemented in two distinct scenarios, both of which may indicate how the departments under consideration may be considered to be relatively efficient. Scenario 1: A generalized VRS output-oriented VRS DEA is executed as specified in Model **M-12**, with $\xi_M + \xi_R = 2$. Scenario 2: A generalized VRS input-oriented DEA is executed as specified in Model **M-12**, with $\xi_M + \xi_R = 1$, with $\xi_R = 1$.

This will provide an LP formulation to evaluate, in relative terms, the efficiency level of each hospital. In each case, the five models for the five different hospitals are solved using GAMS programming language. The results of the relative efficiency estimated for each hospital involved are presented in Table 3 below.

Table 3 Hospital Relative efficiency Results for each Scenario

Hospital	Scenario 1	Scenario 2
H ₁	100%	100%
H ₂	36%	44%
H ₃	100%	100%
H ₄	44%	56%
H ₅	80%	66%

5. Results Comparison and Sensitivity Analysis

Results analysis reveal that the developed models return promising and comparable results. While VRS DEA models consistently identify the efficient and inefficient DMUs without regard to orientation, either to output or input, efficiency scores for inefficient DMUs could be different. This is so because changes in inputs and outputs do not always relate proportionally. Table 3 shows the relative efficiencies score for each hospital in two scenarios. These efficiency scores for the two scenarios have the following implications for the performance of the five hospitals: Hospitals H₁ and H₃ achieved efficient scores (1.0) in both scenarios, indicating they are operating at optimal efficiency levels, and These hospitals serve as benchmarks for the others in the study. For the remaining hospitals, hospital H₂ shows the lowest efficiency scores (0.36 and 0.44), hospital H₄ demonstrates moderate inefficiency (0.44 and 0.56), and hospital H₅ shows better performance but still room for improvement (0.80 and 0.66). We notice that the input-oriented scenario (Scenario 2) generally yielded higher efficiency scores than the output-oriented scenario, this means that hospitals might have more control over their input utilization than their output generation. The variation in scores between the two scenarios depicts the flexibility and adaptability of the proposed DEA model in capturing different efficiency dimensions. Finally, we can recommend:

- Benchmark Hospitals (H₁ and H₃): these hospitals represent best practices and can serve as a model for others. It is true that their current practices need to be maintained, but continuous monitoring is recommended to uphold their efficiency.
- Inefficient Hospitals (H₂, H₄, and H₅): H₂ requires major improvements in both input utilization and output generation. Efforts should emphasize enhancements in operational processes, staff productivity, and patient care outcomes. H₄: the amelioration in Scenario 2 suggests that it needs to focus on its input optimization. It

should try investing in training, utilizing equipment, and budget allocation. H_5 : does better on both H_2 and H_4 , but it must balance input optimization with output improvement. To enhance efficiency, it may use strategies that streamline operations and reallocate resources.

6. Conclusion and Future works

This research developed and demonstrated a strong and flexible DEA framework that had already addressed complex challenges in efficiency assessment under different forms of uncertainty. By implementation on the case study of hospitals, we successfully validated its capabilities to treat deterministic, stochastic, fuzzy, and neutrosophic variables simultaneously in input and output parameters, which was considered an advance in the methodology of DEA. The proposed framework has several key strengths and contributions. First, the ability to handle multiple types of data uncertainty within a model represents a substantial improvement over the traditional DEA approach. Second, the flexibility in handling both input-oriented and output-oriented scenarios, with different returns to scale assumptions, makes the framework a comprehensive tool for efficiency assessment across diverse organizational contexts. The validation of the practical applicability of our framework, as obtained from the hospital case study, indicates that there are significant differences in the efficiency scores of the analyzed units, ranging from 0.36 to 1.0. This application shows that the model is able to highlight both benchmark performers and opportunities for improvement, thus providing valuable insights for decision-makers in performance management and resource allocation.

Moreover, the capability of the framework to provide tangible scores about efficiency while considering multiple forms of uncertainty gives this model considerable strength in terms of supporting any decision. Such features are helpful in complex organizations where conventional deterministic approaches would probably miss operational nuances. The future avenues of this research open with the possibility of additional types and sources of uncertainties or data that may be incorporated within the framework. Longitudinal studies could also be done to assess the change in efficiencies over time and adapt the model to an application in an industrial context; maintain robustness in dealing with uncertainty. The contribution of this research to the field of efficiency assessment is, therefore, immense with an all-encompassing, flexible, and reliable DEA framework. That is where the strength of the model lies: it can handle different forms of uncertainty while retaining practical applicability; hence, it is a useful tool for decision-makers in various sectors, especially in complex organizational settings where uncertainty may be inherent in operational processes.

The aim of this research is developed a DEA model that bear its novelty and generality in considering the different sources of uncertainty—deterministic, fuzzy, stochastic, or neutrosophic—in a common framework. However, its application imposes several limitations. Among these restrictions, enlarged computational complexity can be put as a foremost one. The conjoining of all sorts of evidence in a unified framework LP-based DEA model will put a lot of computational capabilities in stake especially for its generalization to large-scale problems. Another area of concern is scalability. This model has been tried only on a limited number of five hospitals, with performance and scalability to larger databases with many DMUs with complex interdependencies needing to be further duly checked. Also, even if neutrosophic data lend high expressiveness to modeling uncertainty, soul-searching interpretability may prove tough to those practitioners placed in the context of actual healthcare setups, where data clarity and verification must precede everything else. Lastly, a static analysis is conducted with the current model, while the dynamic aspect providing support for performance adjustment commensurate to changing time is the most crucial ingredient in the long-term assessment of care efficiency.

Finally, future research would include working around the above limits to make the model more useful. Creating dynamic DEA models under a neutrosophic framework may be one of them, whereby performance would be captured in time-series trends. Another possible extension could be a generalization of the model to cover multi-sector efficiency analysis, like in education, energy, or even transportation, where uncertainty mostly is manifested in inputs and outputs. Moreover, it could also include machine learning or metaheuristic algorithms in optimizing scale-large problems and increasing computational efficiencies. Future work could also involve developing decision-support frameworks or visualization tools to enhance interpretability of complex fuzzy and neutrosophic results to end-users. Lastly, comparative studies among different paradigms of uncertainty models would go a long way in adding credibility to the robustness and reliability of the proposed framework across different real-world applications.

Data Availability

In this article, no data was used.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Double Framed HyperSoft Set for Studies Factors That Influence and Ways to Improve Vocational College Instruction in Innovation and Entrepreneurship

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Abstract: This study explores the multifaceted dimensions of vocational college instruction in innovation and entrepreneurship. With global economies increasingly relying on innovation as a driver of growth, higher education institutions are called upon to cultivate entrepreneurial mindsets among students. The study identifies and evaluates eight key criteria affecting instructional effectiveness and compares eight instructional strategies (alternatives) using a structured decision-making approach. The methodology integrates both quantitative and qualitative perspectives, ensuring a robust framework for analyzing instructional impact. The MOORA method is used to rank alternatives based on a set of criteria. The Double Framed HyperSoft Set (DFHS) is used to deal with the criteria values by using double framed. The findings highlight which strategies yield the most effective outcomes in fostering innovation-oriented competencies. Results can guide vocational colleges in tailoring their curricula, teaching methods, and institutional support systems to nurture the next generation of entrepreneurs. The research also proposes practical recommendations for enhancing vocational college-based entrepreneurship instruction aligned with the best global practices and student need.

Keywords: Double Framed HyperSoft Set; Vocational college Instruction; Innovation and Entrepreneurship.

1. Introduction

Vocational colleges all around the world have realized in recent years how important it is to equip students with both academic knowledge and real-world skills for the quickly changing workplace. Since they give students the skills and mindset to think creatively, solve challenging problems, and support economic growth, innovation and entrepreneurship have become

essential parts of higher education curricula[1], [2]. One of the most important ways to make sure students are prepared to handle the opportunities and difficulties of the global economy is to incorporate innovation and entrepreneurship into vocational college curricula.

One of the main ways that students can be introduced to the concepts of innovation and entrepreneurship is through vocational college instruction. Students' capacity to think critically, investigate novel ideas, and cultivate entrepreneurial abilities is significantly impacted by the way these subjects are taught[3], [4]. However, there is still a big disconnect between the theoretical information taught in classrooms and the real-world application of that knowledge, even with the increased focus on entrepreneurship in higher education.

How well vocational colleges can teach innovation, and entrepreneurship depends on several factors. These elements include the faculty members' degree of experience and skill, the availability of tools like accelerators and incubators, the general culture of the school, and the backing of business partners[5], [6]. Furthermore, the teaching strategies employed—whether they include practical projects, case studies, or internships—are crucial in closing the knowledge gap between theory and practice. Gaining an understanding of these elements is essential to enhancing vocational college instruction in entrepreneurship and innovation.

The faculty's experience is one of the most important elements in determining how well innovation and entrepreneurship education is shaped. Professors with real-world entrepreneurial or innovative expertise can offer students insightful advice and insights that go beyond conventional academic knowledge[7], [8]. Additionally, faculty development initiatives that focus on enhancing professors' teaching abilities can raise the standard of instruction. The learning experience for students can be greatly improved by making sure that teachers are knowledgeable on both the academic and practical facets of innovation.

The infrastructure and resources required for innovation and entrepreneurship education must be provided by institutions. This includes tangible resources like innovation laboratories, financial support for student-run businesses, and collaborations with professionals in the field who can serve as mentors for students[9], [10]. It's also critical to establish a culture that promotes creativity, risk-taking, and teamwork. Since innovation frequently happens at the nexus of various fields, vocational colleges must also create conditions that stimulate multidisciplinary collaboration.

Student learning outcomes can be significantly impacted by the teaching strategies used in vocational college courses on entrepreneurship and innovation. It's possible that conventional lecture-based methods are insufficient to promote creativity and problem-solving abilities. Project-based learning, experiential learning, and cooperative cooperation might be more successful tactics[11], [12]. These methods assist students in gaining real-world experience, interacting with real-world problems, and comprehending how to use creative thinking in a variety of settings.

Stronger industry-academia cooperation is required to increase the relevance of vocational college education. Vocational colleges can give students access to mentorship, internships, and practical projects that mimic real-world difficulties by collaborating with companies, startups, and other groups. These collaborations further close the gap between education and practice by exposing students to the newest trends, technology, and business difficulties in addition to helping them obtain real-world experience.

Vocational colleges must modify their teaching methods to stay up with the rapidly changing landscape of innovation and entrepreneurship[13], [14]. A static curriculum is insufficient; academic institutions must constantly evaluate and enhance their teaching strategies. Vocational colleges can improve their teaching strategies, make sure they are addressing the needs of their students, and foster the next generation of creative thinkers and entrepreneurs by routinely collecting input from students, faculty, and industry partners.

Criteria weighing plays a vital role in multi-criteria decision making (MCDM) processes, influencing the selection of optimal alternatives. Criteria weights are assigned using various weighting techniques[15], [16]. These techniques ensure a thorough assessment of the options by assisting in identifying the importance of each criterion in the decision-making process. Accurate and trustworthy decision outcomes depend on the proper estimation of criteria weights, particularly in situations where there are several criteria. Inaccurate weights may cause decisions to be less than ideal, possibly ignoring crucial aspects or placing too much emphasis on less significant ones[17], [18]. Because criterion weighting heavily relies on subjective expert assessments, one of the primary issues is the subjectivity involved in weight assignment, which can induce bias and inconsistency.

This reliance may compromise the soundness of the decisions by producing different results depending on the opinions and experiences of the experts. These techniques can also be laborious and time-consuming, involving a great deal of data collecting and analysis. Decision-makers may be discouraged by this complexity from carefully examining all pertinent factors or from revising weights considering fresh information[19], [20]. As a result, choices based on out-of-date or insufficient information may not adequately address pressing issues. All these issues have the potential to greatly affect the results of decisions, especially in domains where accuracy and dependability are essential.

Smarandache presented six new categories of soft sets, including the HyperSoft Set (2018), TreeSoft Set (2022), ForestSoft Set (2024), IndetermSoft Set (2022), IndetermHyperSoft Set (2022), and SuperHyperSoft Set[21], [22], [23].

2. Double Framed HyperSoft Set (DFHS)

The definitions of the DFHS are shown in this section.

Let U be a universal set and $P(U)$ is the powerset of U . Let set of criteria $c_1, c_2, \dots, c_n; n \geq 1$ and the values of crietria $a_1, a_2, \dots, a_n$ and $a_i \cap a_j = \emptyset$ for $i \neq j$

Then the pair $(\omega_1, \omega_2), a_1 \times a_2 \times \dots \times a_n$ the ω_1, ω_2 are mapping from $a_1 \times a_2 \times \dots \times a_n \rightarrow P(U)$ [24]

Example

Let set of alternatives A_1, A_2, A_3, A_4 and set of criteria with values such as:

$$C_1 = \{c_{11}, c_{12}, c_{13}, c_{14}\} \quad (1)$$

$$C_2 = \{c_{21}, c_{22}, c_{23}, c_{24}\} \quad (2)$$

$$C_3 = \{c_{31}, c_{32}, c_{33}, c_{34}\} \quad (3)$$

$$C_4 = \{c_{41}, c_{42}, c_{43}, c_{44}\} \quad (4)$$

Then a mapping $(\omega_1, \omega_2), a_1 \times a_2 \times \dots \times a_n \rightarrow P(U)$ Is called DFHS[25], [26]

$$\omega_1(c_{11}, c_{21}, c_{31}, c_{41}) = \{A_1, A_3\} \quad (5)$$

$$\omega_2(c_{11}, c_{21}, c_{31}, c_{41}) = \{A_2, A_4\} \quad (6)$$

$$\omega_1(c_{11}, c_{21}, c_{31}, c_{41}) = \{A_1\} \quad (7)$$

$$\omega_2(c_{11}, c_{21}, c_{31}, c_{41}) = \{A_2\} \quad (8)$$

We show the steps of the MOORA method under the DFHS to rank the alternatives based on the criteria weights.

Experts created the decision matrix between the criteria using a scale between 0 and 1. The decision matrix is combined into a single matrix. We compute the criteria weights using the average method.

Compute the ratio system such as:

$$Q_{ij} = x_{ij} / \sqrt{\sum_{i=1}^m x_{ij}^2}; i = 1, 2, \dots, m; j = 1, 2, \dots, n; \quad (9)$$

Compute the normalized values for positive and cost criteria such as

$$H_i = \sum_{j=1}^g Q_{ij} - \sum_{j=g+1}^n Q_{ij} \quad (10)$$

Compute the weighted decision matrix

$$R_i = \sum_{j=1}^g w_j H_{ij} - \sum_{j=g+1}^n w_j H_{ij} \quad (11)$$

3. Results

This section shows the results of the DFHS to show the best alternatives based on a set of criteria. This study uses eight criteria and eight alternatives such as:

The criteria with values such as:

- Curriculum Relevance {High, Medium, Low}
- Faculty Expertise {Expert, Competent, Novice}
- Industry Collaboration {Strong, Moderate, Weak}
- Student Engagement {Active, Passive, Inactive}
- Resource Availability {Abundant, Adequate, Scarce}
- Teaching Methodology {Innovative, Traditional, Mixed}
- Institutional Support {Strong, Neutral, Weak}
- Evaluation Techniques {Realistic, Theoretical, Mixed}

The alternatives are: Traditional Lecture-Based Approach, Case-Based Teaching with Real Entrepreneurial Scenarios, Project-Based Learning with Start-Up Development, Industry Mentorship and Internship-Integrated Learning, Hackathons, Innovation Challenges, and Startup Competitions, Interdisciplinary Collaboration-Based Teaching, Online and Blended Entrepreneurship Programs, Problem-Based Learning Using Digital Prototyping Tools.

Three experts created the decision matrices. We combine these decision matrices into a single matrix. We compute the criteria weights using the average method.

C1= 0.1228, This criterion has moderate weight, indicating that aligning the curriculum with current innovation and entrepreneurship trends is important but not the most critical factor.

C2= 0.1238, Slightly more important than C1, this shows that the qualifications and experience of faculty members are key in delivering effective instruction.

C3= 0.1212, Also moderately weighted, this suggests that partnerships with industry are beneficial but may be secondary to internal academic factors.

C4= 0.1446, This is the highest-weighted criterion, indicating that active student participation and interaction are considered crucial for improving instruction.

C5= 0.1157, This has the lowest weight, suggesting that while having sufficient resources (labs, funding, materials) is helpful, it's not as pivotal as other pedagogical or structural elements.

C6= 0.1249, This weight reflects that innovative teaching approaches (like project-based or experiential learning) are a strong factor in instructional effectiveness.

C7= 0.1201, A moderate weight that shows vocational college-level backing (funding, policy, infrastructure) is a meaningful contributor to instructional success.

C8= 0.1270, This relatively high weight emphasizes the importance of effective and realistic assessment methods in measuring and enhancing instructional quality.

We use the definitions of the DFHS to evaluate the alternatives, we divided the alternatives into double frames such as:

First frame, $(\{High\}, \{Expert\}, \{Strong\}, \{Active\}, \{Abundant\}, \{Innovative\}, \{Strong\}, \{Mixed\}) = \{A_1, A_3, A_5, A_7\}$

Second frame, $(\{High\}, \{Expert\}, \{Strong\}, \{Active\}, \{Abundant\}, \{Innovative\}, \{Strong\}, \{Mixed\}) = \{A_2, A_4, A_6, A_8\}$

In the first frame: we show the results of the MOORA method such as:

Compute the ratio system using eq. (9) as shown in Fig 1.

Compute the normalized values for positive and cost criteria using eq. (10).

Compute the weighted decision matrix using eq. (11) as shown in Fig 2. We obtained the final score as shown in Fig 3. We rank the alternatives as shown in Fig 4.

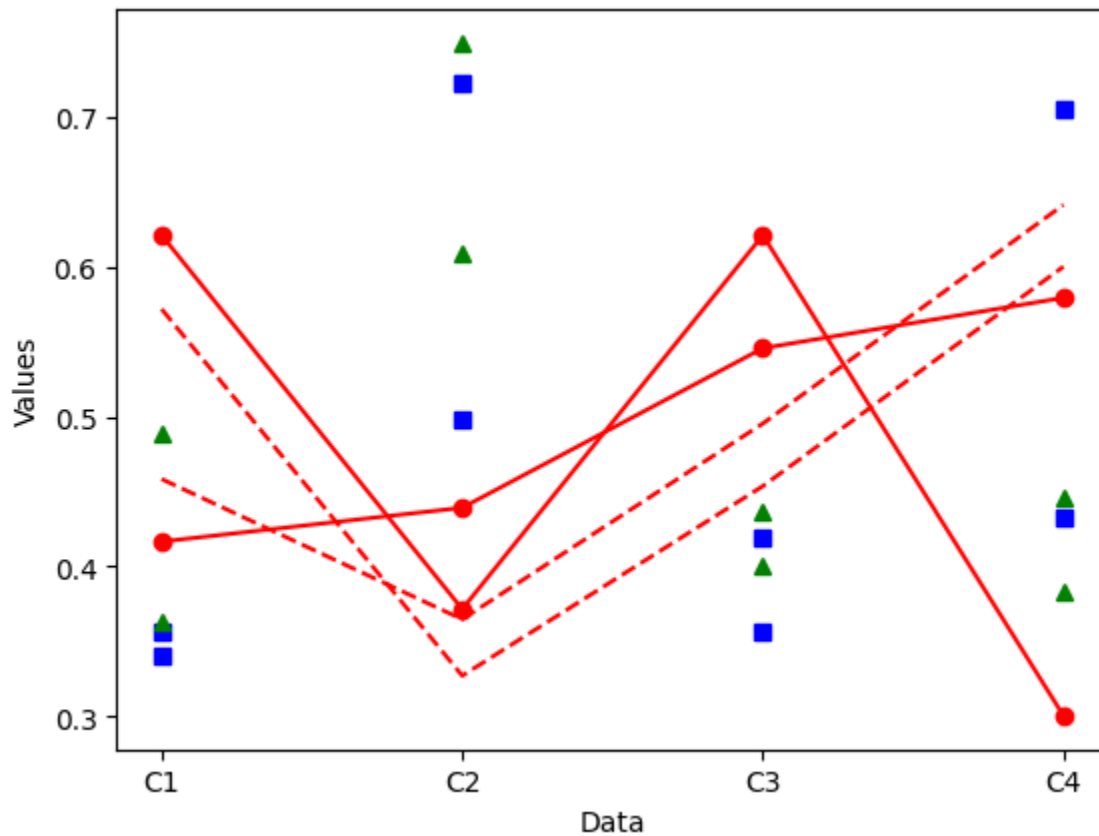


Fig 1. The ratio system values.

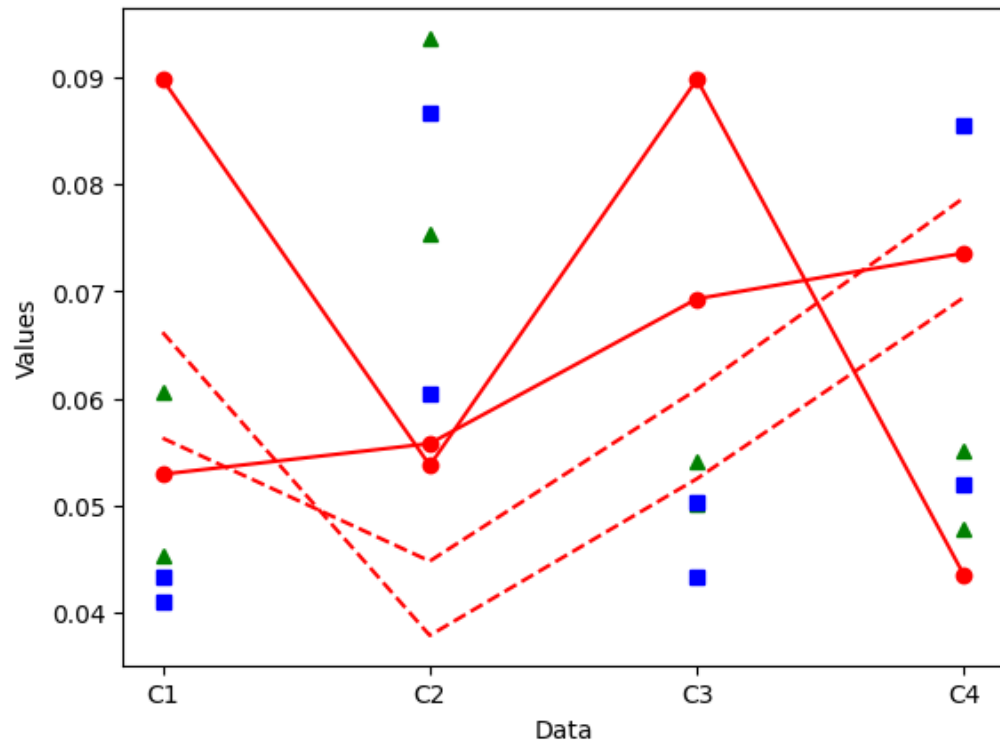


Fig 2. The weighted matrix.

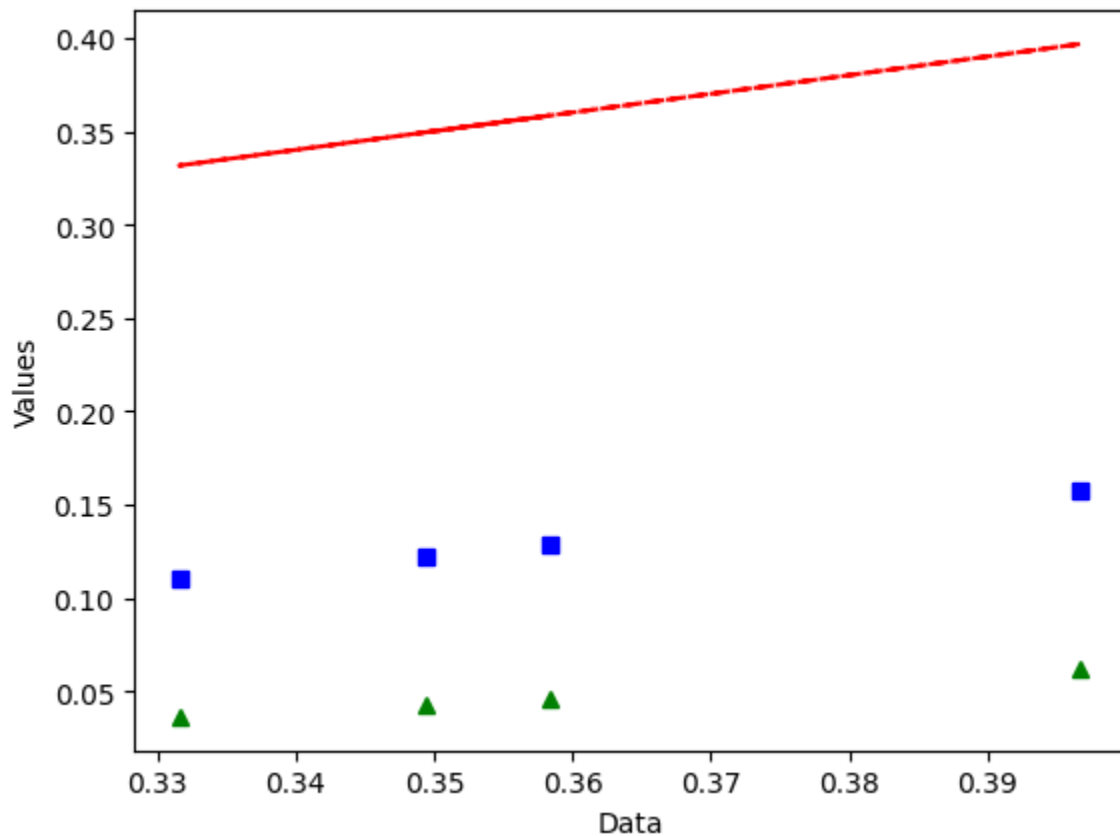


Fig 3. The final score.

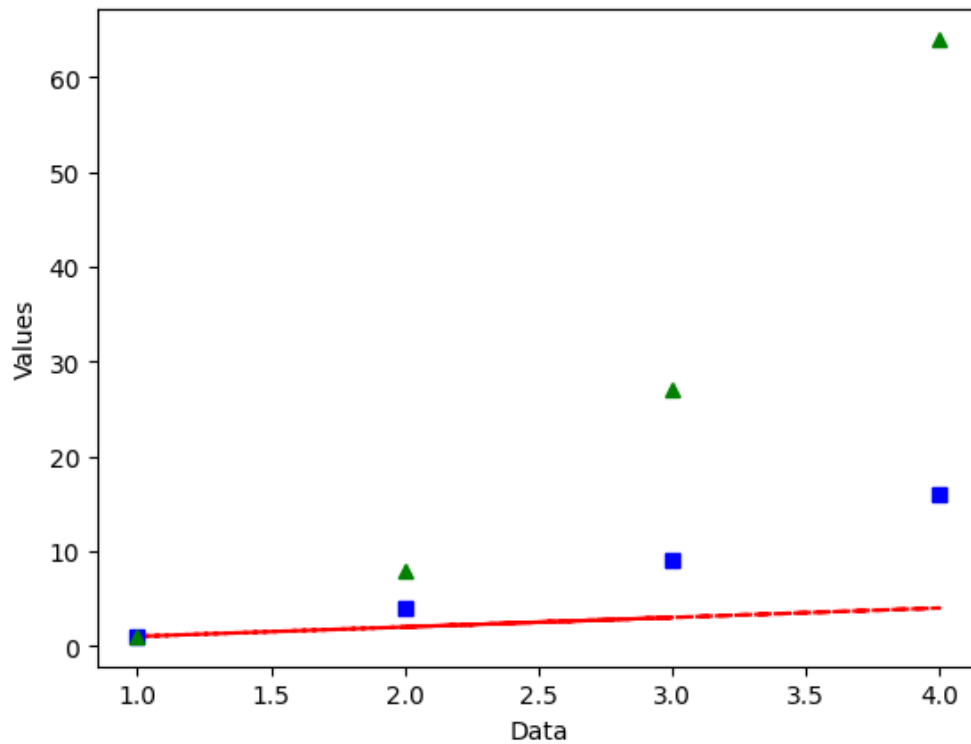


Fig 4. The ranks of alternatives.

In the second frame: we show the results of the MOORA method such as:

Compute the ratio system using eq. (9) as shown in Fig 5.

Compute the normalized values for positive and cost criteria using eq. (10).

Compute the weighted decision matrix using eq. (11) as shown in Fig 6. We obtained the final score as shown in Fig 7. We rank the alternatives as shown in Fig 8.

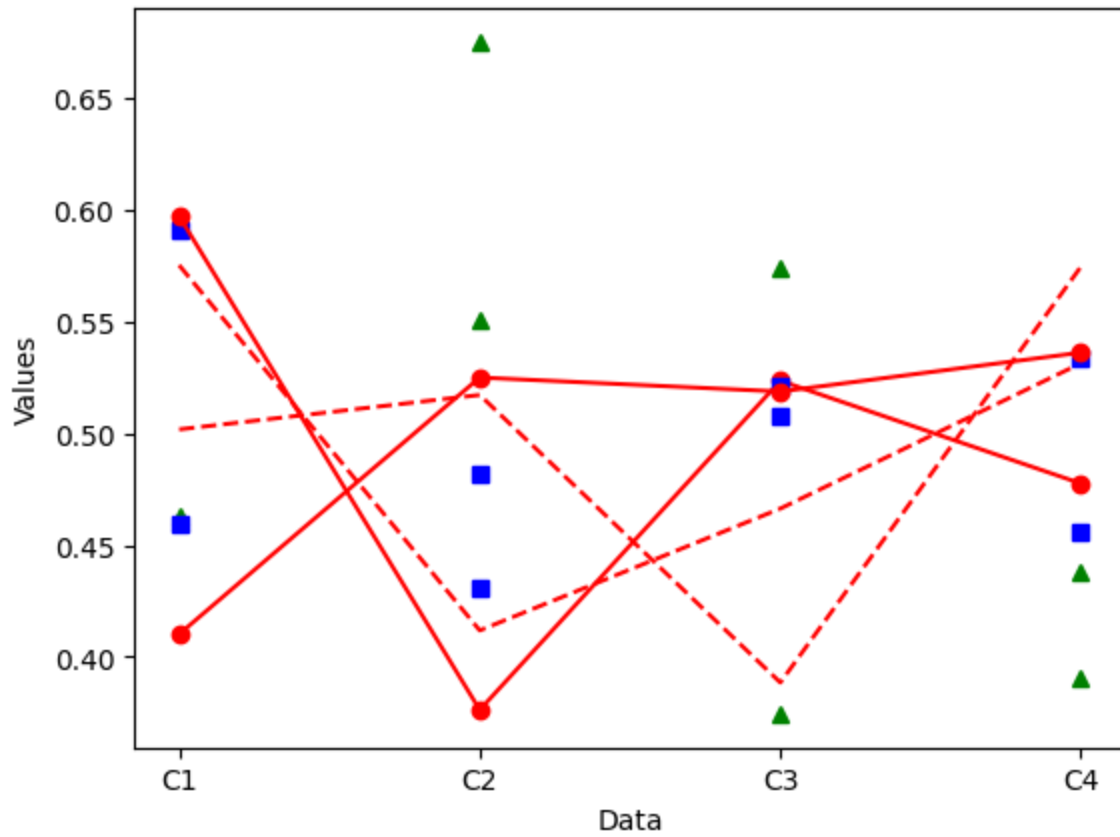


Fig 5. The ratio system values.

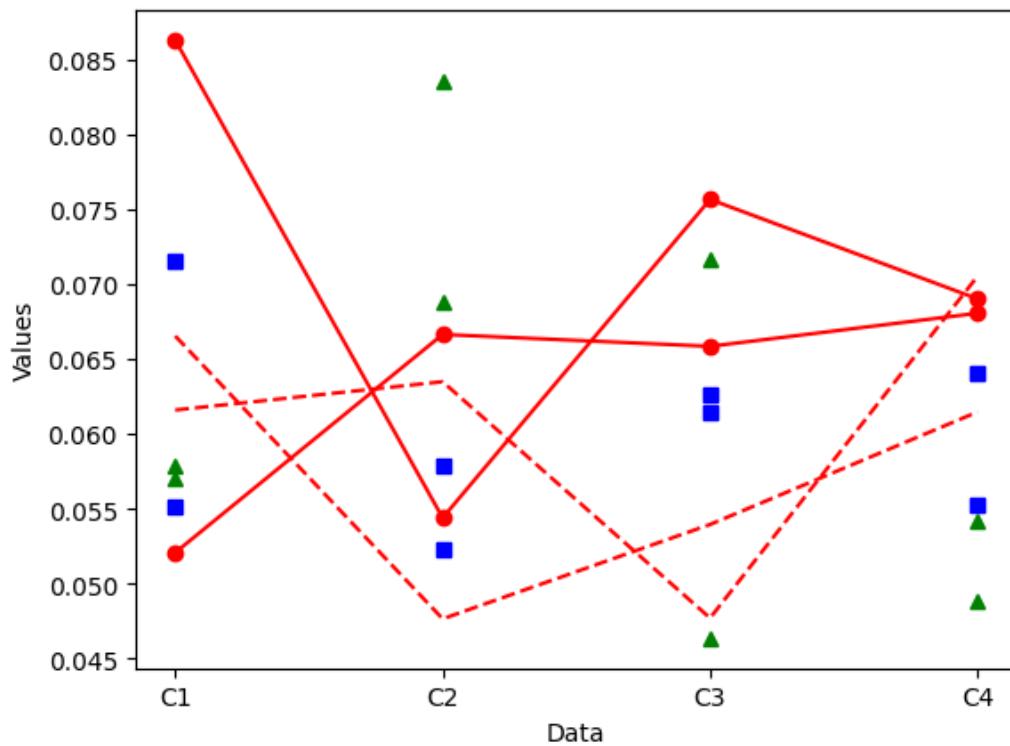


Fig 6. The weighted matrix.

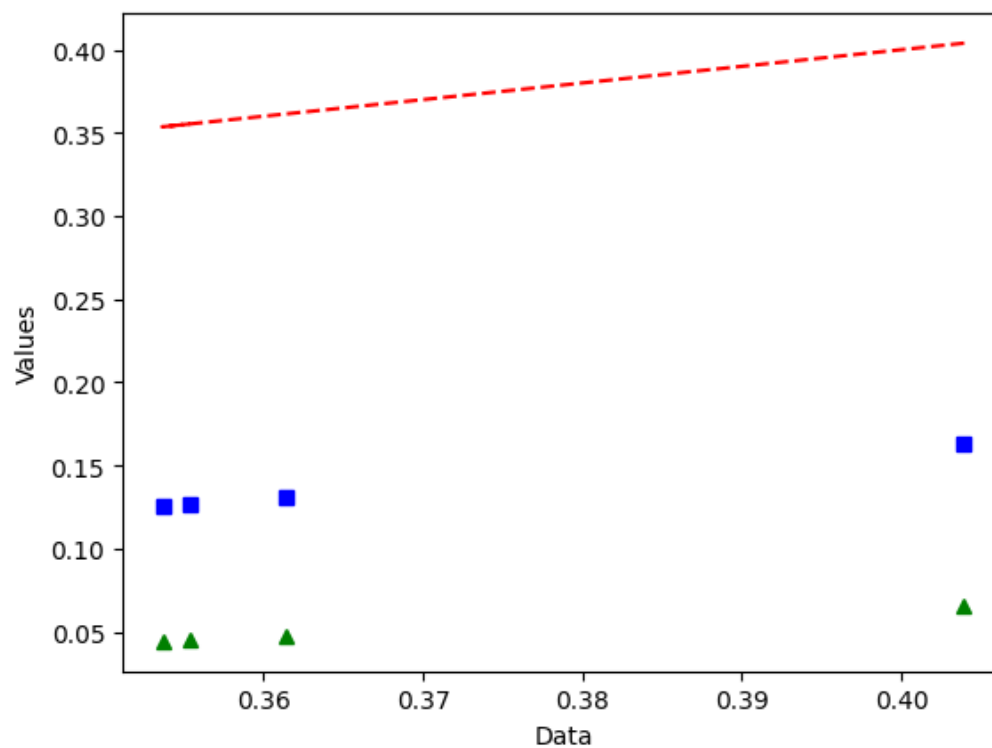


Fig 7. The final score.

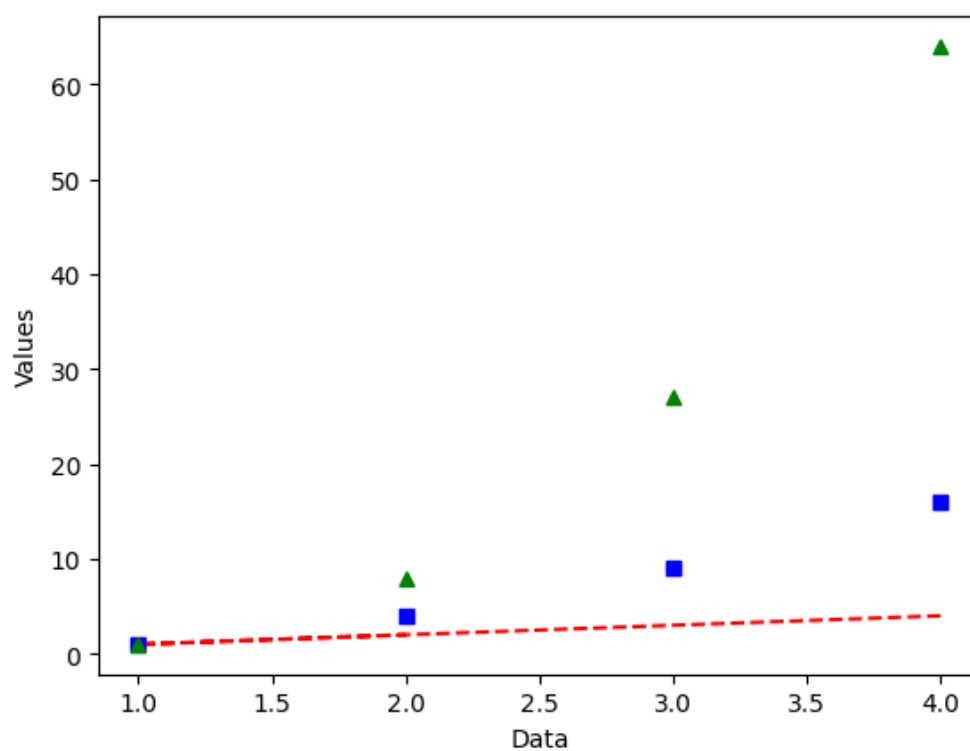


Fig 8. The ranks of alternatives.

We show the analysis of the results of the two frames. In the first frame:

A1, 2, A1 performs below top level, showing good potential but has room for improvement. It is considered a moderately strong choice.

A3, 4, A3 holds the highest rank, indicating it is the most effective or competitive alternative among the four. It demonstrates excellent performance.

A5, 1, A5 is the lowest ranked, suggesting weakness or underperformance in the evaluated criteria. It may require significant improvement.

A7, 3, A7 ranks just below the top, showing strong performance, though not quite at the level of A3. It is a reliable and capable option.

The ranks of the second frame:

A2, 4, A2 is ranked highest, indicating it performs exceptionally well compared to others. It is the top choice with outstanding qualities or results.

A4, 3, A4 is in the third position, reflecting solid performance, but it falls just short of being the best. It's strong, but not quite exceptional.

A6, 1, A6 is the lowest ranked, showing poor performance or underachievement. It needs significant improvement to reach higher ranks.

A8, 2, A8 ranks second, indicating it performs above average but does not quite reach the level of A2. It's a strong alternative, though not the best.

4. Conclusions

The evaluation revealed that innovation and entrepreneurship instruction must move beyond traditional teaching paradigms to stay relevant in today's dynamic landscape. Strategies like project-based learning, industry mentorship, and hackathon-based learning scored the highest in effectiveness, particularly due to their strong alignment with experiential and applied learning models. These methods directly connect students with real-world challenges and entrepreneurial ecosystems. In contrast, conventional methods such as lecture-based instruction were less effective due to limited engagement and practical relevance.

Furthermore, student engagement, curriculum relevance, and industry exposure emerged as top-weighted criteria, indicating that effective instruction must center around immersion, relevance, and real-world application. Vocational colleges are encouraged to invest in platforms that support cross-disciplinary collaboration, provide access to innovation labs, and integrate technological tools to modernize instructional delivery. We used the Double Framed HyperSoft Set to deal with the criteria values. The MOORA method is used to rank the alternatives.

By prioritizing active and experiential methods, institutions can significantly improve student outcomes in innovation and entrepreneurship. This study contributed to a practical decision-making model for administrators and educators seeking to elevate their programs and produce the next generation of changemakers.

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Basic Statistical Concepts On Neutrosophic Soft Sets

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Abstract: This study explores the basic statistical measures—mean, standard deviation, covariance, and correlation—within the framework of neutrosophic soft sets. Neutrosophic soft sets provide a powerful mathematical model for handling uncertainty, indeterminacy, and inconsistency in real-world data analysis. The study defines and formulates these statistical concepts under neutrosophic soft environments, incorporating true, indeterminate, and false memberships. A systematic approach to computing the neutrosophic soft mean and standard deviation is presented to measure central tendency and data dispersion. Furthermore, neutrosophic soft covariance and correlation are examined to analyze relationships between uncertain datasets. Practical applications in decision-making, medical diagnostics, and financial analysis demonstrate the effectiveness of these statistical tools. This research enhances statistical methodologies for uncertain data, offering a robust analytical foundation for fields requiring flexible and reliable data interpretation.

Keywords: Neutrosophic soft set(NSS); Neutrosophic soft mean; Neutrosophic soft covariance; Neutrosophic soft correlation coefficient.

1. Introduction

Making decisions and analyzing data frequently require dealing with ambiguity, uncertainty, and insufficient information. Fuzzy sets, intuitionistic fuzzy sets and soft sets are some of the mathematical models previously created to deal with these kinds of problems. Nevertheless, these models are limited in their ability to handle unclear, inconsistent, and partial data at the same time. Neutrosophic Soft Set(NSS) was pioneered by amalgamating the concepts of Neutrosophic Sets and Soft sets to overcome the limitations. This model effectively represents authenticity, ambiguity and non-authenticity components, making a more flexible and generalized tool for uncertainty modelling. Neutrosophic soft sets are extensively accustomed in decision-making, medical diagnosis, risk assessment, machine learning, and engineering applications where incomplete or conflicting data is involved.

Zadeh [15] initiated the study of uncertainties with the launch of fuzzy sets in 1965. Later, Atanassov pioneered the concept of intuitionistic fuzzy sets. Molodtsov [12] presented the idea of soft sets in 1999 and looked into its applications in a number of fields, including probability theory, theory of measurement, smoothness of functions, game theory, Perron integration, operation research and more. Later, basic soft set fundamentals were specified by Maji et al. [9].

In addition to introducing some new concepts and features Chen [5] identified mistakes in several of the findings of Maji et al. [9]. Many scholars from a variety of fields are currently interested in studying the numerous characteristics and applications of soft set hypothesis. Since then, soft set theory used extensively in numerous scientific and social aspects. Maji et al. [8] evolved fuzzy soft sets which is a hybrid structure with the combination of fuzzy and soft sets. Maji et al later introduced intuitionistic fuzzy soft sets [10]. Neutrosophic set proposal , a computational

tool which involves imprecise, ambiguity, and conflicting data, was first presented by Smarandache [8]. Neutrosophic soft sets were later introduced by Maji [18].

Applications of uncertainty-based statistical concepts pertaining to sociological problems, such as illegal immigration and human trafficking, can be found in Acharjee et al. [4], Mordeson et al. [16], and Acharjee and Mordeson [3]. Furthermore, Acharjee [1], Acharjee and Tripathy [2], and numerous others have hybrid structures associated with soft sets.

We lay groundwork for statistics of neutrosophic soft sets in this study. Here, we develop statistical ideas based on neutrosophic soft set theory. We begin by defining the neutrosophic soft mean, which extends traditional averaging techniques to accommodate neutrosophic values, thereby enabling the calculation of central tendencies in uncertain datasets. Building on this foundation, we developed neutrosophic soft covariance, which measures the degree of variability and co-dependency between neutrosophic soft attributes. This measure is essential for understanding relationships in data characterized by ambiguous membership degrees. Furthermore, we develop neutrosophic soft attribute correlation coefficients to quantify the strength and direction of associations between attributes in neutrosophic soft sets. These coefficients provide a valuable statistical tool for analyzing the interplay between various attributes when dealing with uncertain information and given examples.

Key contributions made by this study include the following to the field of statistical analysis in neutrosophic soft set theory:

Extension of Statistical Measures: It formalizes the computation of fundamental statistical evaluations within the neutrosophic soft set framework, incorporating true, indeterminate, and false memberships.

Handling Uncertainty and Indeterminacy: By integrating statistical tools with neutrosophic soft theory, the study provides a robust approach for analyzing data that involves uncertainty, vagueness, and partial truth, making it applicable in real-world scenarios where classical statistical methods fall short.

Improved Decision-Making Framework: The developed methods enhance decision-making processes in areas such as medical diagnostics, financial risk assessment, and engineering systems, where uncertainty plays a significant role.

Practical Applications and Examples: The study demonstrates the real-world applicability of neutrosophic soft statistical measures through examples and case studies, validating their effectiveness in uncertain environments.

2. Preliminaries

Definition 2.1[7]: A pair (S, A) over the discourse universe η , where S is a mapping provided by $S : A \rightarrow \wp(\eta)$, is called a soft set. A parameterised family of universe subsets is a soft set over η . Here, the approximation of the soft set (S, A) elements is denoted by $S(e)$.

Example 2.2: A university is evaluating candidates for a scholarship based on three parameters:

1. Good in Mathematics
2. Good in Sports
3. Good in Music

Consider the group of applicants be $\eta = \{\chi, \tilde{\lambda}, \gamma, \varpi, \rho\}$

Consider the group of parameters be $E = \{\text{Math, Sports, Music}\}$

Here, soft set F is defined as follows:

- $F(\text{Math}) = \{\chi, \tilde{\lambda}, \gamma\}$ (Candidates χ , $\tilde{\lambda}$, and γ are good at Math)
- $F(\text{Sports}) = \{\tilde{\lambda}, \gamma, \varpi\}$ (Candidates $\tilde{\lambda}$, γ , and ϖ are good at Sports)
- $F(\text{Music}) = \{\chi, \varpi, \rho\}$ (Candidates χ , ϖ , and ρ are good at Music)

So, the **soft set representation** is: $F = \{(\text{Math}, \{\chi, \tilde{\lambda}, \gamma\}), (\text{Sports}, \{\tilde{\lambda}, \gamma, \varpi\}), (\text{Music}, \{\chi, \varpi, \rho\})\}$

Definition 2.3 [8]: When X contains a generic element X represented by $\tilde{\lambda}$, let X be the space of points (objects). Real standard elements of $[0,1]$ are represented by the authentic degree function ξ_A , ambiguity membership function ζ_A , and non-authentic function η_A , which characterise a neutrosophic set A in X .

It is depicted as $A = \left\{ \left(\tilde{\lambda}, (\xi_A(\tilde{\lambda}), \zeta_A(\tilde{\lambda}), \eta_A(\tilde{\lambda})) / \tilde{\lambda} \in E, \xi_A, \zeta_A, \eta_A \in]0^-, 1^+[\right) \right\}$. There are no limitations on the sum of $\xi_A(\tilde{\lambda})$, $\zeta_A(\tilde{\lambda})$ and $\eta_A(\tilde{\lambda})$ and so $0^- \leq \xi_A(\tilde{\lambda}) + \zeta_A(\tilde{\lambda}) + \eta_A(\tilde{\lambda}) \leq 3^+$.

Definition 2.4[18]: Suppose X represents objects in the universe and E represents a collection of parameters. A neutrosophic soft set over X with respect to E is a couple (F, P) , where $F: E \rightarrow P(X)$ is a function that associates each parameter $e \in E$ with a soft set $F(e) \subseteq X$, which represents a subset of X relevant to the parameter e .

Example 2.5: Consider a group of candidates for a job $U = \{\chi, \tilde{\lambda}, \gamma, \varpi\}$.

The parameters are $E = \{\varepsilon_1, \varepsilon_2, \varepsilon_3\} = \{\text{Experience, Skills, Attitude}\}$

Suppose that, $F(\text{Experience}) = \{ \langle \chi, 0.5, 0.6, 0.3 \rangle, \langle \tilde{\lambda}, 0.4, 0.7, 0.6 \rangle, \langle \gamma, 0.6, 0.2, 0.3 \rangle, \langle \varpi, 0.7, 0.3, 0.2 \rangle \}$, $F(\text{skills}) = \{ \langle \chi, 0.6, 0.3, 0.5 \rangle, \langle \tilde{\lambda}, 0.7, 0.4, 0.3 \rangle, \langle \gamma, 0.8, 0.1, 0.2 \rangle, \langle \varpi, 0.7, 0.1, 0.3 \rangle \}$, $F(\text{Attitude}) = \{ \langle \chi, 0.7, 0.4, 0.3 \rangle, \langle \tilde{\lambda}, 0.6, 0.7, 0.2 \rangle, \langle \gamma, 0.7, 0.2, 0.5 \rangle, \langle \varpi, 0.5, 0.2, 0.6 \rangle \}$.

The NSS (F, E) can be depicted tabularly as follows:

Table 1. representation of the aforementioned data in a NSS format

U	Experience	Skills	Attitude
χ	(0.5,0.6,0.3)	(0.6,0.3,0.5)	(0.7,0.4,0.3)
$\tilde{\lambda}$	(0.4,0.7,0.6)	(0.7,0.4,0.3)	(0.6,0.7,0.2)
γ	(0.6,0.2,0.3)	(0.8,0.1,0.2)	(0.7,0.2,0.5)
ϖ	(0.7,0.3,0.2)	(0.7,0.1,0.3)	(0.5,0.2,0.6)

3. Neutrosophic soft mean(NSM) and neutrosophic soft standard deviation(NSSD) of NSS

Definition 3.1: Suppose that U represents objects in the universe and (η, A) represents a NSS over U where $\eta(\varepsilon_i)$ is a NS regarding the characteristic $\varepsilon_i \in A$, then NSM of (η, A) is represented by $\overline{\eta_A} = \{(A, \eta(A))\}$ and $\eta(A)$ is defined as

$$\eta(A) = \left\{ \frac{x_k}{(\min\{a_k^i\}, \min\{b_k^i\}, \max\{c_k^i\})} : k \in \Delta, i \in I \right\}$$

Here a_k^i, b_k^i and c_k^i are authentic value, ambiguity value and non-authentic value of x_k respectively, for the attribute ε_i where $\varepsilon_i \in A$.

If $0 \leq \sigma \leq 1, 0 \leq \tau \leq 1$ and $0 \leq \varsigma \leq 1$ then

$$(\sigma, \tau, \varsigma)_{\overline{\eta_A}} = ((\sigma_1, \tau_1, \varsigma_1), (\sigma_2, \tau_2, \varsigma_2), (\sigma_3, \tau_3, \varsigma_3), \dots, (\sigma_n, \tau_n, \varsigma_n), \dots)$$

Where $\sigma_k = 1$ if $\min\{\sigma_k^i\} \geq \sigma$, otherwise 0,

$\tau_k = 1$ if $\min\{\tau_k^i\} \geq \tau$, otherwise 0 and $\varsigma_k = 0$ if $\max\{\varsigma_k^i\} \leq \varsigma$, otherwise 1 for $i \in I$.

Example 3.2: Consider an NSS (η, A) over a universe U , where

$(\eta, A) = \{(\varepsilon_1, \{[x_1, 0.5, 0.4, 0.2], [x_2, 0.6, 0.5, 0.4], [x_3, 0.9, 0.8, 0.6]\}), \varepsilon_2, \{[x_1, 0.4, 0.6, 0.3], [x_2, 0.7, 0.8, 0.4], [x_3, 0.2, 0.4, 0.6]\}), \varepsilon_3, \{[x_1, 0.5, 0.4, 0.3], [x_2, 0.6, 0.4, 0.8], [x_3, 0.4, 0.6, 0.7]\})\}$

Then $(0.5, 0.6, 0.4) \overline{\eta}_A = ((1, 0, 0), (0, 0, 1), (0, 0, 0));$

Definition 3.3: If (η, A) be an NSS and $\varepsilon_i \in A, i \in I$ then $(\sigma, \tau, \varsigma) \overline{\eta}(\varepsilon_i) = (v_1, v_2, \dots, v_n, \dots)$ where $v_j = (\sigma_j, \tau_j, \varsigma_j)$. In this case $\sigma_j = 1$ if $\eta_j^1(\varepsilon_i)(x_j) \geq \sigma$ and 0; $\tau_j = 1$ if $\eta_j^2(\varepsilon_i)(x_j) \geq \tau$ and 0; otherwise and $\varsigma_j = 0$ if $\eta_j^3(\varepsilon_i)(x_j) \leq \varsigma$ and 1; otherwise. Here $\eta_j^1(\varepsilon_i)(x_j)$ indicates the truth value of x_j in j^{th} position of $\eta(\varepsilon_i) \forall j \in \Delta$. Similarly, $\eta_j^2(\varepsilon_i)(x_j)$ indicates the indeterminacy value of x_j in j^{th} position of $\eta(\varepsilon_i) \forall j \in \Delta$. $\eta_j^3(\varepsilon_i)(x_j)$ Indicates the non-authentic value of x_j in j^{th} position of $\eta(\varepsilon_i) \forall j \in \Delta$.

Example 3.4: Consider Example 3.2, here $(0.6, 0.2, 0.4) \overline{\eta}(\varepsilon_1) = ((0, 1, 0), (1, 1, 0), (1, 1, 1)),$

Definition 3.5: Let (η, A) be a NSS then scale of (η, A) is denoted by h and it is defined as

$$h = \max\{\eta_j^1(\varepsilon_i)(x_j), \eta_j^2(\varepsilon_i)(x_j), \eta_j^3(\varepsilon_i)(x_j) \mid \varepsilon_i \in A, x_j \in U, i \in I, j \in \Delta\}$$

Example 3.6: Let's look at an example 3.2, here $h=0.9$

Definition 3.7: Let U be a universe of objects and a collection of characteristics E ,

where $A \subseteq E$ and $|A| = n$. If (η, A) be a NSS on U , then $(\sigma, \tau, \varsigma)$ -cut NSSD of (η, A) is

represented by $((\sigma, \tau, \varsigma) \overline{\eta}, A)$ and it is interpret as

$$\left(\sqrt{\frac{1}{n}} \sum_{i \in I, k \in \Delta} \left\| (\sigma, \tau, \varsigma) \overline{\eta}_k^1(\varepsilon_i) - (\sigma, \tau, \varsigma) \overline{\eta}_{k,A}^1 \right\|^2}, \sqrt{\frac{1}{n}} \sum_{i \in I, k \in \Delta} \left\| (\sigma, \tau, \varsigma) \overline{\eta}_k^2(\varepsilon_i) - (\sigma, \tau, \varsigma) \overline{\eta}_{k,A}^2 \right\|^2}, \sqrt{\frac{1}{n}} \sum_{i \in I, k \in \Delta} \left\| (\sigma, \tau, \varsigma) \overline{\eta}_k^3(\varepsilon_i) - (\sigma, \tau, \varsigma) \overline{\eta}_{k,A}^3 \right\|^2} \right),$$

Where

$$\left\| (\sigma, \tau, \varsigma) \overline{\eta}_k^1(\varepsilon_i) - (\sigma, \tau, \varsigma) \overline{\eta}_{k,A}^1 \right\|^2 = \left\langle (\sigma, \tau, \varsigma) \overline{\eta}_k^1(\varepsilon_i) - (\sigma, \tau, \varsigma) \overline{\eta}_{k,A}^1, (\sigma, \tau, \varsigma) \overline{\eta}_k^1(\varepsilon_i) - (\sigma, \tau, \varsigma) \overline{\eta}_{k,A}^1 \right\rangle$$

and so on for other part.

Here $(\sigma, \tau, \varsigma) \overline{\eta}_k^1(\varepsilon_i)$ and $(\sigma, \tau, \varsigma) \overline{\eta}_{k,A}^1$ indicate the first coordinate of $(\sigma, \tau, \varsigma) \overline{\eta}_k^1(\varepsilon_i)$ and

$(\sigma, \tau, \varsigma) \overline{\eta}_{k,A}^1$ respectively, where k indicates the k -th ordered pair representation of the authentic, ambiguity and the non-authentic values for $(\sigma, \tau, \varsigma) \overline{\eta}_k^1(\varepsilon_i)$ and $(\sigma, \tau, \varsigma) \overline{\eta}_{k,A}^1$ respectively.

Example 3.8: Let's look at an example 3.2, where

$$(N, A) = \{(\varepsilon_1, \{[x_1, 0.5, 0.4, 0.2], [x_2, 0.6, 0.5, 0.4], [x_3, 0.9, 0.8, 0.6]\}),$$

Let $\sigma=0.3, \tau=0.4, \varsigma=0.2$ then

$$(0.3, 0.4, 0.2) \overline{\eta}(\varepsilon_1) = ((1, 1, 1), (0, 1, 1), (1, 1, 1));$$

$$(0.3, 0.4, 0.2) \overline{\eta}(\varepsilon_2) = ((1, 1, 1), (1, 1, 1), (0, 1, 1));$$

$$(0.3, 0.4, 0.2) \overline{\eta}(\varepsilon_3) = ((1, 1, 1), (1, 1, 1), (1, 1, 1))$$

$$\text{Now } (0.3, 0.4, 0.2) \overline{\eta}_A = ((1, 1, 1), (0, 1, 1), (1, 1, 1))$$

$$\begin{aligned}
& \sqrt{\frac{1}{n} \sum_{i \in I, k \in \Delta} \left\| (0.3, 0.4, 0.2) \overline{\eta_k^1(\varepsilon_i)} - (0.3, 0.4, 0.2) \overline{\eta_{k,A}^1} \right\|^2} \\
&= \sqrt{\frac{1}{3} (0+0+0+1+1+1+0+0+0)} = \sqrt{\frac{3}{3}} = 1 \\
& \sqrt{\frac{1}{n} \sum_{i \in I, k \in \Delta} \left\| (0.3, 0.4, 0.2) \overline{\eta_k^2(\varepsilon_i)} - (0.3, 0.4, 0.2) \overline{\eta_{k,A}^2} \right\|^2} \\
&= \sqrt{\frac{1}{3} (0+0+0+0+0+0+0+0+0)} = 0 \\
& \sqrt{\frac{1}{n} \sum_{i \in I, k \in \Delta} \left\| (0.3, 0.4, 0.2) \overline{\eta_k^3(\varepsilon_i)} - (0.3, 0.4, 0.2) \overline{\eta_{k,A}^3} \right\|^2} \\
&= \sqrt{\frac{1}{3} (1+0+0+0+0+0+0+0+0)} = \sqrt{\frac{1}{3}}
\end{aligned}$$

$$\rho((0.3, 0.2, 0.4)(\eta, A)) = (1, 0, \sqrt{\frac{1}{3}})$$

The above result indicates that standard deviation of $(0.3, 0.2, 0.4)(\eta, A)$ is 1 from truth membership, 0 from indeterminacy and $\sqrt{\frac{1}{3}}$ false membership values.

4. Neutrosophic soft covariance and Neutrosophic soft attribute correlation coefficient with $(\sigma, \tau, \varsigma)$ -cut:

Definition 4.1: Consider the set of attributes E in a universe U. Let (η, ψ) and $(\wp, \phi)$ be NSS where $\psi, \phi \subseteq E$, $|\psi| = n > |\phi| = m$. We extend ϕ to $\phi = \phi \cup \{\hbar_{\ell+1}, \hbar_{\ell+2}, \dots, \hbar_n\}$ such that $\wp_i^1(\hbar_k)(x_i) = 1, \wp_i^2(\hbar_k)(x_i) = 0$ and $\wp_i^3(\hbar_k)(x_i) = 0$ for all $k \in \{\ell+1, \ell+2, \dots, n\}$. Then the $(\sigma, \tau, \varsigma)$ -cut level neutrosophic soft covariance of (η, ψ) and $(\wp, \phi)$ is denoted by

$(\sigma, \tau, \varsigma) \text{NSSCov}((\eta, \psi), (\wp, \phi))$ and is defined as

$$\begin{aligned}
(\sigma, \tau, \varsigma) \text{NSSCov}((\eta, \psi), (\wp, \phi)) &= \left(\frac{1}{n} \{ \|\Delta_1\|^2 + \|\Delta_2\|^2 + \dots + \|\Delta_n\|^2 \}, \frac{1}{n} \{ \|\Delta_1'\|^2 + \|\Delta_2'\|^2 + \dots + \|\Delta_n'\|^2 \}, \right. \\
&\quad \left. \frac{1}{n} \{ \|\Delta_1''\|^2 + \|\Delta_2''\|^2 + \dots + \|\Delta_n''\|^2 \} \right)
\end{aligned}$$

Where

$$\Delta_j = (\min\{\sigma\eta_1^1(\varepsilon_j), \sigma\wp_1^1(\hbar_j)\}, \min\{\sigma\eta_2^1(\varepsilon_j), \sigma\wp_2^1(\hbar_j)\}, \dots, \min\{\sigma\eta_n^1(\varepsilon_j), \sigma\wp_n^1(\hbar_j)\}, \dots),$$

$$\Delta_j' = (\min\{\sigma\eta_1^2(\varepsilon_j), \sigma\wp_1^2(\hbar_j)\}, \min\{\sigma\eta_2^2(\varepsilon_j), \sigma\wp_2^2(\hbar_j)\}, \dots, \min\{\sigma\eta_n^2(\varepsilon_j), \sigma\wp_n^2(\hbar_j)\}, \dots),$$

$$\Delta_j'' = (\min\{\sigma\eta_1^3(\varepsilon_j), \sigma\wp_1^3(\hbar_j)\}, \min\{\sigma\eta_2^3(\varepsilon_j), \sigma\wp_2^3(\hbar_j)\}, \dots, \min\{\sigma\eta_n^3(\varepsilon_j), \sigma\wp_n^3(\hbar_j)\}, \dots),$$

The attributes $\hbar_{\ell+1}, \hbar_{\ell+2}, \dots, \hbar_n$ with $\wp_i^1(\hbar_k)(x_i) = 0, \wp_i^2(\hbar_k)(x_i) = 0$ and $\wp_i^3(\hbar_k)(x_i) = 0$ for

all $k \in \{\ell + 1, \ell + 2, \dots, n\}, i \in \Delta$ are called neutrosophic soft statistical fictitious

characteristics

for φ relative to ψ .

(ii) Let U be a universe of objects and E be a set of attributes E . Let (η, ψ) and $(\wp, \varphi)$ be NSS where $\psi, \varphi \subseteq E, |\psi| = n = |\varphi|$. Then the $(\sigma, \tau, \varsigma)$ -cut level neutrosophic soft covariance of (η, ψ) and $(\wp, \varphi)$ is denoted by $(\sigma, \tau, \varsigma)NSSCov((\eta, \psi), (\wp, \varphi))$ and is defined as

$$(\sigma, \tau, \varsigma)NSSCov((\eta, \psi), (\wp, \varphi)) = \left(\frac{1}{n} \{ \|\Delta_1\|^2 + \|\Delta_2\|^2 + \dots + \|\Delta_n\|^2 \}, \frac{1}{n} \{ \|\Delta'_1\|^2 + \|\Delta'_2\|^2 + \dots + \|\Delta'_n\|^2 \}, \right. \\ \left. \frac{1}{n} \{ \|\Delta''_1\|^2 + \|\Delta''_2\|^2 + \dots + \|\Delta''_n\|^2 \} \right)$$

Where

$$\Delta_j = (\min\{\sigma\eta_1^1(\varepsilon_j), \sigma\wp_1^1(h_j)\}, \min\{\sigma\eta_2^1(\varepsilon_j), \sigma\wp_2^1(h_j)\}, \dots, \min\{\sigma\eta_i^1(\varepsilon_j), \sigma\wp_i^1(h_j)\}, \dots), \\ \Delta'_j = (\min\{\sigma\eta_1^2(\varepsilon_j), \sigma\wp_1^2(h_j)\}, \min\{\sigma\eta_2^2(\varepsilon_j), \sigma\wp_2^2(h_j)\}, \dots, \min\{\sigma\eta_i^2(\varepsilon_j), \sigma\wp_i^2(h_j)\}, \dots), \\ \Delta''_j = (\min\{\sigma\eta_1^3(\varepsilon_j), \sigma\wp_1^3(h_j)\}, \min\{\sigma\eta_2^3(\varepsilon_j), \sigma\wp_2^3(h_j)\}, \dots, \min\{\sigma\eta_i^3(\varepsilon_j), \sigma\wp_i^3(h_j)\}, \dots), \\ \text{and } \varepsilon_j \in A, h_j \in A, i \in \Delta, j \in I.$$

Definition 4.2: Let (η, A) be an NSS with at least two attributes $\varepsilon_1, \varepsilon_2 \in A$, then the neutrosophic soft attribute correlation coefficient of $(\sigma, \tau, \varsigma)\eta(\varepsilon_1)$ and $(\sigma, \tau, \varsigma)\eta(\varepsilon_2)$ is denoted by $NSACC((\sigma, \tau, \varsigma)\eta(\varepsilon_1), (\sigma, \tau, \varsigma)\eta(\varepsilon_2))$ and is defined as

$$NSACC((\sigma, \tau, \varsigma)\eta(\varepsilon_1), (\sigma, \tau, \varsigma)\eta(\varepsilon_2)) = \left(\frac{\sum_{i \in \Delta} \Delta_i}{\|\sigma\eta(\varepsilon_1)\| \cdot \|\sigma\eta(\varepsilon_2)\|}, \frac{\sum_{i \in \Delta} \Delta'_i}{\|\tau\eta(\varepsilon_1)\| \cdot \|\tau\eta(\varepsilon_2)\|}, \frac{\sum_{i \in \Delta} \Delta''_i}{\|\varsigma\eta(\varepsilon_1)\| \cdot \|\varsigma\eta(\varepsilon_2)\|} \right)$$

where $\Delta_i = \min\{\sigma\eta_i(\varepsilon_1), \sigma\eta_i(\varepsilon_2)\}$,

$$\Delta'_i = \min\{\tau\eta_i(\varepsilon_1), \tau\eta_i(\varepsilon_2)\},$$

$$\Delta''_i = \min\{\varsigma\eta_i(\varepsilon_1), \varsigma\eta_i(\varepsilon_2)\}$$

$$\|\sigma\eta(\varepsilon_j)\| = \sqrt{\langle \sigma\eta(\varepsilon_j), \sigma\eta(\varepsilon_j) \rangle} = \sqrt{\sum_{i \in I} (\sigma\eta_i(\varepsilon_j))^2}, \|\sigma\eta(\varepsilon_1)\| \neq 0, \|\sigma\eta(\varepsilon_2)\| \neq 0$$

$$\|\tau\eta(\varepsilon_j)\| = \sqrt{\langle \tau\eta(\varepsilon_j), \tau\eta(\varepsilon_j) \rangle} = \sqrt{\sum_{i \in I} (\tau\eta_i(\varepsilon_j))^2}, \|\tau\eta(\varepsilon_1)\| \neq 0, \|\tau\eta(\varepsilon_2)\| \neq 0$$

$$\|\varsigma\eta(\varepsilon_j)\| = \sqrt{\langle \varsigma\eta(\varepsilon_j), \varsigma\eta(\varepsilon_j) \rangle} = \sqrt{\sum_{i \in I} (\varsigma\eta_i(\varepsilon_j))^2}, \|\varsigma\eta(\varepsilon_1)\| \neq 0, \|\varsigma\eta(\varepsilon_2)\| \neq 0$$

If any one of $\|\sigma\eta(\varepsilon_1)\|, \|\sigma\eta(\varepsilon_2)\|, \|\tau\eta(\varepsilon_1)\|, \|\tau\eta(\varepsilon_2)\|, \|\varsigma\eta(\varepsilon_1)\|, \|\varsigma\eta(\varepsilon_2)\|$ is 0, then

$NSACC((\sigma, \tau, \varsigma)\eta(\varepsilon_1), (\sigma, \tau, \varsigma)\eta(\varepsilon_2))$ is not feasible. Here we employ the notation (∞, ∞, ∞) , regardless what is $\Delta_i, \Delta'_i, \text{ or } \Delta''_i$.

Theorem 4.3: If (η, A) be any NSS with at least two attributes $\varepsilon_1, \varepsilon_2 \in A$ over a universe U , then $(0, 0, 0) \leq NSACC((\sigma, \tau, \varsigma)\eta(\varepsilon_1), (\sigma, \tau, \varsigma)\eta(\varepsilon_2)) \leq (1, 1, 1)$.

Proof: Since

$$\sum_{i \in \Delta} \Delta_i = \sum_{i \in \Delta} \min\{\sigma\eta_i(\varepsilon_1), \sigma\eta_i(\varepsilon_2)\} \leq \sum_{i \in \Delta} \{\sigma\eta_i(\varepsilon_1), \sigma\eta_i(\varepsilon_2)\} \leq \sqrt{\sum_{i \in \Delta} (\sigma\eta_i(\varepsilon_1))^2} \sqrt{\sum_{i \in \Delta} (\sigma\eta_i(\varepsilon_2))^2}$$

$$= \|\overline{\sigma\eta(\varepsilon_1)}\| \cdot \|\overline{\sigma\eta(\varepsilon_2)}\|$$

$$\text{So } \frac{\sum_{i \in \Delta} \Delta_i}{\|\overline{\sigma\eta(\varepsilon_1)}\| \cdot \|\overline{\sigma\eta(\varepsilon_2)}\|} \leq 1.$$

$$\text{Similarly we can show that } \frac{\sum_{i \in \Delta} \Delta_i'}{\|\overline{\tau\eta(\varepsilon_1)}\| \cdot \|\overline{\tau\eta(\varepsilon_2)}\|} \leq 1, \quad \frac{\sum_{i \in \Delta} \Delta_i''}{\|\overline{\varsigma\eta(\varepsilon_1)}\| \cdot \|\overline{\varsigma\eta(\varepsilon_2)}\|} \leq 1$$

$$\text{NSACC}((\sigma, \tau, \varsigma)\eta(\varepsilon_1), (\sigma, \tau, \varsigma)\eta(\varepsilon_2)) \leq (1, 1, 1)$$

$$\text{Again } \sum_{i \in \Delta} \Delta_i \geq 0, \quad \|\overline{\sigma\eta(\varepsilon_1)}\| \cdot \|\overline{\sigma\eta(\varepsilon_2)}\| > 0 \text{ So } \frac{\sum_{i \in \Delta} \Delta_i}{\|\overline{\sigma\eta(\varepsilon_1)}\| \cdot \|\overline{\sigma\eta(\varepsilon_2)}\|} \geq 0.$$

$$\text{Similarly } \frac{\sum_{i \in \Delta} \Delta_i'}{\|\overline{\tau\eta(\varepsilon_1)}\| \cdot \|\overline{\tau\eta(\varepsilon_2)}\|} \geq 0, \quad \frac{\sum_{i \in \Delta} \Delta_i''}{\|\overline{\varsigma\eta(\varepsilon_1)}\| \cdot \|\overline{\varsigma\eta(\varepsilon_2)}\|} \geq 0$$

Hence proved.

Theorem 4.4: If $\eta(\varepsilon_1)$, $\wp(\varepsilon_1)$ and $\mathfrak{Z}(\varepsilon_1)$ are three NSSs of three NSSs (η, A) , $(\wp, A)$ and $(\mathfrak{Z}, A)$ over U such that $\eta(e_1) \subseteq \wp(e_1) \subseteq \mathfrak{Z}(e_1)$ and $e_1 \in A$. Then

$$\text{NSACC}((\sigma, \tau, \varsigma)\overline{\eta(\varepsilon_1)}, (\sigma, \tau, \varsigma)\overline{\mathfrak{Z}(\varepsilon_1)}) \leq \text{NSACC}((\sigma, \tau, \varsigma)\overline{\eta(\varepsilon_1)}, (\sigma, \tau, \varsigma)\overline{\wp(\varepsilon_1)})$$

$$\text{NSACC}((\sigma, \tau, \varsigma)\overline{\eta(\varepsilon_1)}, (\sigma, \tau, \varsigma)\overline{\mathfrak{Z}(\varepsilon_1)}) \leq \text{NSACC}((\sigma, \tau, \varsigma)\overline{\wp(\varepsilon_1)}, (\sigma, \tau, \varsigma)\overline{\mathfrak{Z}(\varepsilon_1)})$$

$$\text{NSACC}((\sigma, \tau, \varsigma)\overline{\eta(\varepsilon_1)}, (\sigma, \tau, \varsigma)\overline{\wp(\varepsilon_1)}) \leq \text{NSACC}((\sigma, \tau, \varsigma)\overline{\mathfrak{Z}(\varepsilon_1)}, (\sigma, \tau, \varsigma)\overline{\wp(\varepsilon_1)})$$

$$\text{if } \|\overline{\sigma\eta(\varepsilon_1)}\| \cdot \|\overline{\sigma\mathfrak{Z}(\varepsilon_1)}\| - \|\overline{\sigma\wp(\varepsilon_1)}\|^2 \leq 0, ,$$

$$\|\overline{\tau\eta(\varepsilon_1)}\| \cdot \|\overline{\tau\mathfrak{Z}(\varepsilon_1)}\| - \|\overline{\tau\wp(\varepsilon_1)}\|^2 \leq 0, \text{ and } \|\overline{\varsigma\wp(\varepsilon_1)}\|^2 - \|\overline{\varsigma\eta(\varepsilon_1)}\| \cdot \|\overline{\varsigma\mathfrak{Z}(\varepsilon_1)}\| \leq 0$$

Proof: Let

$$\text{NSACC}((\sigma, \tau, \varsigma)\overline{\eta(\varepsilon_1)}, (\sigma, \tau, \varsigma)\overline{\mathfrak{Z}(\varepsilon_1)}) = \left(\frac{\sum_{i \in \Delta} \Delta_i^1}{\|\overline{\sigma\eta(\varepsilon_1)}\| \cdot \|\overline{\sigma\mathfrak{Z}(\varepsilon_1)}\|}, \frac{\sum_{i \in \Delta} \Delta_i'^1}{\|\overline{\tau\eta(\varepsilon_1)}\| \cdot \|\overline{\tau\mathfrak{Z}(\varepsilon_1)}\|}, \frac{\sum_{i \in \Delta} \Delta_i''^1}{\|\overline{\varsigma\eta(\varepsilon_1)}\| \cdot \|\overline{\varsigma\mathfrak{Z}(\varepsilon_1)}\|} \right)$$

$$\Delta_i^1 = \min\{\sigma\eta_i(\varepsilon_1), \sigma\mathfrak{Z}_i(\varepsilon_1)\} = \sigma\eta_i(\varepsilon_1),$$

$$\Delta_i'^1 = \min\{\tau\eta_i(\varepsilon_1), \tau\mathfrak{Z}_i(\varepsilon_1)\} = \tau\mathfrak{Z}_i(\varepsilon_1),$$

$$\Delta_i''^1 = \min\{\varsigma\eta_i(\varepsilon_1), \varsigma\mathfrak{Z}_i(\varepsilon_1)\} = \varsigma\mathfrak{Z}_i(\varepsilon_1)$$

$$\|\overline{\sigma\eta(\varepsilon_1)}\| \neq 0, \|\overline{\sigma\mathfrak{Z}(\varepsilon_1)}\| \neq 0, \|\overline{\tau\eta(\varepsilon_1)}\| \neq 0, \|\overline{\tau\mathfrak{Z}(\varepsilon_1)}\| \neq 0, \|\overline{\varsigma\eta(\varepsilon_1)}\| \neq 0, \|\overline{\varsigma\mathfrak{Z}(\varepsilon_1)}\| \neq 0$$

Consider

$$\text{NSACC}((\sigma, \tau, \varsigma)\overline{\eta(\varepsilon_1)}, (\sigma, \tau, \varsigma)\overline{\wp(\varepsilon_1)}) = \left(\frac{\sum_{i \in \Delta} \Delta_i^2}{\|\overline{\sigma\eta(\varepsilon_1)}\| \cdot \|\overline{\sigma\mathfrak{Z}(\varepsilon_1)}\|}, \frac{\sum_{i \in \Delta} \Delta_i'^2}{\|\overline{\tau\eta(\varepsilon_1)}\| \cdot \|\overline{\tau\mathfrak{Z}(\varepsilon_1)}\|}, \frac{\sum_{i \in \Delta} \Delta_i''^2}{\|\overline{\varsigma\eta(\varepsilon_1)}\| \cdot \|\overline{\varsigma\mathfrak{Z}(\varepsilon_1)}\|} \right)$$

$$\Delta_i^1 = \min\{\sigma\eta_i(\varepsilon_1), \sigma\wp_i(\varepsilon_1)\} = \sigma\eta_i(\varepsilon_1),$$

$$\Delta_i^1 = \min\{\tau\eta_i(\varepsilon_1), \tau\wp_i(\varepsilon_1)\} = \tau\mathfrak{Z}_i(\varepsilon_1),$$

$$\Delta_i^1 = \min\{\varsigma\eta_i(\varepsilon_1), \varsigma\mathfrak{Z}_i(\varepsilon_1)\} = \varsigma\mathfrak{Z}_i(\varepsilon_1)$$

$$\|\sigma\eta(\varepsilon_1)\| \neq 0, \|\sigma\mathfrak{Z}(\varepsilon_1)\| \neq 0, \|\tau\eta(\varepsilon_1)\| \neq 0, \|\tau\mathfrak{Z}(\varepsilon_1)\| \neq 0, \|\varsigma\eta(\varepsilon_1)\| \neq 0, \|\varsigma\mathfrak{Z}(\varepsilon_1)\| \neq 0$$

Now

$$\begin{aligned} & \frac{\sum_{i \in \Delta} \Delta_i^1}{\|\sigma\eta(\varepsilon_1)\| \|\sigma\mathfrak{Z}(\varepsilon_1)\|} - \frac{\sum_{i \in \Delta} \Delta_i^2}{\|\sigma\eta(\varepsilon_1)\| \|\sigma\wp(\varepsilon_1)\|} = \frac{\sum_{i \in \Delta} \Delta_i^1}{\|\sigma\eta(\varepsilon_1)\| \|\sigma\mathfrak{Z}(\varepsilon_1)\|} - \frac{\sum_{i \in \Delta} \Delta_i^1}{\|\sigma\eta(\varepsilon_1)\| \|\sigma\wp(\varepsilon_1)\|} \\ & = \frac{\sum_{i \in \Delta} \Delta_i^1}{\|\sigma\eta(\varepsilon_1)\|} \left\{ \frac{1}{\|\sigma\mathfrak{Z}(\varepsilon_1)\|} - \frac{1}{\|\sigma\wp(\varepsilon_1)\|} \right\} \leq 0 \end{aligned}$$

Since $\wp(\varepsilon_1) \subseteq \mathfrak{Z}(\varepsilon_1)$, then $\|\sigma\wp(\varepsilon_1)\| \leq \|\sigma\mathfrak{Z}(\varepsilon_1)\|$

$$\text{Thus } \frac{\sum_{i \in \Delta} \Delta_i^1}{\|\sigma\eta(\varepsilon_1)\| \|\sigma\mathfrak{Z}(\varepsilon_1)\|} \leq \frac{\sum_{i \in \Delta} \Delta_i^2}{\|\sigma\eta(\varepsilon_1)\| \|\sigma\wp(\varepsilon_1)\|}$$

Now $\Delta_i^1 = 0$ or 1 for all $i \in \Delta$. Thus squaring of Δ_i^1

does not alter the sum and so on for other cases

$$\begin{aligned} & \frac{\sum_{i \in \Delta} \Delta_i^1}{\|\tau\eta(\varepsilon_1)\| \|\tau\mathfrak{Z}(\varepsilon_1)\|} - \frac{\sum_{i \in \Delta} \Delta_i^2}{\|\tau\eta(\varepsilon_1)\| \|\tau\wp(\varepsilon_1)\|} = \frac{1}{\|\tau\eta(\varepsilon_1)\|} \left\{ \frac{\sum_{i \in \Delta} \Delta_i^1}{\|\tau\mathfrak{Z}(\varepsilon_1)\|} - \frac{\sum_{i \in \Delta} \Delta_i^2}{\|\tau\wp(\varepsilon_1)\|} \right\} \\ & = \frac{1}{\|\tau\eta(\varepsilon_1)\|} \left\{ \frac{\|\tau\mathfrak{Z}(\varepsilon_1)\|^2}{\|\tau\mathfrak{Z}(\varepsilon_1)\|} - \frac{\|\tau\wp(\varepsilon_1)\|^2}{\|\tau\wp(\varepsilon_1)\|} \right\} = \frac{1}{\|\tau\eta(\varepsilon_1)\|} \{ \|\tau\mathfrak{Z}(\varepsilon_1)\| - \|\tau\wp(\varepsilon_1)\| \} \leq 0 \end{aligned}$$

$$\text{So } \frac{\sum_{i \in \Delta} \Delta_i^1}{\|\tau\eta(\varepsilon_1)\| \|\tau\mathfrak{Z}(\varepsilon_1)\|} \leq \frac{\sum_{i \in \Delta} \Delta_i^2}{\|\tau\eta(\varepsilon_1)\| \|\tau\wp(\varepsilon_1)\|}$$

Hence proved.

Similarly we can prove for remaining parts.

5. Real data application:

Correlation between Customer Satisfaction & Service Quality in a Bank Scenario:

A bank wants to analyze whether **customer satisfaction** is correlated with **service quality** while considering uncertainty in responses. It collects data from five branches, where:

T (Truth Membership): Customers who are satisfied with the service.

I (Indeterminacy Membership): Customers who are neutral or uncertain.

F (Falsity Membership): Customers who are dissatisfied.

The following table shows customer satisfaction and service quality ratings for four branches.

Table 2. Representation of data in a NSS

Branch	Satisfaction	Service Quality
	(T, I, F)	(T, I, F)

A	(0.8,0.1,0.3)	(0.9,0.05,0.25)
B	(0.6,0.2,0.4)	(0.7,0.15,0.35)
C	(0.7,0.1,0.52)	(0.8,0.1,0.6)
D	(0.9,0.45,0.05)	(0.95,0.52,0.03)

Here, **Satisfaction** and **Service Quality** each have three components: authenticity (T), **ambiguity(I)**, and **non-authenticity (F)**.

Here $U = \{\chi, \tilde{\lambda}, \gamma, \varpi\}$ and a set of attributes $E = \{\text{satisfaction, service quality}\}$

Let $e_1 = \text{satisfaction}$, $e_2 = \text{service quality}$

Then

$$(\eta, E) = (\{\varepsilon_1, \{\chi, 0.8, 0.1, 0.3\}, \{\tilde{\lambda}, 0.6, 0.2, 0.4\}, \{\gamma, 0.7, 0.1, 0.52\}, \{\varpi, 0.9, 0.45, .05\}\}, \{\varepsilon_2 \{\chi, 0.9, 0.05, 0.25\}, \{\tilde{\lambda}, 0.7, 0.15, 0.35\}, \{\gamma, 0.8, 0.1, 0.6\}, \{\varpi, 0.95, 0.52, 0.03\}\})$$

$$\text{Let } (\sigma, \tau, \varsigma) = (0.7, 0.2, 0.3)$$

$$(0.7, 0.2, 0.3) \overline{\eta(\varepsilon_1)} = \{(0, 1, 0), (1, 1, 0), (1, 1, 0), (0, 0, 1)\}$$

$$(0.7, 0.2, 0.3) \overline{\eta(\varepsilon_2)} = \{(0, 1, 1), (1, 1, 0), (0, 1, 0), (0, 0, 1)\}$$

$$\text{Then } \Delta_1 = \min\{0, 0\} = 0, \Delta_2 = \min\{1, 1\} = 1, \Delta_3 = \min\{1, 0\} = 0, \Delta_4 = \min\{0, 0\} = 0$$

$$\Delta'_1 = \min\{1, 1\} = 1, \Delta'_2 = \min\{1, 1\} = 1, \Delta'_3 = \min\{1, 1\} = 1, \Delta'_4 = \min\{0, 0\} = 0$$

$$\Delta''_1 = \min\{0, 1\} = 0, \Delta''_2 = \min\{0, 0\} = 0, \Delta''_3 = \min\{0, 0\} = 0, \Delta''_4 = \min\{1, 1\} = 1$$

$$\text{Then NSACC}((0.7, 0.2, 0.3) \overline{\eta(\varepsilon_1)}, (0.7, 0.2, 0.3) \overline{\eta(\varepsilon_2)}) = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{2}}) = (0.7, 0.6, 0.7)$$

6. Conclusion

In this study, we developed some fundamental aspects on neutrosophic soft sets using cut sets. The application of statistical concepts within the framework of neutrosophic soft sets delivers a powerful methodology for handling uncertainty, indeterminacy, and inconsistency in data analysis. Neutrosophic soft mean and standard deviation offer a structured approach to measuring central tendency and dispersion in uncertain datasets, while neutrosophic soft covariance and correlation facilitate the evaluation of relationships between variables in imprecise environments. These measures enhance decision-making processes in fields such as health care, finance, and engineering, where data often contain elements of vagueness and partial truth. By integrating statistical analysis with neutrosophic soft set theory, This study advances the creation of more efficient and adaptable tools for real-world data analysis. Future research can further optimize these methods and extend their applications to complex, high-dimensional datasets.

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Measuring Teaching Success in College Foreign Literature Programs: An Evaluation Perspective using Double Framed SuperHyperSoft Set

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Abstract: To assess effectiveness teaching in college foreign literature programs, a complex, multifaceted methodology that considers both student outcomes and pedagogical quality is needed. Since literary courses naturally develop analytical abilities, cultural awareness, and critical thinking, it is more important than ever to create efficient evaluation frameworks. Using both quantitative and qualitative data to gauge efficacy, this study investigates the major markers and variables affecting instructional performance in these programs. Through an analysis of teaching methodologies, curricular alignment, learner engagement, and assessment tactics, the study provides a well-founded viewpoint on how to improve student happiness and academic performance in foreign literature training. We use the multi-criteria decision making (MCDM) approach to deal with different criteria. The CoCoSo method is used to rank alternatives. The Double Framed SuperHyperSoft set is used to deal with criteria values.

Keywords: Double Framed SuperHyperSoft Set; Teaching Success; College Foreign Literature Programs; Evaluation Perspective.

1. Introduction

Courses in foreign literature are essential to liberal arts education because they provide students with an introduction to ideas, cultures, and historical accounts from throughout the world. In addition to improving language proficiency, these classes foster cultural knowledge, empathy, and intellectual curiosity[1], [2]. The effectiveness of instruction in these programs necessitates a thorough and meticulous assessment due to their complex effects.

Teaching foreign literature effectively necessitates a blend of student-centered involvement, pedagogical flexibility, and in-depth subject understanding. Because of the variety of texts,

authors, and cultural settings, teaching is a challenging undertaking that mostly relies on teachers' capacity to generate links between literature and students' real-world experiences.

The whole success of instruction in literary programs cannot be adequately captured by conventional evaluation criteria like exam results and attendance rates[3], [4]. This shortcoming necessitates the use of more flexible assessment techniques that consider a variety of factors, such as the caliber of the discussions, interpretive abilities, cross-cultural competency, and student reflections.

Furthermore, student feedback has become a more significant factor in assessing classroom effectiveness. The opinions of students provide important information about the efficacy of teaching strategies, classroom dynamics, and the general learning environment. By incorporating these viewpoints, assessments are guaranteed to be inclusive of students' opinions in addition to being objective.

The fitness between learning outcomes and course objectives is a crucial factor in assessing teaching effectiveness[5], [6]. The richness of cultural material must be preserved as foreign literature programs constantly adjust to international academic norms. Thus, the attainment of both verbal proficiency and cultural awareness by pupils is a prerequisite for effective instruction.

The teaching of foreign literature has also changed because of technological integration. More interactive and varied information distribution is made possible by digital tools and internet resources[7], [8]. Any strong evaluation system must take into consideration the effects of these advances, particularly as digital literacy emerges as a crucial skill in higher education.

In determining the quality of instruction, institutional support and faculty growth are equally important. Peer observations, pedagogical training, and professional development opportunities all support the enhancement of education[9], [10]. Therefore, the larger ecosystem in which teaching takes place, including departmental objectives and administrative rules, must be considered by evaluation systems.

The purpose of this study is to offer an assessment viewpoint that considers the difficulties of college-level foreign literature instruction. It aims to create a more thorough framework for evaluating teaching success by establishing important performance metrics, looking at stakeholder input, and studying pedagogical outcomes—one that emphasizes both academic rigor and the transformational potential of literature.

The criteria weights and the alternatives' performance in relation to the specified criteria are the two primary data sets that constitute the basis of MCDM problems. Various MCDM techniques, also referred to as ranking methods, are used to aggregate these sets for the purpose of evaluating alternatives. External assessments, whether they be objective assessments based on information gathered from various evaluations, observations, or monitoring, or subjective assessments based on expert judgments, are the primary basis for evaluating how well the alternatives perform in

relation to the criteria[11], [12]. The process by which the weights of the criteria are determined, however, typically takes a different turn.

The weights of criteria are crucial in ranking or choosing the best option in an MCDM situation, when the alternatives' performance values are evenly distributed, and no alternative is completely dominating[13], [14]. Accurately calculating the weights of criteria—whether based on expert opinions, evaluations, judgments, expectations, interpretations, objective values, or a combination of these crucial given the influence. This will subsequently result in more dependable conclusions.

The weights of the criterion are established using MCDM weighting techniques. The MCDM weighing techniques, according to Singh & Pant, can be divided into two primary categories: indirect criteria weighing techniques, which incorporate theories and mathematical functions or models, and direct criteria weighing techniques, which comprise ranking-weight, rating, scaling, and point-allocation procedures.

2. Double-Framed SuperHyperSoft Set (DFSHS)

Between 2018-2024 Smarandache introduced six new types of soft sets: HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, SuperHyperSoft Set, TreeSoft Set, ForestSoft Set[15], [16], [17].

Let U be a universal set and the $P(U)$ is the power set of U . Set of attributes $a_1, a_2, \dots, a_n; n \geq 1$ and satisfy the conditions $A_i \cap A_j = \emptyset$ for all $i \neq j$. Let $P(A_i)$ the power set of A_i . The cartesian production of the power set can be defined as[18], [19]:

$$C = P(A_1) \times P(A_2) \times \dots \times P(A_n) \quad (1)$$

The DFSHS over U is then a triple

$$(d_1, d_2; C) \quad (2)$$

$$d_1: C \rightarrow P(U), \quad (3)$$

$$d_2: C \rightarrow P(U), \quad (4)$$

d_1 and d_2 can map each element of C

d_1 and d_2 are two frames for positive and negative or lower and upper criteria.

The DFSHS is used with the MCDM method such as CoCoSo method to rank the alternatives. The steps of this method are shown as:

Create the decision matrix between the criteria and alternatives. Combine the decision matrix and computing the criteria weights using the average method.

Normalize the decision matrix such as:

$$q_{ij} = \frac{x_{ij} - \min_i x_{ij}}{\max_i x_{ij} - \min_i x_{ij}} \quad (5)$$

$$q_{ij} = \frac{\max_i x_{ij} - x_{ij}}{\max_i x_{ij} - \min_i x_{ij}} \quad (6)$$

Determine the power weighted matrix.

$$P_i = \sum_{j=1}^n w_j q_{ij} \quad (7)$$

Compute the total weighted matrix.

$$T_i = \sum_{j=1}^n (q_{ij})^{w_j} \quad (8)$$

Determine the relative weight of every alternative.

$$Z_{ia} = \frac{P_i + T_i}{\sum_{i=1}^m (P_i + T_i)} \quad (9)$$

$$Z_{ib} = \frac{P_i}{\min_i P_i} + \frac{T_i}{\min_i T_i} \quad (10)$$

$$Z_{ic} = \frac{h(P_i) + (1-h)(T_i)}{(h \max_i P_i + (1-h) \max_i T_i)} \quad 0 \leq h \leq 1 \quad (11)$$

Determine the final value of each alternative.

$$Z_i = (Z_{ia} Z_{ib} Z_{ic})^{\frac{1}{3}} + \frac{1}{3} (Z_{ia} + Z_{ib} + Z_{ic}) \quad (12)$$

Rank the alternatives.

3. Case Study

This section shows the case study for Measuring Teaching Success in College Foreign Literature Programs: An Evaluation Perspective using 12 criteria and 8 alternatives.

The criteria and alternatives are:

Teaching Effectiveness {High, Low}; Measures the effectiveness of teaching methods, content delivery, and student engagement.

Student Satisfaction {Very Satisfied, Satisfied}; Reflects how content students are with the course structure, teaching methods, and learning outcomes.

Student Learning Outcomes {Excellent, Good, Average, Poor}; Assesses how well students have met the course objectives and improved their knowledge of foreign literature.

Classroom Participation {Active, Moderate, Passive, None}; Measures the level of student interaction and participation in discussions, debates, and group activities.

Course Material Quality {Excellent, Good, Average, Below Average}; Evaluates the relevance, depth, and clarity of the course materials used, such as textbooks, articles, and online resources.

Instructor Availability {Always Available, Often Available, Occasionally Available, Rarely Available}; Reflects the ease with which students can approach the instructor for help and clarification.

Assessment and Feedback Quality {Timely, Detailed, Constructive, Poor}; Assesses how promptly and effectively the instructor provides feedback on assignments and exams.

Student Engagement with Course Content {Highly Engaged, Moderately Engaged, Low Engagement, No Engagement}; Measures how actively students engage with course materials outside of class (e.g., through independent reading, research, or extra assignments).

Instructor Knowledge of Subject Matter {Expert, Knowledgeable, Adequate, Insufficient}; Evaluates the depth of the instructor's knowledge and their ability to effectively communicate complex ideas in foreign literature.

Course Organization {Well-Organized, Organized, Disorganized, Poorly Organized}; Measures how clearly the course schedule, objectives, and expectations are communicated and followed.

Use of Technology in Teaching {Innovative, Effective, Moderate, None}; Assesses how effectively the instructor uses technology (e.g., multimedia, online platforms) to enhance the learning experience.

Cultural Relevance of Literature {Highly Relevant, Relevant, Moderately Relevant, Irrelevant}; Evaluates how well the course content reflects global perspectives and the cultural diversity inherent in foreign literature.

The alternatives are: Traditional Lecture-Based Approach, Student-Centered Learning, Blended Learning, Socratic Method, Project-Based Learning, Flipped Classroom, Peer Review and Collaborative Learning, Experiential Learning.

Three decision makers created the decision matrix using a scale between 0 and 1. We aggregate the decision matrix and obtain the criteria weights using the average method.

C1: 0.076915883

- This criterion represents a relatively low influence in the overall evaluation, indicating that it has a minor impact compared to other criteria. It may pertain to a supporting aspect of the evaluation, contributing to the overall assessment but not a determining factor in the outcomes.

C2: 0.077285973

- Slightly higher than C1, this criterion still represents a moderate weight in the evaluation process. It suggests that the factor plays an important role but does not dominate the overall evaluation. It could be related to aspects such as class participation or attendance.

C3: 0.078485778

- This criterion is similar to C2 but with a slightly stronger impact. It likely pertains to a factor that contributes more significantly to teaching success or learning outcomes, such as engagement with course materials or application of learning in real-world contexts.

C4: 0.093462453

- C4 holds a substantial weight in the evaluation, indicating that this criterion plays a more prominent role in the overall assessment. It may be linked to a critical element of the teaching process, such as the clarity of course objectives or the effectiveness of assessment methods.

C5: 0.08419132

- This criterion represents a moderately high influence. It may relate to factors like the instructor's responsiveness to student needs or the adaptability of the course content. While important, it is not the most significant factor but still contributes considerably to the overall evaluation.

C6: 0.086176073

- Slightly higher than C5, this criterion also has a moderate-high impact. It could be related to the quality of feedback provided to students, the support for learning outside of class, or the use of technology to enhance learning experiences.

C7: 0.072072888

- This criterion has a relatively low weight, suggesting it may be a supportive factor that is not as central to the evaluation. It may address factors like classroom atmosphere, minor aspects of instructional delivery, or peripheral elements that impact the overall learning experience but do not stand out significantly.

C8: 0.098894408

- C8 is the highest-weighted criterion, indicating that it is one of the most significant aspects in the evaluation process. It may relate to critical areas such as overall teaching effectiveness, alignment of course objectives with learning outcomes, or the ability of the instructor to engage and inspire students.

C9: 0.086710315

- This criterion holds a moderate-high weight and could pertain to factors like the extent to which students feel prepared for exams or assignments or the alignment of teaching methods with student learning styles. It plays a crucial role but is not the highest priority.

C10: 0.076813412

- Similar to C1, C10 has a lower weight, indicating that it is a supplementary factor. It might involve less direct elements of the teaching experience, such as course administration or logistics, that still impact the student experience but do not heavily influence overall teaching success.

C11: 0.088652288

- This criterion is significant, contributing a notable weight to the overall evaluation. It may relate to factors such as the extent to which students' critical thinking and creativity are fostered or how well the course promotes independent research and inquiry.

C12: 0.080339209

- This criterion holds a moderate weight and could be related to factors such as the diversity and relevance of the course materials or the opportunities for students to apply theoretical knowledge in practical settings. While important, it is not a dominant factor in the overall assessment.

The weights indicate the relative importance of each criterion in the evaluation process, with C8 having the highest influence and C7 having the lowest.

We use DFSHS to divide the alternatives into two frames as lower and upper such as:

First frame

{{High, Low}, {Very Satisfied, Satisfied}, {Excellent}, {Active}, {Excellent}, {Always Available}, {Timely}, {Highly Engaged}, {Expert}, {Well-Organized}, {Innovative}, {Highly Relevant}}={A1,A2,A3,A4}

Second frame

{{High, Low}, {Very Satisfied, Satisfied}, {Excellent}, {Active}, {Excellent}, {Always Available}, {Timely}, {Highly Engaged}, {Expert}, {Well-Organized}, {Innovative}, {Highly Relevant}}={A5,A6,A7,A8}

In each frame, we have uncertainty in the first and second criteria. So, we have four groups of each frame such as:

- ✓ {{High}, {Very Satisfied}, {Excellent}, {Active}, {Excellent}, {Always Available}, {Timely}, {Highly Engaged}, {Expert}, {Well-Organized}, {Innovative}, {Highly Relevant}}={A1,A2,A3,A4}

- ✓ {{High}, {Satisfied}, {Excellent}, {Active}, {Excellent}, {Always Available}, {Timely}, {Highly Engaged}, {Expert}, {Well-Organized}, {Innovative}, {Highly Relevant}}={A1,A2,A3,A4}
- ✓ {{Low}, {Very Satisfied}, {Excellent}, {Active}, {Excellent}, {Always Available}, {Timely}, {Highly Engaged}, {Expert}, {Well-Organized}, {Innovative}, {Highly Relevant}}={A1,A2,A3,A4}
- ✓ {{Low}, {Satisfied}, {Excellent}, {Active}, {Excellent}, {Always Available}, {Timely}, {Highly Engaged}, {Expert}, {Well-Organized}, {Innovative}, {Highly Relevant}}={A1,A2,A3,A4}

Second frame

- {{High}, {Very Satisfied}, {Excellent}, {Active}, {Excellent}, {Always Available}, {Timely}, {Highly Engaged}, {Expert}, {Well-Organized}, {Innovative}, {Highly Relevant}}={A5,A6,A7,A8}
- {{High}, {Satisfied}, {Excellent}, {Active}, {Excellent}, {Always Available}, {Timely}, {Highly Engaged}, {Expert}, {Well-Organized}, {Innovative}, {Highly Relevant}}={A5,A6,A7,A8}
- {{Low}, {Very Satisfied}, {Excellent}, {Active}, {Excellent}, {Always Available}, {Timely}, {Highly Engaged}, {Expert}, {Well-Organized}, {Innovative}, {Highly Relevant}}={A5,A6,A7,A8}
- {{Low}, {Satisfied}, {Excellent}, {Active}, {Excellent}, {Always Available}, {Timely}, {Highly Engaged}, {Expert}, {Well-Organized}, {Innovative}, {Highly Relevant}}={A5,A6,A7,A8}

First frame and first group:

Normalize the decision matrix using eqs. (5 and 6) as shown in Fig 1. Determine the power weighted matrix using eq. (7) as shown in Fig 2. Compute the total weighted matrix using eq. (8) as shown in Fig 3. Determine the relative weight of every alternative using eqs. (9-11). Determine the final value of each alternative using eq. (12).

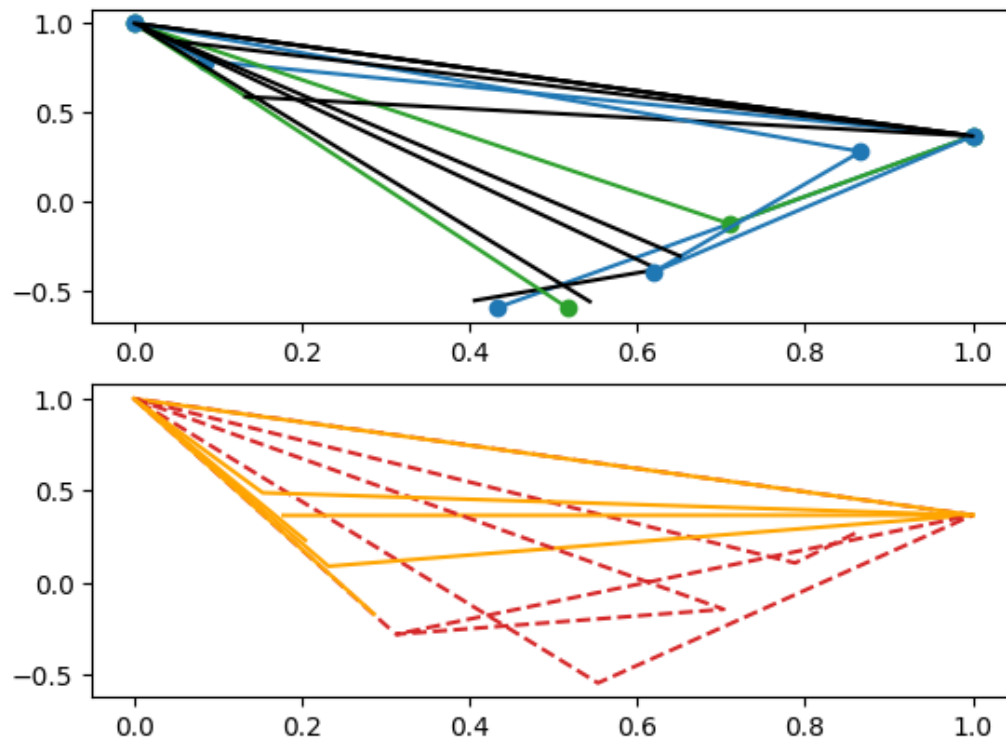


Fig 1. The normalization values.

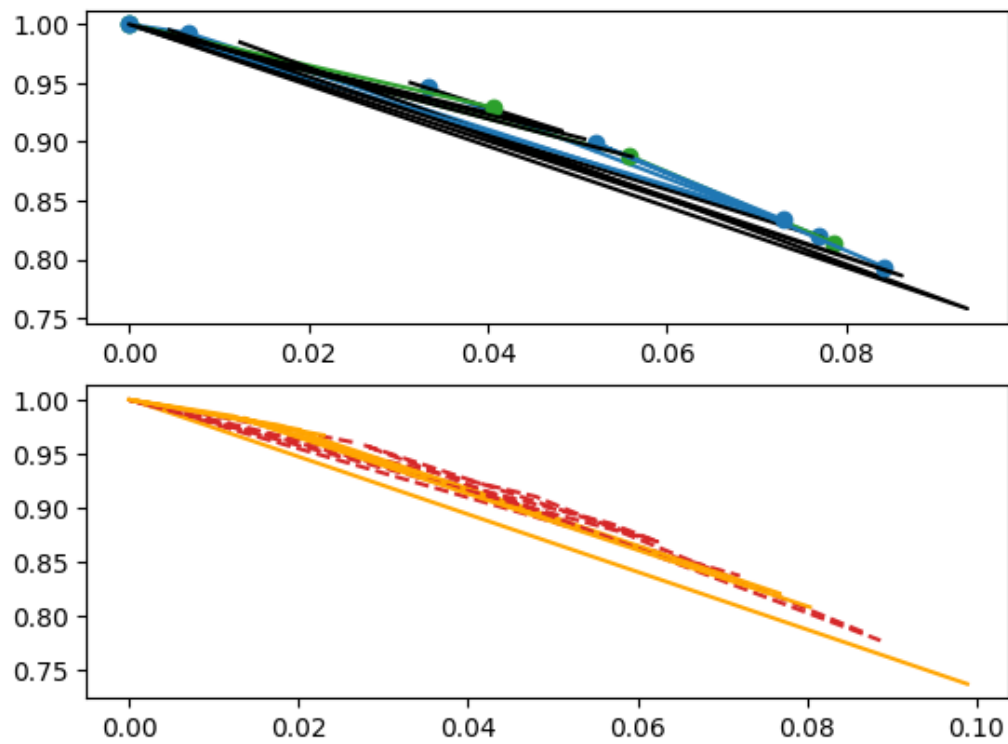


Fig 2. The power weighted matrix.

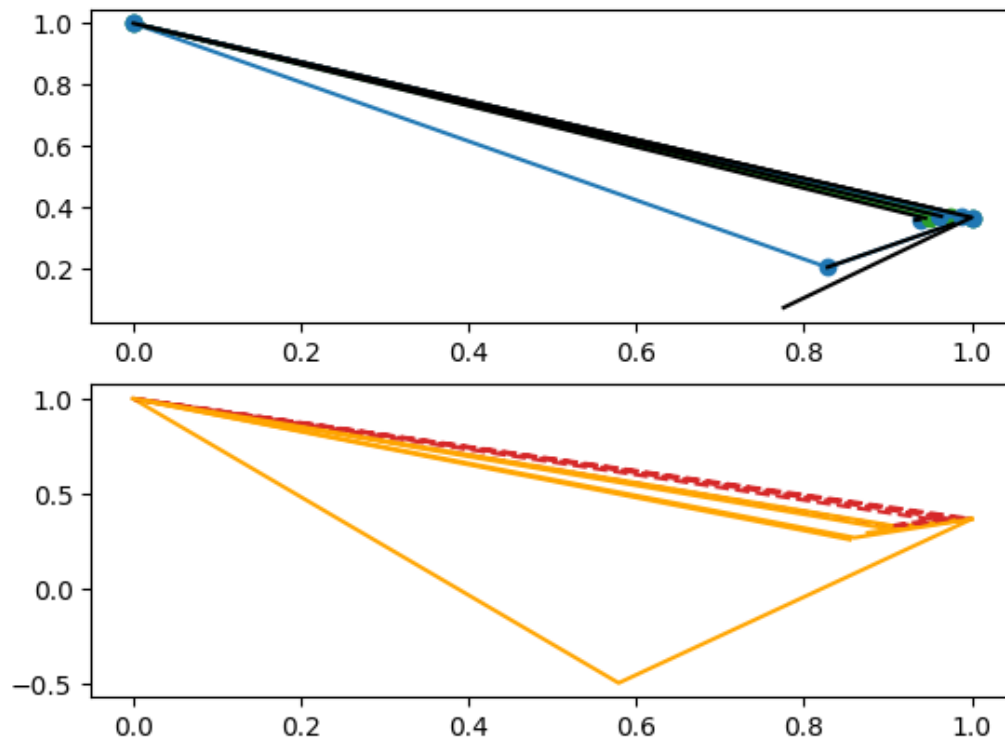


Fig 3. The total weighted matrix.

First frame and second group:

Normalize the decision matrix using eqs. (5 and 6) as shown in Fig 4. Determine the power weighted matrix using eq. (7) as shown in Fig 5. Compute the total weighted matrix using eq. (8) as shown in Fig 6. Determine the relative weight of every alternative using eqs. (9-11). Determine the final value of each alternative using eq. (12).

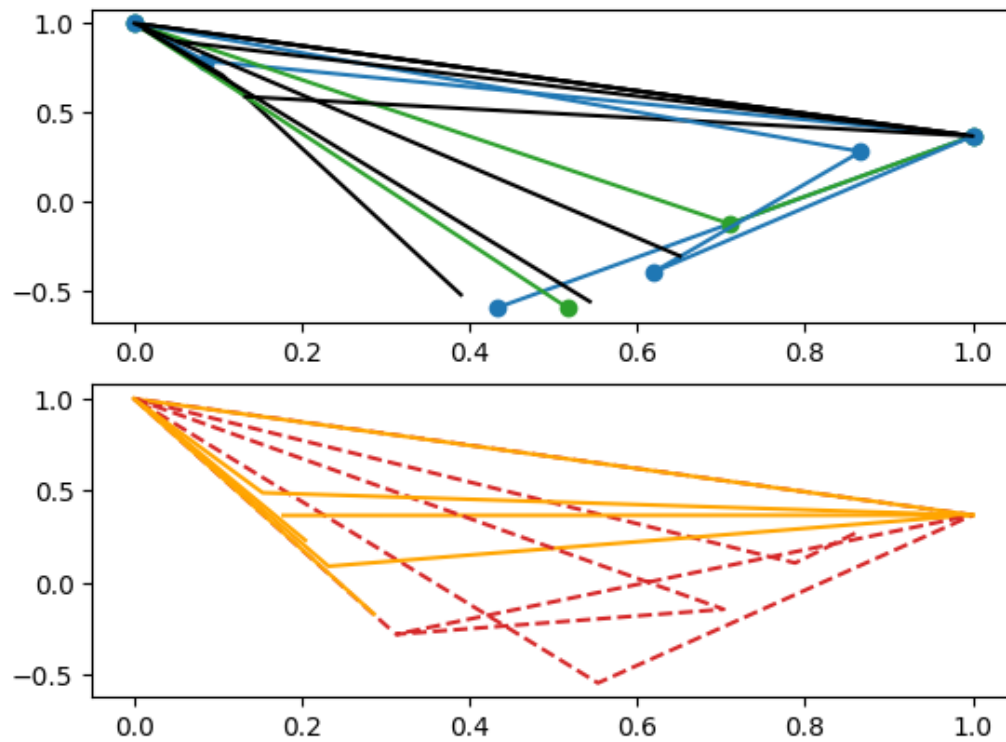


Fig 4. The normalization values.

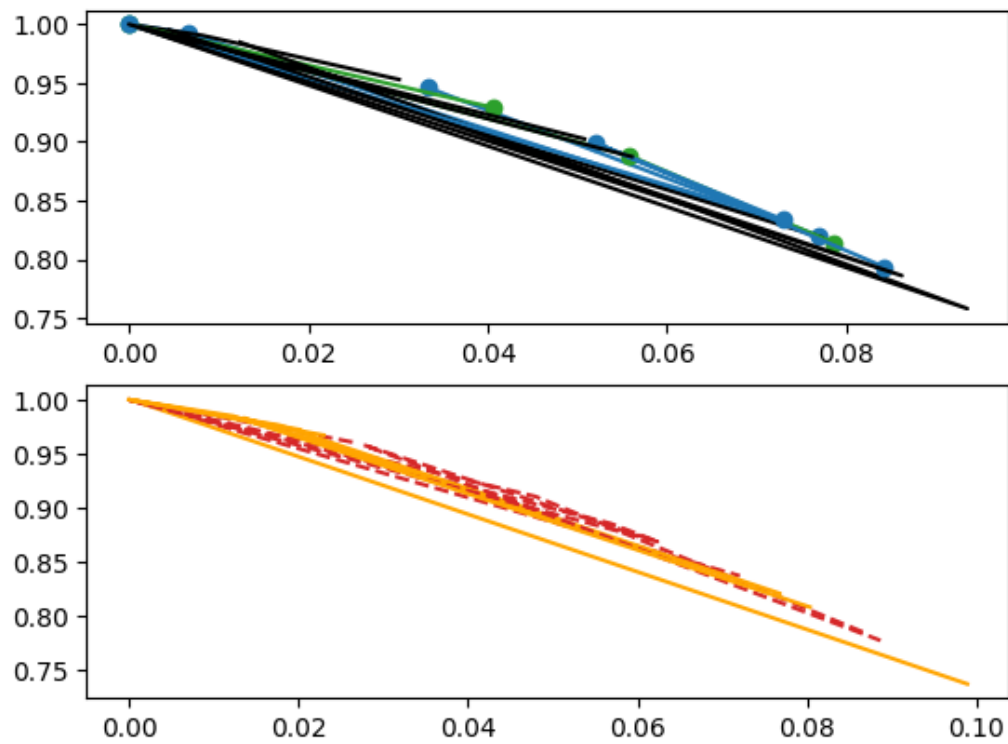


Fig 5. The power weighted matrix.

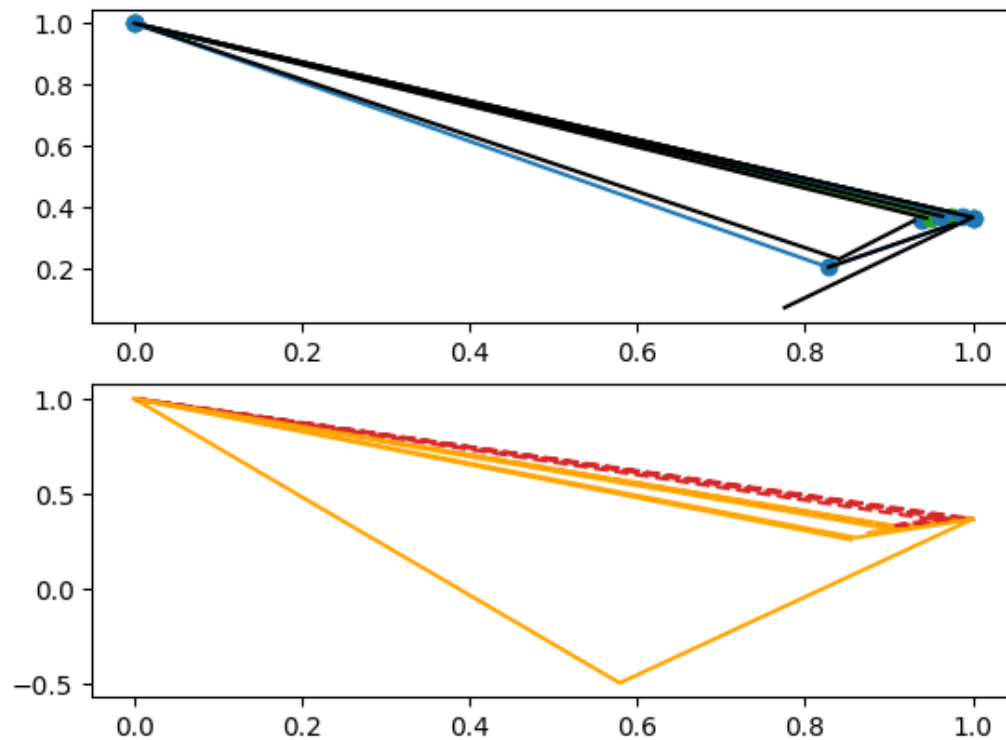


Fig 6. The total weighted matrix.

First frame and third group:

Normalize the decision matrix using eqs. (5 and 6) as shown in Fig 7. Determine the power weighted matrix using eq. (7) as shown in Fig 8. Compute the total weighted matrix using eq. (8) as shown in Fig 9. Determine the relative weight of every alternative using eqs. (9-11). Determine the final value of each alternative using eq. (12).

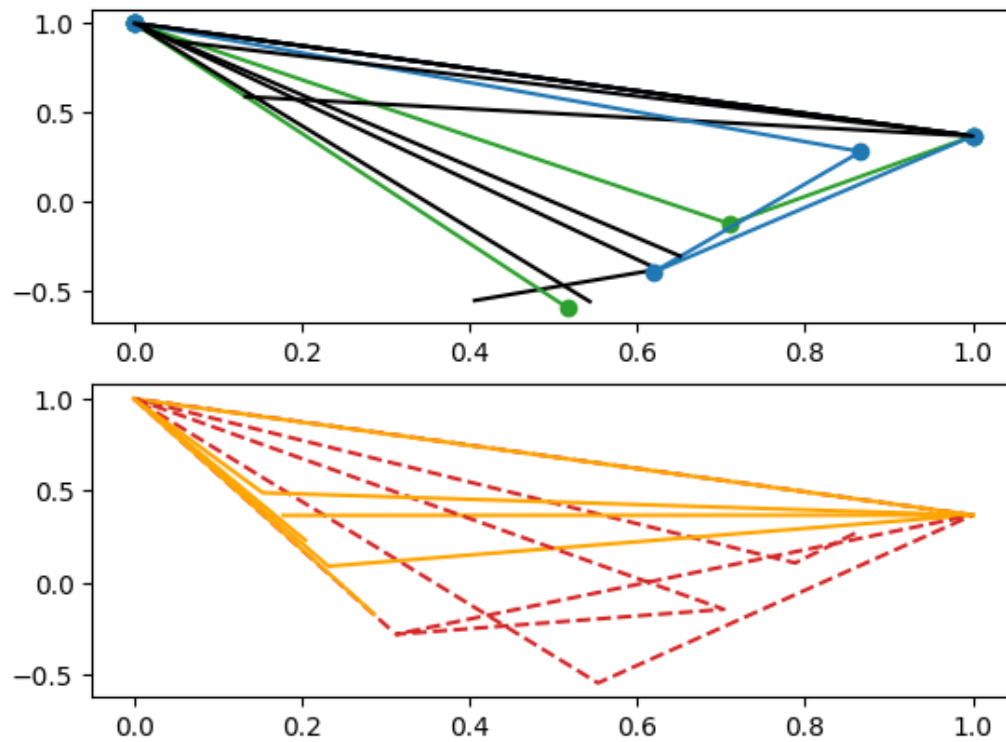


Fig 7. The normalization values.

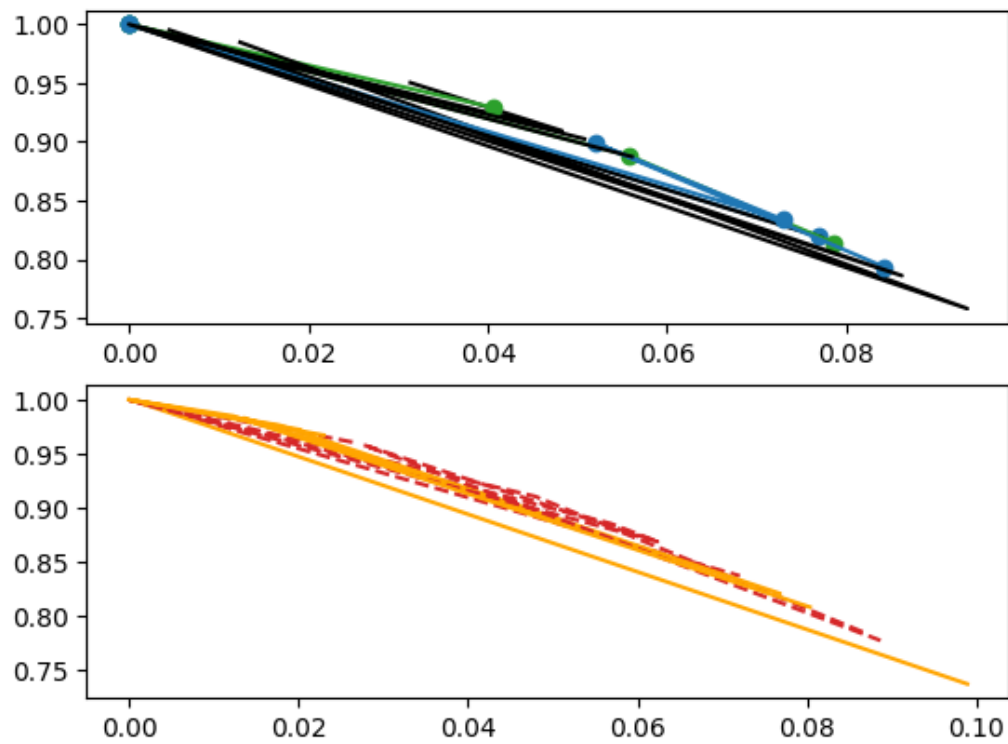


Fig 8. The power weighted matrix.

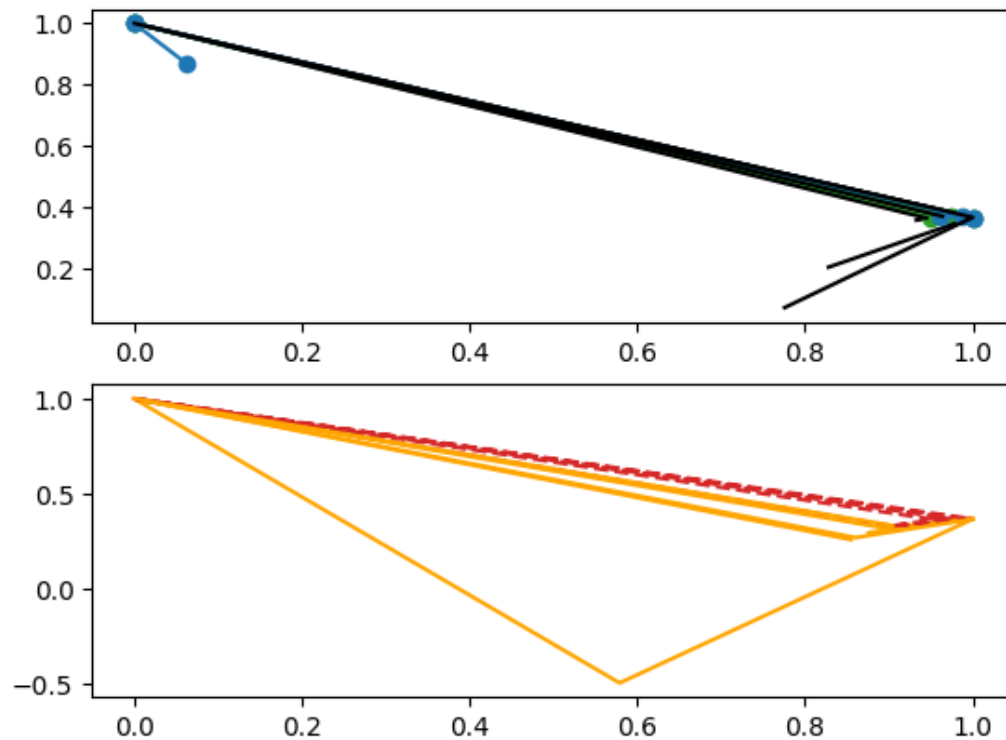


Fig 9. The total weighted matrix.

First frame and fourth group:

Normalize the decision matrix using eqs. (5 and 6) as shown in Fig 10. Determine the power weighted matrix using eq. (7) as shown in Fig 11. Compute the total weighted matrix using eq. (8) as shown in Fig 12. Determine the relative weight of every alternative using eqs. (9-11). Determine the final value of each alternative using eq. (12). The final ranks of the first frame are shown in Fig 13.

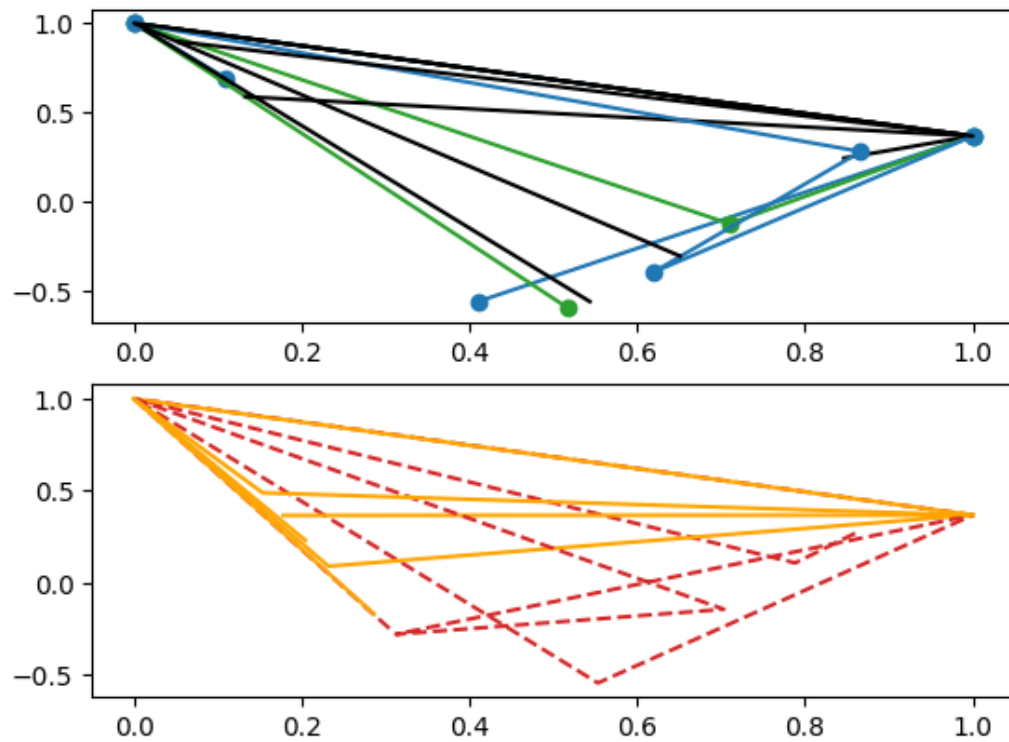


Fig 10. The normalization values.

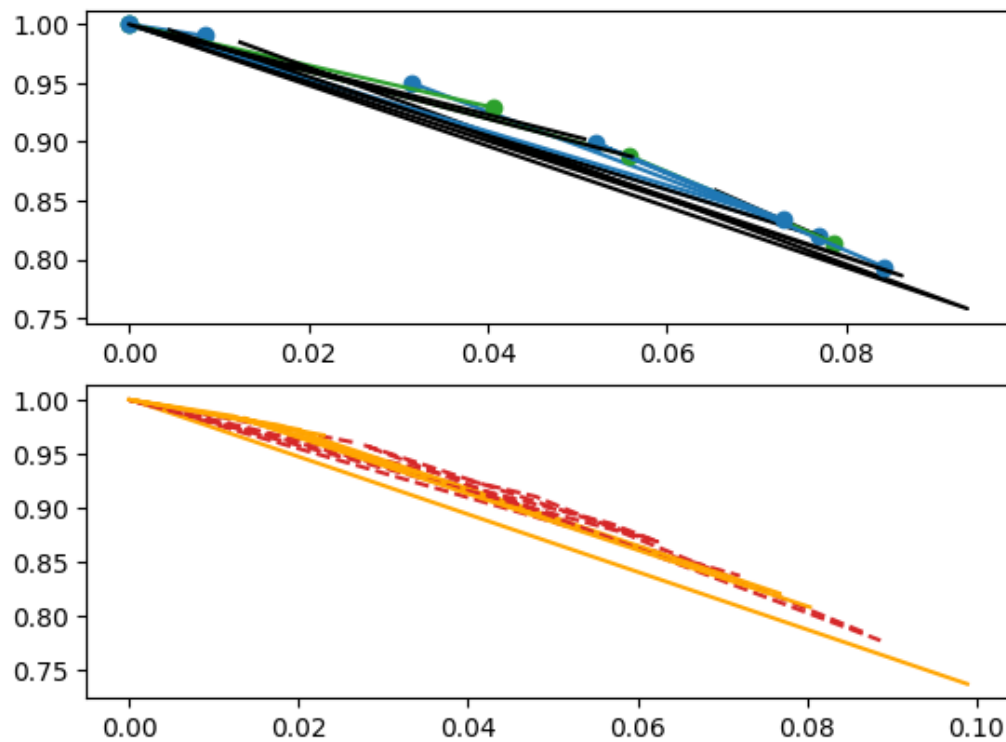


Fig 11. The power weighted matrix.

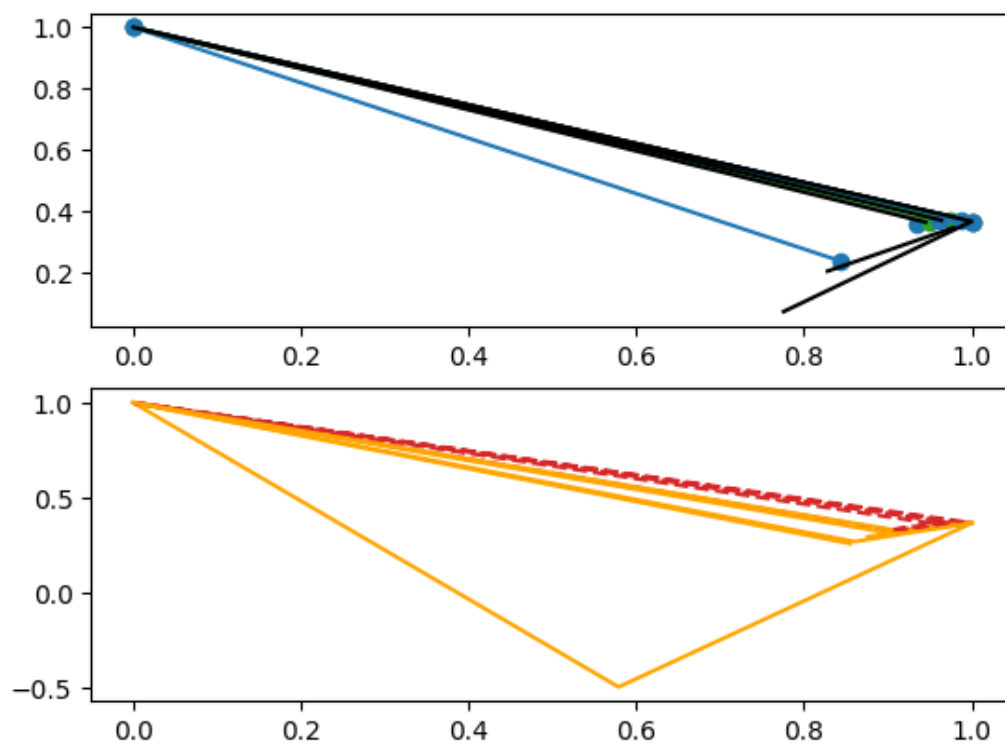


Fig 12. The total weighted matrix.

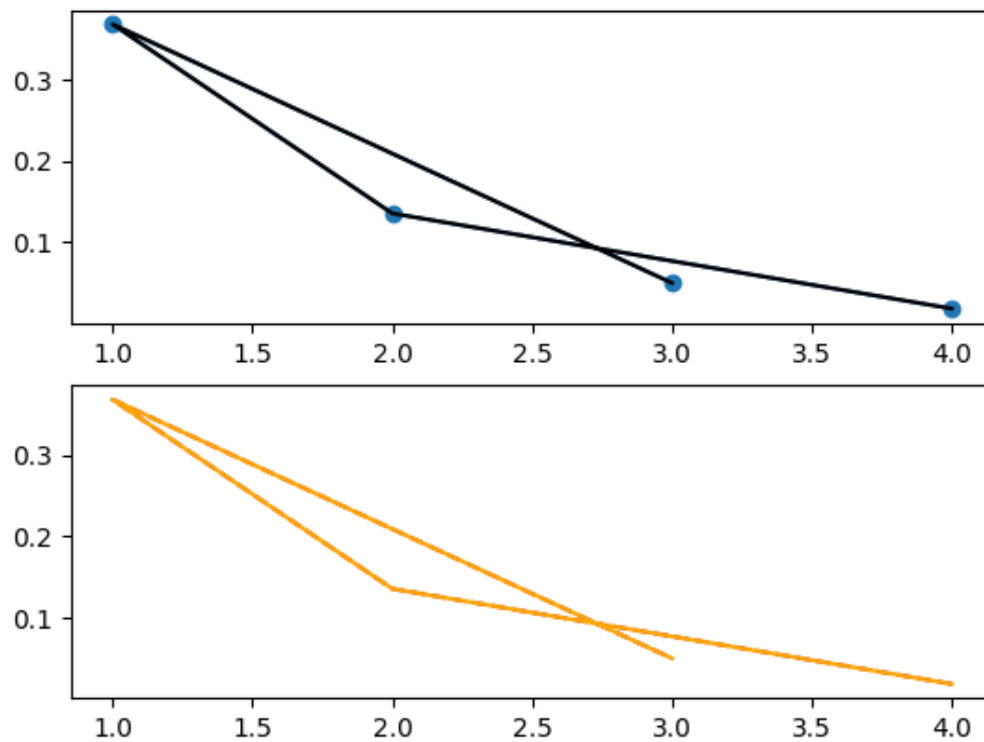


Fig 13. The final ranks.

Second frame and first group:

Normalize the decision matrix using eqs. (5 and 6) as shown in Fig 14. Determine the power weighted matrix using eq. (7) as shown in Fig 15. Compute the total weighted matrix using eq. (8) as shown in Fig 16. Determine the relative weight of every alternative using eqs. (9-11). Determine the final value of each alternative using eq. (12).

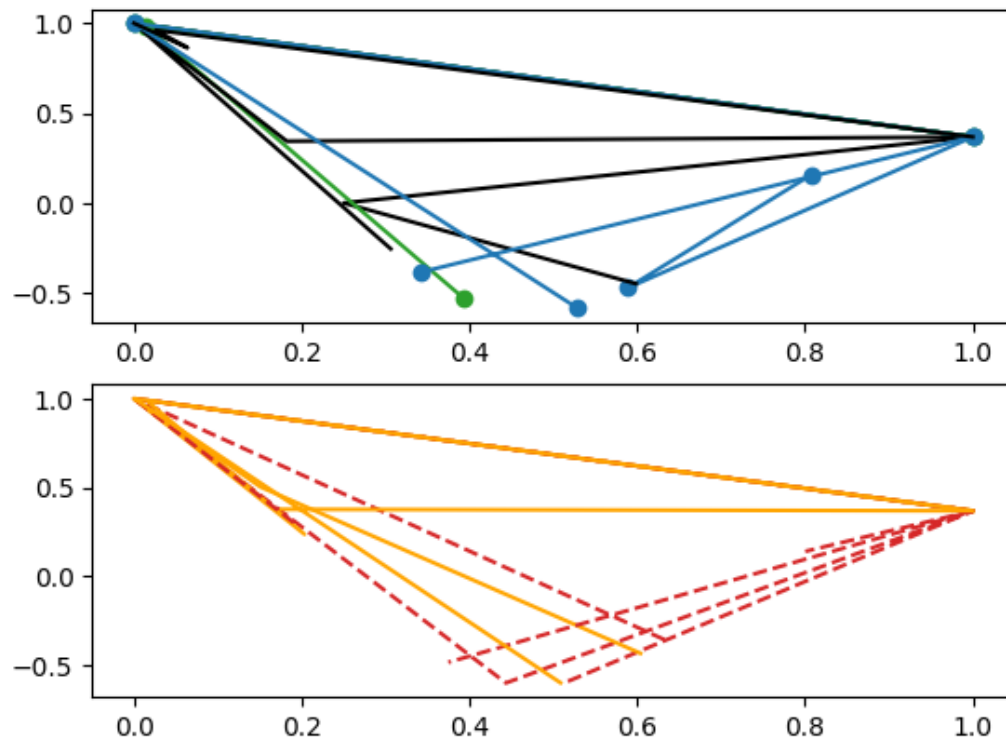


Fig 14. The normalization values.

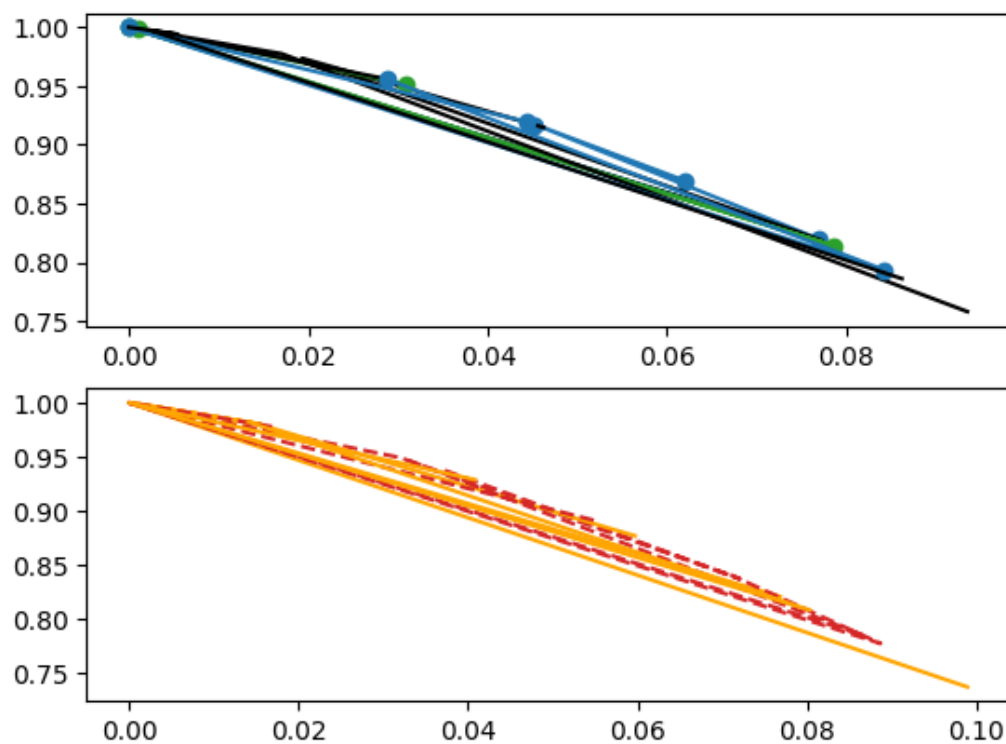


Fig 15. The power weighted matrix.

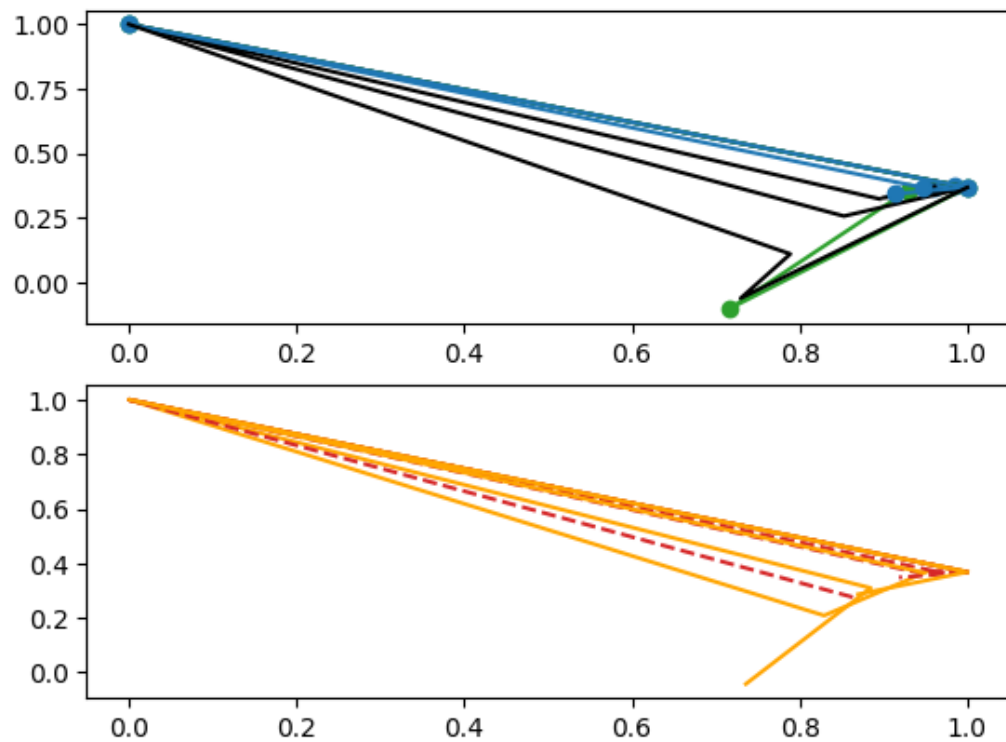


Fig 16. The total weighted matrix.

Second frame and second group:

Normalize the decision matrix using eqs. (5 and 6) as shown in Fig 17. Determine the power weighted matrix using eq. (7) as shown in Fig 18. Compute the total weighted matrix using eq. (8) as shown in Fig 19. Determine the relative weight of every alternative using eqs. (9-11). Determine the final value of each alternative using eq. (12).

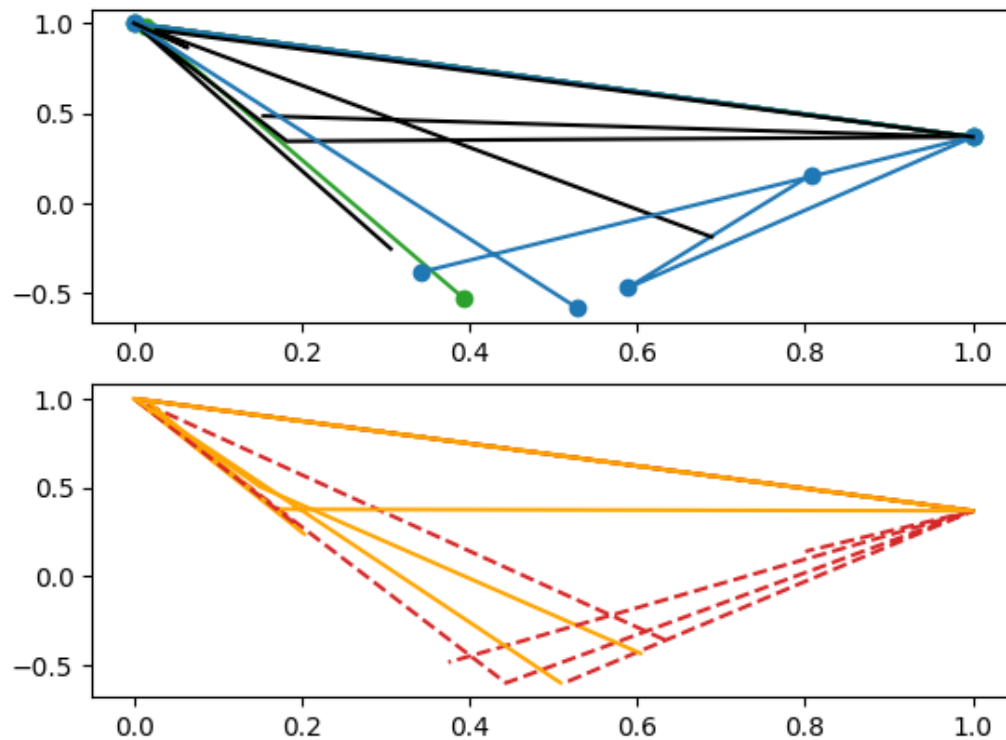


Fig 17. The normalization values.

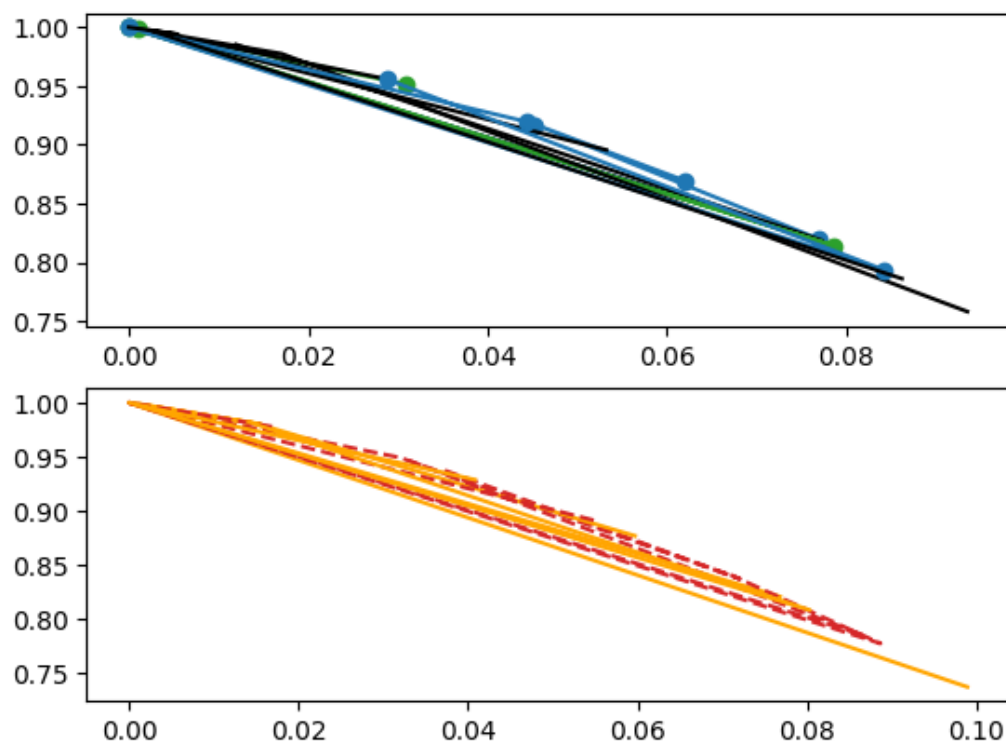


Fig 18. The power weighted matrix.

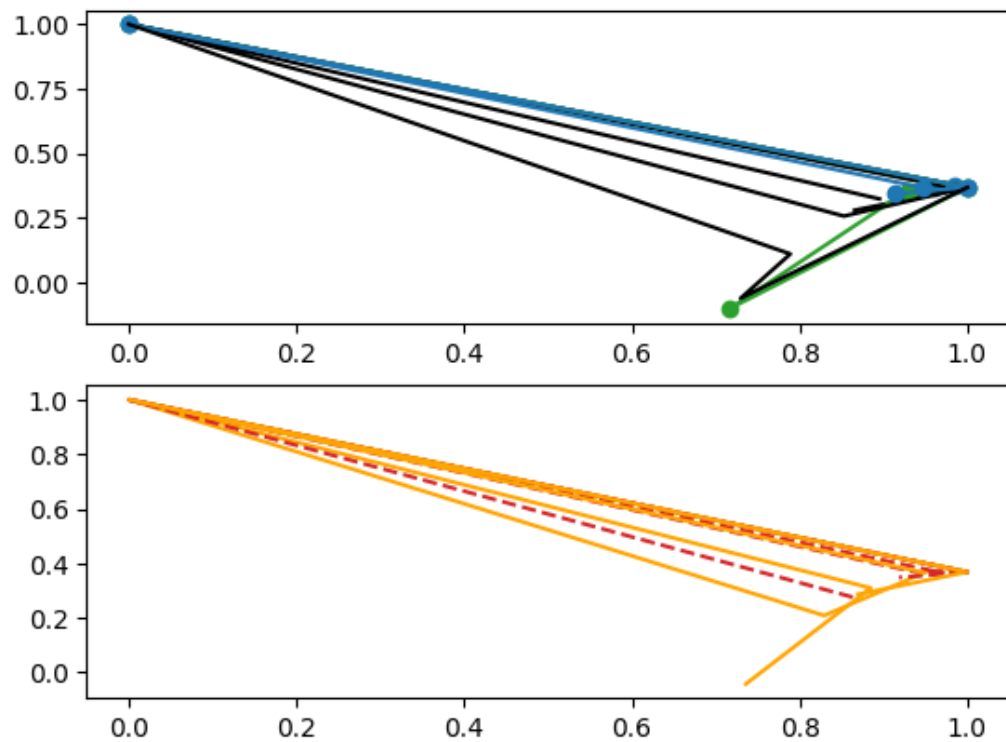


Fig 19. The total weighted matrix.

Second frame and third group:

Normalize the decision matrix using eqs. (5 and 6) as shown in Fig 20. Determine the power weighted matrix using eq. (7) as shown in Fig 21. Compute the total weighted matrix using eq. (8) as shown in Fig 22. Determine the relative weight of every alternative using eqs. (9-11). Determine the final value of each alternative using eq. (12).

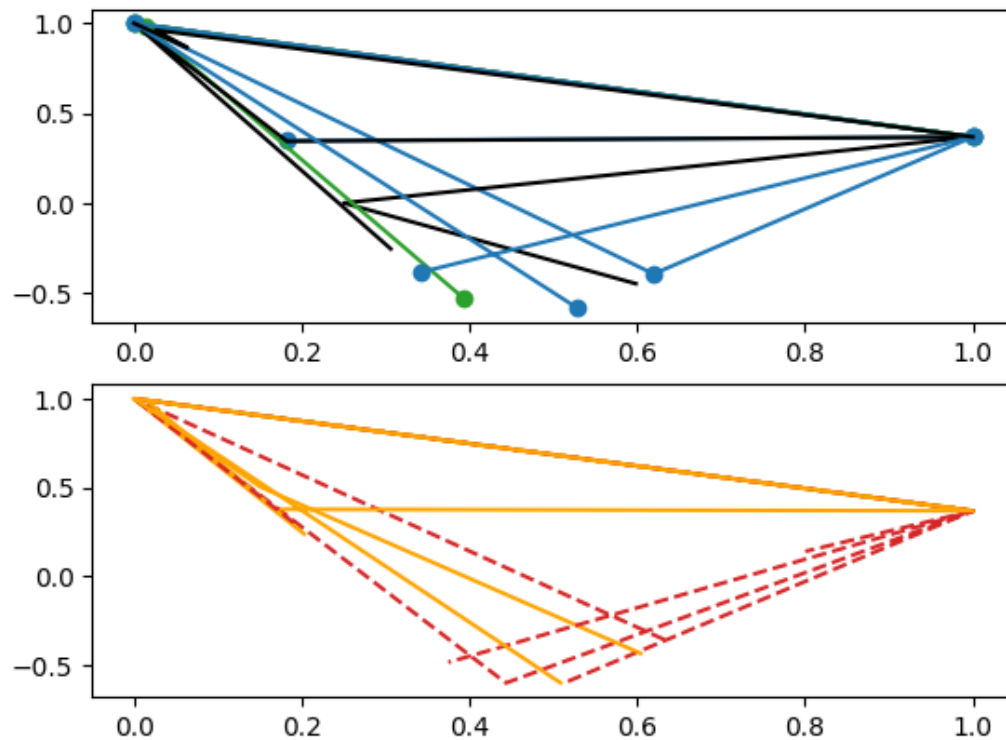


Fig 20. The normalization values.

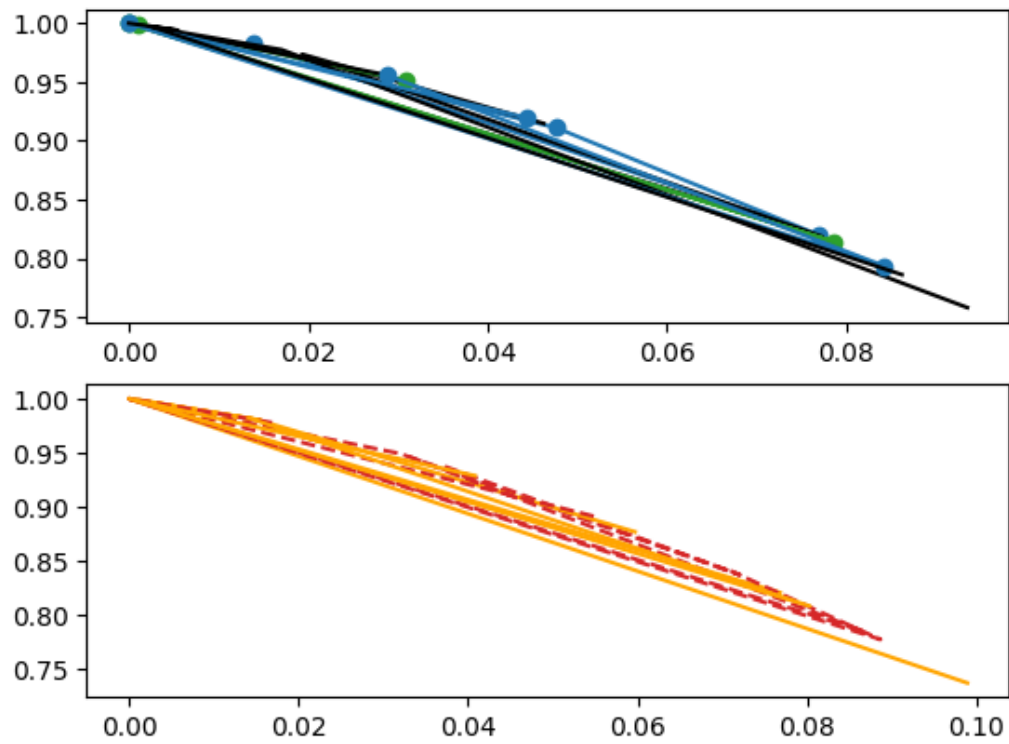


Fig 21. The power weighted matrix.

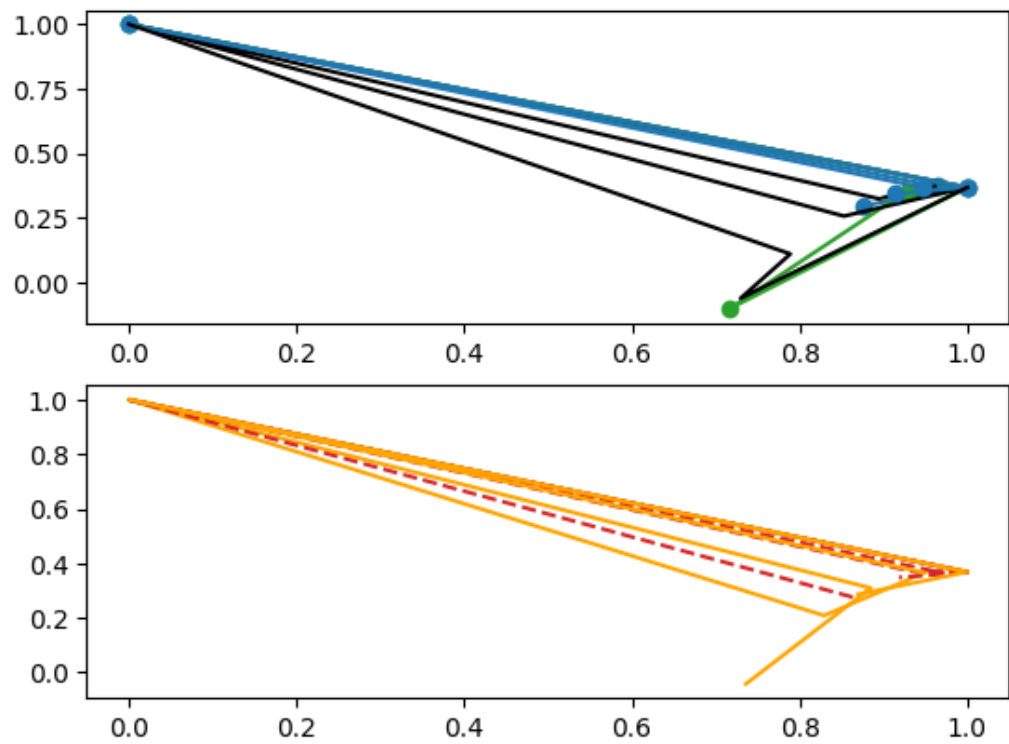


Fig 22. The total weighted matrix.

Second frame and fourth group:

Normalize the decision matrix using eqs. (5 and 6) as shown in Fig 23. Determine the power weighted matrix using eq. (7) as shown in Fig 24. Compute the total weighted matrix using eq. (8) as shown in Fig 25. Determine the relative weight of every alternative using eqs. (9-11). Determine the final value of each alternative using eq. (12). The final ranks of the second frame are shown in Fig 26.

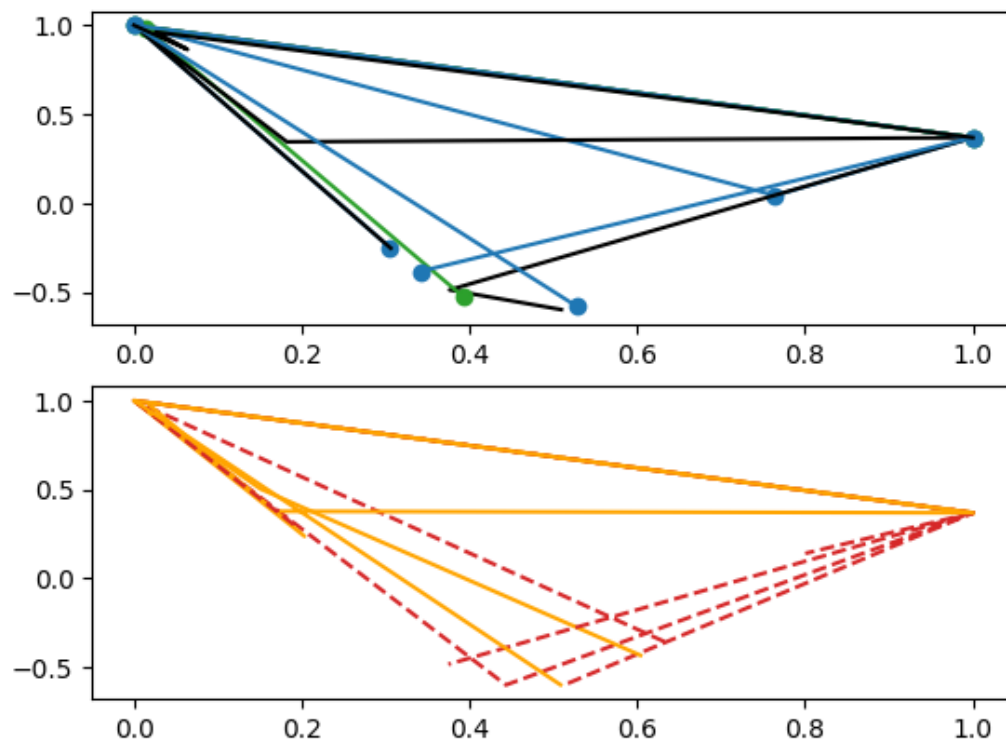


Fig 23. The normalization values.

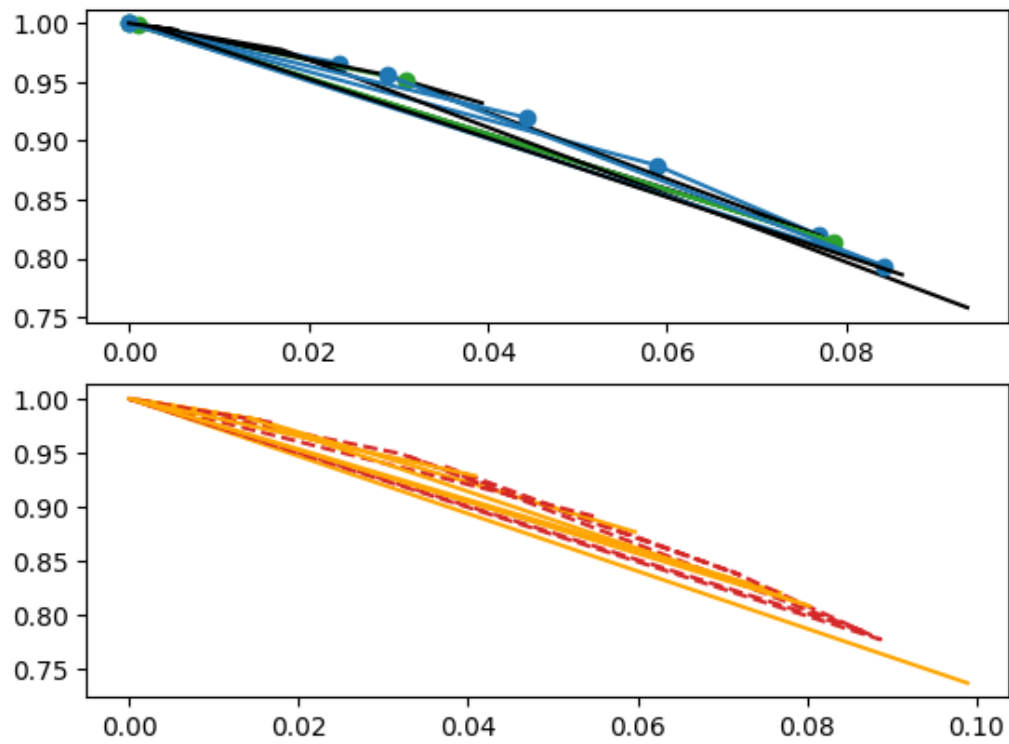


Fig 24. The power weighted matrix.

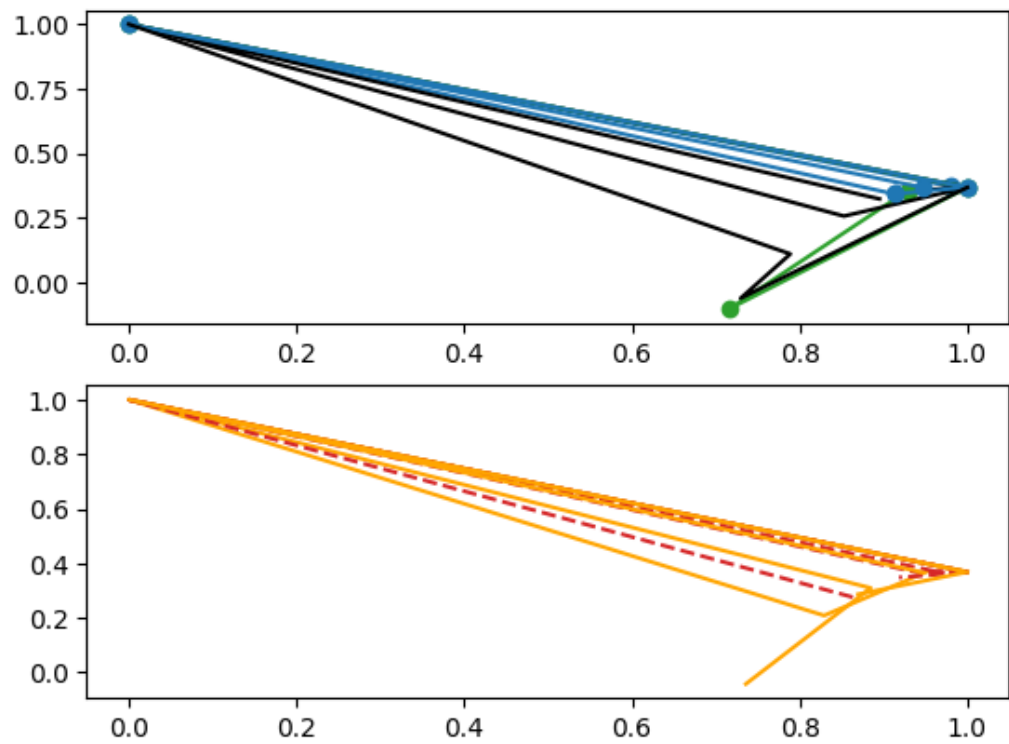


Fig 25. The total weighted matrix.

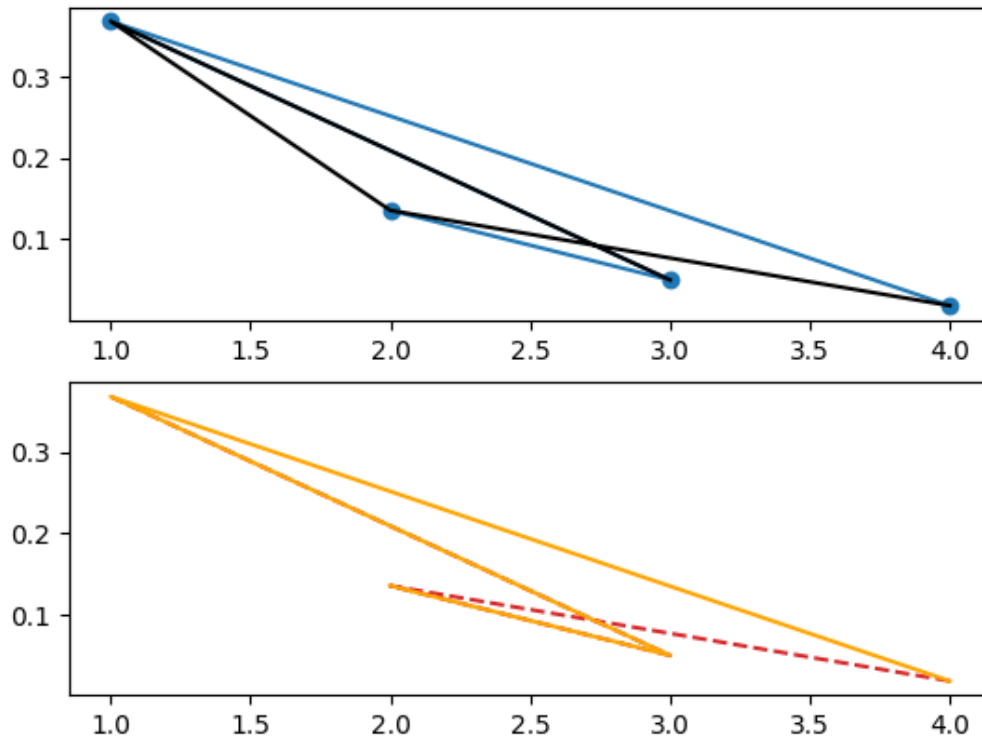


Fig 26. The final ranks.

4. Conclusions

There is no one lens that can adequately describe the effectiveness of teaching foreign literature in college programs. It calls for a well-rounded assessment mechanism that considers student growth, cultural relevance, and academic excellence. This study emphasizes the necessity of multifaceted evaluation techniques that consider curricular efficacy, student feedback, and innovative pedagogy. Improving the instruments and frameworks for assessing educational accomplishment becomes more than just an academic priority—it becomes a cultural necessity as foreign literature continues to play a significant role in forming global citizens. We used the CoCoSo method to rank the alternatives. The Double Framed SuperHyperSoft Set is used to deal with different criteria values.

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A Combined GIS-MCDM Approach to Site Selection of Temporary Shelter: A Case Study in Dahab, Egypt

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Abstract

In this study, a model based on the Geographic Information System (GIS) and MCDM method: the Criteria Importance through Inter-Criteria Correlation (CRITIC) based type-2 neutrosophic numbers (T2NN) is used to give researchers and stakeholders a simple and reliable spatial decision-making tool. First the model focuses on determining a group of experts with experience related to the study. The selected group of experts then determines effective objectives and attributes concerning the study and estimates the relative importance of each objective and its related set of attributes using a set of linguistic variables. The T2NN-CRITIC method was used to evaluate the objectives and attribute weights and importance. Second, the spatial attribute layers were integrated with the global weights evaluated using ArcGIS Pro to rank and choose the most suitable site for the study. The proposed model was implemented through a case study to select the most suitable site for temporary shelter in case of floods in Dahab, Egypt. According to the analysis of the current study, 5.16% of the study area (17,550 Km²) is highly suitable as a site for a temporary shelter in the event of a flood in Dahab, Egypt, (126,570 Km²) of the study area classified as moderately suitable, (161,850 Km²) of the study area classified as marginally suitable and, finally (34,080 Km²) classified as not suitable. Finally, sensitivity analysis has been used to confirm the stability of the model results.

Keywords: Temporary shelter site selection; Disaster management; Geographic information system; CRITIC; T2NN.

1. Introduction

Communities must prepare for disasters, whether they are man-made or natural [1]. A disaster may have a number of damaging consequences depending on its nature, its scope, and its location, especially if it occurs in a developing country [2]. Floods are one of the natural occurrences that, if overlooked, will harm society greatly [3]. More than ever in recent decades, heavy rainfall resulting in floods has impacted Egyptian coastal areas along the Mediterranean Sea and the Red Sea, as well as arid and semi-arid areas such as Upper Egypt (e.g., Luxor, Aswan, and Assiut) and the Sinai Peninsula [4]. The increase in the intensity of rainfall rates can be disastrous, wreaking havoc on lives, infrastructures, resources, and rich cultural heritage [5]. In recent events during the period from March 12 to 14, 2020, Egypt witnessed heavy rains leading to floods that affected more than 20,000 people, according to the International Federation of Red Cross and Red Crescent Societies flood emergency plan of action report, which was published on July 14, 2020. The report clarified that only 2,258 people were assisted out of the 12,950 targets. This increases interest in disaster management activities including the site selection of temporary shelters in events of flood. A temporary shelter is defined as a location that contains a large group of affected people from a disaster and provides them with a temporary roof, meals, clothing, water supplies, healthcare, and security for a limited time [6]. In the case of providing additional resources, existing public buildings such as schools, community centers, and fitness centers can serve as temporary shelters in emergencies [7].

Recent literature has illustrated that the need for temporary shelters has increased in the last decade due to the major role that they play in disaster management [8]–[14]. In the last decade, temporary shelter selection has become a popular topic among scholars due to its significant role in assisting people during and after a disaster [11]. Key objectives and attributes for temporary shelter site selection have been established and categorized differently in previous literature [11], [15], [24]–[33], [16], [33]–[35], [17]–[23]. Objectives and attributes may differ, but basically may include accessibility (distance from roads, distance from river, distance from drainage networks), topography (Elevation, slope), land use (Bare-land, urban, green spaces), environmental considerations (distance to hazard sites, distance to exclusion zone), and safety considerations (altitude, distance to the inundation and evacuation) [30]. In this study, five main objectives and eighteen attributes are selected and used by experts for the evaluation of the presented case study. Since site selection for a temporary shelter is a complex process multi-criteria decision-making (MCDM) methods plays a major role in helping decision-makers in the selection process [36, 37]. The selection of an appropriate site for a temporary shelter is not a simple, straightforward task; it depends on a large set of dependent objectives and attributes [38]. Through the assessment and

comparison of the distinguishing characteristics of the alternatives, the MCDM tool offers a suitable choice. Recently, several studies used different MCDM methods to address the problem of site selection for temporary shelters in the event of flood. Some of these methods focused on dealing with uncertainty in decision making process. For instance, a fuzzy The technique for order preference by similarity to ideal solution (TOPSIS) model has been presented to prioritize identified criteria and evaluate the candidate alternatives for emergency shelter [39]. In an intelligent multi-agent system for refugee sitting, a comparative analysis of fuzzy multi-attribute decision-making (MADM) methods has been presented [40]. Additionally, multi-objective decision-making framework integrated goal programming with fuzzy sets and the analytic hierarchy process (AHP) method to prioritize emergency shelter areas [41].

In MCDM problems where geo-referenced information is crucial, Geographic Information System (GIS) is a useful tool. The GIS framework provides the capacity for geo-referenced data computations, management, analysis, and visualization. GIS mostly transforms irrelevant, raw data into a comprehensible form when paired with expert perception. In this study, the best sites for temporary shelters in Dahab, Egypt, were chosen using a GIS and MCDM based methodologies. Thus, a special and coherent framework that can deal with challenging spatial planning issues is made possible by combining the two separate approaches of GIS and MCDM. There are several studies in the literature that use MCDM methods based on GIS to assess suitable locations [38], [42]–[44] but lack of studies in the literature discussed the site selection problem for temporary shelters using an integrated neutrosophic MCDM method in spatial environment. The Criteria Importance through Inter-Criteria Correlation (CRITIC) method was created by Diakoulaki et al. [45] to establish objective weights while simultaneously taking into account the variances and correlations among various criteria. Neutrosophic set theory was developed by Smarandache [46] to address real-world decision-making issues. The fuzzy theory is extended by neutrosophic sets. Neutrosophic numbers have proven to be a reliable area of research for locating incongruent and ambiguous data. Type-1 neutrosophic numbers (T1NN) are represented as a triplet (T, I, F) , where each member of the triplet falls inside the range $[0, 1]$. They are referred to as membership or truth values (T), neutral values (I), and falsehood values (F), respectively. Each neutrosophic component in Type-2 neutrosophic numbers (T2NN) is divided into its truth, indeterminacy, and falsehood subparts in the form of (T_T, T_I, T_F) , (I_T, I_I, I_F) , and (F_T, F_I, F_F) . The T2NN is an advanced form of neutrosophic methodology. It is a useful technique for addressing the incompleteness or imprecision of expert knowledge. The T2NN-CRITIC method developed by [47] is used in this study along with GIS tools to help decision-makers locate the most suitable sites for temporary shelter. The T2NN-CRITIC is used in

this study due to its various advantages. First, it is possible to successfully represent how real-world preferences favor experience over expertise and vice versa. To be able to clarify the objective significance of evaluation criteria, it is possible to simultaneously evaluate their differences and correlations. Third, choosing between two built-in factors gives you more freedom to choose your preferred temporary shelter scheme.

Due to the important role of the neutrosophic set in handling uncertainty via considering truth, indeterminacy, and falsity degrees and then simulating the natural decision-making process, the main aim of this research, is to use the T2NN-CRITIC method in a raster GIS environment to select the appropriate site of temporary shelters in Dahab city, located along the south-eastern coast of the Sinai Peninsula, Egypt, to support people in the event of flooding. There is no such literature available on the systematic evaluation of suitable sites of temporary shelters in Dahab, Egypt as far as the authors' knowledge allows.

1.1 Study contributions

Given the previously mentioned points of encouragement and the discussion of the related studies that came before it, the main aims of this study are to:

- To use the T2NN-CRITIC method in a spatial environment for the first time which is divided into four main steps: Step 1: The primary goal of the study is accurately identified, and whether a single or group-based study will be conducted is also decided by the stakeholders. The selected expert/s then determines the appropriate set of objectives and attributes related to the study. Step 2: The spatial data for each attribute is gathered in a GIS environment, and a spatial analysis is then carried out to create a map layer for each attribute. Then, standardization is used to develop standardized attribute values ranging from 0 to 1. Step 3: Experts use the T2NN-CRITIC method to calculate the local and global weights of the objectives and attributes. Step 4: The resulting standardized attribute map layers from Step 2 and the global weights of the main objectives and attributes in Step 3 are then integrated and overlay operation is done to produce a final weight map using GIS tools, which is then classified to rank and select the most suitable sites. The amount of land that is available is divided into four categories based on its suitability: "highly suitable", "moderately suitable", "Marginally Suitable", and "not suitable".
- To apply GIS and MCDM to assist stakeholders locating suitable sites for temporary shelters in Dahab, Egypt in case of floods.
- To perform the sensitivity analysis to assess the persistence of the priority rating.

The innovative aspect of the current study is to combine the GIS tool and the T2NN-CRITIC approach, which have not before been used to find suitable locations for temporary shelters in case of floods in Dahab, Egypt. The study is viable as a result of the numerous suggestions made by area experts, stakeholders, and specialists from various environmental organizations, medical experts, and planning authorities.

1.2 Study structure

The remaining parts of the study are demonstrated to achieve the aims of the study and consist of the following: Section 2 briefly discusses the relative literature. Section 3 shows the fundamental methodology. In Section 4, the actual case study results and discussion for the site selection of temporary shelters in Dahab, Egypt is discussed. A sensitivity analysis is presented in Section 5. Section 6 shows the conclusion and offers suggestions for future work and the study's limitations.

2. Literature review

In the second section of this research, we will review related studies previously published and clarify basic terminologies.

2.1 GIS with MCDM

This section discusses the application of GIS in MCDM methods and its significance in real-life scenarios. The theoretical advancement and recent research on these strategies are shown in Table 1 of this section.

Table 1. Literature concerning different spatial MCDM methods and applications.

Ref.	Year	Spatial MCDM method	Area of application
[48]	2021	AHP + suitability analysis using WLC	Optimal sites for photovoltaic (PV) plants
[49]	2021	integrated index method (IIM), AHP-TOPSIS, and AHP-VIKOR	Landslide susceptibility maps (LSMs)
[50]	2021	AHP with Weighted overlay analysis (WOA)	Potential landfill site selection
[51]	2021	AHP + suitability maps	Railway design with spatial environmental considerations
[52]	2021	AHP and ANP with TOPSIS	Location selection of multi-purpose utility tunnels
[53]	2021	Fuzzy method, AHP with CA-Markov model	Forecasting drought susceptibility
[54]	2021	AHP with GIS	Drought vulnerability assessment and mapping
[55]	2021	TOPSIS with GIS	Sustainable development of territorial units
[56]	2021	F-AHP with GIS	Assessment of solar and wind farm locations
[57]	2022	AHP with GIS	Potential facility locations for park-and-ride facilities along transit corridors

[57]	2022	AHP, VIKOR, TOPSIS, WEIGHTED SUPERPOSITION, PSI, ARAS, OCRA, SMART,	Locate appropriate sites for installing photovoltaic solar farms
[58]	2022	Pythagorean fuzzy AHP, TOPSIS	Pandemic hospital site selection
[59]	2022	AHP-TOPSIS and fuzzy-GIS	Optimal off-shore wind location selection and assessment
[60]	2022	F-AHP, VIKOR and Psychometric-VIKOR	Bike-sharing station site selection
[61]	2022	OWA, MOLA module	Sustainable urban land-use optimization
[62]	2022	GIS-AHP	Site selection for radioactive waste disposal facility
[63]	2022	AHP and TOPSIS	Dam site suitability mapping
[64]	2023	AHP with WOSA	Vulnerability to water erosion and mapping potential control sites
[65]	2023	AHP with Fuzzy-VIKOR	Risk analysis, Identifying the regions with urban vulnerability to potential fire hazards
[66]	2023	AHP-GIS, interval-FAHP-GIS and ANP-GIS	Assess multi-hazard risks such as floods, muddy-water flows and landslides induced by rainstorms
[67]	2023	GIS-SMCA	Prioritize potential areas for TOD
[68]	2023	FGAHP with CRITIC	Site selection for tidal current power plant
[69]	2023	AHP – FMCDM	Traffic management, site selection of car parking
[70]	2023	GIS-AHP	Waste disposal site selection
[71]	2023	GIS-AHP	Sustainable mango production
[72]	2023	GIS-AHP	Potential sites for housing development

2.2 Neutrosophic sets with MCDM

A neutrosophic set, first introduced by Smarandache, is defined by a membership function for truth, a function for indeterminacy, and a function for falsity [73]. Recent articles on these strategies are shown in Table 2.

Table 2. Literature concerning neutrosophic sets with MCDM methods and applications.

Ref.	Year	Neutrosophic used	MCDM Method	Area of application
[74]	2021	Bipolar Neutrosophic	TOPSIS, EDAS, WSM, VICKOR	Select optimal wind turbine
[75]	2021	Single valued Neutrosophic set	fuzzy TOPSIS	Supplier selection in the production industry
[76]	2021	Quadripartitioned single-valued neutrosophic set (QSVNS)	QSVNPDOWAA QSVNPDOWGA	Buy a smartphone

[77]	2021	Rough Neutrosophic Set	cross entropy; VIKOR	Planning of remediation for historic pedestrian bridges
[78]	2021	Single valued neutrosophic set theory	BWM, VIKOR	Evaluating the performance of IoT based supply chain
[79]	2021	Single-valued neutrosophic set (SVNS) and the interval-valued neutrosophic set (IVNS)	MCDM method based on proposed SF and AF under IVNSs	Selecting a pre-school for the first time, by the parents of a kindergarten child
[80]	2021	Neutrosophic Soft Set	TOPSIS, WSM, and WPM	The selection of LASER as surgical instrument
[81]	2022	Pythagorean neutrosophic set	PNG-based MCDM method	Best investing option for a company
[82]	2022	m-generalized q-neutrosophic sets (mGqNNs)	CoCoSomGqNN	civil engineering industry
[83]	2022	Neutrosophic sets	a novel neutrosophic MCDM	Fourth party logistics firm assessment
[84]	2022	The neutrosophic hyper-soft set	NHSS-MCDM	Optimal civil engineer for construction firm
[85]	2022	Multi-Valued Multi-Polar Neutrosophic Sets	mPIVNSWA-MCDM	associate professor selection for university
[86]	2023	Single-valued neutrosophic(SVN) -based rough sets	SVN-MCDM	Find best diagnosis for patients
[87]	2023	Trapezoidal Neutrosophic Sets	WASPAS	Risk assessment
[88]	2023	Multi-Valued Multi-Polar Neutrosophic Sets	MCDM based on MVmNSS by using TOPSIS	Optimum fuzzy soft constants
[89]	2023	Triangular neutrosophic sets	COCOSO MCDM	select best strategy in higher education
[90]	2023	Interval Valued Neutrosophic	IVN-AHP and VIKOR based MCDM method	Evaluation of universities based on student perspective
[91]	2023	Single-valued neutrosophic	CRITIC-MULTIMOORA	Warehouse manager selection
[92]	2023	Interval Valued Neutrosophic	AHP	Training assignment
[93]	2023	Single Valued Neutrosophic	TOPSIS	Interval valued neutrosophic

2.3 Site selection for temporary shelters

Site selection for temporary shelters is problematic because it depends on several contradictory criteria. In Table 3, a comparison of several temporary shelter site selection methodologies is covered.

Table 3. Literature of site selection methodologies for temporary shelters.

Ref.	Year	Methodology	Disaster Type
[11]	2021	Suitability assessment	Earthquake
[94]	2021	Multi-criteria decision analysis	Reduce human casualties during airstrike
[95]	2021	NSGA-II algorithm	Landslide
[28]	2021	P-Center Model	Flood
[96]	2022	Rapid visual screening, a geographic information system, and fuzzy analytic hierarchy process	Earthquake
[97]	2022	Index-based model	Earthquake
[98]	2022	Developed geographically weighted regression	Earthquake
[99]	2023	F-AHP	Volcano mount eruption
[42]	2023	Large Group Decision-Making (LGDM)	Earthquakes

3. Methodology

This study uses a neutrosophic-based MCDM method along with GIS tools to locate suitable sites for temporary shelters in Dahab, Egypt. The four steps of this study's framework are depicted in Figure_1. Step1: the essential aim of the study is accurately determined, and the decision of whether to conduct a single or group-based study is also made. After identifying the goal and expert(s), the selected expert(s) uses their knowledge and previous literature concerning the problem to identify appropriate objectives and associated sets of attributes. Step2: all the attributes have their spatial data collected in a GIS environment, and then a spatial analysis is conducted to create a map layer for each attribute. Standardization is then applied to prepare standardized attribute values ranging between 0 and 1 to create a common scale for allowing the map layers to be comparable. Step3: experts use the linguistic terms to evaluate both objectives and attributes. In the case of group decision-making, expert evaluations are aggregated, and then using the T2NN-CRITIC method, the local and global weights of the objectives and attributes are calculated. Step4: the resulting standardized attribute map layers from Step 2 and the global weights of the main objectives and attributes in Step 3 are then integrated using GIS tools. Overlay operation is done to produce a final weight map, which is classified to rank and select the most suitable sites. The amount of land that is available is divided into four categories based on its suitability: "highly suitable", "moderately suitable", "marginally suitable", and "not suitable". Finally, sensitivity analysis is conducted to help stakeholders explore the range of possible outcomes and identify the key drivers and risks of the selected decision.

3.1 Study area

The aim of this study is to locate suitable sites for temporary shelters in Dahab City, Egypt. As illustrated in Figure_ 2, the city of Dahab is approximately 80 kilometers (50 miles) northeast of Sharm el-Sheikh on the southernmost tip of Egypt's Sinai. Dahab is a region that is well-known around the world, particularly for its gorgeous coral reefs, uncommon marine animals, and alluring diving activities. In the south, it is likewise regarded as an emerging and developing metropolis.

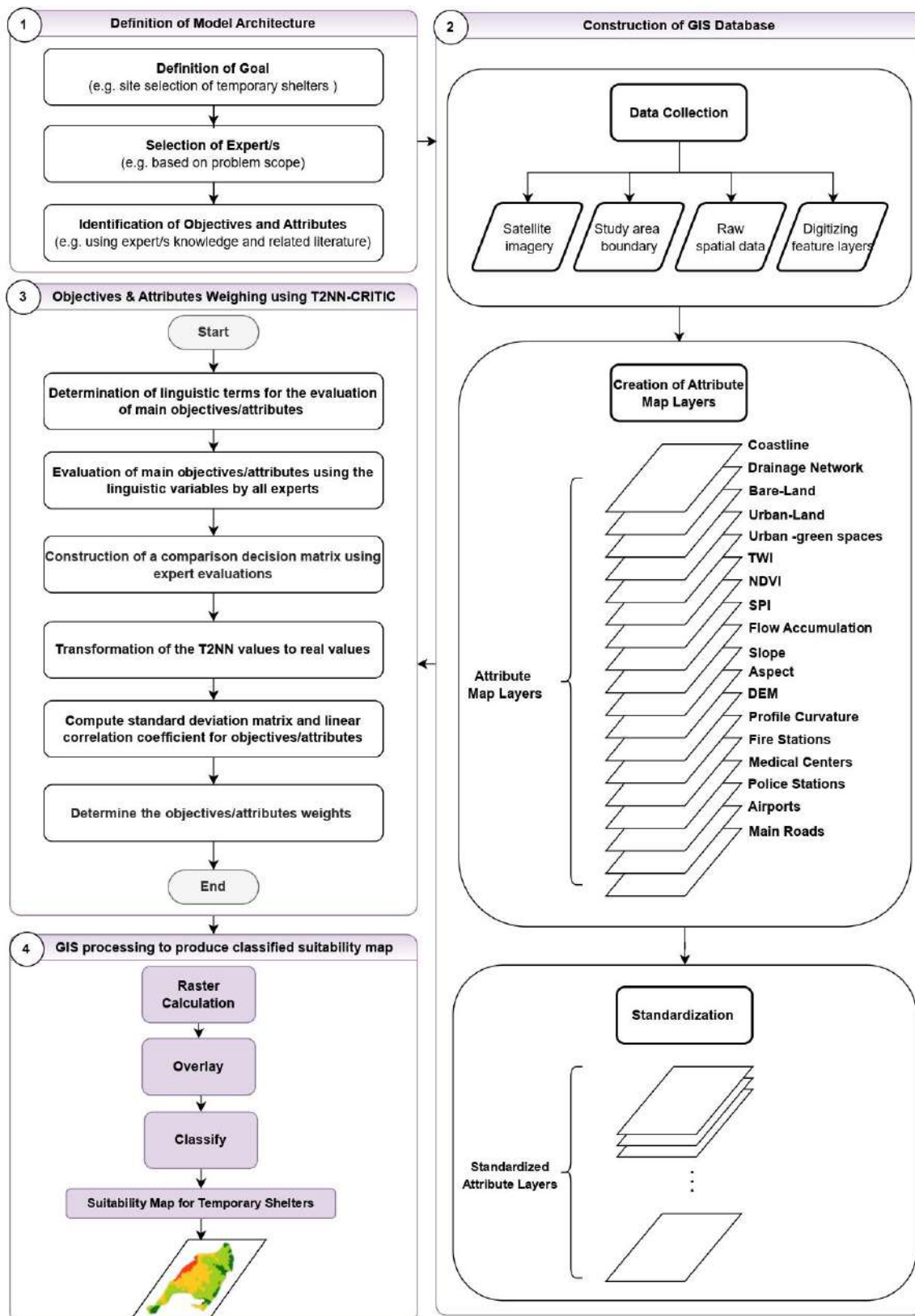


Figure_ 1. The proposed spatial T2NN-CRITIC framework for suitable site selection of temporary shelters.

Dahab City is around 1130 km², and it is situated near the "mouth" of the Wadi Dahab Basin, a sizable hydrographic basin. Wadi Dahab Basin carries a lot of water during the flood seasons, which flows quickly through high, steeply sloping rocky ravines and feeds the outlet of the basin that encircles the city of Dahab [100]. One of the world's most dangerous natural disasters is flooding. They have a large-scale effect, especially in arid/semiarid regions, resulting in thousands of human deaths and massive economic losses. Recent frequent flash floods [101], the geographic location, the semi-arid nature of Dahab City, and its tourism and economic importance are the reasons why it is chosen as the study area [100]. The source of data used in this study was collected and processed from the Egyptian central agency for public mobilization and statistics using ArcGIS Pro 2.6 software by ESRI.

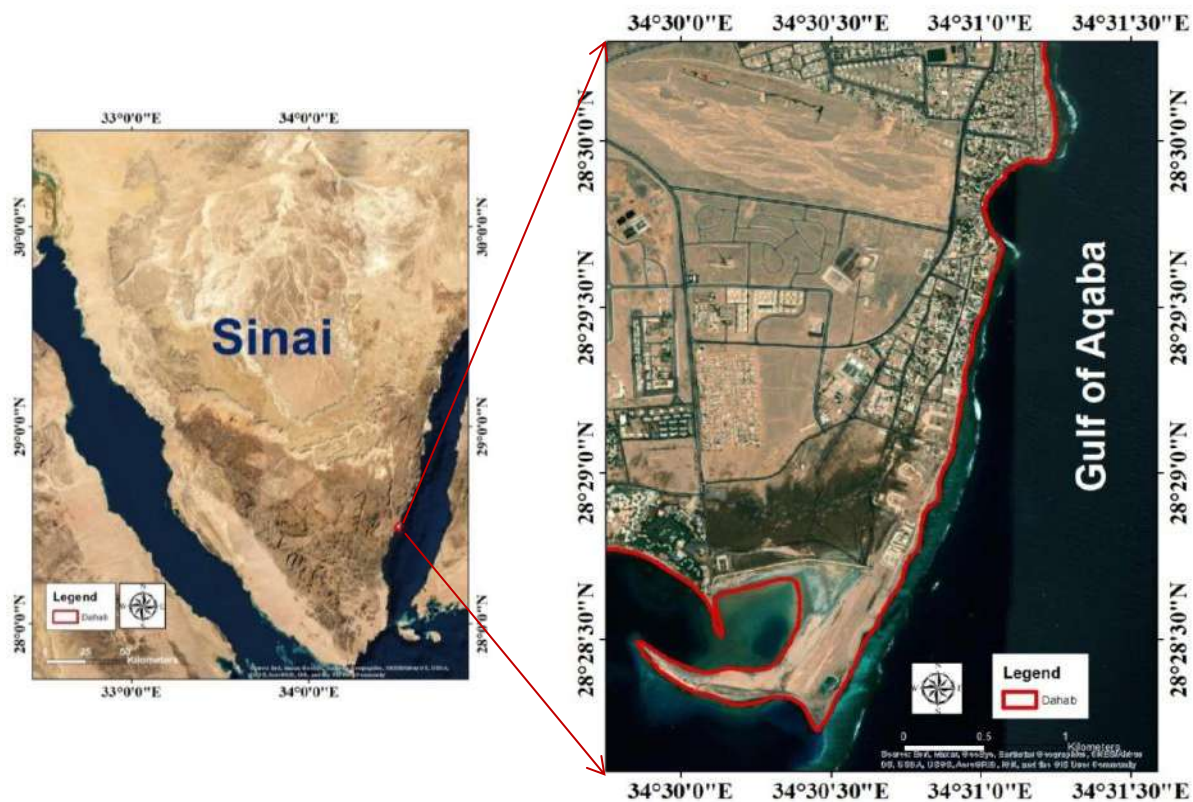


Figure_ 2. Location map of Dahab City (ESRI, 2023).

3.2 Panel of experts

Group decision-making is more appropriate in this study to cover all fields and aspects of the predefined goal. According to the selected study aim to identify suitable sites for temporary shelters in

case of floods in Dahab, Egypt a set of experts with high experience and knowledge of the aim were selected as shown in Table 4. Since experts are people who possess both knowledge and experience in a very specialized field, our model treated all experts equally important and did not prioritize them.

Table .4. Details on the participants of the panel of experts.

Expert	Degree	Occupation	Profession	Gender
E ₁	PhD	Industry	Civil Engineering	Male
E ₂	PhD	Academia	Data Science	Female
E ₃	M.Sc.	Industry	Urban Planning	Male
E ₄	M.Sc.	Academia	Construction Technology and Management	Male
E ₅	M.Sc.	Industry	Medical doctor	Female

3.3 Data Selection, Collection, and Standardization

This study's spatial and attribute data were gathered from secondary data sources. The following sentences go into further depth on the various data sources utilized for the identification of temporary shelter sites in Dahab, Egypt. United States Geological Survey (USGS) generated global digital elevation model (DEM) was used along with ArcGIS Pro 2.6 software tools to generate slope and aspect. The Land-use maps were produced using supervised classification on a Landsat-8 image collected from the United States Geological Survey (USGS) website. Both road and coastline are collected from the Egyptian national authority for remote sensing and space sciences (NARSS). Medical centers, fire stations, airports, and police department's data are collected from the Egyptian Central Agency for Public Mobilization and Statistics (CAPMAS). The remaining data are produced using different analysis tools available in ArcGIS Pro 2.6 software such as raster calculator, curvature, aspect, slope, and flow accumulation. In this study, the highest and lowest values of each attribute map layer were used to determine the standardization of each attribute map layer using a linear scale transformation method. Depending on whether the objective is to be maximized (i.e., a higher value indicates greater desire) or reduced (i.e., a lower value shows more desirability), Eq. (1) for benefit attributes and Eq. (2) for cost attributes, are used respectively. W_{ij} stands for the standardized values of the attribute that were maximized or decreased [102]. Moreover, W_{ij}^{min} is the j th attribute's lowest value, and W_{ij}^{max} is its highest value. Attributes of standardized map layers are shown in Figure_4.

$$W_{ij} = \frac{W_{ij} - W_{ij}^{min}}{W_{ij}^{max} - W_{ij}^{min}} \quad (1)$$

$$W_{ij} = \frac{W_{ij}^{max} - W_{ij}}{W_{ij}^{max} - W_{ij}^{min}} \quad (2)$$

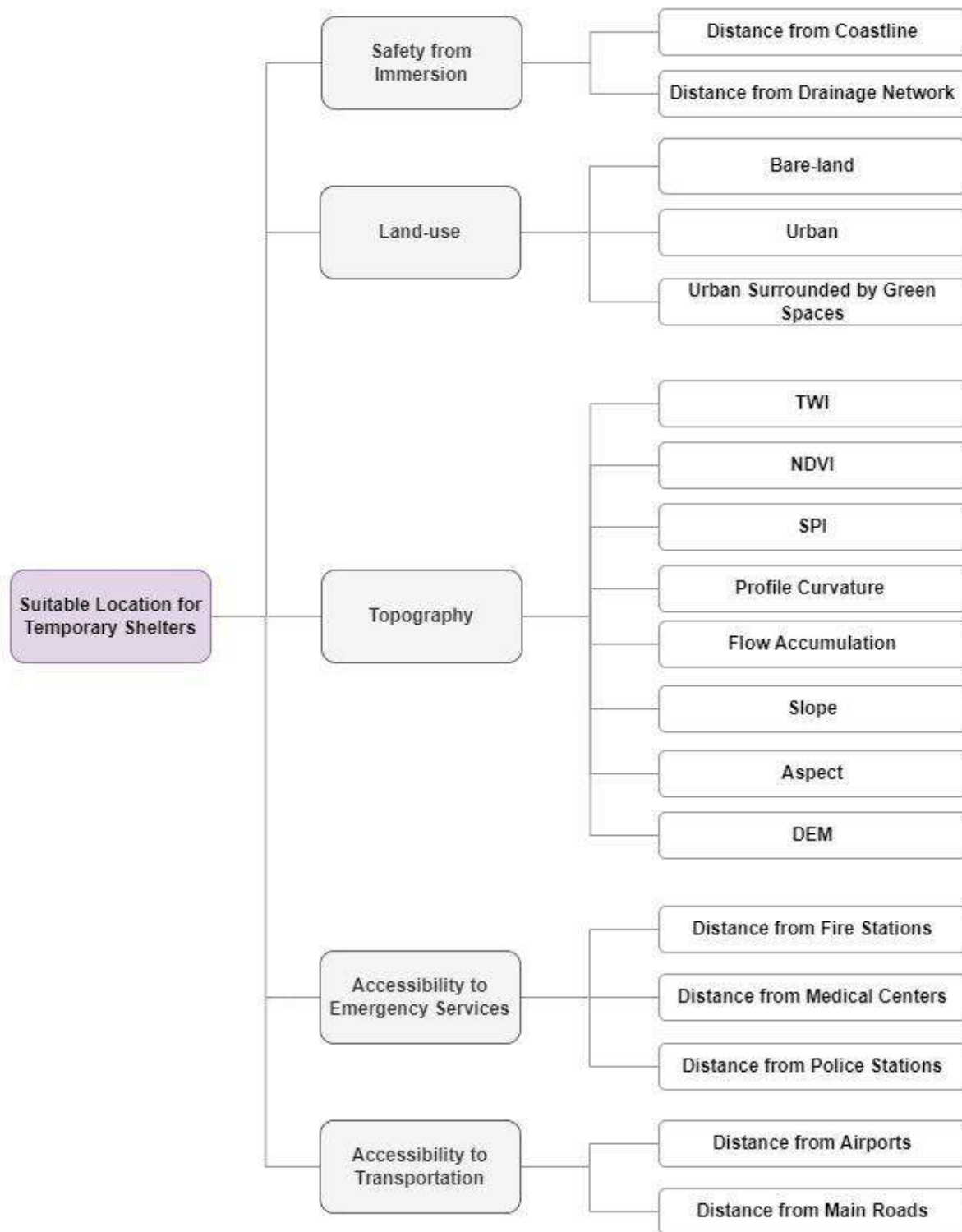


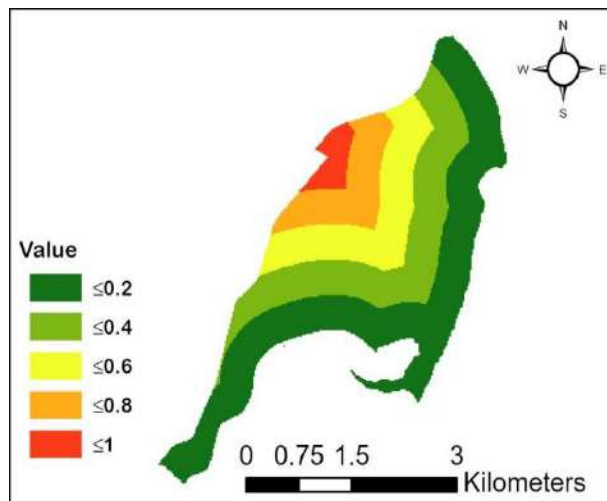
Figure_3. The hierarchy structure of objectives and attributes.

Table 5. Details of the evaluation objectives, attributes and spatial analysis used in this study.

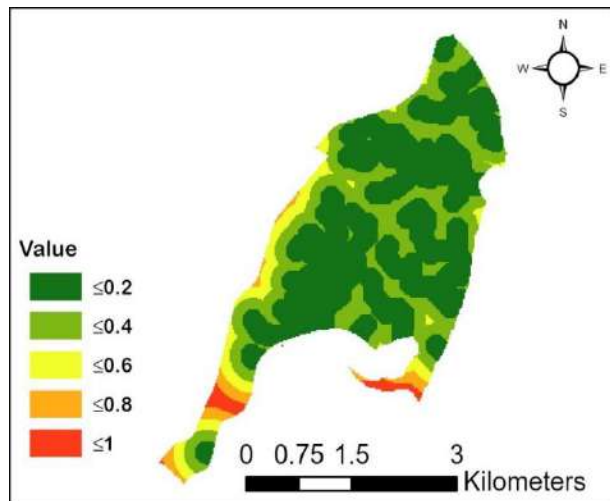
Objectives	Attributes	Description	Spatial Analysis	Standardization (Cost or Benefit)
Safety from immersion (O1)	Distance from coastline (A11)	Euclidean distance from the coastline is a significant element in flood risk; as the distance increases, the probability of flooding decreases Figure_5.1.	Euclidean distance	Minimize (Cost)
	Distance from drainage network (A12)	Together with the sub-main roadways, drainage networks are a typical method for water collection and disposal systems. Flood risk decreases as distance (network distance) from the urban drainage network increases Figure_5.2.	Euclidean distance	Minimize (Cost)
Land-use (O2)	Bare-land (A21)	The general sense described as land that is undeveloped and sits idle in one location without a building Figure_5.3.	Supervised classification + Euclidean distance	Maximize (Benefit)
	Urban-land (A22)	The broad notion shown by villages, buildings that are both commercial and residential, paved surfaces, and roadways [22], [103]–[105] Figure_5.4.		
	Urban surrounded by green spaces (A23)	Any open, public space with vegetation and preservation, such as parks, is referred to as being surrounded by green spaces in an urban area. There is a correlation between the size of these areas and a lower risk of flooding. Some locations have had basic amenities, including restrooms and access to drinking water. This kind of land use is suitable for usage as temporary shelters [106] Figure_5.5.		
Topography (O3)	TWI (A31)	Topographic wetness index (TWI) indicates the propensity for water to migrate downslope due to gravity forces and the spatial distribution of wetness conditions. TWI is crucial for controlling surface runoff since the more wet a region is, the more runoff it will produce [107] Figure_5.6.	Raster Calculator	Minimize (Cost)
	NDVI (A32)	The normalized differential vegetation index is The Landsat 8 satellite image with a resolution of 30 m was used to collect the images from which the NDVI was computed. The ratio of the red (band	Raster Calculator	Maximize (Benefit)

		4) and near infrared (band 5) readings is used to determine NDVI [106] Figure_5.7.		
	SPI (A33)	stream power index (SPI) It serves as a metaphor for the erosional force of water movement [108] Figure_5.8.	Raster Calculator	Minimize (Cost)
	Flow accumulation (A34)	Using the flow direction raster, it was estimated. In this scenario, an increase in flow accumulation should coincide with an increase in flood sensitivity [22] Figure_5.9.	Flow accumulation	Minimize (Cost)
	Slope (A35)	The slope or gradient of a surface determines how steep and incline it is, as well as what percentage of height change there is between any two points on the surface Figure_5.10.	Slope	Maximize (Benefit)
	Aspect (A36)	It affects both the soil's humidity and the direction that flooded water flows [109] 5.11.	Aspect	Minimize (Cost)
	DEM (A37)	With increasing elevation, the flood risk will decrease. Usually, the highest areas are less prone to water login and flood [30] Figure_5.12.	-	Maximize (Benefit)
	Profile Curvature (A38)	Depict the flow convergence, diversion, and slope variation as it changes direction along a contour [110] Figure_5.13.	Profile curvature	Minimize (Cost)
Accessibility to emergency services (O4)	Distance from fire stations (A41)	The distance between the temporary shelter's site and the nearest fire-station. The closer, the better for rapid response 5.14.	Euclidean distance	Maximize (Benefit)
	Distance from medical centers (A42)	The distance between the temporary shelter's site and the nearest hospital, or medical centers. The closer, the better survival rates for injured or citizens require medical care 5.15.	Euclidean distance	Maximize (Benefit)
	Distance from police stations (A43)	The distance between the temporary shelter's site and the nearest police-station. The closer, the better safety and management purposes 5.16.	Euclidean distance	Maximize (Benefit)
Accessibility to transportation (O5)	Distance from airport (A51)	The distance between the temporary shelter's site and the nearest airport, either international or local. The closer to airports, the better for evacuation purposes 5.17.	Euclidean distance	Maximize (Benefit)

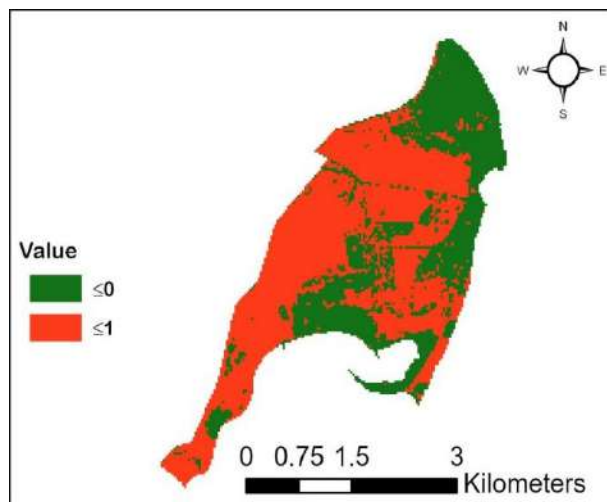
Distance from main roads (A52)	It is important to keep temporary shelters as close as possible to major roadways in order to reduce arrival times 5.18.	Euclidean distance	Maximize (Benefit)
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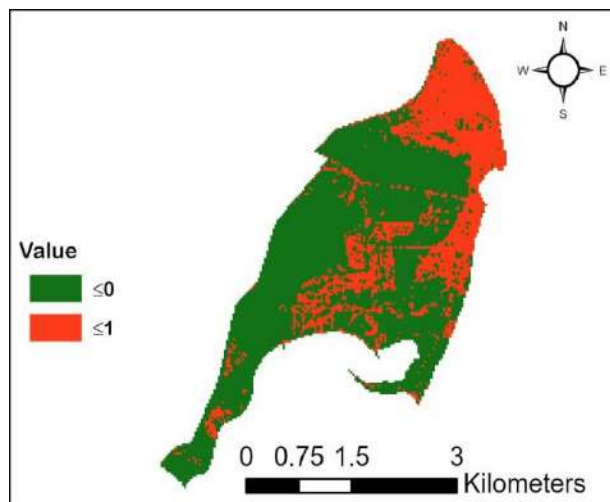
4.1 Distance from coastline.



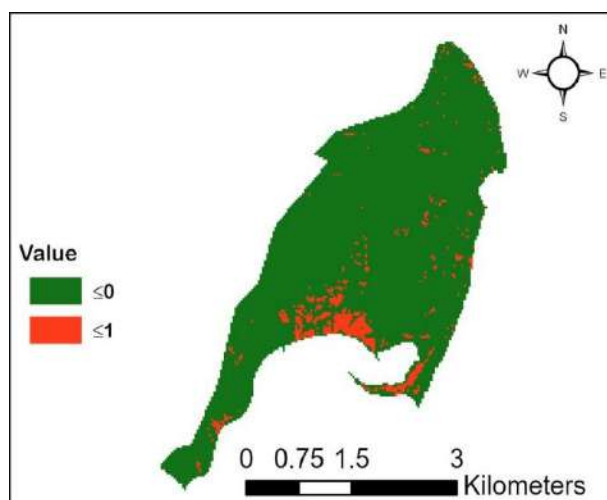
4.2 Distance from drainage network.



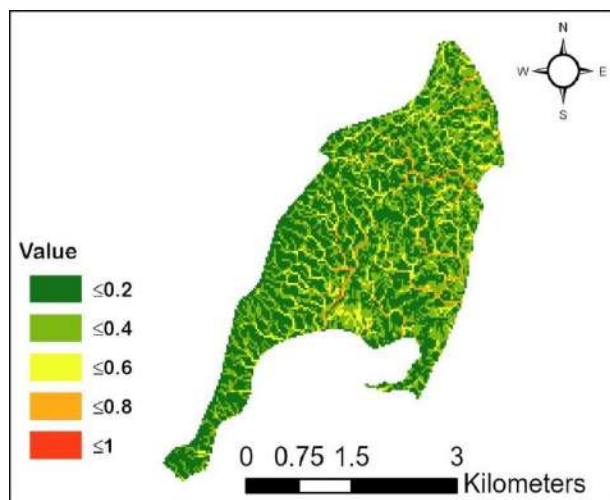
4.3 Bare-land.



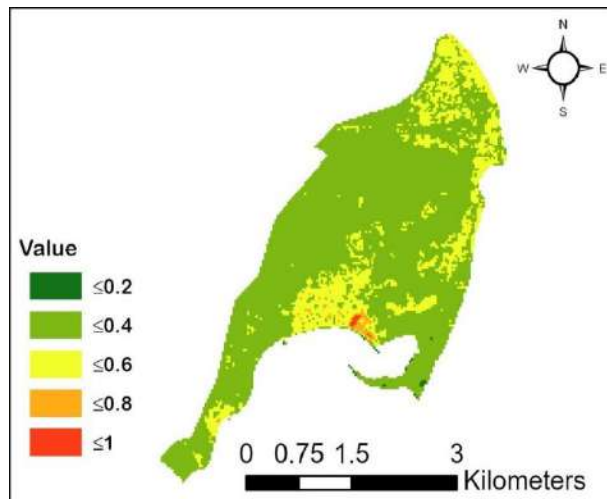
4.4 Urban-land.



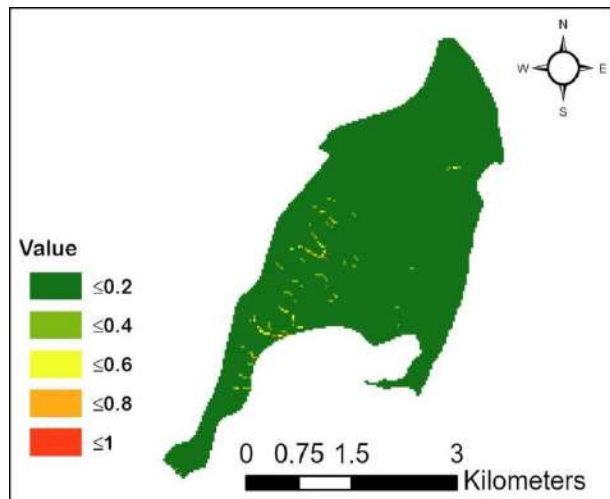
4.5 Urban surrounded by green spaces.



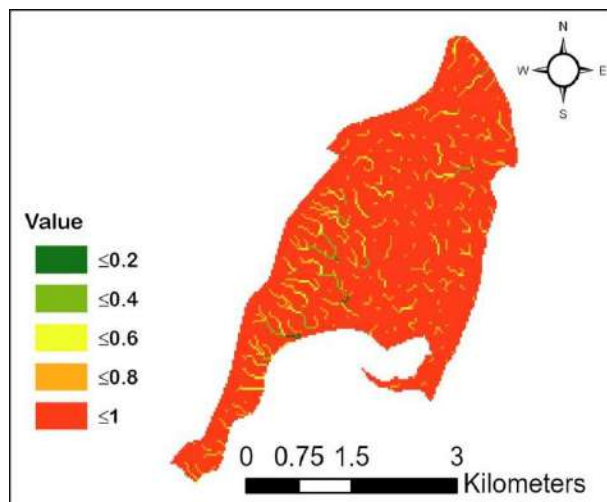
4.6 TWI.



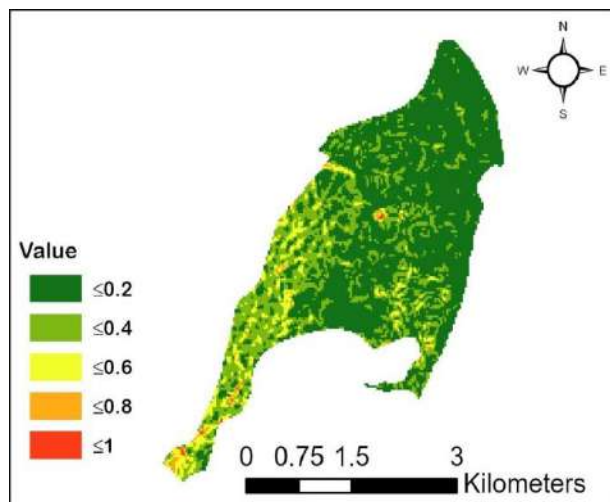
4.7 NDVI.



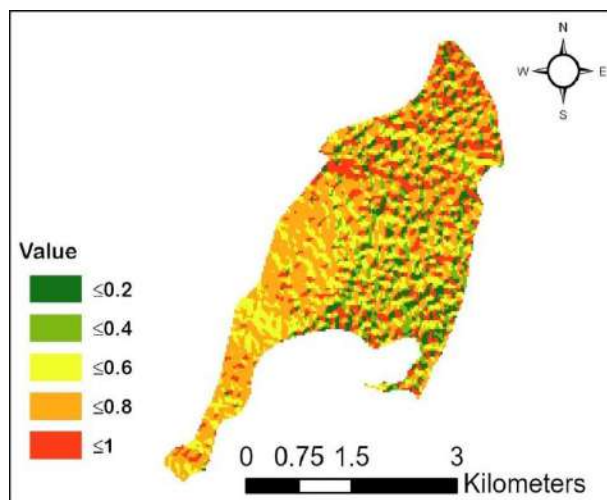
4.8 SPI.



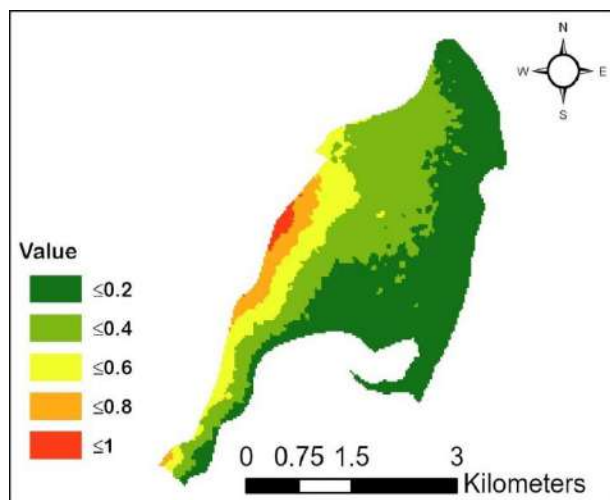
4.9 Flow accumulation.



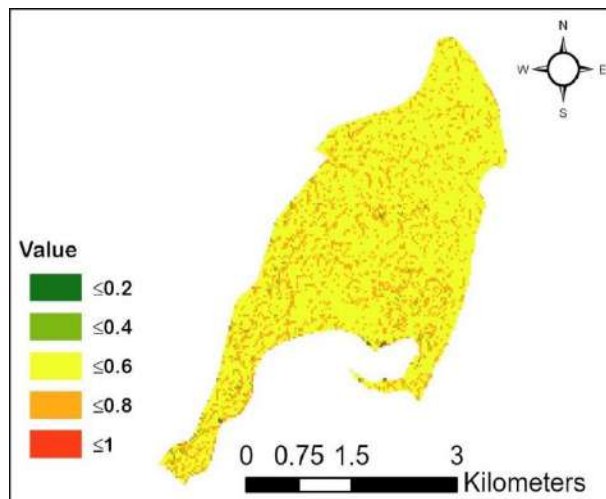
4.10 Slope.



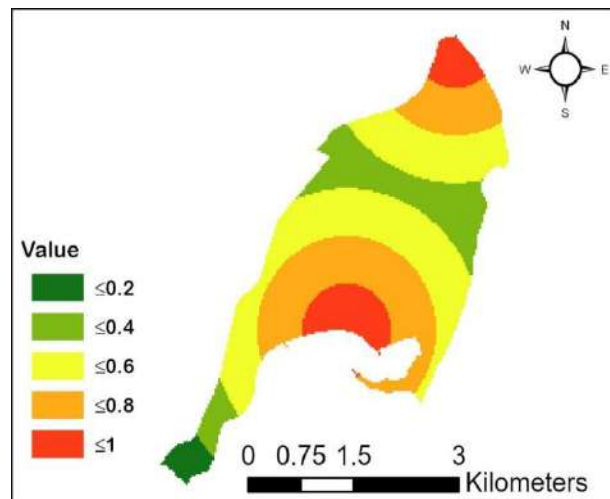
4.11 Aspect.



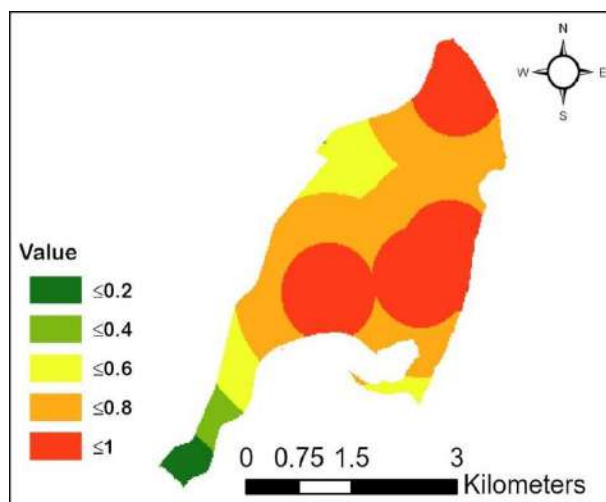
4.12 DEM.



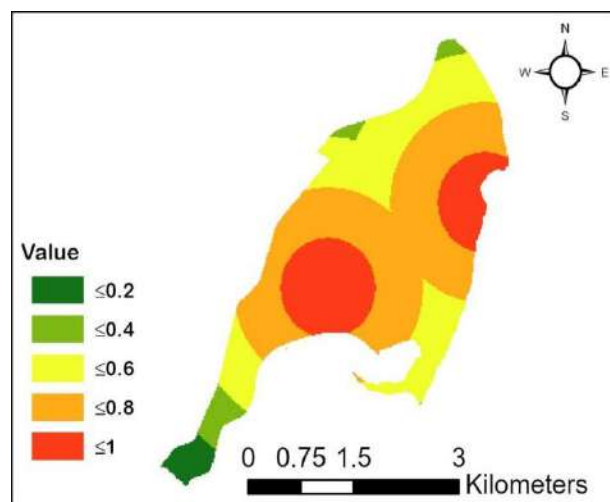
4.13 Profile curvature.



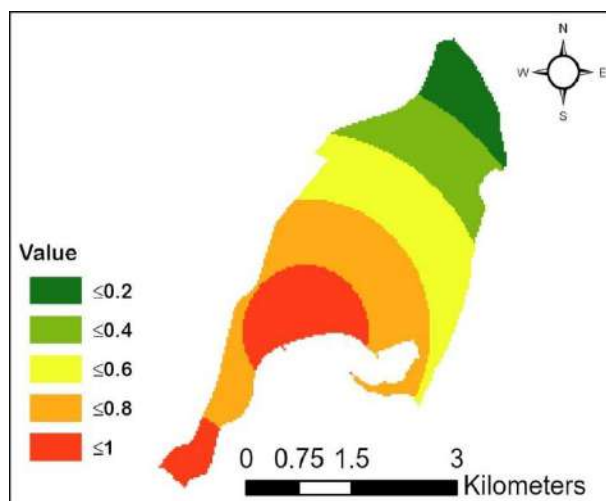
4.14 Distance from fire stations.



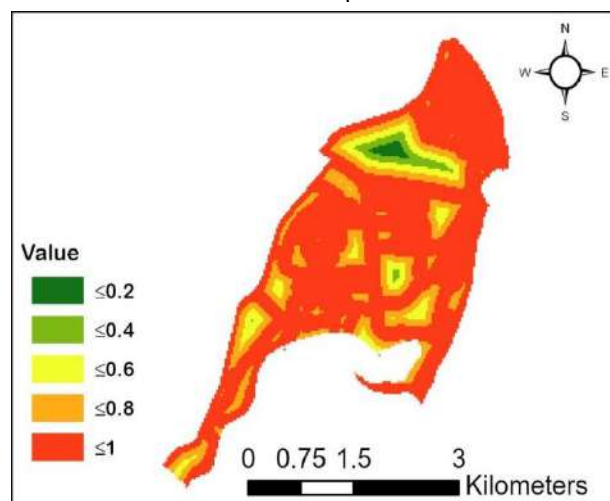
4.15 Distance from medical centers.



4.16 Distances from police stations.



4.17 Distance from airports.



4.18 Distance from main roads.

Figure_ 4. Standardized attribute map layers using ArcGIS Pro 2.6.

3.4 T2NN-CRITIC

The evaluation of potential locations for temporary shelters involves a number of complicated difficulties and contradictory factors, necessitating the use of advanced decision-making techniques. Applying the T2NN-CRITIC method is one of the options that have a strong ability to handle complex and contradictory problems with various criteria [111]. Simic et al. [111] developed the T2NN-CRITIC method in 2022. The following are its benefits: It is possible to effectively depict municipal bodies' actual preferences for favoring experience over competence and vice versa. To clarify the objective significance of evaluation criteria, differences and correlations among them might be taken into account simultaneously. Greater flexibility is provided by two built-in parameters. The process of the T2NN-CRITIC method can be described as follows:

1. Construct a problem hierarchy structure that contains identifying the main objectives and attributes.
2. Consider a set of x attributes is denoted by $A = \{A_1, A_2, \dots, A_x\}$ and a set of m objectives is represented by $O = \{O_1, O_2, \dots, O_m\}$. Let $E = \{E_1, E_2, \dots, E_t\}$ be a set of experts who offered their evaluation report for each attribute $A_i (i = 1, 2, \dots, x)$ against their objective $O_j (j = 1, 2, \dots, m)$. Let $s = (s_1, s_2, \dots, s_m)^T$ be the vector of weights for objective s_j and $p = (p_1, p_2, \dots, p_t)^T$ be the global weight vector for each attribute $p_t (t = 1, 2, \dots, t)$ such that $\sum_{j=1}^m s_j = 1, \sum_{t=1}^m p_t = 1$.
3. Construct a pairwise comparison matrix for objectives/attributes by all experts to express their preferences for these objectives/attributes. The comparison decision matrix is constructed applying the linguistic terms presented in Table 6 by the experts. The expert must make the rank of objectives and attributes in the form of T2NNs (T_T, T_I, T_F) , (I_T, I_I, I_F) , and (F_T, F_I, F_F) as an example, if the expert will rank the first objective as the best one and give it an "Extremely high" linguistic variable, then the final evaluation value will take the following form of the T2NNs $((0.95, 0.90, 0.95); (0.10, 0.10, 0.05); (0.05, 0.05, 0.05))$.

$$A_1/O_1 \quad \dots \quad A_x/O_y$$

$$\tilde{R} = \begin{matrix} E_1 \\ \vdots \\ E_t \end{matrix} \begin{bmatrix} \left(T_{\tilde{R}_1(1)}(x/y), I_{\tilde{R}_1(1)}(x/y), F_{\tilde{R}_1(1)}(x/y) \right), \\ \left(I_{\tilde{R}_1(1)}(x/y), I_{\tilde{R}_1(1)}(x/y), I_{\tilde{R}_1(1)}(x/y) \right), \\ \left(F_{\tilde{R}_1(1)}(x/y), F_{\tilde{R}_1(1)}(x/y), F_{\tilde{R}_1(1)}(x/y) \right) \\ \vdots \\ \left(T_{\tilde{R}_1(1)}(x/y), I_{\tilde{R}_1(1)}(x/y), F_{\tilde{R}_1(1)}(x/y) \right), \\ \left(I_{\tilde{R}_1(1)}(x/y), I_{\tilde{R}_1(1)}(x/y), I_{\tilde{R}_1(1)}(x/y) \right), \\ \left(F_{\tilde{R}_1(1)}(x/y), F_{\tilde{R}_1(1)}(x/y), F_{\tilde{R}_1(1)}(x/y) \right) \end{bmatrix} \cdots \begin{bmatrix} \left(T_{\tilde{R}_t(t)}(x/y), I_{\tilde{R}_t(t)}(x/y), F_{\tilde{R}_t(t)}(x/y) \right), \\ \left(I_{\tilde{R}_t(t)}(x/y), I_{\tilde{R}_t(t)}(x/y), I_{\tilde{R}_t(t)}(x/y) \right), \\ \left(F_{\tilde{R}_t(t)}(x/y), F_{\tilde{R}_t(t)}(x/y), F_{\tilde{R}_t(t)}(x/y) \right) \\ \vdots \\ \left(T_{\tilde{R}_t(t)}(x/y), I_{\tilde{R}_t(t)}(x/y), F_{\tilde{R}_t(t)}(x/y) \right), \\ \left(I_{\tilde{R}_t(t)}(x/y), I_{\tilde{R}_t(t)}(x/y), I_{\tilde{R}_t(t)}(x/y) \right), \\ \left(F_{\tilde{R}_t(t)}(x/y), F_{\tilde{R}_t(t)}(x/y), F_{\tilde{R}_t(t)}(x/y) \right) \end{bmatrix} \quad (3)$$

Table 6. T2NN linguistic terms for weighing objectives, and attributes.

Linguistic terms	Acronyms	Type-2 neutrosophic numbers
Extremely low	ELW	$\langle (0.20, 0.20, 0.10); (0.65, 0.80, 0.85); (0.45, 0.80, 0$
Low	LOW	$\langle (0.35, 0.35, 0.10); (0.50, 0.75, 0.80); (0.50, 0.75, 0$
Medium low	MDL	$\langle (0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.40, 0$
Medium	MEM	$\langle (0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.30, 0$
Medium high	MDH	$\langle (0.60, 0.45, 0.50); (0.20, 0.15, 0.25); (0.10, 0.25, 0$
High	HHH	$\langle (0.70, 0.75, 0.80); (0.15, 0.15, 0.25); (0.10, 0.15, 0$
Extremely high	EXH	$\langle (0.95, 0.90, 0.95); (0.10, 0.10, 0.05); (0.05, 0.05, 0$

4. Transform the T2NNs to real values by applying the Eq. (4) [47].

$$S(\tilde{R}) = \frac{1}{12} \left\{ 8 + \left(T_{\tilde{R}}(x/y) + 2 \left(T_{\tilde{R}}(x/y) \right) + T_{\tilde{R}}(x/y) \right) - \left(I_{\tilde{R}}(x/y) + 2 \left(I_{\tilde{R}}(x/y) \right) + I_{\tilde{R}}(x/y) \right) - \left(F_{\tilde{R}}(x/y) + 2 \left(F_{\tilde{R}}(x/y) \right) + F_{\tilde{R}}(x/y) \right) \right\} \quad (4)$$

5. Compute the standardized decision matrix for objectives/attributes according to Eq. (5) and Eq. (6).

For advantage objectives/attributes:

$$x/y_{ij}^* = \frac{x/y_{ij} - \min(x/y_{ij})}{\max(x/y_{ij}) - \min(x/y_{ij})} \quad i = 1, 2 \dots x \text{ and } j = 1, 2 \dots y. \quad (5)$$

For disadvantage objectives/attributes:

$$x/y_{ij}^* = 1 - \frac{x/y_{ij} - \min(x/y_{ij})}{\max(x/y_{ij}) - \min(x/y_{ij})} \quad i = 1, 2 \dots x \text{ and } j = 1, 2 \dots y. \quad (6)$$

6. Computation of the values of the matrix's standard deviation and linear correlation per column. Then, identify the amount of information for the objectives/attributes by applying the Eq. (7).

$$q_j = \sigma_j \cdot \sum_{v=1}^m (1 - r_{jv}) \quad (7)$$

where σ_j is the standard deviation of the objectives/attributes, and r_{jv} is the linear correlation coefficient for the objectives/attributes.

7. Determine the objectives/attributes weights by applying the Eq. (8).

$$w_j = \frac{q_j}{\sum_{v=1}^m q_v} \quad (8)$$

3.5 GIS processing

According to the proposed framework in Figure_.1. Step number 4 focus on the use of GIS tools to produce the final suitability map as follow:

1. Integration operation is conducted using Eq. (9) to combine each attribute map layer with its global weight. Let $L = \{l_1, l_2, \dots, l_d\}$ denote set of attribute map layers, $p = (p_1, p_2, \dots, p_t)^T$ be the global weight vector for each attribute, and $w = (w_1, w_2, \dots, w_b)^T$ be the weight vector for integrated attribute map layer w_b :

$$w_b = l_d * p_t \quad (9)$$

where $b = 1, \dots, d$.

2. Using GIS tools overlay operation is computed to aggregate all attribute map layers and to calculate final weight by applying the Eq. (10). Let f be the final overlay attribute weight map layer:

$$f = \sum_{b=1}^d w_b \quad (10)$$

where $b = 1, \dots, d$.

3. Rank sites based on their final overlaid value divided into four categories based on their suitability: "highly suitable", "moderately suitable", "Marginally Suitable", and "not suitable". Stakeholders can then take a suitable decision.
4. To evaluate the longevity of the priority rating and to ascertain the effectiveness of the suggested model, a sensitivity analysis of the results is carried out. Eight different sensitivity scenarios tested the rank of objectives.

4. Results and discussions

In this section, the analysis of the findings applying the framework is presented. This study used GIS tools and T2NN-CRITIC method to assess the suitability of sites for temporary shelters in Dahab, Egypt in case of floods.

The T2NN-CRITIC method was used to develop the weights for the objectives and attributes, which were based on the views expressed in the interviews by the experts. In this regard, the main objectives were assessed by all experts using linguistic variables as presented in Table 7. The chosen set of objectives' correlation coefficients is calculated to determine the evaluation scores for each objective as exhibited in

Table 8. Table A.1-A.12 (Appendix A) presents the evaluation of attributes related to each main objective by all experts and the normalized decision matrix for attributes related to each main objective.

Table 7. Evaluation of main objectives using linguistic variables by all experts

Experts	Main Objectives				
	O ₁	O ₂	O ₃	O ₄	O ₅
E ₁	HHH	MDL	ELW	MEM	LOW
E ₂	ELW	MEM	LOW	MDH	MDL
E ₃	MDL	MDH	ELW	MEM	MEM
E ₄	LOW	MDL	LOW	LOW	LOW
E ₅	ELW	MEM	MDH	MEM	MDL

Using the CRITIC method the relative weight of each objective is calculated. Table 8 shows the correlation coefficient of the relationship between the main objectives and final weights. In Figure_5, we can note that "safety from immersion" is the highest objective in weight with a score of 0.332 followed by topography with a score of 0.205.

Table 8. Correlation coefficient of the relationship among the main objectives and final weights.

Obj.	O ₁	O ₂	O ₃	O ₄	O ₅	q_j	w_j
O ₁	1.000	-0.286	-0.431	0.102	-0.357	2.093	0.332
O ₂	-0.286	1.000	0.662	0.426	0.993	0.915	0.145
O ₃	-0.431	0.662	1.000	0.094	0.687	1.291	0.205
O ₄	0.102	0.426	0.094	1.000	0.480	1.070	0.170
O ₅	-0.357	0.993	0.687	0.480	1.000	0.938	0.149



Figure_5. Final weights of main objectives.

Through Tables 9-18, all attributes for each main objective are evaluated by all experts using T2NN linguistic variables. The main objective attributes' correlation coefficients are also calculated to determine the evaluation scores for each attribute.

Table 9. Evaluation of safety from immersion attributes using linguistic variables by all experts.

Experts	Safety from immersion attributes	
	A_{11}	A_{12}
E_1	HHH	MDL
E_2	ELW	MEM
E_3	MDL	MDH
E_4	MDH	MDL
E_5	ELW	ELW

Table 10. Correlation coefficient of the relationship among safety from immersion attributes and final weights.

	A_{11}	A_{12}	q_j	w_j
A_{11}	1.000	0.097	0.415	0.555
A_{12}	0.097	1.000	0.333	0.445

Table 11. Evaluation of land-use attributes using linguistic variables by all experts.

Experts	Land-use attributes		
	A_{21}	A_{22}	A_{23}
E_1	MDL	ELW	MEM
E_2	MEM	EXH	MDH
E_3	MDH	ELW	MEM
E_4	MDL	LOW	HHH
E_5	MEM	MDH	MEM

Table 12. Correlation coefficient of the relationship among land-use attributes and final weights.

	A_{21}	A_{22}	A_{23}	q_j	w_j
A_{21}	1.000	0.578	-0.526	0.808	0.309
A_{22}	0.578	1.000	-0.131	0.641	0.245
A_{23}	-0.526	-0.131	1.000	1.165	0.446

Table 13. Evaluation of topography attributes using linguistic variables by all experts.

Experts	Topography attributes							
	A_{31}	A_{32}	A_{33}	A_{34}	A_{35}	A_{36}	A_{37}	A_{38}
E_1	HHH	MEM	ELW	MDL	MEM	ELW	MDL	LOW
E_2	ELW	MDH	EXH	MEM	MDH	LOW	MEM	MDL
E_3	MDL	MEM	ELW	MDH	MEM	ELW	MDH	MEM
E_4	LOW	LOW	LOW	MDL	HHH	LOW	ELW	LOW
E_5	ELW	MEM	MDH	MEM	MEM	MDH	MEM	MDL

Table 14. Correlation coefficient of the relationship among topography attributes and final weights.

	A_{31}	A_{32}	A_{33}	A_{34}	A_{35}	A_{36}	A_{37}	A_{38}	q_j	w_j
A_{31}	1.000	0.102	-0.696	-0.286	-0.371	0.431	0.036	0.357	3.820	0.192
A_{32}	0.102	1.000	0.638	0.426	-0.594	0.094	0.769	0.480	1.877	0.095
A_{33}	-0.696	0.638	1.000	0.578	-0.131	0.392	0.606	0.664	2.042	0.103
A_{34}	-0.286	0.426	0.578	1.000	-0.526	0.662	0.879	0.993	1.773	0.089

A_{35}	-0.371	-0.594	-0.131	-0.526	1.000	0.578	0.784	0.524	4.609	0.232
A_{36}	-0.431	0.094	0.392	0.662	-0.578	1.000	0.573	0.687	2.420	0.122
A_{37}	-0.036	0.769	0.606	0.879	-0.784	0.573	1.000	0.894	1.537	0.077
A_{38}	-0.357	0.480	0.664	0.993	-0.524	0.687	0.894	1.000	1.778	0.090

Table 15. Evaluation of accessibility to emergency services attributes using linguistic variables by all experts.

Experts	Accessibility to emergency services attributes		
	A_{41}	A_{42}	A_{43}
E_1	MEM	ELW	LOW
E_2	MEM	EXH	MDH
E_3	MDH	HHH	MEM
E_4	MDL	LOW	HHH
E_5	MEM	MDH	EXH

Table 16. Correlation coefficient of the relationship among accessibility to emergency services attributes and final weights.

	A_{41}	A_{42}	A_{43}	q_j	w_j
A_{41}	1.000	0.572	-0.346	0.628	0.337
A_{42}	0.572	1.000	0.370	0.473	0.254
A_{43}	-0.346	0.370	1.000	0.761	0.409

Table 17. Evaluation of accessibility to transportation attributes using linguistic variables by all experts.

Experts	Accessibility to transportation attributes	
	A_{51}	A_{52}
E_1	HHH	MDL
E_2	EXH	MEM
E_3	MDL	ELW
E_4	MDH	MDL
E_5	MEM	ELW

Table 18. Correlation coefficient of the relationship among accessibility to transportation attributes and final weights.

	A_{51}	A_{52}	q_j	w_j
A_{51}	1.000	0.952	0.019	0.472
A_{52}	0.952	1.000	0.021	0.528

In Table.19, A_{11} (i.e., Distance from coastline) is the attribute with the highest weight value of 0.184 followed by A_{12} (i.e., Distance from drainage network) with global weight of 0.148. The attribute with the least weight value is A_{37} (i.e., DEM) with a global weight of 0.016.

Table 19. Global weights of main objectives and their attributes.

Main Objectives	Weights of main objectives	Attributes	Local weight of Attributes	Rank	Global weight of Attributes	Rank global weights
Safety from Immersion O_1	0.332	A_{11}	0.555	1	0.184	1
		A_{12}	0.445	2	0.148	2
Land-use O_2	0.145	A_{21}	0.309	2	0.045	9
		A_{22}	0.245	3	0.036	12
		A_{23}	0.446	1	0.065	6
		A_{31}	0.192	2	0.039	11
Topography O_3	0.205	A_{32}	0.095	5	0.019	15
		A_{33}	0.103	4	0.021	14
		A_{34}	0.089	7	0.018	17
		A_{35}	0.232	1	0.048	8
		A_{36}	0.122	3	0.025	13
		A_{37}	0.077	8	0.016	18
		A_{38}	0.090	6	0.018	16
		A_{41}	0.337	2	0.057	7
Accessibility to Emergency Services O_4	0.170	A_{42}	0.254	3	0.043	10
		A_{43}	0.409	1	0.070	5
Accessibility to Transportation O_5	0.149	A_{51}	0.472	2	0.070	4
		A_{52}	0.528	1	0.079	3

In Figure_6 ArcGIS Pro 2.6 Model Builder all attribute map layers are integrated with each attribute's global weigh and all the integrated attribute map layers are overlaid using an overlay operation.

The final overlaid attribute map layer is classified into the 4 main classes found in Table 20. The final site suitability map is shown in Figure_7 classified into 4 main classes, the first class value ranges between 0 and 25% and represents sites that are not suitable for temporary shelters. The second class ranges between 25% and 50% and represents marginally suitable sites for the selected problem. The third class ranges between 50% and 75% and represents moderately suitable sites. Finally, the fourth class ranges between 75% and 100% and indicates highly desired sites for temporary shelters in case of flood in Dahab, Egypt.

Table 20. Suitability classes.

Category	Value Range	Interpretation	Color Scheme
Not Suitable	0 % - 25%	Absolute ineligibility for concession	
Marginally Suitable	25% - 50%	An inconvenient or somewhat acceptable	
Moderately Suitable	50% - 75%	Suitable in the majority	

Highly Suitable	75% - 100%	Perfectly acceptable for concession	
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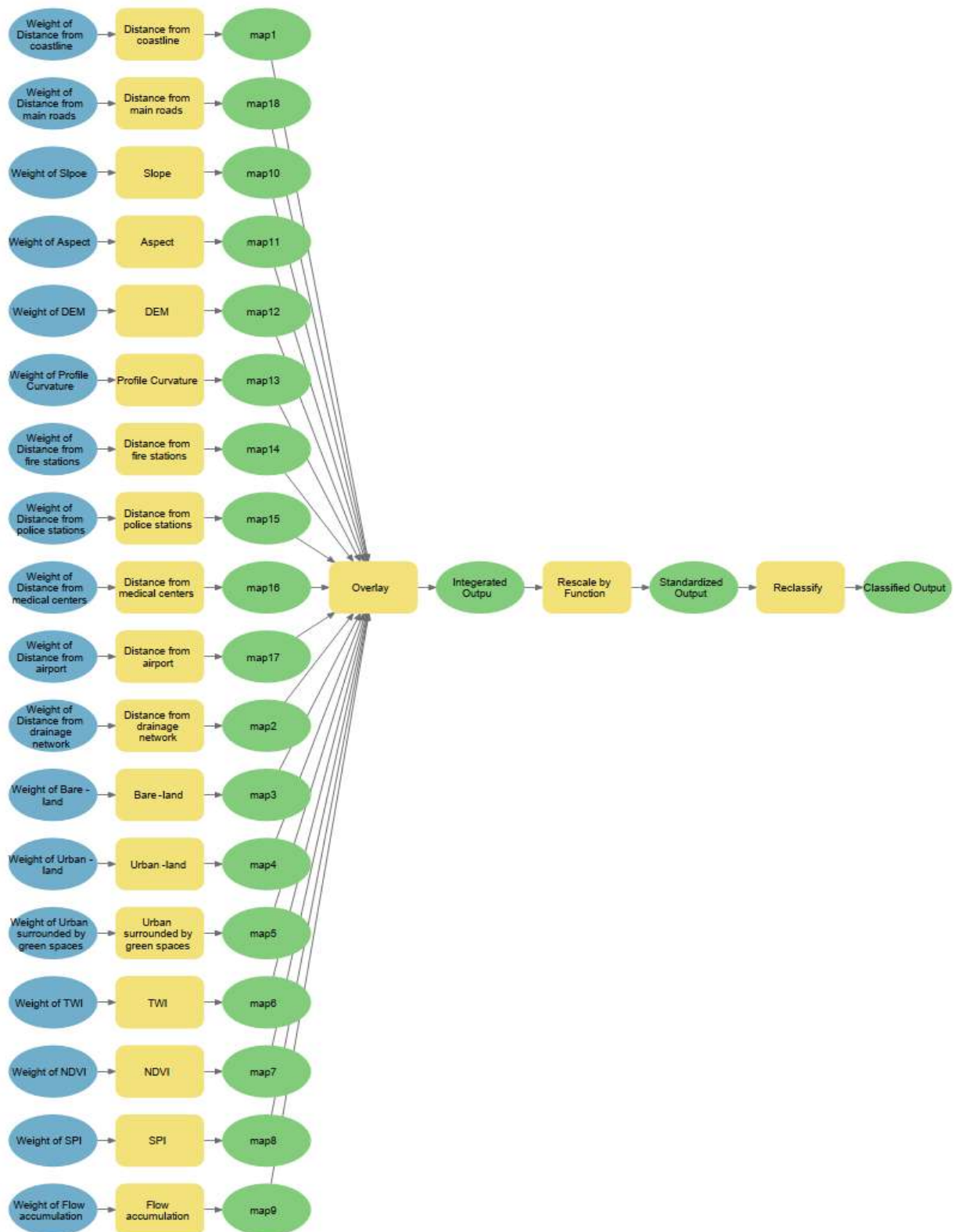


Figure_6. Model Builder for classified suitability map.

The selection of temporary shelters is one of the main issues in the disaster management process. Although Egyptian authorities have been trying to enhance planning in case of floods, still, according to late reports, the effects of floods on people and properties are high and require further action. For these reasons, MCDM methods integrated with GIS tools are considered an important tool for improving the quality and success of the disaster management process, including temporary shelter selection in case of floods. The research community and authorities looking for the best locations for temporary shelters in case of floods will benefit greatly from this study, which is the first of its kind to be conducted in Dahab, Egypt. The study's findings will aid in better disaster planning and management in the future.

This paper seeks to provide an adequate method for locating and classifying suitable sites of temporary shelters in case of floods. The results show that "safety from immersion" is the most important objective, with a weight of 0.332. Second in rank comes the "topography" objective with a score of 0.205. The "Accessibility to emergency services" comes third in rank with a score of 0.17. The least two objectives in ranks are the "Accessibility to transportation" with a score of 0.149 and the "Land-use" objective with a score of 0.145. According to the results of the study 5.16% of the study area (17,550 Km²) is highly suitable as a site for a temporary shelter in the event of a flood in Dahab, Egypt, (126,570 Km²) of the study area classified as moderately suitable, (161,850 Km²) of the study area classified as marginally suitable and, finally (34,080 Km²) classified as not suitable.

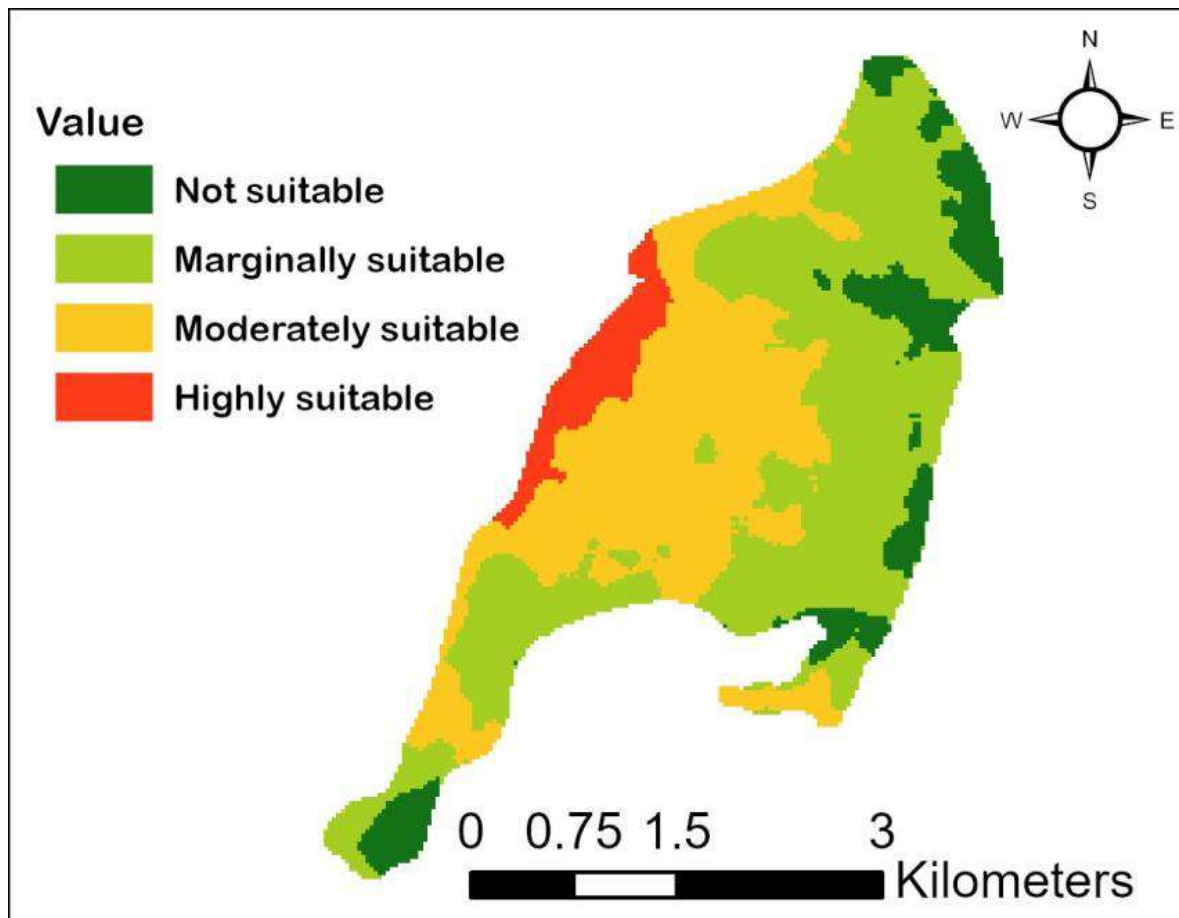


Figure_7. Site suitability for temporary shelters in case of flood in Dahab, Egypt.

5. Sensitivity analysis

In this section, the sensitivity analysis of the results is conducted to assess the persistence of the priority rating and it can be an efficient way to determine the proposed approach's efficiency. The rank of objectives was subjected to a sensitivity analysis. Eight different sensitivity scenarios were evaluated as found in Table 21. The obtained objective weights are taken into account in Scenario 1. Scenario 2 has equal weights for each objective. In Scenario 3, the O_1 objective's weight is raised by 20%. In Scenario 4, the O_2 objective has a 20% higher weight. In Scenario 5, the O_3 objective's weight is raised by 20%. In Scenario 6, the O_4 objective's weight is increased by 20%. In Scenario 7, the O_5 objective has a 20% higher weight. In Scenario 8, the weight of the O_1 and O_3 objectives—which actually have the highest weight of the five objectives used—is increased by 10%. Figure_8 shows the resulting suitability maps for each sensitivity scenario. As seen in the figure, adjusting the ranking of the attributes has a direct influence on the weights of the objectives.

Table 21. Sensitivity analysis objective weight scenarios.

Scenarios	O1	O2	O3	O4	O5
Scenario 1	-	-	-	-	-
Scenario 2	20%	20%	20%	20%	20%
Scenario 3	+20%				
Scenario 4		+20%			
Scenario 5			+20%		
Scenario 6				+20%	
Scenario 7					+20%
Scenario 8	+10%		+10%		

The findings of the sensitivity analysis indicate that scenarios 6 and 7 propose the most change in the percentage of suitability classes for site selection of temporary shelters, as found in figures from Figure_8, the results are either stable or slightly changing throughout weight differences in the remaining scenarios.

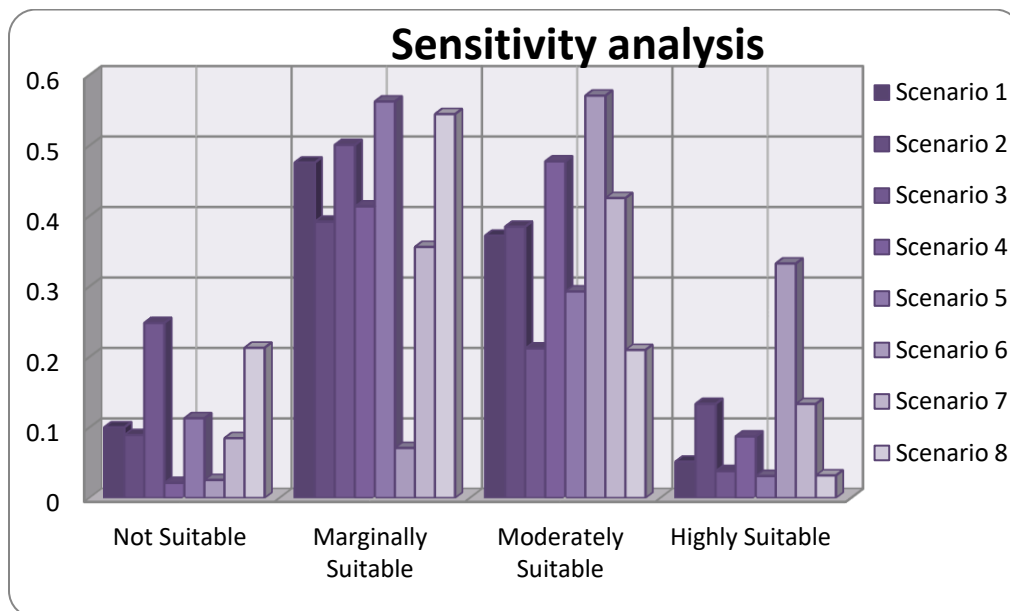
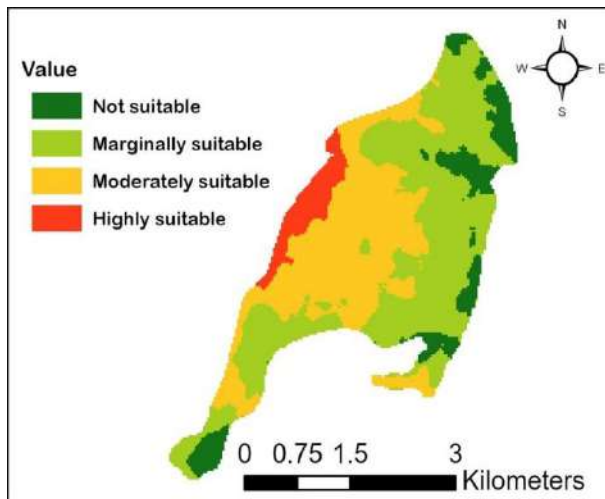
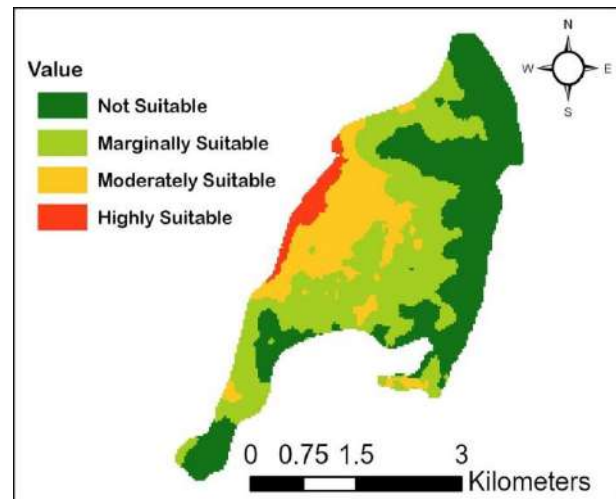


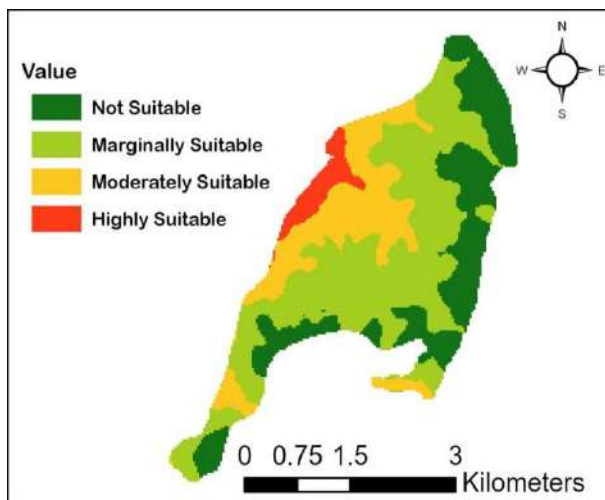
Figure_8. Suitability classes for sensitivity scenarios.



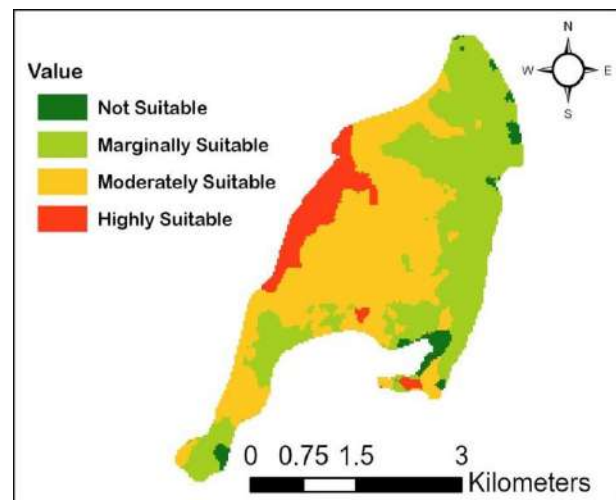
9.1 SA-Scenario 1.



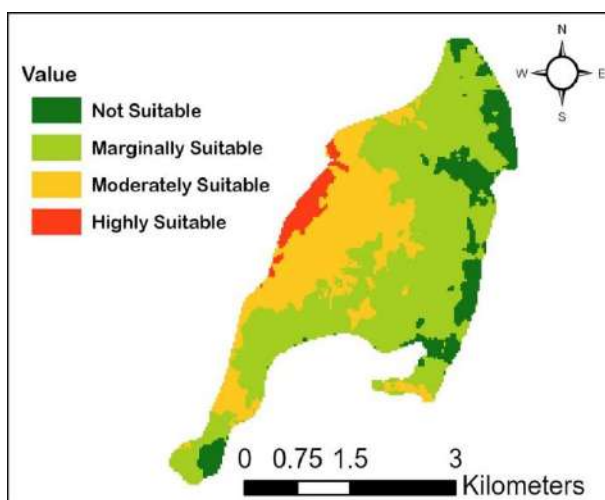
9.2 SA-Scenario 2.



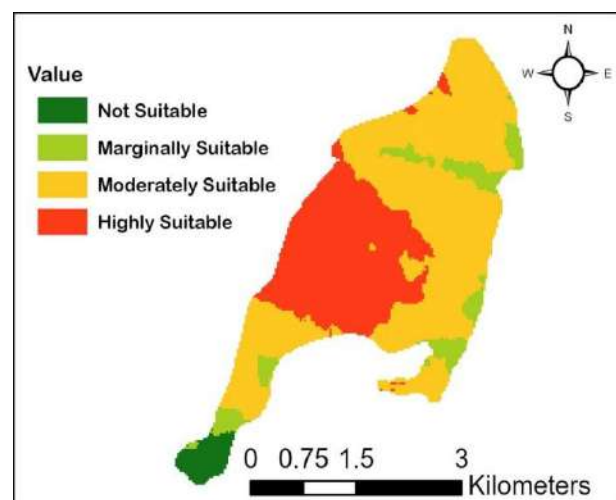
9.3 SA-Scenario 3.



9.4 SA-Scenario 4.



9.5 SA-Scenario 5.



9.6 SA-Scenario 6.

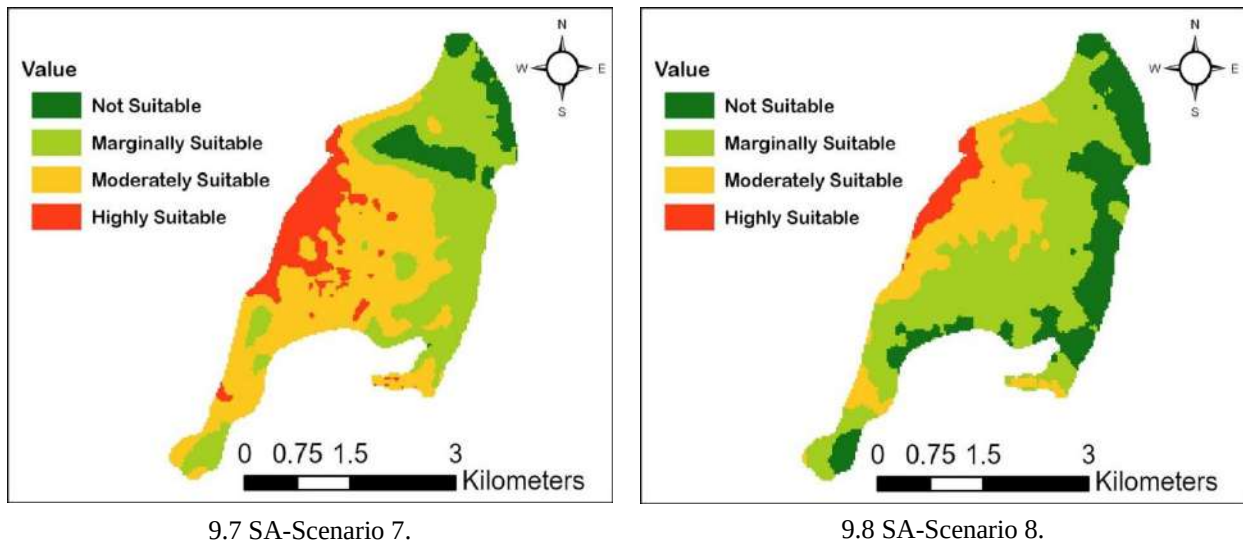


Figure_9. Sensitivity analysis scenarios of main objectives.

6. Conclusions

The study's goal is to identify potential locations in Dahab, Egypt, for emergency flood shelters. The first and most important step in the proper planning and management of disasters includes assessing suitable sites for temporary shelters. For the purpose of finding suitable sites for temporary shelters, a combination of GIS and MCDM techniques has been used, taking into account objectives such as land use, topography, and accessibility to emergency services and safety from immersion. The study includes eighteen significant attributes that were chosen from the literature and considered by the experts. The evaluation attributes were given varying degrees of importance using the T2NN-CRITIC method. The GIS approach created the final four suitability classes of "highly suitable", "moderately suitable", "marginally suitable", and "not suitable" by preparing a spatial dimension of the decision objectives and elaborating on them. The study reveals that 5.16% of the study area ($17,550 \text{ Km}^2$) is highly suitable as a site for a temporary shelter in the event of a flood in Dahab, Egypt, ($126,570 \text{ Km}^2$) of the study area is classified as moderately suitable, ($161,850 \text{ Km}^2$) of the study area classified as marginally suitable and, finally ($34,080 \text{ Km}^2$) classified as not suitable. The five objectives' sensitivity analyses were completed. There were eight different sensitivity scenarios tested. The first scenario objective weights each objective is equally weighted. In Scenario 2 the weight of the O_1 objective is increased by 20% in Scenario 3. The O_2 objective has a 20% higher weight in Scenario 4. The weight of the O_3 objective is increased by 20% in Scenario 5. The O_4 objective's weight is raised by 20% in Scenario 6. The O_5 objective has a 20% higher weight in Scenario 7. The weight of the O_1 and O_3 objectives, which are actually the two with the highest weights

out of the five used, is increased by 10% in Scenario 8. Sensitivity analysis concluded that the output of suitable land is sensitive to changes in the weights of the objectives. The study limitation centered about a certain amount of the study was done using data that is freely accessible from various trustworthy sources, but these data are less updated and accurate than privately accessible data. With the help of the ground measurement data, the outcomes can be further enhanced. However, by combining the developed methodology with MCDM ideas, open-source GIS tools, and presumptions, the study is making it easier to replicate improved results. In the future, we plan to use different MCDM methods to solve the problem to compare the accuracy of the results also use various MCDM methods with GIS to solve other disaster management problems to facilitate the complex dependencies between objectives and attributes.

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Appendix A

Table A.1. Evaluation of main objectives using T2NNs by all experts.

Experts	Main objectives	
	O_1	O_2
E ₁	$\langle(0.95, 0.90, 0.95); (0.10, 0.10, 0.05); (0.05, 0.05, 0.05)\rangle$	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.40, 0.35)\rangle$
E ₂	$\langle(0.20, 0.20, 0.10); (0.65, 0.80, 0.85); (0.45, 0.45, 0.45)\rangle$	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.30, 0.25)\rangle$
E ₃	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.45, 0.45)\rangle$	$\langle(0.60, 0.45, 0.50); (0.20, 0.15, 0.25); (0.10, 0.05, 0.05)\rangle$
E ₄	$\langle(0.35, 0.35, 0.10); (0.50, 0.75, 0.80); (0.50, 0.50, 0.50)\rangle$	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.40, 0.35)\rangle$
E ₅	$\langle(0.20, 0.20, 0.10); (0.65, 0.80, 0.85); (0.45, 0.45, 0.45)\rangle$	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.30, 0.25)\rangle$
Experts	Main objectives	
	O_3	O_4
E ₁	$\langle(0.20, 0.20, 0.10); (0.65, 0.80, 0.85); (0.45, 0.45, 0.45)\rangle$	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.30, 0.25)\rangle$
E ₂	$\langle(0.35, 0.35, 0.10); (0.50, 0.75, 0.80); (0.50, 0.50, 0.50)\rangle$	$\langle(0.60, 0.45, 0.50); (0.20, 0.15, 0.25); (0.10, 0.05, 0.05)\rangle$
E ₃	$\langle(0.20, 0.20, 0.10); (0.65, 0.80, 0.85); (0.45, 0.45, 0.45)\rangle$	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.30, 0.25)\rangle$
E ₄	$\langle(0.35, 0.35, 0.10); (0.50, 0.75, 0.80); (0.50, 0.50, 0.50)\rangle$	$\langle(0.35, 0.35, 0.10); (0.50, 0.75, 0.80); (0.50, 0.50, 0.50)\rangle$
E ₅	$\langle(0.60, 0.45, 0.50); (0.20, 0.15, 0.25); (0.10, 0.05, 0.05)\rangle$	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.30, 0.25)\rangle$
Experts	Main objectives	
	O_5	
E ₁	$\langle(0.35, 0.35, 0.10); (0.50, 0.75, 0.80); (0.50, 0.75, 0.65)\rangle$	
E ₂	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.40, 0.60)\rangle$	
E ₃	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.30, 0.45)\rangle$	
E ₄	$\langle(0.35, 0.35, 0.10); (0.50, 0.75, 0.80); (0.50, 0.75, 0.65)\rangle$	
E ₅	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.40, 0.60)\rangle$	

Table A.2. Normalized decision matrix based on main objectives by all experts.

Experts	Main objectives				
	O_1	O_2	O_3	O_4	O_5
E ₁	1.000	0.000	0.000	0.730	0.000
E ₂	0.000	0.468	0.149	1.000	0.562
E ₃	0.383	1.000	0.717	0.668	1.000
E ₄	0.122	0.000	0.149	0.000	0.000
E ₅	0.000	0.468	1.000	0.668	0.562
Standard deviation	0.421	0.415	0.432	0.369	0.427

Table A.3. Evaluation of safety from immersion attributes using T2NNs by all experts.

Experts	safety from immersion attributes	
	A_{11}	A_{12}
E ₁	$\langle(0.95, 0.90, 0.95); (0.10, 0.10, 0.05); (0.05, 0.05, 0.05)\rangle$	$\langle(0.20, 0.20, 0.10); (0.65, 0.80, 0.85); (0.45, 0.45, 0.45)\rangle$
E ₂	$\langle(0.20, 0.20, 0.10); (0.65, 0.80, 0.85); (0.45, 0.45, 0.45)\rangle$	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.30, 0.25)\rangle$
E ₃	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.45, 0.45)\rangle$	$\langle(0.60, 0.45, 0.50); (0.20, 0.15, 0.25); (0.10, 0.05, 0.05)\rangle$
E ₄	$\langle(0.60, 0.45, 0.50); (0.20, 0.15, 0.25); (0.10, 0.05, 0.05)\rangle$	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.40, 0.35)\rangle$
E ₅	$\langle(0.20, 0.20, 0.10); (0.65, 0.80, 0.85); (0.45, 0.45, 0.45)\rangle$	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.30, 0.25)\rangle$

Table A.4. Normalized decision matrix based on safety from immersion attributes by all experts.

Experts	Safety from immersion attributes	
	A_{11}	A_{12}
E_1	1.000	0.468
E_2	0.000	0.717
E_3	0.383	1.000
E_4	0.817	0.468
E_5	0.000	0.000
Standard deviation	0.460	0.369

Table A.5. Evaluation of land-use attribute using T2NNs by all experts.

Experts	Land-use attributes	
	A_{21}	A_{22}
E_1	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.30, 0.45)\rangle$	$\langle(0.20, 0.20, 0.10); (0.65, 0.80, 0.85); (0.45, 0.30, 0.45)\rangle$
E_2	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.30, 0.45)\rangle$	$\langle(0.95, 0.90, 0.95); (0.10, 0.10, 0.05); (0.05, 0.05, 0.05)\rangle$
E_3	$\langle(0.60, 0.45, 0.50); (0.20, 0.15, 0.25); (0.10, 0.05, 0.05)\rangle$	$\langle(0.20, 0.20, 0.10); (0.65, 0.80, 0.85); (0.45, 0.30, 0.45)\rangle$
E_4	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.30, 0.45)\rangle$	$\langle(0.35, 0.35, 0.10); (0.50, 0.75, 0.80); (0.50, 0.35, 0.35)\rangle$
E_5	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.30, 0.45)\rangle$	$\langle(0.60, 0.45, 0.50); (0.20, 0.15, 0.25); (0.10, 0.05, 0.05)\rangle$
Experts	Land-use attributes	
	A_{23}	
E_1	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.30, 0.45)\rangle$	
E_2	$\langle(0.60, 0.45, 0.50); (0.20, 0.15, 0.25); (0.10, 0.25, 0.15)\rangle$	
E_3	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.30, 0.45)\rangle$	
E_4	$\langle(0.70, 0.75, 0.80); (0.15, 0.15, 0.25); (0.10, 0.15, 0.15)\rangle$	
E_5	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.30, 0.45)\rangle$	

Table A.6. Normalized decision matrix based on land-use attributes by all experts.

Experts	Land-use attributes		
	A_{21}	A_{22}	A_{23}
E_1	0.000	0.000	0.105
E_2	0.468	1.000	0.559
E_3	1.000	0.488	0.000
E_4	0.000	0.101	1.000
E_5	0.468	0.680	0.000
Standard deviation	0.415	0.413	0.439

Table A.7. Evaluation of topography attributes using T2NNs by all experts.

Experts	Topography attributes	
	A_{31}	A_{32}
E_1	$\langle(0.70, 0.75, 0.80); (0.15, 0.15, 0.25); (0.10, 0.05, 0.05)\rangle$	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.30, 0.45)\rangle$
E_2	$\langle(0.20, 0.20, 0.10); (0.65, 0.80, 0.85); (0.45, 0.30, 0.45)\rangle$	$\langle(0.60, 0.45, 0.50); (0.20, 0.15, 0.25); (0.10, 0.05, 0.05)\rangle$
E_3	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.30, 0.45)\rangle$	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.30, 0.45)\rangle$
E_4	$\langle(0.35, 0.35, 0.10); (0.50, 0.75, 0.80); (0.50, 0.35, 0.35)\rangle$	$\langle(0.35, 0.35, 0.10); (0.50, 0.75, 0.80); (0.50, 0.35, 0.35)\rangle$
E_5	$\langle(0.20, 0.20, 0.10); (0.65, 0.80, 0.85); (0.45, 0.30, 0.45)\rangle$	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.30, 0.45)\rangle$

Experts	A_{33}	A_{34}
E ₁	$\langle(0.20, 0.20, 0.10); (0.65, 0.80, 0.85); (0.45, 0.00, 0.00)\rangle$	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.00, 0.00)\rangle$
E ₂	$\langle(0.95, 0.90, 0.95); (0.10, 0.10, 0.05); (0.05, 0.00, 0.00)\rangle$	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.00, 0.00)\rangle$
E ₃	$\langle(0.20, 0.20, 0.10); (0.65, 0.80, 0.85); (0.45, 0.00, 0.00)\rangle$	$\langle(0.60, 0.45, 0.50); (0.20, 0.15, 0.25); (0.10, 0.00, 0.00)\rangle$
E ₄	$\langle(0.35, 0.35, 0.10); (0.50, 0.75, 0.80); (0.50, 0.00, 0.00)\rangle$	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.00, 0.00)\rangle$
E ₅	$\langle(0.60, 0.45, 0.50); (0.20, 0.15, 0.25); (0.10, 0.00, 0.00)\rangle$	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.00, 0.00)\rangle$
Experts	A_{35}	A_{36}
E ₁	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.00, 0.00)\rangle$	$\langle(0.20, 0.20, 0.10); (0.65, 0.80, 0.85); (0.45, 0.00, 0.00)\rangle$
E ₂	$\langle(0.60, 0.45, 0.50); (0.20, 0.15, 0.25); (0.10, 0.00, 0.00)\rangle$	$\langle(0.35, 0.35, 0.10); (0.50, 0.75, 0.80); (0.50, 0.00, 0.00)\rangle$
E ₃	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.00, 0.00)\rangle$	$\langle(0.20, 0.20, 0.10); (0.65, 0.80, 0.85); (0.45, 0.00, 0.00)\rangle$
E ₄	$\langle(0.70, 0.75, 0.80); (0.15, 0.15, 0.25); (0.10, 0.00, 0.00)\rangle$	$\langle(0.35, 0.35, 0.10); (0.50, 0.75, 0.80); (0.50, 0.00, 0.00)\rangle$
E ₅	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.00, 0.00)\rangle$	$\langle(0.60, 0.45, 0.50); (0.20, 0.15, 0.25); (0.10, 0.00, 0.00)\rangle$
Experts	A_{37}	A_{38}
E ₁	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.00, 0.00)\rangle$	$\langle(0.35, 0.35, 0.10); (0.50, 0.75, 0.80); (0.50, 0.00, 0.00)\rangle$
E ₂	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.00, 0.00)\rangle$	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.00, 0.00)\rangle$
E ₃	$\langle(0.60, 0.45, 0.50); (0.20, 0.15, 0.25); (0.10, 0.00, 0.00)\rangle$	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.00, 0.00)\rangle$
E ₄	$\langle(0.20, 0.20, 0.10); (0.65, 0.80, 0.85); (0.45, 0.00, 0.00)\rangle$	$\langle(0.35, 0.35, 0.10); (0.50, 0.75, 0.80); (0.50, 0.00, 0.00)\rangle$
E ₅	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.00, 0.00)\rangle$	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.00, 0.00)\rangle$

Table A.8. Normalized decision matrix based on topography attributes by all experts.

Experts	Topography attributes							
	A_{31}	A_{32}	A_{33}	A_{34}	A_{35}	A_{36}	A_{37}	A_{38}
E ₁	1.000	0.730	0.000	0.000	0.105	0.000	0.468	0.000
E ₂	0.000	1.000	1.000	0.468	0.559	0.149	0.717	0.562
E ₃	0.383	0.668	0.488	1.000	0.000	0.717	1.000	1.000
E ₄	0.122	0.000	0.101	0.000	1.000	0.149	0.000	0.000
E ₅	0.000	0.668	0.680	0.468	0.000	1.000	0.717	0.562
Standard deviation	0.421	0.369	0.413	0.415	0.439	0.432	0.375	0.427

Table A.9. Evaluation of accessibility to emergency services attributes using T2NNs by all experts.

Experts	Accessibility to emergency services attributes	
	A_{41}	C_{42}
E ₁	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.00, 0.00)\rangle$	$\langle(0.20, 0.20, 0.10); (0.65, 0.80, 0.85); (0.45, 0.00, 0.00)\rangle$
E ₂	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.00, 0.00)\rangle$	$\langle(0.95, 0.90, 0.95); (0.10, 0.10, 0.05); (0.05, 0.00, 0.00)\rangle$
E ₃	$\langle(0.60, 0.45, 0.50); (0.20, 0.15, 0.25); (0.10, 0.00, 0.00)\rangle$	$\langle(0.70, 0.75, 0.80); (0.15, 0.15, 0.25); (0.10, 0.00, 0.00)\rangle$
E ₄	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.00, 0.00)\rangle$	$\langle(0.35, 0.35, 0.10); (0.50, 0.75, 0.80); (0.50, 0.00, 0.00)\rangle$
E ₅	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.00, 0.00)\rangle$	$\langle(0.60, 0.45, 0.50); (0.20, 0.15, 0.25); (0.10, 0.00, 0.00)\rangle$
Experts	A_{43}	
E ₁	$\langle(0.35, 0.35, 0.10); (0.50, 0.75, 0.80); (0.50, 0.75, 0.65)\rangle$	
E ₂	$\langle(0.60, 0.45, 0.50); (0.20, 0.15, 0.25); (0.10, 0.25, 0.15)\rangle$	
E ₃	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.30, 0.45)\rangle$	

E_4	$\langle(0.70, 0.75, 0.80); (0.15, 0.15, 0.25); (0.10, 0.15, 0.15)\rangle$
E_5	$\langle(0.95, 0.90, 0.95); (0.10, 0.10, 0.05); (0.05, 0.05, 0.05)\rangle$

Table A.10. Normalized decision matrix based on accessibility to emergency services attributes by all experts.

Experts	Accessibility to emergency services attributes			
	A_{41}	A_{42}	A_{43}	
E_1	0.468	0.000	0.000	
E_2	0.468	1.000	0.644	
E_3	1.000	0.832	0.430	
E_4	0.000	0.101	0.813	
E_5	0.468	0.680	1.000	
Standard deviation	0.354	0.447	0.385	

Table A.11. Evaluation of accessibility to transportation attributes using T2NNs by all experts.

Experts	Accessibility to transportation attributes	
	A_{51}	A_{52}
E_1	$\langle(0.70, 0.75, 0.80); (0.15, 0.15, 0.25); (0.10, 0.15, 0.15)\rangle$	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.40, 0.35)\rangle$
E_2	$\langle(0.95, 0.90, 0.95); (0.10, 0.10, 0.05); (0.05, 0.05, 0.05)\rangle$	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.30, 0.40)\rangle$
E_3	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.40, 0.35)\rangle$	$\langle(0.20, 0.20, 0.10); (0.65, 0.80, 0.85); (0.45, 0.40, 0.35)\rangle$
E_4	$\langle(0.60, 0.45, 0.50); (0.20, 0.15, 0.25); (0.10, 0.15, 0.15)\rangle$	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.40, 0.35)\rangle$
E_5	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.30, 0.40)\rangle$	$\langle(0.20, 0.20, 0.10); (0.65, 0.80, 0.85); (0.45, 0.40, 0.35)\rangle$

Table A.12. Normalized decision matrix based on accessibility to transportation attributes by all experts.

Experts	Accessibility to transportation attributes	
	A_{51}	A_{52}
E_1	0.754	0.653
E_2	1.000	1.000
E_3	0.000	0.000
E_4	0.531	0.653
E_5	0.248	0.000
Standard deviation	0.396	0.444

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Toward Better Physical Education Outcomes Assessing the Effectiveness of Blended Learning in Middle School Physical Education using TreeSoft Set

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Abstract: Physical education (PE) teaching methods have changed because of blended learning, which combines traditional in-person training with online learning resources. The implementation of blended learning in physical education in middle schools, when kids are going through major developmental changes, necessitates a thorough assessment to ascertain its actual efficacy. By looking at both technology integration and physical activity outcomes, this study investigates the critical elements that make blended learning in middle school physical education settings successful. The study assesses how environmental conditions, instructor involvement, student engagement, and instructional design affect overall learning effectiveness using a multi-criteria assessment approach. We use the TreeSoft set to divide the criteria and sub criteria as Tree and we compute the criteria weights in each local criteria and in global criteria. We rank the alternatives using the WSM method. For educators and policymakers looking to maximize physical education in a changing digital educational environment, the findings provide insightful information.

Keywords: TreeSoft Set; Blended Learning; Middle School; Physical Education; MCDM Problem.

1. Introduction

Globally, educational systems are undergoing fast change, and technology is becoming a more significant factor in determining how pupils learn. Physical education is one of the topics where blended learning, an instructional approach that combines traditional classroom experiences with online components, is becoming more and more popular. This change in pedagogy calls for a reevaluation of teaching methods, especially in middle school physical education, where active participation and physical growth are crucial[1], [2].

Physical education necessitates active engagement and skill development in physical activities, in contrast to academic topics that mostly rely on theoretical knowledge. Thus, there are special

opportunities and problems associated with implementing blended learning in physical education. It challenges teachers to reconsider how digital resources can support or improve in-person instruction and the growth of students' fitness[3], [4].

A student's middle school years are a time of formative development, characterized by emotional, cognitive, and physical maturation. At this level, successful PE programs can promote healthy behaviors that last a lifetime. In this situation, blended learning strategies need to be thoroughly evaluated to make sure they support students' physical literacy while also meeting their developmental needs.

Blended physical education programs now frequently incorporate interactive apps, online fitness tracking, virtual evaluations, and video demonstrations. Although these tools offer flexibility and customization, it is yet unclear how well they will encourage participation and physical activity in the real world[5], [6]. To achieve the best results, it is imperative to investigate how these technologies are used in conjunction with in-person instruction.

Teachers in blended physical education settings frequently face challenges such as inadequate technology infrastructure, a lack of training, and trouble remotely tracking students' progress[7], [8]. Understanding these issues and how they affect student outcomes and instructional quality is necessary for evaluating effectiveness.

Any physical education program's success depends heavily on the engagement and motivation of its students. Because of the combination of digital content and physical duties, it becomes more difficult to retain engagement in blended situations[9], [10]. As a result, one of the most important factors in determining how successful blended physical education programs are should be involvement.

To evaluate the efficacy of blended learning in middle school physical education, a strong framework with several assessment criteria is necessary. These could include learning environment, teacher support, student results, technological access, and instructional design. Stakeholders can determine areas for improvement as well as strengths through a multi-criteria review.

By looking at important success variables from both a qualitative and quantitative perspective, this study seeks to offer a thorough evaluation of blended learning in middle school physical education. It gives teachers and decision-makers a better idea of what makes blended learning effective in a topic that has historically relied on in-person interaction and presence.

Decision-makers (DMs) directly determine the relative weights of criteria in this procedure by weighing their relative importance. Either a group decision-making process or individual direct weighing can be used[11], [12]. Objective weighting techniques use various mathematical techniques to obtain the weights of criteria from the decision matrix. Among these, the significance of CRITIC and Shannon's entropy

In contrast to the previous category, several subjective weighting methods have been developed thus far to optimize the process of translating human thought into criteria weights. The primary basis of these methods is pairwise comparison, but there are other methods that use different approaches. Subjective weighting methods are more commonly used than objective weighting methods[13], [14]. They are primarily developed to translate DMs' opinions, expectations, assessments, judgments, and interpretations of the importance of criteria into weights for those criteria.

The HyperSoft Set (2018), IndetermSoft Set (2022), IndetermHyperSoft Set (2022), SuperHyperSoft Set, TreeSoft Set (2022), and ForestSoft Set (2024) are the six new varieties of Soft Sets that Smarandache announced[15], [16], [17], [18].

2. Proposed Model

A TreeSoft Set maps subsets of a universal set to the power set of $Tree(A)$, a hierarchical tree-like structure of attributes. It is well known that MultiSoft Sets and comparable frameworks can be generalized using ideas like TreeSoft Sets[19], [20].

Let U be a universal set of discourse, and H non-empty set of U and the power set is $P(U)$. Let set attributes or criteria or factors such as: $A = \{A_1, A_2, \dots, A_n\}; n \geq 1$, each attribute is considered a first-level attribute. Every first level attribute has different sub criteria

$$A_i = \{A_{i,1}, A_{i,2}, \dots\} \quad (1)$$

$$A_1 = \{A_{1,1}, A_{1,2}, \dots\} \quad (2)$$

$$A_2 = \{A_{2,1}, A_{2,2}, \dots\} \quad (3)$$

$$A_3 = \{A_{3,1}, A_{3,2}, \dots\} \quad (4)$$

$$A_4 = \{A_{4,1}, A_{4,2}, \dots\} \quad (5)$$

$$A_5 = \{A_{5,1}, A_{5,2}, \dots\} \quad (6)$$

$$A_6 = \{A_{6,1}, A_{6,2}, \dots\} \quad (7)$$

$$A_7 = \{A_{7,1}, A_{7,2}, \dots\} \quad (8)$$

$$A_8 = \{A_{8,1}, A_{8,2}, \dots\} \quad (9)$$

$$A_9 = \{A_{9,1}, A_{9,2}, \dots\} \quad (10)$$

$$A_{10} = \{A_{10,1}, A_{10,2}, \dots\} \quad (11)$$

The TreeSoft set is defined as:

$$F: P(Tree(A)) \rightarrow P(H) \quad (12)$$

We show an example of the TreeSoft Set as shown in Fig 1. In the first level, we show the six criteria, in the second level we show three criteria with values.

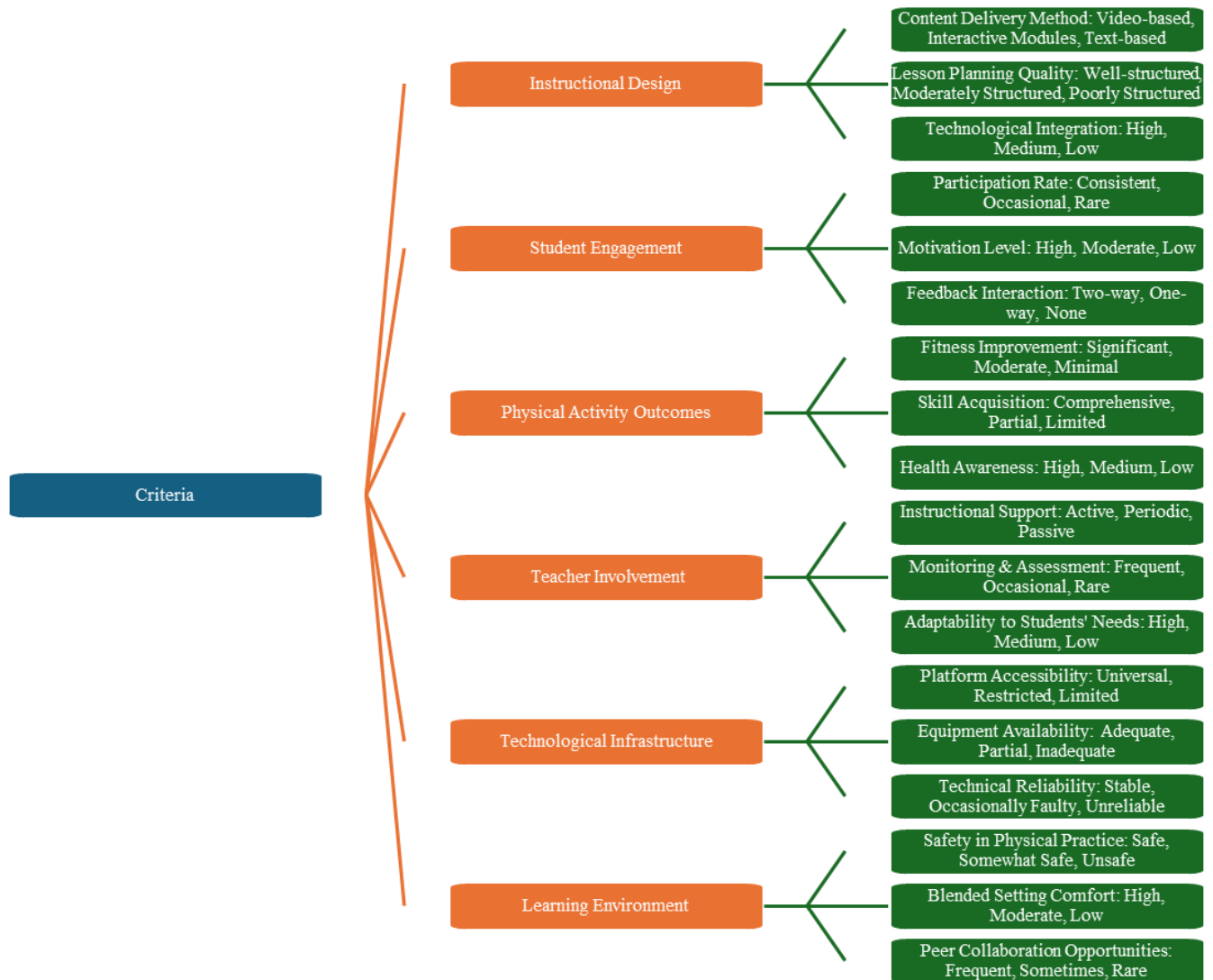


Fig 1. The TreeSoft graph.

We show the steps of the WSM method to rank the alternatives.

Create the decision matrix.

The decision makers create the decision matrix based on their opinions.

$$X = \begin{pmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{pmatrix} \quad (13)$$

Aggregate the decision matrix into a single matrix.

Compute the criteria weights.

The weights of the criteria are computed using the average method from the decision matrix between the criteria and the alternatives.

Compute the weighted decision matrix.

The weighted decision matrix is computed by multiplying the weights of criteria by the decision matrix.

$$H_{ij} = w_j x_{ij} \quad (14)$$

Compute the total sum of weighted decision matrix.

$$k_i = \sum_{j=1}^n H_{ij} \quad (15)$$

Rank the alternatives.

The alternatives are ranked based on the highest value on the k_i .

3. Application

We present an application of the proposed approach to computing the criteria weights and ranking alternatives. This study uses six criteria as shown in Fig 1. We use ten alternatives as: Flipped Classroom with Fitness Apps, Hybrid Weekly Online/Offline PE Sessions, Game-based Learning with Motion Sensors, Virtual Reality Assisted PE, Interactive Live-Streamed PE Lessons, Peer-Coached Blended Fitness Program, Pre-recorded Instruction with Daily Activity Logs, Project-Based Movement Challenges, Community Integrated Outdoor, AI-Personalized PE Instruction & Tracking.

We use the TreeSoft set to divide the criteria and sub criteria into a tree. We compute the weights of criteria in each level and then compute the global weights of criteria. In the first, we compute the weights of the main criteria. Three experts created the decision matrix between the main criteria and alternatives. We combine the decision matrix as shown in Fig 2. We compute the criteria weights using the average method.

C1– 0.1617, This criterion carries a moderate weight, highlighting the importance of how well the blended PE curriculum is structured, including the clarity of learning objectives, alignment with standards, and coherence in content delivery.

C2– 0.1652, With a slightly higher weight, student engagement is recognized as a critical factor in ensuring the success of blended PE. This includes motivation, participation, and active interaction with the platform and peers.

C3– 0.1803, This criterion holds the highest weight among all, reflecting the primary goal of physical education—improving students’ fitness, physical skills, and health literacy. Its prominence emphasizes outcome-based success measurement.

C4– 0.1653, Teacher involvement is considered highly influential, almost equal to student engagement, emphasizing the role of instructors in delivering, adapting, and monitoring blended learning for maximum effectiveness.

C5– 0.1607, This factor is moderately significant, indicating that while technology is essential to enable blended learning, its impact is largely dependent on the quality of instruction and engagement strategies built around it.

C6– 0.1669, This weight suggests a substantial role of the physical and psychological environment where blended learning takes place. It encompasses factors like student comfort, safety, and opportunities for collaboration

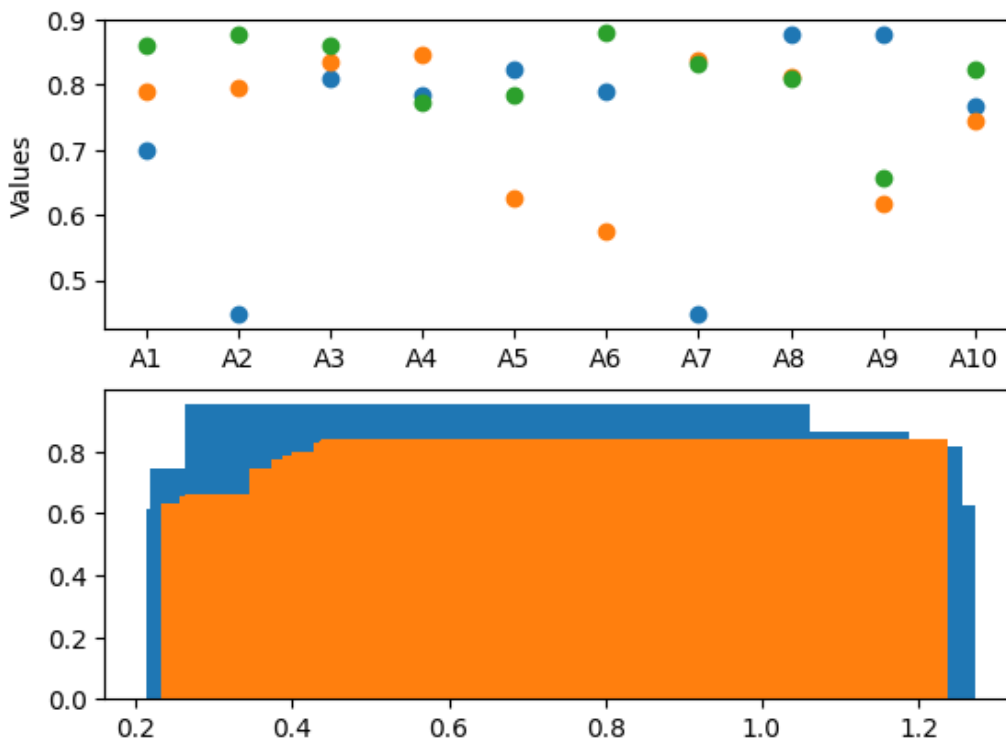


Fig 2. The combined decision matrix for main criteria.

Then we compute the criteria weights in the first sub criteria. We create the decision matrix between the criteria and alternatives. We combine the decision matrix as shown in Fig 3. We compute the criteria weights using the average method as shown in Fig 4.

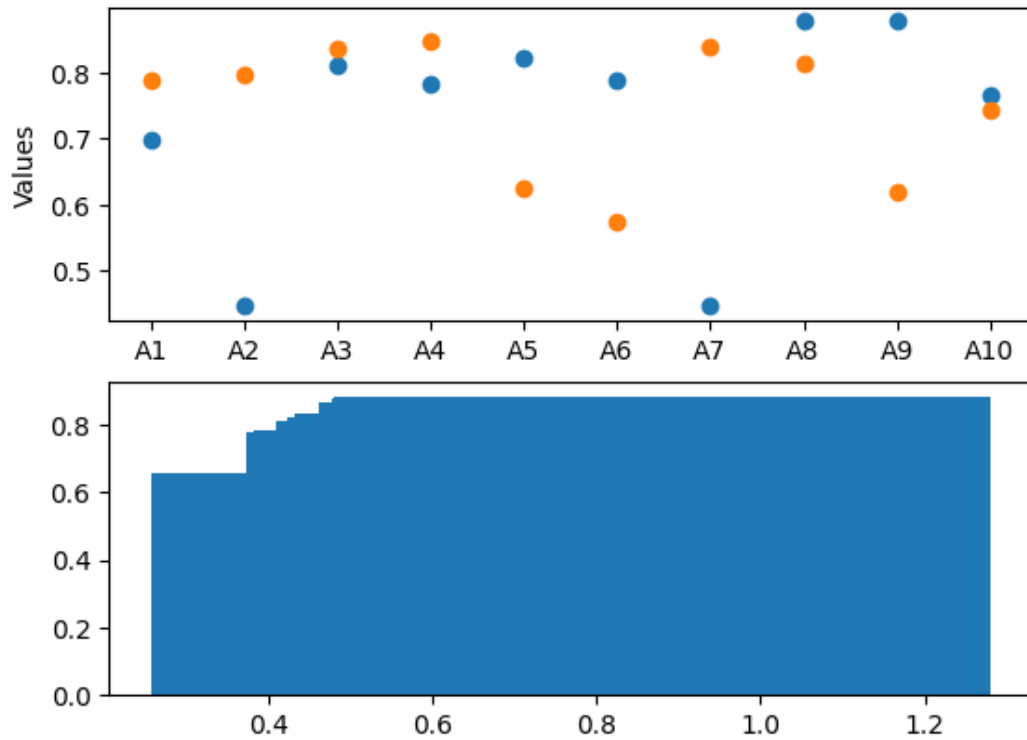


Fig 3. Combined decision matrix for first sub criterion.

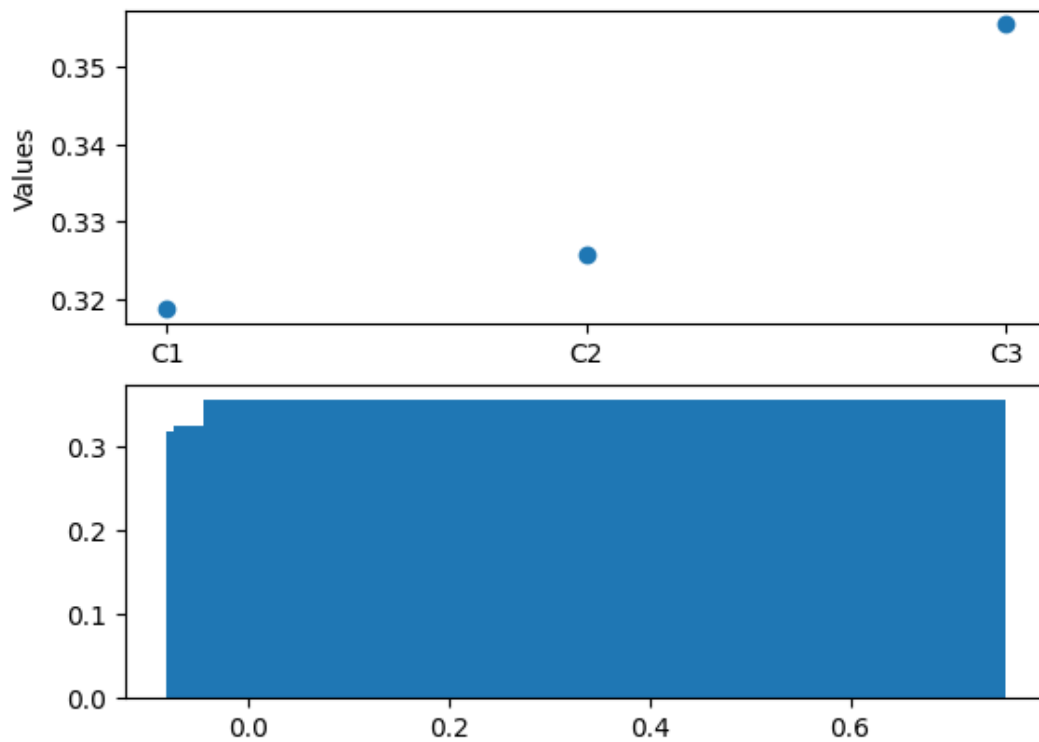


Fig 4. Weights of first sub criterion.

Then we compute the criteria weights in the second sub criteria. We create the decision matrix between the criteria and alternatives. We combine the decision matrix as shown in Fig 5. We compute the criteria weights using the average method as shown in Fig 6.

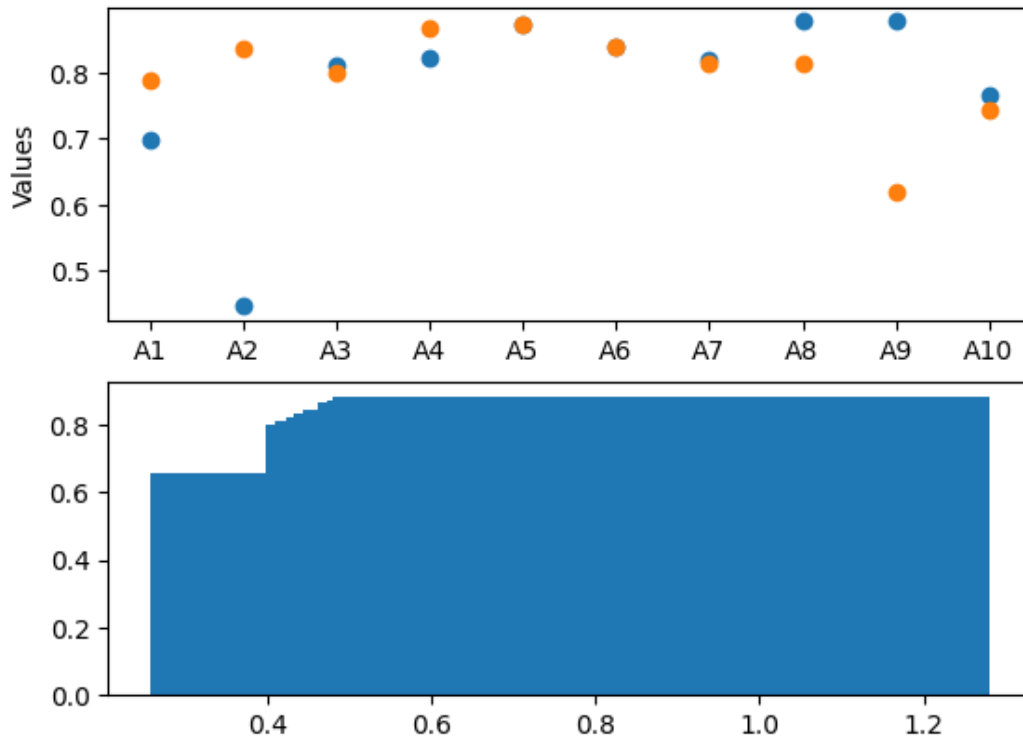


Fig 5. Combined decision matrix for second sub criterion.

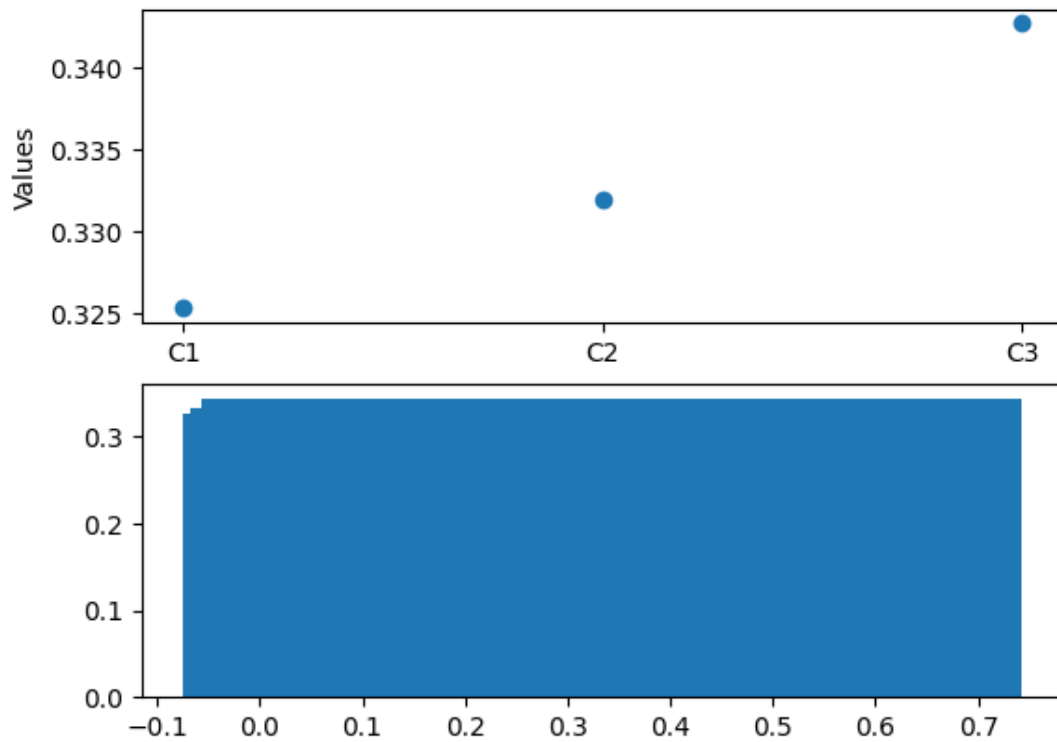


Fig 6. Weights of second sub criterion.

Then we compute the criteria weights in the third sub criteria. We create the decision matrix between the criteria and alternatives. We combine the decision matrix as shown in Fig 7. We compute the criteria weights using the average method as shown in Fig 8.

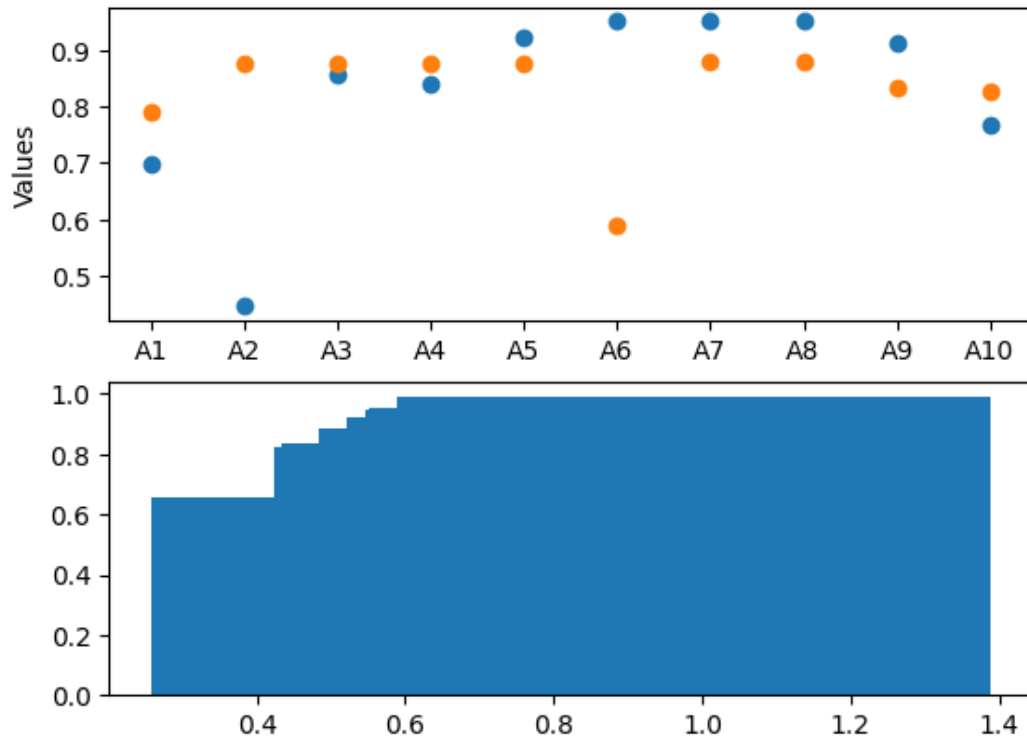


Fig 7. Combined decision matrix for third sub criterion.

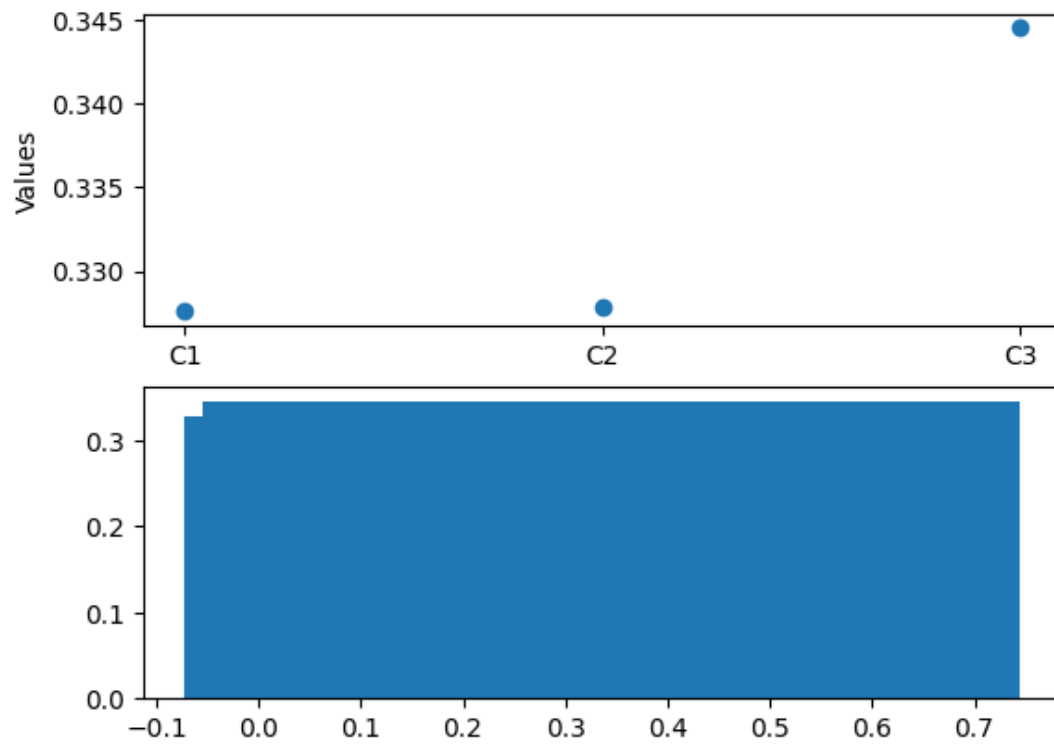


Fig 8. Weights of third sub criterion.

Then we compute the criteria weights in the fourth sub criteria. We create the decision matrix between the criteria and alternatives. We combine the decision matrix as shown in Fig 9. We compute the criteria weights using the average method as shown in Fig 10.

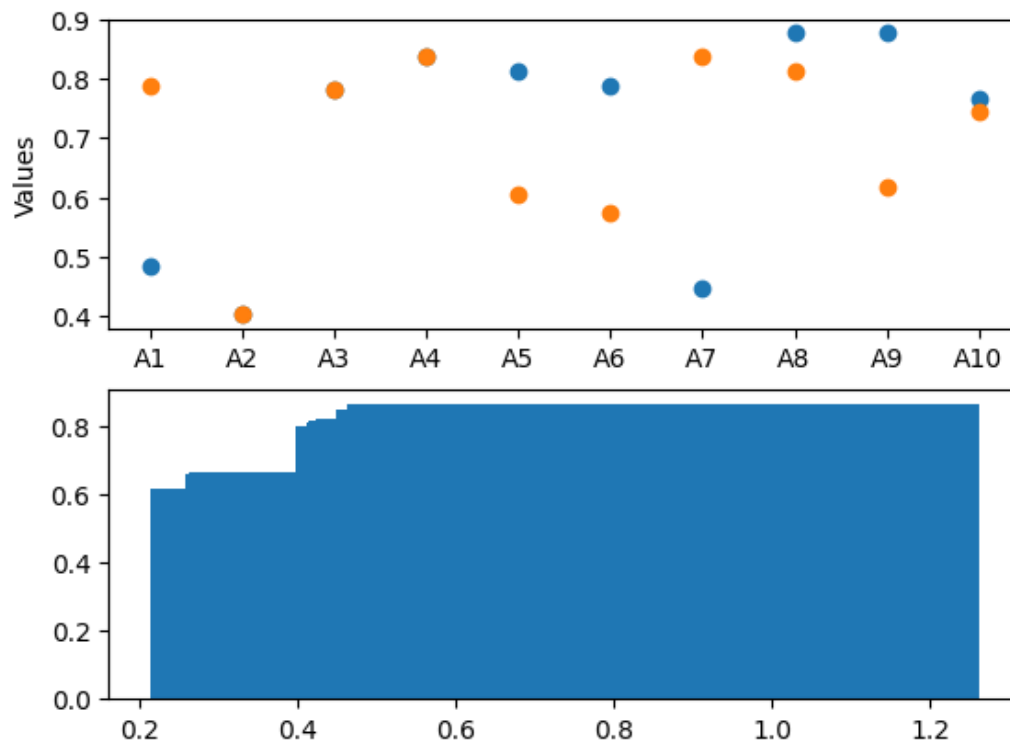


Fig 9. Combined decision matrix for fourth sub criterion.

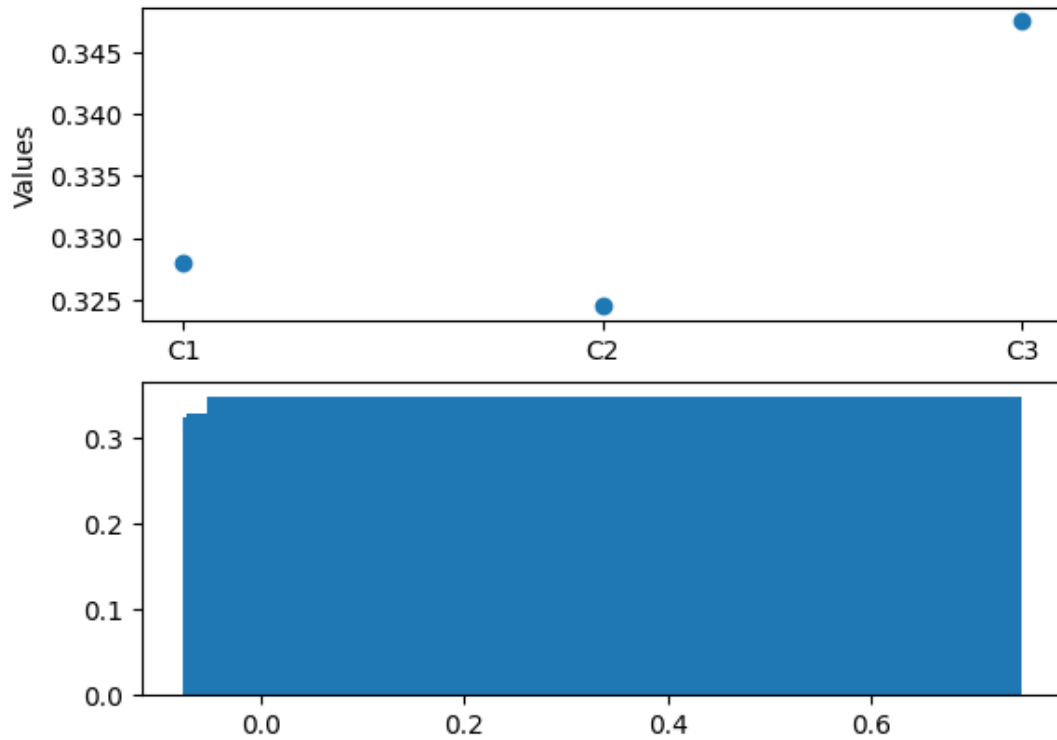


Fig 10. Weights of fourth sub criterion.

Then we compute the criteria weights in the fifth sub criteria. We create the decision matrix between the criteria and alternatives. We combine the decision matrix as shown in Fig 11. We compute the criteria weights using the average method as shown in Fig 12.

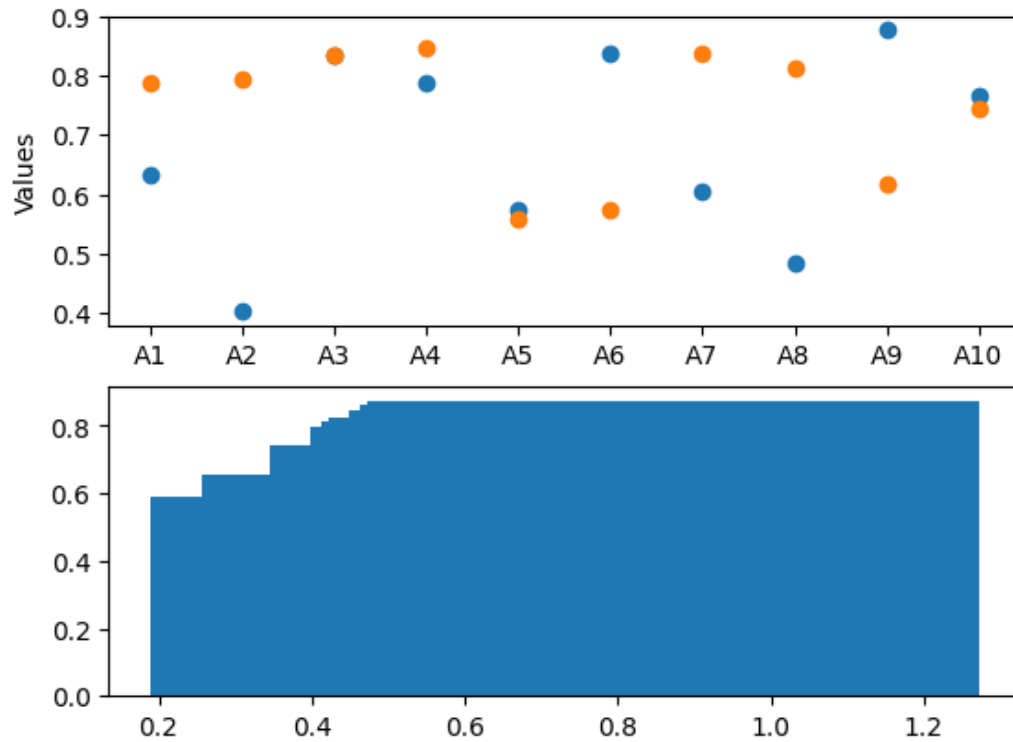


Fig 11. Combined decision matrix for fifth sub criterion.

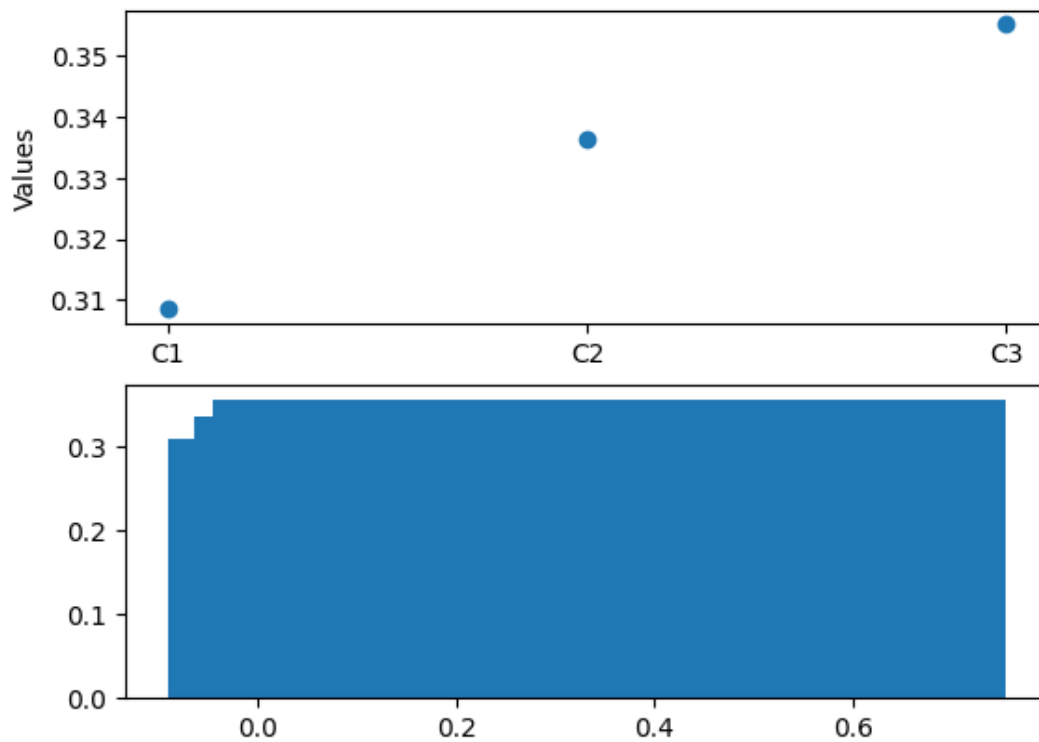


Fig 12. Weights of fifth sub criterion.

Then we compute the criteria weights in the sixth sub criteria. We create the decision matrix between the criteria and alternatives. We combine the decision matrix as shown in Fig 13. We compute the criteria weights using the average method as shown in Fig 14.

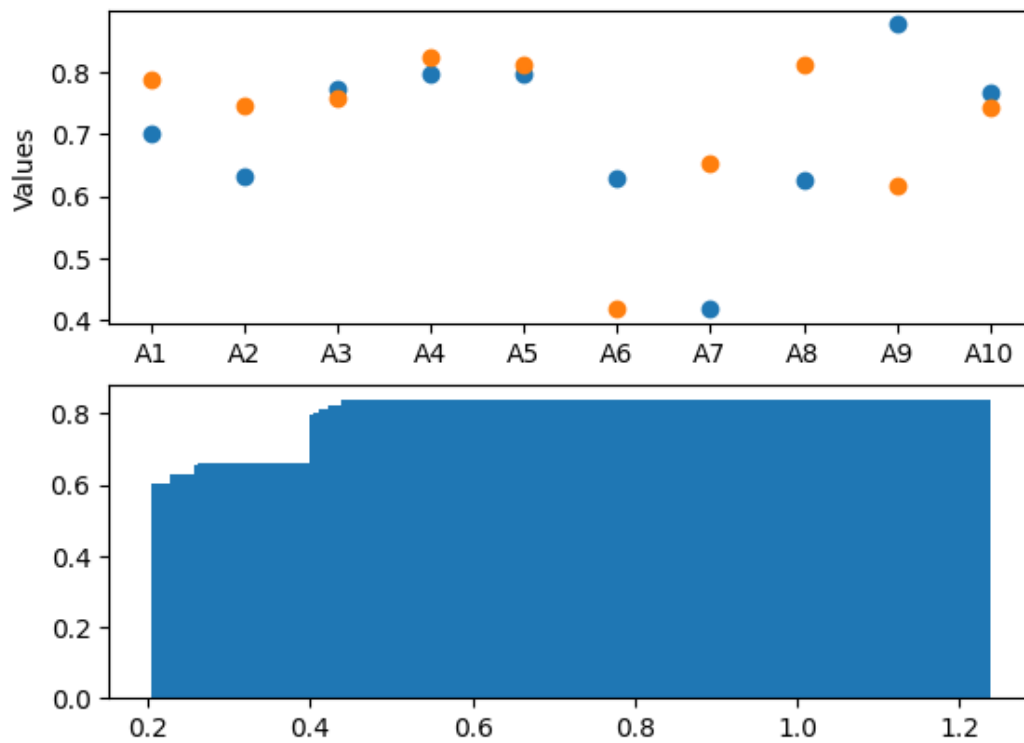


Fig 13. Combined decision matrix for sixth sub criterion.

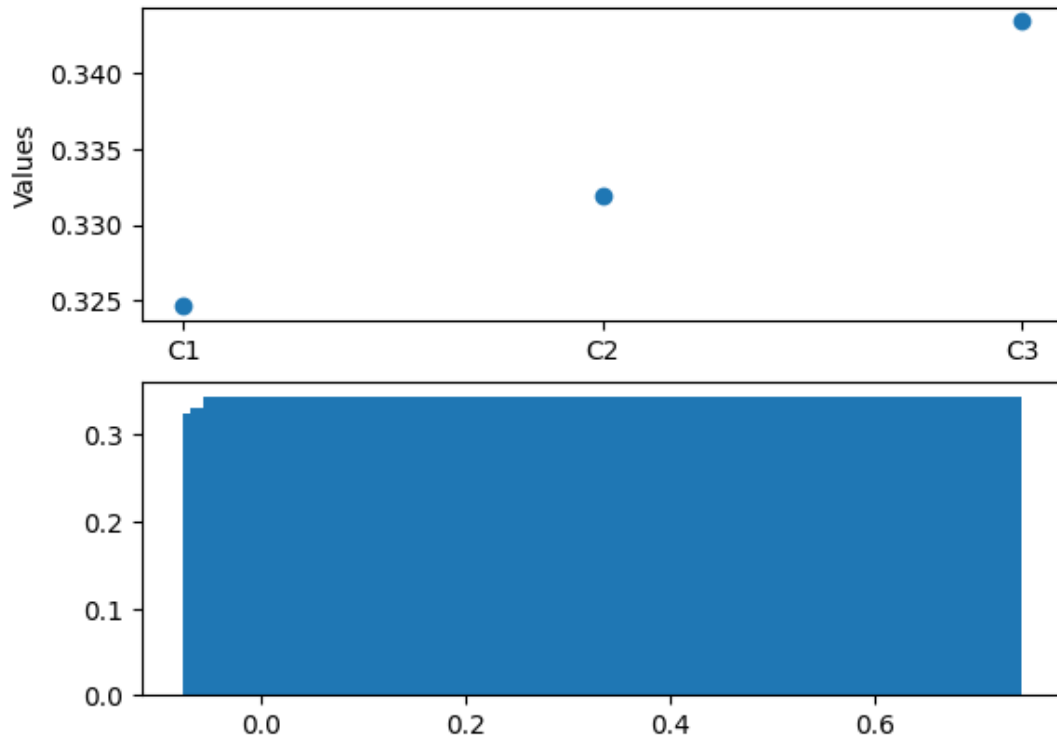


Fig 14. Weights of sixth sub criterion.

Then we compute the weights of global criteria by multiplying the weights of main criteria by the weights of sub criteria as shown in Fig 15. Then we rank the alternatives using the WSM method.

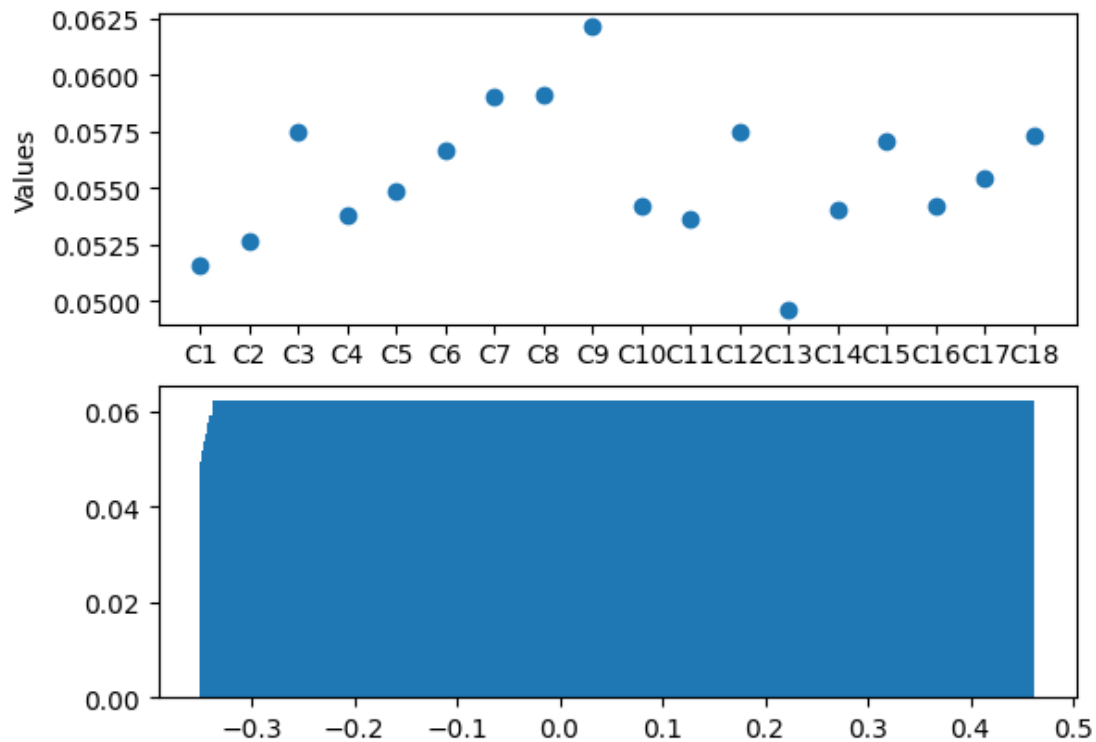


Fig 15. The weights of global criteria.

We show the steps of the WSM method to rank the alternatives.

We compute the weighted decision matrix using eq. (14) as shown in Fig 16.

We compute the total sum of weighted decision matrix using eq. (15) as shown in Fig 17.

We rank the alternatives as shown in Fig 18.

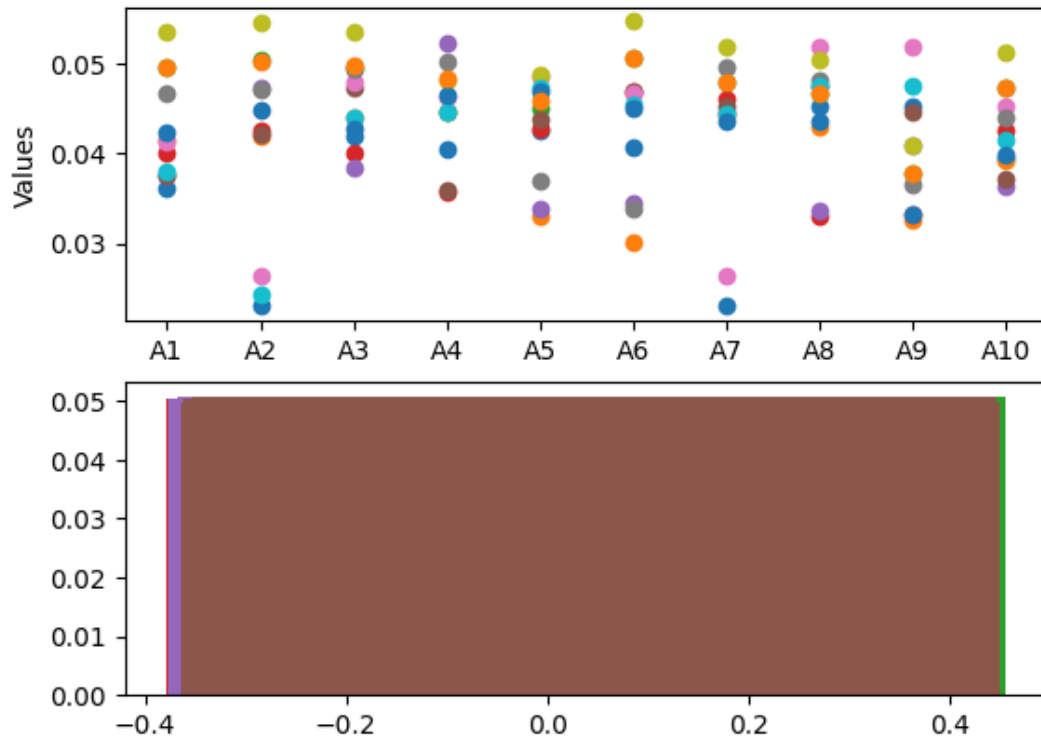


Fig 16. The weighted decision matrix.

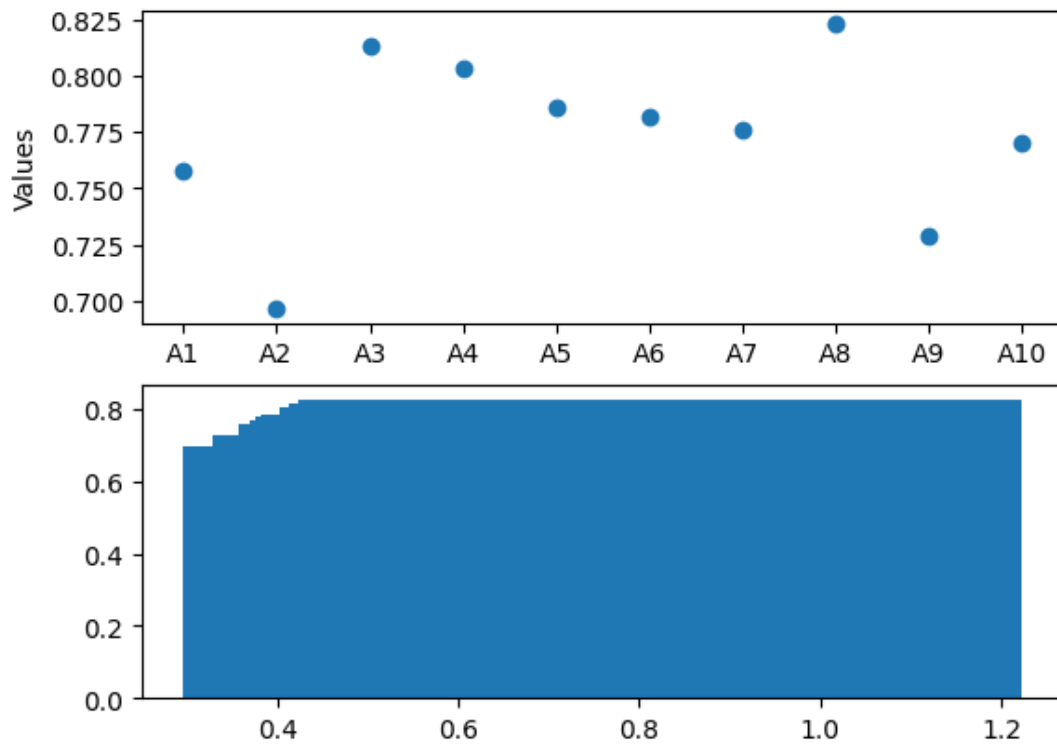


Fig 17. The total sum weight.

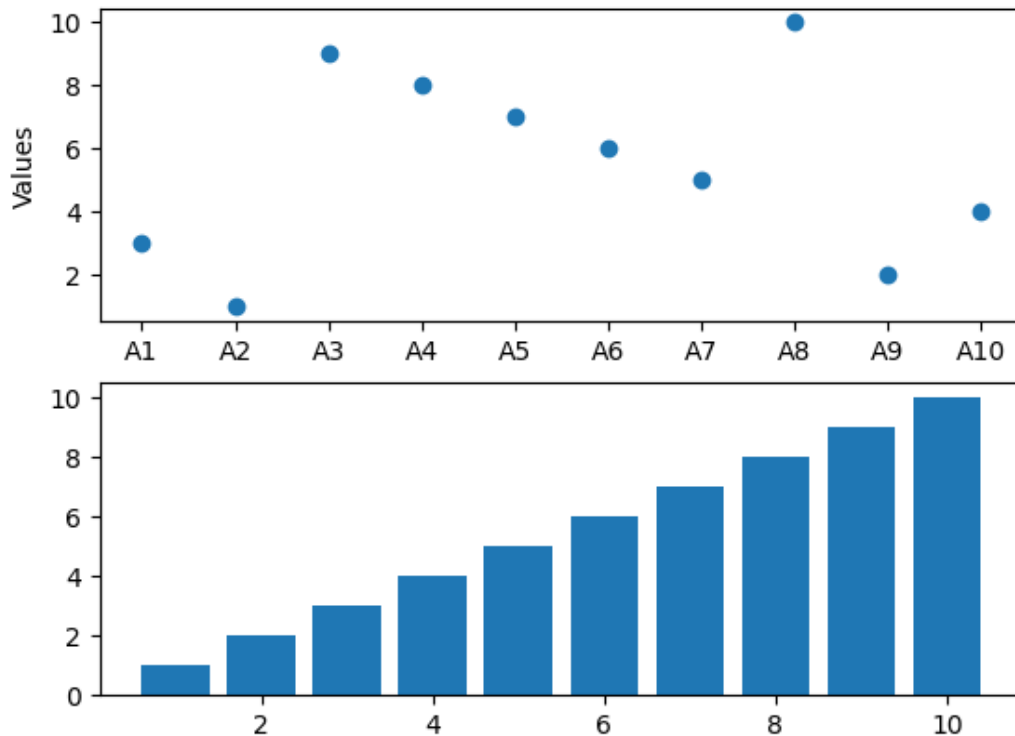


Fig 18. The rank of alternatives.

5. Conclusions

There are opportunities as well as challenges associated with integrating blended learning into middle school physical education. Although it could make physical education more adaptable, inclusive, and customized, its effectiveness depends on several interconnected elements, including the caliber of instruction, student participation, teacher support, and technological preparedness. This study highlights how crucial it is to assess integrated physical education programs holistically and according to clear standards to make sure that learning objectives are in line with developmental and health objectives. The evaluation's conclusions can direct the creation and execution of more successful physical education programs, which will ultimately result in student populations that are healthier and more active. We used the TreeSoft Set to divide the criteria as a tree and we computed the weights in each level. We rank the alternatives using the WSM method.

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Improved neutrosophic exponential-ratio estimator of population mean with simple random sampling

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Abstract: In this paper we present a neutrosophic exponential-ratio estimator for calculating the population mean using simple random sampling. In sampling methods classical statistics always depends on exact and complete data, but when we are dealing with unclear data these all become insufficient. By managing ambiguous and indeterminate data, neutrosophic statistics an extension of fuzzy and classical statistics addresses this drawback. The bias and mean square error (MSE) of proposed estimator are derived up to the first approximation order. Comparative study shows that it is more efficient than existing estimators, especially when we are working with data that is imprecise or of the neutrosophic kind. The proposed approach produces interval-based estimations in contrast to traditional estimators, which summarizes the unknown population mean with minimal MSE, improving reliability. The effectiveness of the estimator is confirmed by simulations and neutrosophic data sets, highlighting its potential in situations where uncertainty is common in the real world.

Keywords: Neutrosophic statistics, population mean, exponential-ratio estimator, bias, mean squared error, simple random sampling.

1. Introduction

In sample surveys, the statistical technique that works with indeterminate data and inference techniques with varying degrees of indeterminacy is called neutrophilic statistics. Based on interval analysis, it is an expansion of interval statistics. All sets and intervals, including finite discrete sets, are covered by neutrophilic statistics, which is based on set analysis. Classical and interval statistics assume that each member of the sample or population belongs 100% of the time, but neutrosophic statistics can account for individuals who are partially belonging or not belonging. The accuracy of data obtained by neutrosophic statistics is higher than that of classical and interval statistics. It works with data and inference methods and is more flexible than traditional statistics. Neutrosophic and interval statistics overlap if all sets are intervals, each individual is 100% representative of the sample or population, and there is only one probability distribution curve. However, more non-classical statistical techniques are required than classical ones since our environment contains more unclear data.

In traditional statistics, data are known and created by discrete numerical values. Numerous writers developed several estimators for calculating the finite population mean under classical statistics when auxiliary data was included. Instead of using the study variable alone, the study

revealed that when there is a good correlation between the study variable and the auxiliary variable, we obtain considerably lower sampling error for the ratio and, as a result, we may use less sampling for ratio estimation. One paper [1] included a thorough overview of ratio estimation, along with examples and its features. Alternatively, the ratio estimation approach lowers the sample size while maintaining the same level of precision [2]. Classic ratio, regression, and exponential approaches are easy to compute population mean estimates. These methods use auxiliary variable parameters like coefficients of variation, skewness, and kurtosis, and require knowledge of the auxiliary variable's population characteristics.

The author of [3] considered supplementary data and suggested the best linear unbiased (BLU) estimator, a typical regression estimate of the population mean of the study variable. The traditional ratio estimate of population mean under SRS was presented by [2]. Using supplementary data, [4] proposed a product and ratio estimate of the population mean under SRS. A ratio type estimate for the population mean under SRS was proposed by [5]. An exponential ratio estimate of population mean under SRS was suggested by [6]. Using known auxiliary variable parameters, [7-10] proposed various modified ratio estimators of population mean under SRS. A population mean estimator of the generalized exponential ratio type was proposed by [11] and improved by [12] using [13] method. While [14] considered the co-efficient of variation and median of an auxiliary variable and developed an estimation procedure of population mean, [15] used the coefficient of skewness of the auxiliary variable and presented a ratio method of estimation of population mean. A modified ratio estimator of population mean was developed by [16] using a linear combination of the quartile deviation and the co-efficient of skewness. [17] proposed a modified ratio estimator of population mean using the size of the sample selected from the population. An improved ratio cum exponential ratio estimate of population mean was proposed by [18]. To create a new ratio estimator for predicting population mean under SRS, [19] used a coefficient of variation, correlation coefficient, and regression coefficient. In order to estimate population mean using SRS, [20] proposed a few improved ratio estimators. Using a variety of auxiliary data, [21] created a class of population mean estimators under SRS. Neutrosophic statistics specifically addresses unpredictability and indeterminacy. [22] introduced the idea of neutrosophic statistics, which was then expanded upon by [23,24], who saw it as a generalization of both classical and neutrosophic statistics. Additionally, [25] provided a basic overview of neutrosophic statistics. For the first time in the history of neutrosophic statistics, [26] have created the neutrosophic ratio-type estimators to estimate the mean of the inadequate population using auxiliary information to resolving the problem of estimating the population mean of neutrosophic data. Using Neutrosophic studies and an auxiliary variable, [27] provides a Neutrosophic exponential-type estimator for population mean estimation in the presence of uncertainty.

Inspired from all previous work, we suggested an improved neutrosophic exponential-ratio estimator which extend the work of [26] to estimate population mean with auxiliary information, addressing sample uncertainty. Although several estimators have been developed based on classical statistics, but they do not meet the requirements arising from uncertainty and indeterminacy of a real data. While some neutrosophic statistics have been introduced for estimating the parameters, such estimators either do not achieve high MSE and PRE efficiencies or are not sufficiently adaptive to the auxiliary information. There are few studies in the literature considering exponential-ratio estimators in neutrosophic environment. These limitations open up a research gap to propose various efficient neutrosophic estimators that can provide better estimations as compared to existing neutrosophic methods and also in situations having imperfect information or interval values.

To fill this gap, the current study suggests a new neutrosophic exponential-ratio estimator for estimating population mean under simple random sampling. To optimize performance, the proposed method combines existing estimators by taking advantage of the coefficient of variation as auxiliary variable, and a balancing parameter is introduced. The new estimator is derived in a way that can be handled analytically and theoretically compared with existing neutrosophic estimator and classical estimator. The practical utility of the new construction is showcased with a real dataset and

benchmark simulation study where it is associated with reductions in mean squared error and percentage relative efficiency.

There are several essential differences between the proposed estimator and the already available classical and neutrosophic estimators. The first is that the classical estimators assume that the data is precise and do not consider the uncertainty or indeterminacy that often arises in real-life conditions. The proposed estimator is developed based on neutrosophic statistics, allowing it to be suitable for processing imprecise and interval-valued data. The second is that most of the already developed neutrosophic estimators are either performable with the use of only simple ratio or product form or do not use auxiliary variable for the estimator construction. To generalize, a novel method introduces an exponential-ratio form of the new estimator, which includes the coefficient of variation of an auxiliary variable and applies a balancing parameter that optimizes efficiency. Such design not only increases the accuracy of the estimation but also enhances flexibility in an uncertain environment, making the estimator more robust and applicable. Additionally, we compare our neutrosophic exponential ratio estimator to other Existing estimators which demonstrate its superiority under certain conditions and efficiency criteria are developed, as described in Section 3. Section 4 presents quantifiable data on the efficacy of the recommended strategies. Finally, Section 5 provides classical and neutrosophic real and simulated data sets, section 6 focused on conclusion.

2. Materials & Methodology

Consider a finite population of 'N' units $K = (K_1, K_2, \dots, K_N)$ that contains a neutrosophic random sample of size $n_N \in [n_L, n_U]$. For the i^{th} ($i=1, 2, \dots, N$) unit from P , let $y_{N(i)}$ and $x_{N(i)}$ stand for the study variable and auxiliary variable.

Let $y_{N(i)}$ is the i^{th} representative observation of our neutrosophic data, originating from $y_{N(i)} \in [y_L, y_U]$ and in the same way for the auxiliary variable $x_{N(i)} \in [x_L, x_U]$. Let $\bar{y}_{N(i)} \in [\bar{y}_L, \bar{y}_U]$ is our key neutrosophic variable and $\bar{x}_{N(i)} \in [\bar{x}_L, \bar{x}_U]$ is associated with our study variable and serves as our auxiliary neutrosophic variable $\bar{y}_N$. Furthermore, $Y_{N(i)} \in [Y_L, Y_U]$ and $X_{N(i)} \in [X_L, X_U]$ are the statistics for the neutrosophic data. Also let $C_{yN} \in [C_{yL}, C_{yU}]$ and $C_{xN} \in [C_{xL}, C_{xU}]$ are coefficients of variation for neutrosophic Y_N , X_N and $\rho_{yxN} \in [\rho_{yxL}, \rho_{yxU}]$ is the association between Y_N and X_N that is neutrosophic.

Assume that the neutrosophic mean error terms $\Delta_{yN} \in [\Delta_{yL}, \Delta_{yU}]$ and $\Delta_{xN} \in [\Delta_{xL}, \Delta_{xU}]$. Let the study's neutrosophic sets be calculated as

$$\bar{y}_{N(i)} = \bar{Y}_N + \Delta_{yN(i)} \text{ and } \bar{x}_{N(i)} = \bar{X}_N + \Delta_{xN(i)}$$

such that

$$\begin{aligned} E(\Delta_{yN}) &= E(\Delta_{xN}) = 0; \\ E(\Delta_{yN}^2) &= \delta_N C_{yN}^2; \quad E(\Delta_{xN}^2) = \delta_N C_{xN}^2; \\ E(\Delta_{yN} \Delta_{xN}) &= \delta_N C_{yxN}; \end{aligned}$$

Where $C_{yxN} = \rho_{yxN} C_{yN} C_{xN}$; $\delta_N = \left[\frac{1}{n_N} - \frac{1}{N_N} \right]$; $\delta_N \in [\delta_L, \delta_U]$;

$$\begin{aligned} \Delta_{yN}^2 &\in [\Delta_{yL}^2, \Delta_{yU}^2]; \quad \Delta_{xN}^2 \in [\Delta_{xL}^2, \Delta_{xU}^2]; \quad \Delta_{yxN} \in [\Delta_{yxL}, \Delta_{yxU}]; \\ S_{yN} &\in [S_{yL}, S_{yU}]; \quad S_{xN} \in [S_{xL}, S_{xU}]; \quad S_{yxN} \in [S_{yxL}, S_{yxU}]; \\ C_{yN} &\in [C_{yL}, C_{yU}]; \quad C_{xN} \in [C_{xL}, C_{xU}]; \quad C_{yxN} \in [C_{yxL}, C_{yxU}]; \\ \rho_{yxN} &\in [\rho_{yxL}, \rho_{yxU}] \text{ respectively.} \end{aligned}$$

We may use a range of existing estimators, such as the unbiased estimator, ratio estimator, and exponential estimator for population mean estimation under indeterminacy, to estimate the performance of our suggested estimator. These estimators are represented as follows

Typically, the unbiased neutrosophic estimator is provided by

$$\hat{Y}_N = \bar{y}_n$$

With the MSE,

$$MSE(\hat{Y}_N) = \delta_N \bar{Y}_N^2 C_{yN}^2 \quad (1)$$

[26] suggested neutrosophic ratio estimator for determining the finite population mean in the presence of auxiliary variables is as follows:

$$\hat{Y}_{RN} = \bar{X}_N \left(\frac{\bar{Y}_N}{\bar{X}_N} \right)$$

$$\hat{Y}_{RN} = \bar{X}_N \left(\frac{\bar{Y}_N + \Delta_{yN}}{\bar{X}_N + \Delta_{xN}} \right)$$

Where, $\hat{Y}_{RN} \in [\hat{Y}_{RL}, \hat{Y}_{RU}]$, $\Delta_{yN} \in [\Delta_{yL}, \Delta_{yU}]$ and $\Delta_{xN} \in [\Delta_{xL}, \Delta_{xU}]$

The bias and MSE of the $\hat{Y}_{RN}$ up to the first order of approximation are given below,

$$\begin{aligned} Bias(\hat{Y}_{RN}) &= \delta_N \bar{Y}_N [C_{xN}^2 - C_{yxN}] \\ MSE(\hat{Y}_{RN}) &= \delta_N \bar{Y}_N^2 [C_{yN}^2 + C_{xN}^2 - 2C_{yxN}] \end{aligned} \quad (2)$$

Where, $\delta_N \in [\delta_{NL}, \delta_{NU}]$, $C_{xN}^2 \in [C_{xL}^2, C_{xU}^2]$, $C_{yN}^2 \in [C_{yL}^2, C_{yU}^2]$, $C_{yxN} \in [C_{yxL}, C_{yxU}]$

The mean for a finite population in the presence of auxiliary variables is estimated by [26] using a neutrosophic exponential-type estimator inspired from [6]:

$$\hat{Y}_{expN} = \bar{y}_N \exp \left(\frac{\bar{X}_N - \bar{x}_N}{\bar{X}_N + \bar{x}_N} \right)$$

$$\hat{Y}_{expN} = (\bar{Y}_N + \Delta_{yN}) \exp \left(-\frac{\Delta_{xN}}{2\bar{X}_N} \left(1 + \frac{\Delta_{xN}}{2\bar{X}_N} \right)^{-1} \right)$$

Where $\hat{Y}_{expN} \in [\hat{Y}_{expL}, \hat{Y}_{expU}]$, $\bar{y}_N \in [\bar{y}_L, \bar{y}_U]$, $\bar{Y}_N \in [\bar{Y}_L, \bar{Y}_U]$, $\bar{x}_N \in [\bar{x}_L, \bar{x}_U]$, $\bar{X}_N \in [\bar{X}_L, \bar{X}_U]$, $\Delta_{yN} \in [\Delta_{yL}, \Delta_{yU}]$, $\Delta_{xN} \in [\Delta_{xL}, \Delta_{xU}]$.

The bias and MSE of $\hat{Y}_{expN}$ up to first-degree of approximation are

$$\begin{aligned} Bias(\hat{Y}_{expN}) &= \delta_N \bar{Y}_N \left[\frac{3}{8} C_{xN}^2 - \frac{1}{2} C_{yxN} \right] \\ MSE(\hat{Y}_{expN}) &= \delta_N \bar{Y}_N^2 \left[C_{yN}^2 + \frac{1}{4} C_{xN}^2 - C_{yxN} \right] \end{aligned} \quad (3)$$

Where $\delta_N \in [\delta_L, \delta_U]$, $n_N \in [n_L, n_U]$, $C_{xN}^2 \in [C_{xL}^2, C_{xU}^2]$, $C_{yN}^2 \in [C_{yL}^2, C_{yU}^2]$, $C_{yxN} \in [C_{yxL}, C_{yxU}]$.

Inspired by reference [9], [26] have created an altered version of the neutrosophic ratio estimator, utilizing the coefficient of variation as an auxiliary variable.

$$\hat{Y}_{SDrN} = \bar{y}_N \left(\frac{\bar{X}_N + C_{xN}}{\bar{x}_N + C_{xN}} \right)$$

$$\hat{Y}_{SDrN} = (\bar{Y}_N + \Delta_{yN}) \left(1 + \frac{\Delta_{xN}}{\bar{X}_N + C_{xN}} \right)^{-1} \left(\frac{\bar{X}_N + C_{xN}}{\bar{X}_N + C_{xN} + \Delta_{xN}} \right)$$

Where $\hat{Y}_{SDrN} \in [\hat{Y}_{SDrL}, \hat{Y}_{SDrU}]$, $\bar{y}_N \in [\bar{y}_L, \bar{y}_U]$, $\bar{Y}_N \in [\bar{Y}_L, \bar{Y}_U]$, $\bar{x}_N \in [\bar{x}_L, \bar{x}_U]$, $\bar{X}_N \in [\bar{X}_L, \bar{X}_U]$, $\Delta_{yN} \in [\Delta_{yL}, \Delta_{yU}]$, $\Delta_{xN} \in [\Delta_{xL}, \Delta_{xU}]$, $C_{xN} \in [C_{xL}, C_{xU}]$

For the Bias and MSE of $\hat{Y}_{SDrN}$ up to the first order of approximation we get,

$$\begin{aligned} Bias(\hat{Y}_{SDrN}) &= \delta_N \bar{Y}_N \left[\left(\frac{\bar{X}_N}{\bar{X}_N + C_{xN}} \right)^2 C_{xN}^2 - \left(\frac{\bar{X}_N}{\bar{X}_N + C_{xN}} \right) C_{yxN} \right] \\ MSE(\hat{Y}_{SDrN}) &= \delta_N \bar{Y}_N^2 \left[C_{yN}^2 + \left(\frac{\bar{X}_N}{\bar{X}_N + C_{xN}} \right)^2 C_{xN}^2 - \left(\frac{\bar{X}_N}{\bar{X}_N + C_{xN}} \right) C_{yxN} \right] \end{aligned} \quad (4)$$

Where $\delta_N \in [\delta_L, \delta_U]$, $\bar{y}_N \in [\bar{y}_L, \bar{y}_U]$, $n_N \in [n_L, n_U]$, $C_{xN}^2 \in [C_{xL}^2, C_{xU}^2]$, $C_{yN}^2 \in [C_{yL}^2, C_{yU}^2]$, $C_{yxN} \in [C_{yxL}, C_{yxU}]$.

Inspired by [10], employing the neutrosophic ratio-type estimator provided as well as the coefficient of variation and kurtosis

$$\hat{Y}_{US1rN} = \bar{y}_N \left(\frac{\bar{X}_N \beta_{2(x)N} + C_{xN}}{\bar{x}_N \beta_{2(x)N} + C_{xN}} \right)$$

$$\hat{Y}_{US1rN} = (\bar{Y}_N + \Delta_{yN}) \left(1 + \frac{\beta_{2(x)N} \Delta_{xN}}{\bar{X}_N \beta_{2(x)N} + C_{xN}} \right)^{-1}$$

$$\hat{Y}_{US2rN} = \bar{y}_N \left(\frac{\bar{X}_N C_{xN} + \beta_{2(x)N}}{\bar{x}_N C_{xN} + \beta_{2(x)N}} \right)$$

$$\hat{Y}_{US2rN} = (\bar{Y}_N + \Delta_{yN}) \left(1 + \frac{C_{xN} \Delta_{xN}}{\bar{X}_N C_{xN} + \beta_{2(x)N}} \right)^{-1}$$

Where $\hat{Y}_{US1rN} \in [\hat{Y}_{US1rL}, \hat{Y}_{US1rU}]$, $\hat{Y}_{US2rN} \in [\hat{Y}_{US2rL}, \hat{Y}_{US2rU}]$, $\bar{y}_N \in [\bar{y}_L, \bar{y}_U]$, $\bar{Y}_N \in [\bar{Y}_L, \bar{Y}_U]$, $\bar{x}_N \in [\bar{x}_L, \bar{x}_U]$, $\bar{X}_N \in [\bar{X}_L, \bar{X}_U]$, $\Delta_{yN} \in [\Delta_{yL}, \Delta_{yU}]$, $\Delta_{xN} \in [\Delta_{xL}, \Delta_{xU}]$, $C_{xN} \in [C_{xL}, C_{xU}]$, $\beta_{2(x)N} \in [\beta_{2(x)L}, \beta_{2(x)U}]$

For the Bias and MSE of $\hat{Y}_{SKrN}$ up to the first order of approximation we get,

$$\begin{aligned} Bias(\hat{Y}_{US1rN}) &= \delta_N \bar{Y}_N \left[\left(\frac{\bar{X}_N \beta_{2(x)N}}{\bar{X}_N \beta_{2(x)N} + C_{xN}} \right)^2 C_{xN}^2 - \left(\frac{\bar{X}_N \beta_{2(x)N}}{\bar{X}_N \beta_{2(x)N} + C_{xN}} \right) C_{yxN} \right] \\ MSE(\hat{Y}_{US1rN}) &= \delta_N \bar{Y}_N^2 \left[C_{yN}^2 + \left(\frac{\bar{X}_N \beta_{2(x)N}}{\bar{X}_N \beta_{2(x)N} + C_{xN}} \right)^2 C_{xN}^2 - 2 \left(\frac{\bar{X}_N \beta_{2(x)N}}{\bar{X}_N \beta_{2(x)N} + C_{xN}} \right) C_{yxN} \right] \end{aligned} \quad (5)$$

$$\begin{aligned} Bias(\hat{Y}_{US2rN}) &= \delta_N \bar{Y}_N \left[\left(\frac{\bar{X}_N C_{xN}}{\bar{X}_N C_{xN} + \beta_{2(x)N}} \right)^2 C_{xN}^2 - \left(\frac{\bar{X}_N C_{xN}}{\bar{X}_N C_{xN} + \beta_{2(x)N}} \right) C_{yxN} \right] \\ MSE(\hat{Y}_{US2rN}) &= \delta_N \bar{Y}_N^2 \left[C_{yN}^2 + \left(\frac{\bar{X}_N C_{xN}}{\bar{X}_N C_{xN} + \beta_{2(x)N}} \right)^2 C_{xN}^2 - 2 \left(\frac{\bar{X}_N C_{xN}}{\bar{X}_N C_{xN} + \beta_{2(x)N}} \right) C_{yxN} \right] \end{aligned} \quad (6)$$

Where $\delta_N \in [\delta_L, \delta_U]$, $\bar{Y}_N \in [\bar{Y}_L, \bar{Y}_U]$, $\bar{X}_N \in [\bar{X}_L, \bar{X}_U]$, $n_N \in [n_L, n_U]$, $C_{xN}^2 \in [C_{xL}^2, C_{xU}^2]$, $C_{yN}^2 \in [C_{yL}^2, C_{yU}^2]$, $C_{yxN} \in [C_{yxL}, C_{yxU}]$, $\beta_{2(x)N} \in [\beta_{2(x)L}, \beta_{2(x)U}]$.

[26] suggested an additional neutrosophic estimator in which the coefficient of kurtosis is considered as an auxiliary variable.

$$\begin{aligned} \hat{Y}_{SKrN} &= \bar{y}_N \left(\frac{\bar{X}_N + \beta_{2(x)N}}{\bar{x}_N + \beta_{2(x)N}} \right) \\ \hat{Y}_{SKrN} &= (\bar{Y}_N + \Delta_{yN}) \left(1 + \frac{\Delta_{xN}}{\bar{X}_N + \beta_{2(x)N}} \right)^{-1} \end{aligned}$$

Where $\hat{Y}_{SKrN} \in [\hat{Y}_{SKrL}, \hat{Y}_{SKrU}]$, $\bar{y}_N \in [\bar{y}_L, \bar{y}_U]$, $\bar{Y}_N \in [\bar{Y}_L, \bar{Y}_U]$, $\bar{x}_N \in [\bar{x}_L, \bar{x}_U]$, $\bar{X}_N \in [\bar{X}_L, \bar{X}_U]$, $\Delta_{yN} \in [\Delta_{yL}, \Delta_{yU}]$, $\Delta_{xN} \in [\Delta_{xL}, \Delta_{xU}]$, $\beta_{2(x)N} \in [\beta_{2(x)L}, \beta_{2(x)U}]$.

For the Bias and MSE of $\hat{Y}_{SKrN}$ up to the first order of approximation we get,

$$\begin{aligned} Bias(\hat{Y}_{SKrN}) &= \delta_N \bar{Y}_N \left[\left(\frac{\bar{X}_N}{\bar{X}_N + \beta_{2(x)N}} \right)^2 C_{xN}^2 - \left(\frac{\bar{X}_N}{\bar{X}_N + \beta_{2(x)N}} \right) C_{yxN} \right] \\ MSE(\hat{Y}_{SKrN}) &= \delta_N \bar{Y}_N^2 \left[C_{yN}^2 + \left(\frac{\bar{X}_N}{\bar{X}_N + \beta_{2(x)N}} \right)^2 C_{xN}^2 - 2 \left(\frac{\bar{X}_N}{\bar{X}_N + \beta_{2(x)N}} \right) C_{yxN} \right] \end{aligned} \quad (7)$$

Where $\delta_N \in [\delta_L, \delta_U]$, $n_N \in [n_L, n_U]$, $C_{xN}^2 \in [C_{xL}^2, C_{xU}^2]$, $C_{yN}^2 \in [C_{yL}^2, C_{yU}^2]$, $C_{yxN} \in [C_{yxL}, C_{yxU}]$.

[26] proposed a generalized exponential-type neutrosophic estimator to calculate a finite population mean:

$$\begin{aligned} \hat{Y}_{KNN} &= \bar{y}_N \exp \left[\alpha \left(\frac{\bar{X}_N^{\frac{1}{h}} - \bar{x}_N^{\frac{1}{h}}}{\bar{X}_N^{\frac{1}{h}} + (a-1)\bar{x}_N^{\frac{1}{h}}} \right) \right] \\ \hat{Y}_{KNN} &= (\bar{Y}_N + \Delta_{yN}) \exp \left[-\frac{\alpha \Delta_{xN}}{ah \bar{X}_N^{\frac{1}{h}}} \left(1 + \frac{\Delta_{xN}}{h \bar{X}_N^{\frac{1}{h}}} - \frac{\Delta_{xN}}{ah \bar{X}_N^{\frac{1}{h}}} \right)^{-1} \right] \end{aligned}$$

Where $\hat{Y}_{KNN} \in [\hat{Y}_{KNNL}, \hat{Y}_{KNNU}]$, $\bar{y}_N \in [\bar{y}_L, \bar{y}_U]$, $\bar{Y}_N \in [\bar{Y}_L, \bar{Y}_U]$, $\bar{x}_N \in [\bar{x}_L, \bar{x}_U]$, $\bar{X}_N \in [\bar{X}_L, \bar{X}_U]$, $\Delta_{yN} \in [\Delta_{yL}, \Delta_{yU}]$, $\Delta_{xN} \in [\Delta_{xL}, \Delta_{xU}]$, $(-\infty < \alpha < \infty)$ and $h(h > 0)$ are real constant, $a(a \neq 0)$ is another constant.

For the Bias and MSE of $\hat{Y}_{KNN}$ up to the first order of approximation we get,

$$\begin{aligned} Bias(\hat{Y}_{KNN}) &= \delta_N \bar{Y}_N \left[\frac{\alpha C_{xN}^2}{ah^2} - \frac{\alpha C_{xN}^2}{a^2 h^2} + \frac{\alpha^2 C_{xN}^2}{2a^2 h^2} - \frac{\alpha C_{yxN}}{ah} \right] \\ MSE(\hat{Y}_{KNN}) &= \delta_N \bar{Y}_N^2 \left[C_{yN}^2 + \frac{\alpha^2 C_{xN}^2}{a^2 h^2} - \frac{2\alpha C_{yxN}}{ah} \right] \end{aligned} \quad (8)$$

Where $\delta_N \in [\delta_L, \delta_U]$, $n_N \in [n_L, n_U]$, $C_{xN}^2 \in [C_{xL}^2, C_{xU}^2]$, $C_{yN}^2 \in [C_{yL}^2, C_{yU}^2]$, $C_{yxN} \in [C_{yxL}, C_{yxU}]$.

3. Proposed Estimator

To address the issue of data indeterminacy with neutrosophic data, several preexisting estimators were converted into neutrosophic estimators. Using the coefficient of variation as an auxiliary variable, we have created a modified version of the neutrosophic exponential ratio estimator, inspired by [4,26].

$$\hat{Y}_{propN(SRS)} = \alpha_N \left(\frac{\bar{y}_N}{\bar{x}_N} \right) \bar{X}_N + (1 - \alpha_N) \bar{Y}_N \exp \left(\frac{\bar{X}_N - \bar{x}_N}{\bar{x}_N} \right) \quad (9)$$

$\bar{x}_N$ & $\bar{y}_N$ are the sample means and $\bar{X}_N$ & $\bar{Y}_N$ are the population means of the auxiliary and study variables, respectively.

α_N is a balancing parameter to balance the contribution of the two parts of the estimator: the ratio term and the exponential term.

Now expand and simplifying the equation (9), we get

$$(\hat{Y}_{propN(SRS)} - \bar{Y}_N) = \bar{Y}_N \left(-\Delta_{xN} + \frac{3}{2} \Delta_{xN}^2 + \alpha_N (\Delta_{yN} - \Delta_{yN} \Delta_{xN} - \frac{1}{2} \Delta_{xN}) \right) \quad (10)$$

Now take the expectation on equation (10) we get

$$Bias(\hat{Y}_{propN(SRS)}) = \bar{Y}_N \delta_N \left(\frac{3}{2} C_{xN}^2 - \alpha_N C_{yxN} \right)$$

Now for the MSE take the square on both the sides of equation (10), we get

$$(\hat{Y}_{propN(SRS)} - \bar{Y}_N)^2 = \bar{Y}_N^2 \left(\Delta_{xN}^2 + \alpha_N^2 \Delta_{yN}^2 - 2\alpha_N \Delta_{yN} \Delta_{xN} \right) \quad (11)$$

Now take the expectations on both the sides of equation (11), we get

$$MSE(\hat{Y}_{propN(SRS)}) = \delta_N \bar{Y}_N^2 (C_{xN}^2 + \alpha_N^2 C_{yN}^2 - 2\alpha_N C_{yxN})$$

Now to optimize the value of α_N take the derivation with respect to α_N of $MSE(\hat{Y}_{propN(SRS)})$ we get

$$\begin{aligned} \frac{d}{d\alpha_N} MSE(\hat{Y}_{propN(SRS)}) &= 0 \\ \frac{d}{d\alpha_N} \left\{ \bar{Y}_N^2 \theta_N (C_{xN}^2 + \alpha_N^2 C_{yN}^2 - 2\alpha_N \rho_{yxN} C_{yN} C_{xN}) \right\} &= 0 \\ \bar{Y}_N^2 \theta_N [0 + 2\alpha_N C_{yN}^2 - 2\rho_{yxN} C_{yN} C_{xN}] &= 0 \\ 2\alpha_N C_{yN}^2 &= 2\rho_{yxN} C_{yN} C_{xN} \\ \alpha_N &= \frac{\rho_{yxN} C_{xN}}{C_{yN}} = \varphi_N \\ MSE(\hat{Y}_{propN(SRS)}) &= \delta_N \bar{Y}_N^2 (C_{xN}^2 + \varphi_N^2 C_{yN}^2 - 2\varphi_N C_{yxN}) \end{aligned} \quad (12)$$

4. Proposed Estimator Theoretical Comparison in Stratified Random Sampling

The following observations will be made when we compare the suggested estimator $\hat{Y}_{prop(SRS)}$ with the existing estimators covered in Sect. (2).

Observation 1: from equation (1) and (12)

$$MSE(\hat{Y}_{propN(SRS)}) < MSE(\hat{Y}_0), \quad \text{if and only if } MSE(\hat{Y}_0) - MSE(\hat{Y}_{propN(SRS)}) > 0,$$

$$\text{Or if } \delta_N \bar{Y}_N^2 (C_{yN}^2 (1 - \varphi_N^2) - C_{xN}^2 + 2\varphi_N C_{yxN}) > 0.$$

Observation 2: from equation (2) and (12)

$$MSE(\hat{Y}_{propN(SRS)}) < MSE(\hat{Y}_{RN}), \quad \text{if and only if } MSE(\hat{Y}_{RN}) - MSE(\hat{Y}_{propN(SRS)}) > 0,$$

$$\text{Or if } \delta_N \bar{Y}_N^2 [C_{yN}^2 (1 - \varphi_N^2) - 2C_{yxN} (1 + \varphi_N)] > 0.$$

Observation 3: from equation (3) and (12)

$$MSE(\hat{Y}_{propN(SRS)}) < MSE(\hat{Y}_{expN}), \quad \text{if and only if } MSE(\hat{Y}_{expN}) - MSE(\hat{Y}_{propN(SRS)}) > 0,$$

$$\text{Or if } \delta_N \bar{Y}_N^2 \left[C_{yN}^2 (1 - \varphi_N^2) - \frac{3}{4} C_{xN}^2 - C_{yxN} (1 + 2\varphi_N) \right] > 0.$$

Observation 4: from equation (4) and (12)

$$MSE(\hat{Y}_{propN(SRS)}) < MSE(\hat{Y}_{SDrN}), \quad \text{if and only if } MSE(\hat{Y}_{SDrN}) - MSE(\hat{Y}_{propN(SRS)}) > 0,$$

$$\text{Or if } \delta_N \bar{Y}_N^2 \left[C_{yN}^2 (1 - \varphi_N^2) + \left(\left(\frac{\bar{X}_N}{\bar{X}_N + C_{xN}} \right)^2 - 1 \right) C_{xN}^2 - \left(\frac{\bar{X}_N}{\bar{X}_N + C_{xN}} + 2\varphi_N \right) C_{yxN} \right] > 0.$$

Observation 5: from equation (5) and (12)

$$MSE(\hat{Y}_{propN(SRS)}) < MSE(\hat{Y}_{US1rN}), \quad \text{if and only if } MSE(\hat{Y}_{US1rN}) - MSE(\hat{Y}_{propN(SRS)}) > 0,$$

$$\text{Or if } \delta_N \bar{Y}_N^2 \left[C_{yN}^2 (1 - \varphi_N^2) + \left(\left(\frac{\bar{X}_N \beta_{2(x)N}}{\bar{X}_N \beta_{2(x)N} + C_{xN}} \right)^2 - 1 \right) C_{xN}^2 - 2C_{yxN} \left(\frac{\bar{X}_N \beta_{2(x)N}}{\bar{X}_N \beta_{2(x)N} + C_{xN}} + \varphi_N \right) \right] > 0.$$

Observation 6: from equation (6) and (12)

$$MSE(\hat{Y}_{propN(SRS)}) < MSE(\hat{Y}_{US2rN}), \quad \text{if and only if } MSE(\hat{Y}_{US2rN}) - MSE(\hat{Y}_{propN(SRS)}) > 0,$$

$$\text{Or if } \delta_N \bar{Y}_N^2 \left[C_{yN}^2 (1 - \varphi_N^2) + \left(\left(\frac{\bar{X}_N C_{xN}}{\bar{X}_N C_{xN} + \beta_{2(x)N}} \right)^2 - 1 \right) C_{xN}^2 - 2C_{yxN} \left(\frac{\bar{X}_N C_{xN}}{\bar{X}_N C_{xN} + \beta_{2(x)N}} + \varphi_N \right) \right] > 0.$$

Observation 7: from equation (7) and (12)

$$MSE(\hat{Y}_{propN(SRS)}) < MSE(\hat{Y}_{SKrN}), \quad \text{if and only if } MSE(\hat{Y}_{SKrN}) - MSE(\hat{Y}_{propN(SRS)}) > 0,$$

$$\text{Or if } \delta_N \bar{Y}_N^2 \left[C_{yN}^2 (1 - \varphi_N^2) + \left(\left(\frac{\bar{x}_N}{\bar{x}_N + \beta_{2(x)N}} \right)^2 - 1 \right) C_{xN}^2 - 2C_{yxN} \left(\frac{\bar{x}_N}{\bar{x}_N + \beta_{2(x)N}} + \varphi_N \right) \right] > 0.$$

Observation 8: from equation (8) and (12)

$$MSE(\hat{Y}_{propN(SRS)}) < MSE(\hat{Y}_{KNN}), \text{ if and only if } MSE(\hat{Y}_{KNN}) - MSE(\hat{Y}_{propN(SRS)}) > 0,$$

$$\text{Or if } \delta_N \bar{Y}_N^2 \left[C_{yN}^2 (1 - \varphi_N^2) + C_{xN}^2 \left(\frac{\alpha^2}{a^2 h^2} - 1 \right) - 2C_{yxN} \left(\frac{\alpha}{ah} + \varphi_N \right) \right] > 0.$$

5. Numerical Study

5.1 Numerical Comparison for the real and simulated data set:

Table 1. Neutrosophic and Classical real data for the population parameter.

Neutrosophic Parameter	Neutrosophic Interval	Classical Parameter	Classical Value
N_N	(750, 850) countries/districts	N	800
n_N	(75, 80) regions	n	75
$\bar{X}_N$	(140000, 160000) tests	$\bar{X}$	150,000
$\bar{Y}_N$	(25000, 35000) cases	$\bar{Y}$	30,000
S_{xN}^2	(30000, 40000)	S_x^2	35,000
S_{yN}^2	(10000, 15000)	S_y^2	12,500
ρ_{yxN}	(0.5, 0.6)	ρ_{yx}	0.55
C_{xN}	(0.001237, 0.00125)	C_x	0.0012472
C_{yN}	(0.0004, 0.003499271)	C_y	0.0037
$\beta_{2(x)N}$	(3.0, 3.4)(slightly leptokurtic)	$\beta_{2(x)}$	3.2

5.2 Empirical study

To assess the effectiveness and efficiency of the suggested neutrosophic ratio-type estimators, we empirically investigated the situation using actual COVID-19 data. The number of confirmed cases in different locations as well as the overall number of COVID-19 tests performed are included in this dataset. The study variable (Y) indicates the number of verified COVID-19 cases, whereas the auxiliary variable (X) shows the number of tests performed.

5.3 Description of the Data

The COVID-19 dataset was gathered from publicly accessible and validated sources, such as Johns Hopkins University (JHU) and the World Health Organization (WHO). The information was taken out for the time frame of March 2020–December 2021. India is the region chosen for this study, which includes data from several states and union territories.

Data Sources

- **World Health Organization (WHO):** <https://covid19.who.int>
- **Johns Hopkins University (JHU) COVID-19 Dashboard:** <https://coronavirus.jhu.edu/map.html>

Table 2. MSE of Neutrosophic and Classical estimators for the real population.

Estimator	MSE Neutrosophic	Estimator	MSE Classical
$\bar{y}_N$	(129.52381, 169.852941)	$\bar{y}$	151.0416667
$\bar{y}_{RN}$	(101.853441, 118.717603)	$\bar{y}_R$	112.3553024
$\bar{y}_{expN}$	(112.590958, 138.866786)	$\bar{y}_{exp}$	127.4693179
$\bar{y}_{SDrN}$	(101.853441, 118.717603)	$\bar{y}_{SDr}$	112.3553026
$\bar{y}_{US1rN}$	(101.853441, 118.717603)	$\bar{y}_{US1r}$	112.3553025

$\tilde{y}_{US2rN}$	(102.117179, 119.2161305)	$\tilde{y}_{US2r}$	112.7261892
$\tilde{y}_{SKrN}$	(101.853768, 118.718229)	$\tilde{y}_{SKr}$	112.3557668
$\tilde{y}_{KNN}$	(97.1428571, 108.705882)	$\tilde{y}_{KN}$	105.3515625
$\tilde{y}_{PropN}$	(9.29300292, 13.8713235)	$\tilde{y}_{Prop}$	11.799375

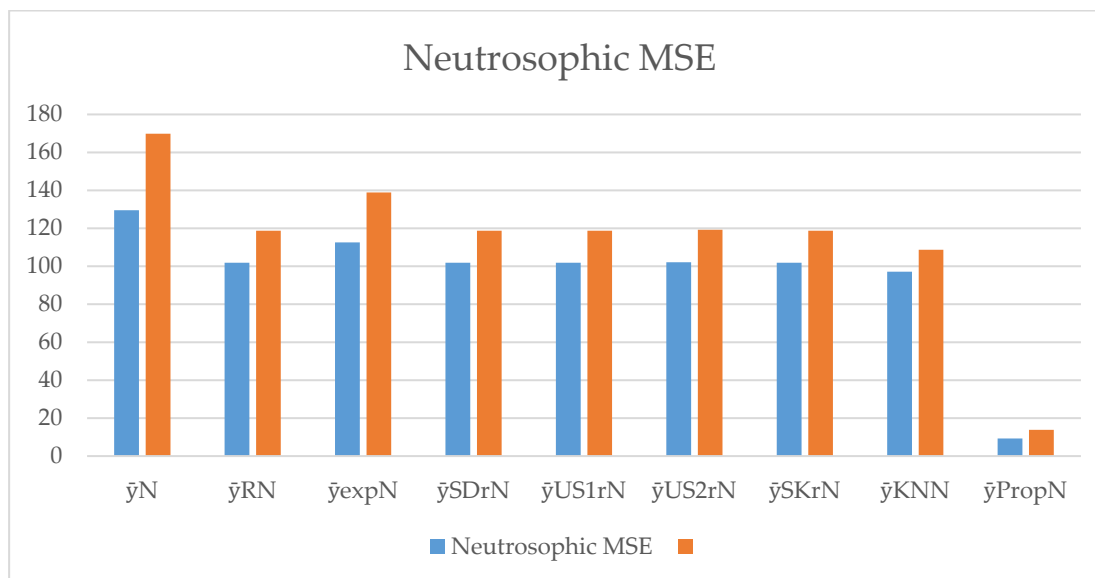


Figure 1. Neutrosophic MSE of estimators for the real population.

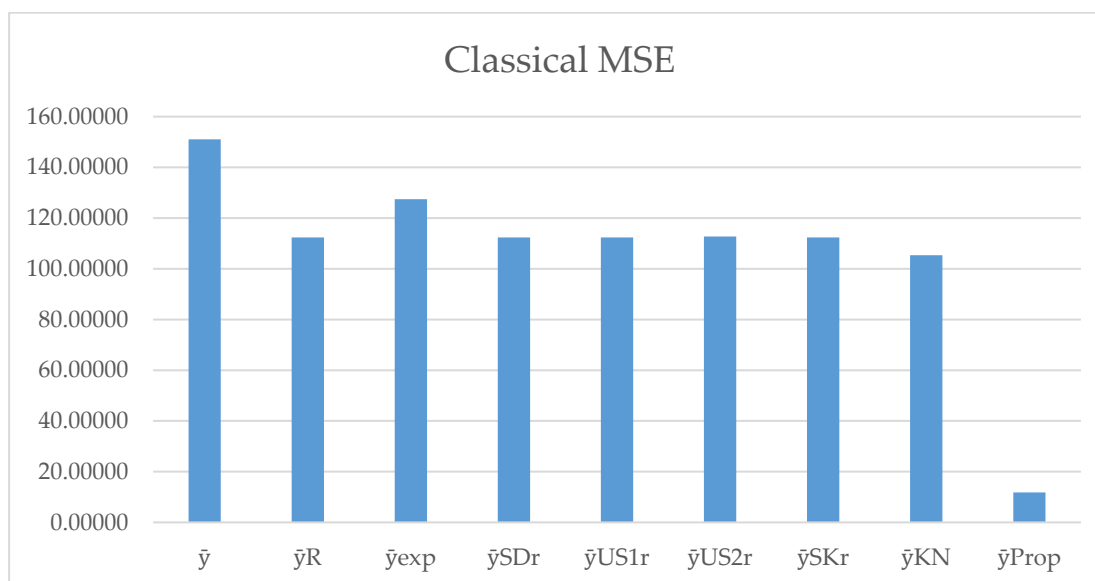


Figure 2. Classical MSE of estimators for the real population.

Table 3. PRE of Neutrosophic and Classical estimators for the real population.

Estimator	PRE Neutrosophic	Estimator	PRE Classical
$\tilde{y}_N$	(100, 100)	$\tilde{y}$	100
$\tilde{y}_{RN}$	(127.17, 143.07)	$\tilde{y}_R$	134.432166
$\tilde{y}_{expN}$	(115.04, 122.31)	$\tilde{y}_{exp}$	118.4925668
$\tilde{y}_{SDrN}$	(127.17, 143.07)	$\tilde{y}_{SDr}$	134.4321658
$\tilde{y}_{US1rN}$	(127.17, 143.07)	$\tilde{y}_{US1r}$	134.432166
$\tilde{y}_{US2rN}$	(126.84, 142.47)	$\tilde{y}_{US2r}$	133.9898632

$\tilde{y}_{SKrN}$	(127.17, 143.07)	$\tilde{y}_{SKr}$	134.4316104
$\tilde{y}_{KNN}$	(133.33, 156.25)	$\tilde{y}_{KN}$	143.3691756
$\tilde{y}_{PropN}$	(1224.49, 1393.78)	$\tilde{y}_{Prop}$	1280.081925

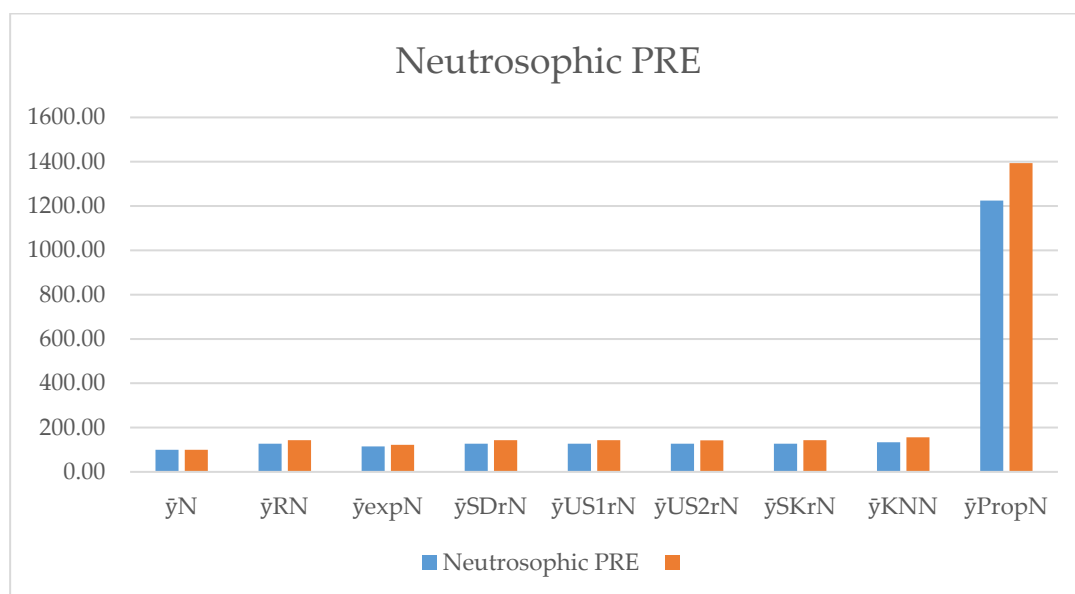


Figure 3. Neutrosophic PRE of estimators for the real population.

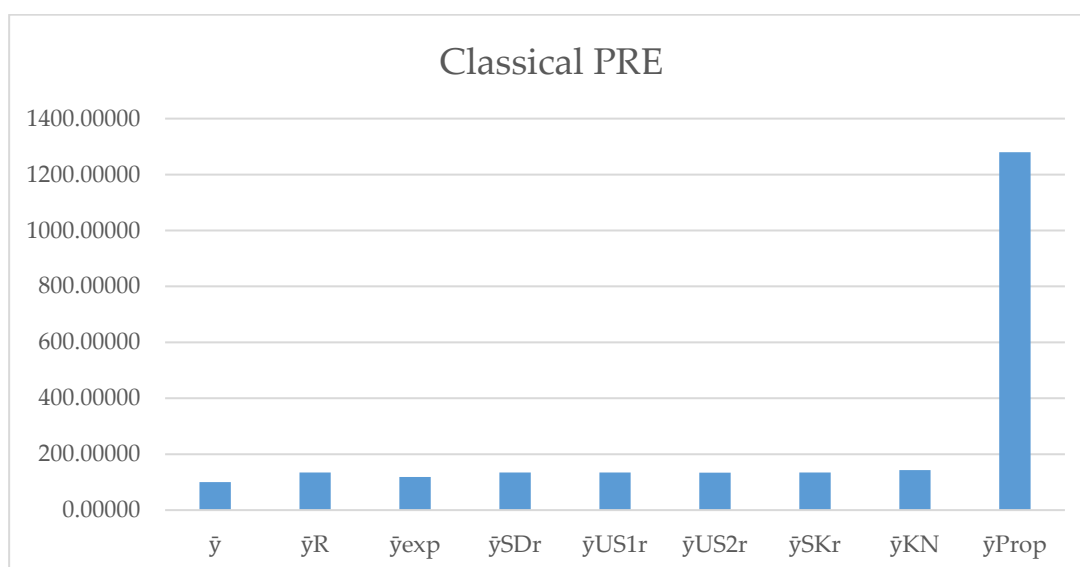


Figure 4. Classical PRE of estimators for the real population.

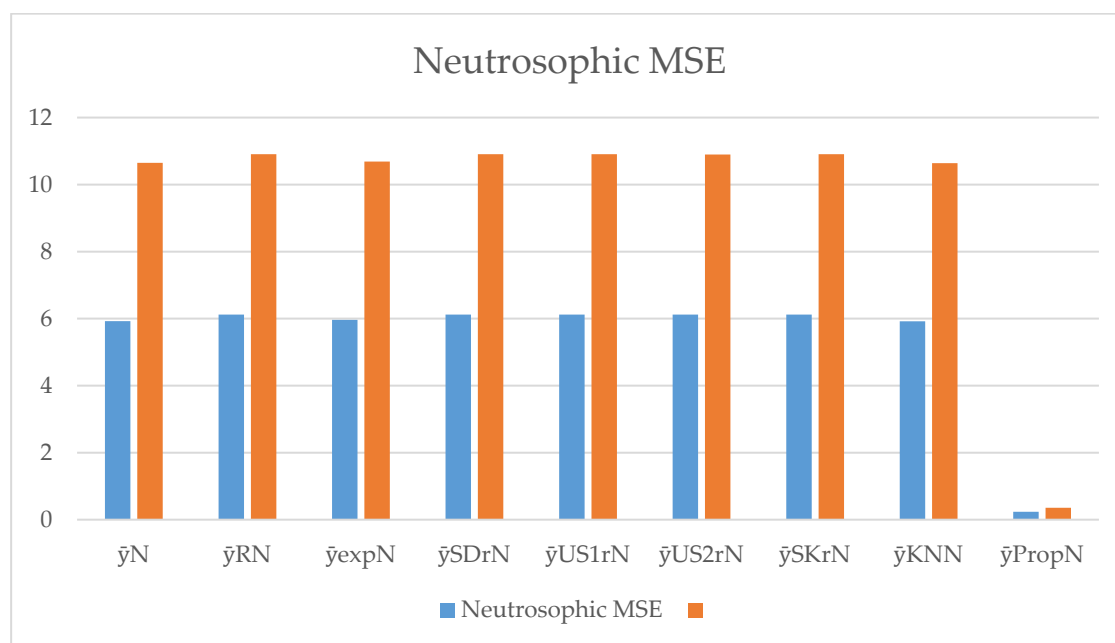
Table 4. Neutrosophic and Classical simulated data for the population parameter.

Neutrosophic Parameter	Neutrosophic Interval	Classical Parameter	Classical Value
N_N	(1200, 1200)	N	1200
n_N	(30, 30)	n	30
$\bar{X}_N$	(160.5, 175.3)	$\bar{X}$	169.222
$\bar{Y}_N$	(70.5, 80.7)	$\bar{Y}$	76.2299
S_{xN}^2	(37.21, 51.84)	S_x^2	41.119774
S_{yN}^2	(182.25, 327.61)	S_y^2	183.23519

ρ_{yxN}	(0.015, 0.025)	ρ_{yx}	0.0245
C_{xN}	(0.38006, 0.041072)	C_x	0.379
C_{yN}	(0.1914894, 0.2242875)	C_y	0.1776
$\beta_{2(x)N}$	(0.07, 0.21)	$\beta_{2(x)}$	0.1306

Table 5. MSE of Neutrosophic and Classical estimators for the simulated population.

Estimator	MSE Neutrosophic	Estimator	MSE Classical
$\bar{y}_N$	(5.92, 10.65)	$\bar{y}$	5.96
$\bar{y}_{RN}$	(6.12, 10.91)	$\bar{y}_R$	6.16
$\bar{y}_{expN}$	(5.964, 10.69)	$\bar{y}_{exp}$	5.99
$\bar{y}_{SDrN}$	(6.12, 10.91)	$\bar{y}_{SDr}$	6.164
$\bar{y}_{US1rN}$	(6.12, 10.91)	$\bar{y}_{US1r}$	6.16
$\bar{y}_{US2rN}$	(6.12, 10.9)	$\bar{y}_{US2r}$	6.16
$\bar{y}_{SKrN}$	(6.121, 10.906)	$\bar{y}_{SKr}$	6.164
$\bar{y}_{KNN}$	(5.922, 10.641)	$\bar{y}_{KN}$	5.95
$\bar{y}_{PropN}$	(0.23, 0.36)	$\bar{y}_{Prop}$	0.27

**Figure 5.** Neutrosophic MSE of estimators for the simulated population.

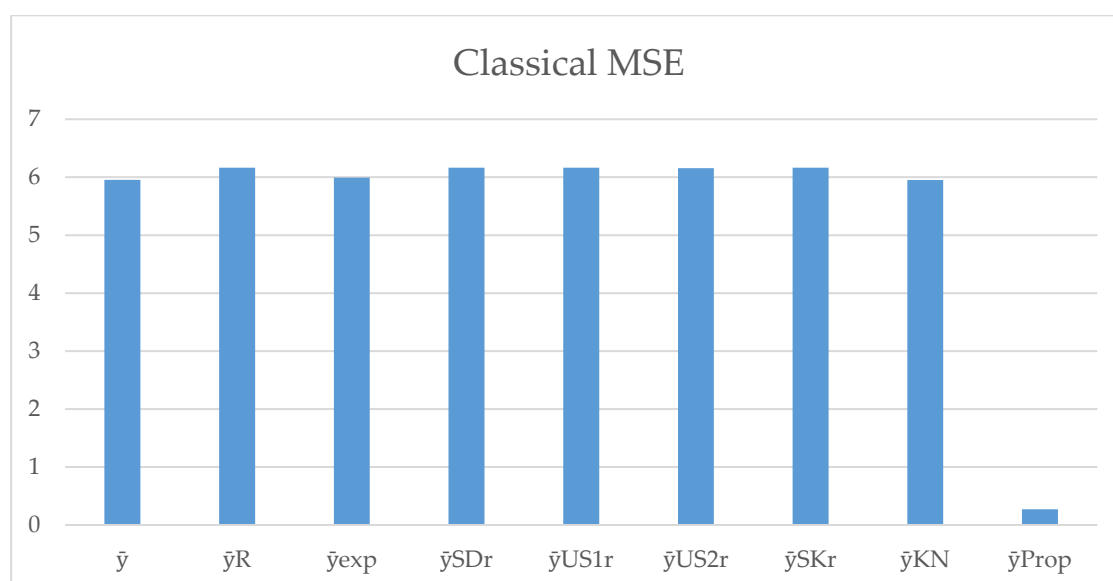


Figure 6. Classical MSE of estimators for the simulated population.

Table 6. PRE of Neutrosophic and Classical estimators for the simulated population.

Estimator	PRE Neutrosophic	Estimator	PRE Classical
$\bar{y}_N$	(100, 100)	$\bar{y}$	100
$\bar{y}_{RN}$	(96.76, 97.62)	$\bar{y}_R$	96.61
$\bar{y}_{expN}$	(99.32, 99.62)	$\bar{y}_{exp}$	99.39
$\bar{y}_{SDrN}$	(96.77, 97.62)	$\bar{y}_{SDr}$	96.61
$\bar{y}_{US1rN}$	(96.79, 99.63)	$\bar{y}_{US1r}$	96.63
$\bar{y}_{US2rN}$	(96.8, 97.69)	$\bar{y}_{US2r}$	96.76
$\bar{y}_{SKrN}$	(96.77, 97.63)	$\bar{y}_{SKr}$	96.62
$\bar{y}_{KNN}$	(100.02, 100.06)	$\bar{y}_{KN}$	100.06
$\bar{y}_{PropN}$	(2539.09, 2983.87)	$\bar{y}_{Prop}$	2197.26

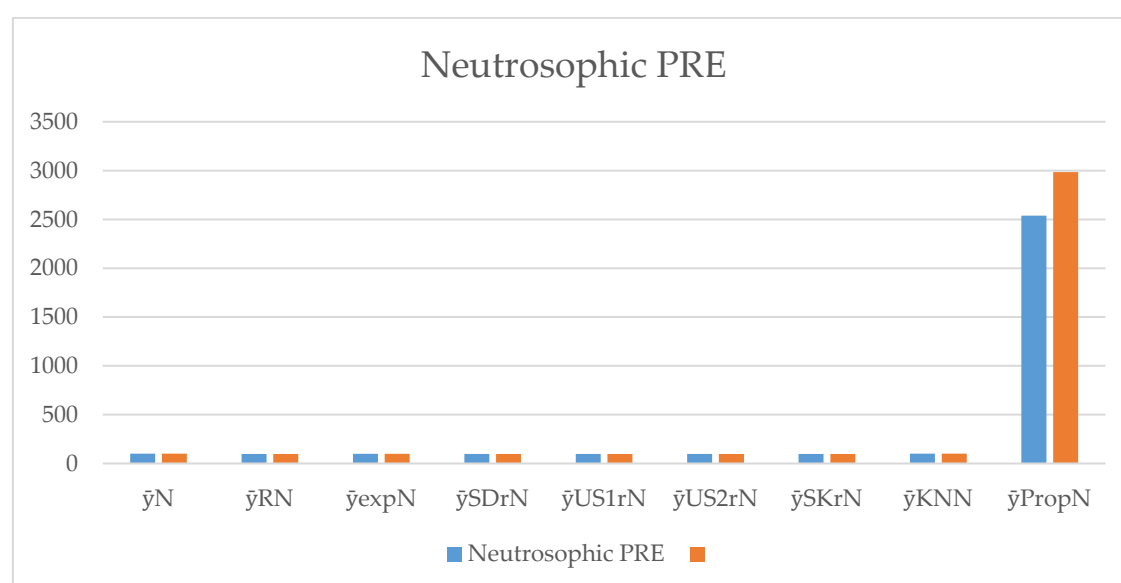


Figure 7. Neutrosophic PRE of estimators for the simulated population.

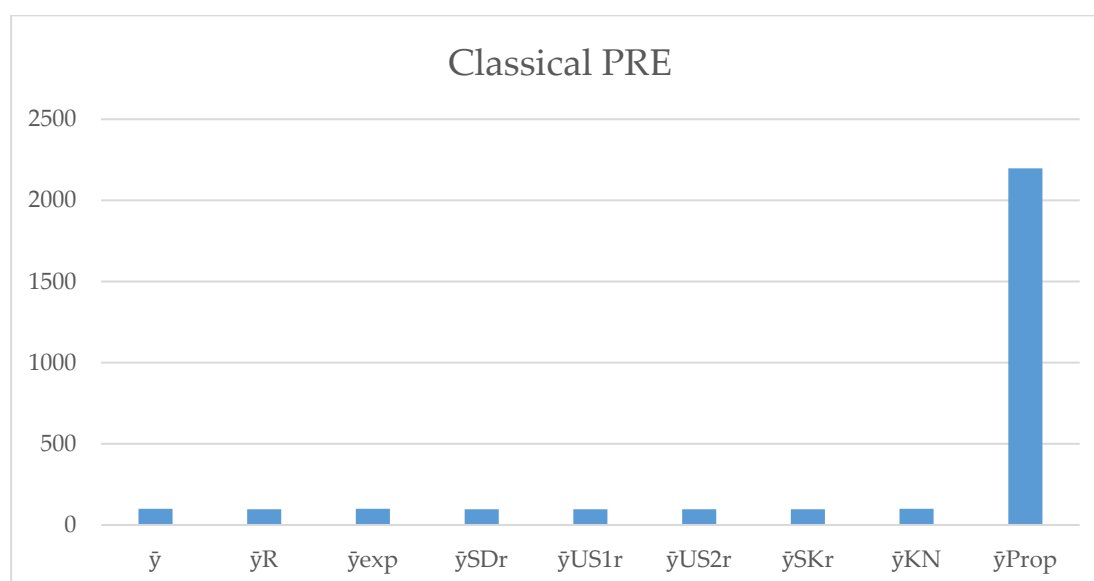


Figure 8. Classical PRE of estimators for the simulated population.

6. Result Analysis and Conclusion

Real Data Results: The findings from the COVID-19 real dataset, as summarized in Table 2, along with the Figure 1 & 2 shows that the proposed neutrosophic exponential-ratio estimator has the least MSE value when compared to the classical and alternative neutrosophic estimators. Specifically, the suggested estimator is having superior MSE intervals, giving more precision. as per shown in table 2, the range of the MSE of the suggested estimator is (9.293, 13.871), while the traditional estimator has an MSE of 11.799. Similarly, The PRE values shown in Table 3 along with the Figure 3 & 4 that is also demonstrate considerable efficiency gains. The suggested estimator gets the PRE values ranging from 1224.49% to 1393.78% when compared to the traditional estimator, demonstrating its strength and effectiveness in handling neutrosophic data.

Simulated Data Results: The simulated dataset also given in Tables 4. The suggested estimator consistently performs better than the traditional classical and alternative neutrosophic estimators in terms of MSE and PRE. As per shown in Table 5 along with Figure 5 & 6, the MSE range for the suggested estimator is (0.23, 0.36), while the standard version has an MSE of 0.27. By observing the Table 6 and Figure 7 & 8 the suggested estimator has the range of PRE values as high as 2539.09% to 2983.87%, indicating remarkable performance in contexts with varied levels of uncertainty.

7. Conclusion

This study introduces an improved neutrosophic exponential-ratio estimator for estimating population mean using simple random sampling. The data uncertainties were effectively resolved and the proposed method was found to outperform with the existing estimators in terms of MSE and PRE. Regarding types of data, neutrosophic statistics show more scope and flexibility than traditional statistics enabling them to work with complex data better. This approach must be considered in further research in connection with other designs of sampling such as stratified or cluster sampling and in other domains where uncertainty is an issue with the data.

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Painting Effects Evaluation Based on Artificial Intelligence Technology using the Triangular OverNorm

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Abstract:

The field of art, especially digital painting, has seen a radical change since the introduction of artificial intelligence (AI). As AI-generated art grows more popular, it is more important than ever to assess the usefulness and aesthetic appeal of these pieces. To assess painting effects using AI technology, this study offers a thorough framework that combines artistic sensibility with technical accuracy. To examine the effectiveness and impact of seven different AI-based painting techniques, eight assessment criteria are used. By providing insights into how machines understand, mimic, and possibly enhance human creativity in the visual arts, the study seeks to bridge the gap between computational developments and artistic interpretation. This study uses the multi-criteria decision-making approach (MCDM). The AROMAN method is used to rank the alternatives. The numerical example is provided in this study with eight criteria and seven alternatives. The sensitivity analysis is provided in this study to show the stability of the ranks. The results show the ranks of alternatives are stable in different cases.

Keywords: Triangular OverNorm; Painting Effects; Artificial Intelligence; Multi-Criteria Decision making.

1. Introduction

A new creative frontier has emerged from the nexus of art and artificial intelligence. Conventional ideas of authorship and aesthetics are challenged by AI-generated paintings, which range from neural style transfers to GAN-based art generation. As these technologies develop, a systematic assessment is required to determine their aesthetic and perceptual worth[1], [2].

Subjective interpretation is a major component of traditional art appraisal techniques. To guarantee consistency and quality, however, objective and repeatable measures are also required when AI is involved. A comprehensive strategy that blends algorithmic evaluation with human aesthetic judgment is made possible using computational analysis[3], [4].

AI systems are being trained to mimic well-known artists' brushstrokes, color schemes, and textures in the context of digital painting. Although there are great opportunities for entertainment and education, there are also difficulties with originality and authenticity[5], [6].

A set of eight standards for assessing AI-generated painting effects is presented in this study. These encompass both artistic elements like color harmony and emotional expressiveness as well as technical ones like resolution and style coherence. The goal is to provide a comprehensive model for evaluating AI-assisted art's efficacy[7], [8].

Seven painting techniques or alternatives have been chosen for comparison. These cover a variety of AI models and methodologies, such as diffusion models, generative adversarial networks (GANs), and neural style transfer. Every technique has its own advantages and visual results.

To provide fair assessments across multiple metrics, the evaluation methodology makes use of multi-criteria decision-making (MCDM) methodologies. Ranking techniques like AROMAN could be applied, enabling a more methodical investigation of each painting model's performance in relation to the criteria.

In addition to evaluating current painting technologies, one of the study's objectives is to provide developers and artists with guidance on enhancing the creative potential of AI tools. Better interface design and algorithmic improvement can be guided by the input from these assessments[9], [10].

This study aims to codify the assessment of AI-generated art, a field that is still in its early stages. It seeks to encourage more thorough research in the area by providing a formal framework, thereby establishing previously unheard-of connections between creativity and technology.

Finding and assessing elements is a common issue in decision-making. One popular decision-making technique that works well for this task is MCDM technology. Many disciplines have made extensive use of MCDM approaches. MCDM techniques have been widely used, especially in Painting Effects Evaluation[11], [12].

The ability of MCDM approaches to thoroughly assess and balance a variety of factors, gives them distinct advantages[13]. Furthermore, MCDM techniques combine quantitative and qualitative data to guarantee comprehensive analysis. MCDM techniques help decision-makers better manage important risks and, in the end, improve the Painting Effects Evaluation by facilitating an open and explicable decision-making process[14], [15].

Nevertheless, there hasn't been much research done on using MCDM methodologies to examine Painting Effects Evaluation. All possible structural elements that can affect regular operations

were not taken into consideration in the investigations. Furthermore, there hasn't been a clear explanation of how to translate research findings into practical maintenance plans in the literature. This study will methodically examine the structural factor analysis problem of Painting Effects Evaluation to enhance assessment performance[16], [17]. This article's main contribution is the creation of a novel MCDM framework for evaluating Painting Effects Evaluation. The results of the analysis improve MCDM methodology.

Smarandache (2007) expanded the uncertain set to include uncertain OverSet (when some component is > 1) because he noted that, for instance, an employee who works overtime should have a degree of membership > 1 , whereas an employee who works regular full-time and has a degree of membership $= 1$ [18], [19]; uncertain UnderSet (when some neutrosophic component is < 0), because, for instance, an employee who causes more harm than good to his company should have a degree of membership < 0 , whereas an employee who benefits the company and has a degree of membership > 0 ; and uncertain OffSet (when some neutrosophic components are off the interval $[0, 1]$, i.e. some neutrosophic component > 1 and some neutrosophic component < 0)[20], [21], [22].

Subsequently, the concepts of uncertain logic, measure, probability, statistics, etc. were expanded to include uncertain over-, under-, and off-logic, measure, probability, statistics, etc. They use the term "uncertain" to refer to all fuzzy and fuzzy-extensions, including plithogenic, neutrosophic, spherical, intuitive, and others.

The following is a list of the more particular points:

The structural elements causing Painting Effects Evaluation have never been thoroughly identified and assessed until now. To thoroughly examine all potential components and their interactions, a specialized MCDM framework has been created.

The Triangular OverNorm assesses the relative significance of structural components in terms of occurrence probability and severity. Furthermore, the important components' cause-and-effect linkages are examined by the Triangular OverNorm average method.

The practical case study for Painting Effects Evaluation is carried out in this paper. Data sources include expert opinions gathered from qualified personnel and maintenance records reported at bus depots. This study offers practical Painting Effects Evaluation based on the assessment results.

By further improving roadworthiness, the results of this endeavor can guarantee more dependable Painting Effects Evaluation.

The work's maintenance recommendations have the potential to improve both financial performance and operational efficiency.

The structure of this document is as follows: Section shows the Triangular OverNorm definitions. The suggested approach for factor selection and relationship analysis is described in Section 3. By

using the suggested framework in the context of Painting Effects Evaluation, Section 4 provides specifics about the case study. Section 5 shows the sensitivity analysis. Section 6 concludes by summarizing the consequences and outlining possible lines of inquiry.

2. Triangular OverNorm

We show the definitions of Triangular OverNorm (t-OverNorm)[22].

t-OverNorm on interval $[0, \pi]$ and $\pi > 1$. (1)

$T_{overNorm}: [0, \pi] \times [0, \pi] \rightarrow [0, \pi]$ and $\pi > 1$ (2)

$T_{overNorm}(C, D) = T_{overNorm}(D, C)$, commutativity. (3)

$T_{overNorm}(C, T_{overNorm}(D, E)) = T_{overNorm}(T_{overNorm}(C, D), E)$ associativity. (4)

if $D \geq E$, then $T_{overNorm}(C, D) \leq T_{overNorm}(C, E)$ monotonicity. (5)

$T_{overNorm}(C, \pi) = X$ or the neutral components is π (6)

$T_{overNorm}(C, D) = \min(C, D)$ (7)

3. The AROMAN Method

We show the steps of the AROMAN method to rank the alternatives based on the opinions of experts and decision makers.

Build the decision matrix

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix} \quad (8)$$

Combine the decision matrix using the $T_{overNorm}$

Compute the criteria weights.

The criteria weights are computed using the average method.

Calculate the linear normalization

$$U_{ij} = \frac{x_{ij} - \min x_{ij}}{\max x_{ij} - \min x_{ij}} \quad (9)$$

Calculate the vector normalization

$$Q_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}} \quad (10)$$

Calculate the normalization matrix

$$H_{ij} = \frac{\delta U_{ij} + (1-\delta) Q_{ij}}{2} \quad (11)$$

The value of δ among 0 and 1.

Calculate the weighted normalized decision matrix.

$$N_{ij} = W_j H_{ij} \quad (12)$$

Compute the positive and negative values

$$A_i = \sum_{j=1}^n (N_{ij})^{\max} \quad (13)$$

$$B_i = \sum_{j=1}^n (N_{ij})^{\min} \quad (14)$$

Calculate the R values

$$R_i = A_i^\omega + B_i^{(1-\omega)} \quad (15)$$

The value of ω among 0 and 1

Rank the alternatives

4. Numerical Example

This study shows the numerical example of the proposed approach. We use eight criteria and seven alternatives. The criteria are:

Style Consistency (C1) – Alignment between original style and generated image.

Resolution Quality (C2) – Clarity and detail level of the final output.

Color Harmony (C3) – Aesthetic blending and use of color schemes.

Brushstroke Realism (C4) – Accuracy and naturalness of stroke patterns.

Emotional Expressiveness (C5) – Ability to evoke emotion or mood.

Content Preservation (C6) – Retention of subject integrity during transformation.

Rendering Speed (C7) – Time taken to generate the painting.

User Satisfaction (C8) – Subjective user ratings on overall impression.

The alternatives: Neural Style Transfer, CycleGAN, StyleGAN , DALL·E, Stable Diffusion, DeepArt.io , RunwayML's Art Generator.

Four experts created the decision matrix as shown in Table 1. Then we combine the decision matrix using the $T_{overNorm}$.

The criteria weights are computed using the average method.

C1= 0.07027027, C2= 0.108108108, C3=0.124324324, C4= 0.118918919, C5 = 0.145945946, C6 =0.145945946, C7= 0.113513514, C8= 0.172972973.

To ensure an effective and comprehensive evaluation of AI-generated painting effects, the following eight criteria are assigned normalized weights (summing to 1). These weights reflect the relative importance of each criterion in the overall assessment process.

C1 – Style Consistency (Weight: 0.0703)

This criterion evaluates how well the AI-generated painting retains the intended artistic style. With a lower weight, it suggests that while style consistency is valuable, it is considered less critical compared to other aspects in the evaluation framework.

C2 – Resolution Quality (Weight: 0.1081)

Resolution quality assesses the sharpness, clarity, and detail of the final painting. A moderate weight reflects its importance in ensuring visual fidelity, particularly for professional or print-ready outputs.

C3 – Color Harmony (Weight: 0.1243)

Color harmony refers to the aesthetic balance and compatibility of colors used in the AI-generated image. This moderate-high weight highlights its artistic significance in conveying mood and visual coherence.

C4 – Brushstroke Realism (Weight: 0.1189)

This measures how convincingly the AI mimics natural brushstrokes. With a balanced weight, it indicates an appreciation for artistic authenticity, especially in simulations of classical painting styles.

C5 – Emotional Expressiveness (Weight: 0.1459)

This criterion captures the AI painting's ability to evoke emotions or atmosphere. A high weight underlines the critical role of emotional depth and engagement in evaluating artistic value.

C6 – Content Preservation (Weight: 0.1459)

Content preservation assesses how accurately the main elements or subjects are retained during the transformation or stylization process. Sharing the highest weight with C5, this criterion emphasizes clarity and recognizability in AI artworks.

C7 – Rendering Speed (Weight: 0.1135)

This refers to how quickly the AI can produce a final image. While not the highest priority, its decent weight reflects the practical importance of time efficiency, particularly in commercial or iterative workflows.

C8 – User Satisfaction (Weight: 0.1730)

User satisfaction aggregates subjective feedback from users about the overall appeal, usability, and artistic fulfillment. As the most heavily weighted criterion, it reflects the goal of AI painting: to please and inspire its audience.

These weights provide a balanced and user-centered evaluation model, combining technical, aesthetic, and experiential aspects of AI-generated paintings. The highest emphasis is placed on user satisfaction, emotional expressiveness, and content preservation, highlighting the importance of human impact and artistic integrity in AI art.

Table 1. The decision matrix.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	[0.1,2]	[0.3,1.4]	[0.5,1.5]	[0.6,0.1.6]	[0.7,0.1.7]	[0.8,1.8]	[0.9,1.9]	[0.5,1.5]
A_2	[0.1,2]	[0.9,1.9]	[0.8,1.8]	[0.7,0.1.7]	[0.6,0.1.6]	[0.5,1.5]	[0.1,2]	[0.6,0.1.6]
A_3	[0.3,1.4]	[0.5,1.5]	[0.6,0.1.6]	[0.7,0.1.7]	[0.8,1.8]	[0.3,1.4]	[0.3,1.4]	[0.7,0.1.7]
A_4	[0.6,0.1.6]	[0.5,1.5]	[0.3,1.4]	[0.1,2]	[0.9,1.9]	[0.1,2]	[0.5,1.5]	[0.8,1.8]
A_5	[0.7,0.1.7]	[0.6,0.1.6]	[0.5,1.5]	[0.3,1.4]	[0.1,2]	[0.9,1.9]	[0.6,0.1.6]	[0.9,1.9]
A_6	[0.7,0.1.7]	[0.7,0.1.7]	[0.6,0.1.6]	[0.7,0.1.7]	[0.9,1.9]	[0.8,1.8]	[0.7,0.1.7]	[0.8,1.8]
A_7	[0.8,1.8]	[0.8,1.8]	[0.5,1.5]	[0.8,1.8]	[0.8,1.8]	[0.7,0.1.7]	[0.8,1.8]	[0.7,0.1.7]
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	[0.6,0.1.6]	[0.3,1.4]	[0.5,1.5]	[0.6,0.1.6]	[0.7,0.1.7]	[0.8,1.8]	[0.9,1.9]	[0.5,1.5]
A_2	[0.5,1.5]	[0.9,1.9]	[0.8,1.8]	[0.6,0.1.6]	[0.6,0.1.6]	[0.5,1.5]	[0.6,0.1.6]	[0.6,0.1.6]
A_3	[0.3,1.4]	[0.5,1.5]	[0.6,0.1.6]	[0.5,1.5]	[0.6,0.1.6]	[0.3,1.4]	[0.5,1.5]	[0.7,0.1.7]
A_4	[0.1,2]	[0.5,1.5]	[0.3,1.4]	[0.3,1.4]	[0.5,1.5]	[0.1,2]	[0.3,1.4]	[0.8,1.8]
A_5	[0.9,1.9]	[0.6,0.1.6]	[0.5,1.5]	[0.1,2]	[0.3,1.4]	[0.9,1.9]	[0.1,2]	[0.7,0.1.7]
A_6	[0.6,0.1.6]	[0.6,0.1.6]	[0.6,0.1.6]	[0.9,1.9]	[0.1,2]	[0.6,0.1.6]	[0.9,1.9]	[0.8,1.8]
A_7	[0.5,1.5]	[0.5,1.5]	[0.5,1.5]	[0.8,1.8]	[0.6,0.1.6]	[0.5,1.5]	[0.8,1.8]	[0.7,0.1.7]
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	[0.1,2]	[0.3,1.4]	[0.5,1.5]	[0.6,0.1.6]	[0.7,0.1.7]	[0.8,1.8]	[0.9,1.9]	[0.5,1.5]
A_2	[0.9,1.9]	[0.9,1.9]	[0.8,1.8]	[0.7,0.1.7]	[0.6,0.1.6]	[0.5,1.5]	[0.1,2]	[0.6,0.1.6]
A_3	[0.8,1.8]	[0.1,2]	[0.6,0.1.6]	[0.7,0.1.7]	[0.8,1.8]	[0.3,1.4]	[0.9,1.9]	[0.7,0.1.7]
A_4	[0.7,0.1.7]	[0.9,1.9]	[0.1,2]	[0.1,2]	[0.1,2]	[0.1,2]	[0.8,1.8]	[0.8,1.8]
A_5	[0.6,0.1.6]	[0.8,1.8]	[0.9,1.9]	[0.1,2]	[0.9,1.9]	[0.1,2]	[0.7,0.1.7]	[0.9,1.9]
A_6	[0.5,1.5]	[0.7,0.1.7]	[0.8,1.8]	[0.9,1.9]	[0.8,1.8]	[0.9,1.9]	[0.6,0.1.6]	[0.8,1.8]
A_7	[0.3,1.4]	[0.6,0.1.6]	[0.7,0.1.7]	[0.8,1.8]	[0.7,0.1.7]	[0.8,1.8]	[0.5,1.5]	[0.7,0.1.7]
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	[0.8,1.8]	[0.9,1.9]	[0.3,1.4]	[0.9,1.9]	[0.7,0.1.7]	[0.6,0.1.6]	[0.9,1.9]	[0.3,1.4]
A_2	[0.9,1.9]	[0.1,2]	[0.1,2]	[0.1,2]	[0.6,0.1.6]	[0.5,1.5]	[0.1,2]	[0.9,1.9]
A_3	[0.9,1.9]	[0.3,1.4]	[0.9,1.9]	[0.3,1.4]	[0.5,1.5]	[0.3,1.4]	[0.3,1.4]	[0.5,1.5]
A_4	[0.3,1.4]	[0.3,1.4]	[0.3,1.4]	[0.7,0.1.7]	[0.5,1.5]	[0.3,1.4]	[0.7,0.1.7]	[0.7,0.1.7]
A_5	[0.1,2]	[0.1,2]	[0.1,2]	[0.1,2]	[0.3,1.4]	[0.1,2]	[0.3,1.4]	[0.1,2]
A_6	[0.9,1.9]	[0.9,1.9]	[0.9,1.9]	[0.3,1.4]	[0.1,2]	[0.9,1.9]	[0.1,2]	[0.3,1.4]
A_7	[0.1,2]	[0.5,1.5]	[0.6,0.1.6]	[0.7,0.1.7]	[0.6,0.1.6]	[0.7,0.1.7]	[0.3,1.4]	[0.9,1.9]

We calculate the linear normalization using eq. (9) as shown in Fig 1.

We calculate the vector normalization using eq. (10) as shown in Fig 2.

We Calculate the normalization matrix using eq. (11) as shown in Fig 3.

We calculate the weighted normalized decision matrix using eq. (12) as shown in Fig 4.

We compute the positive and negative values using eqs. (13 and 14).

We calculate the R values using eq. (15) as shown in Fig 5.

We rank the alternatives as shown in Fig 6.

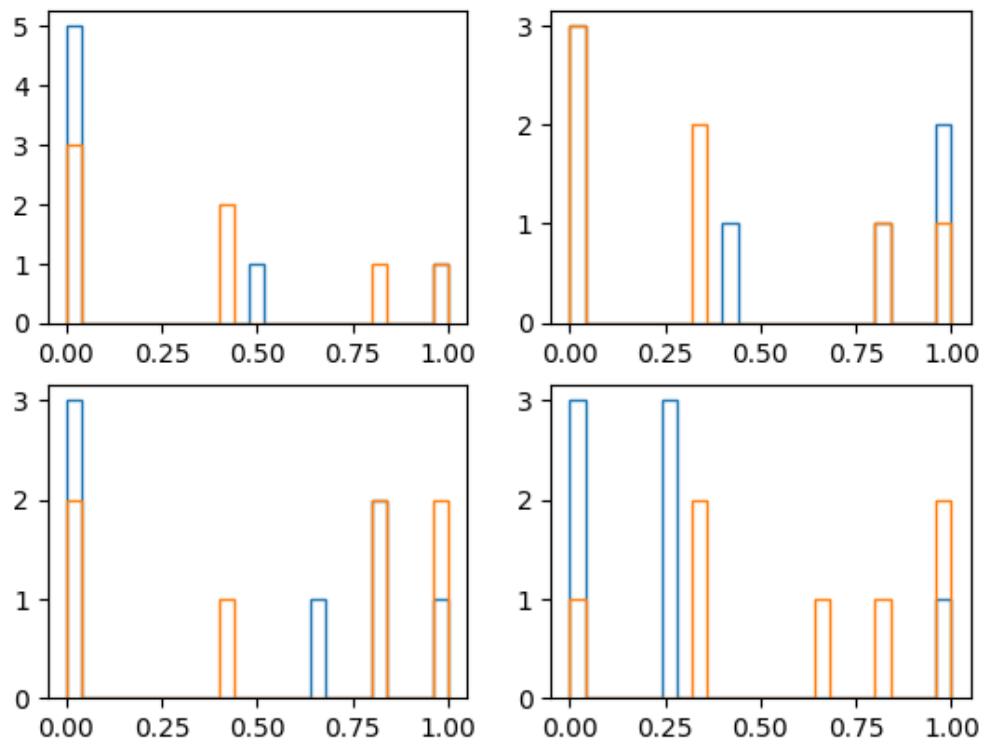


Fig 1. The linear normalization.

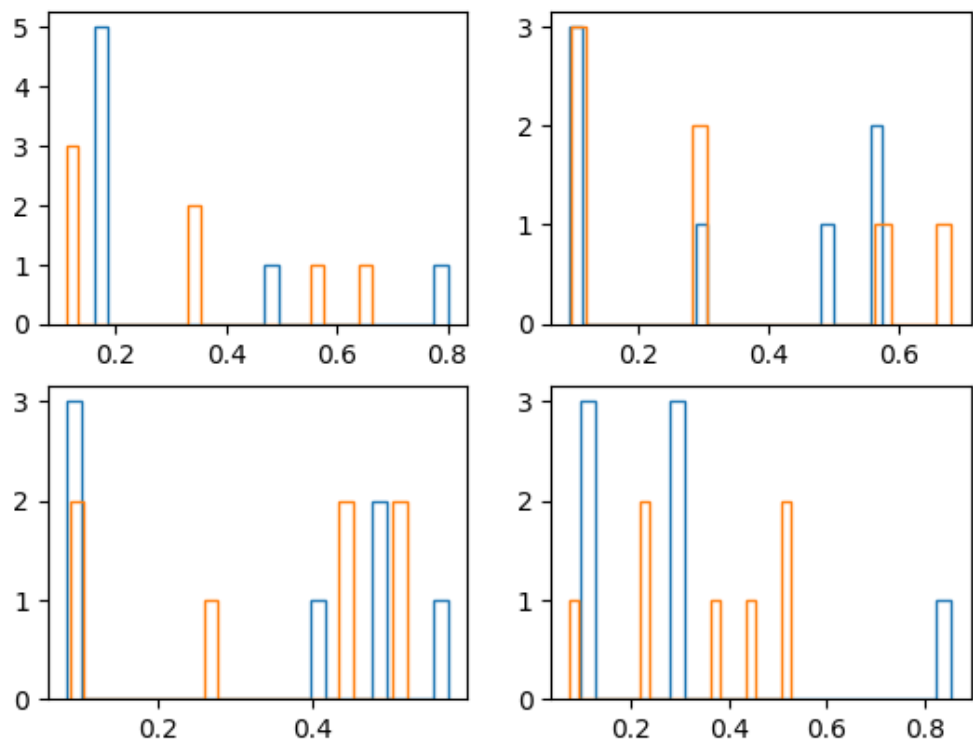


Fig 2. The vector normalization.

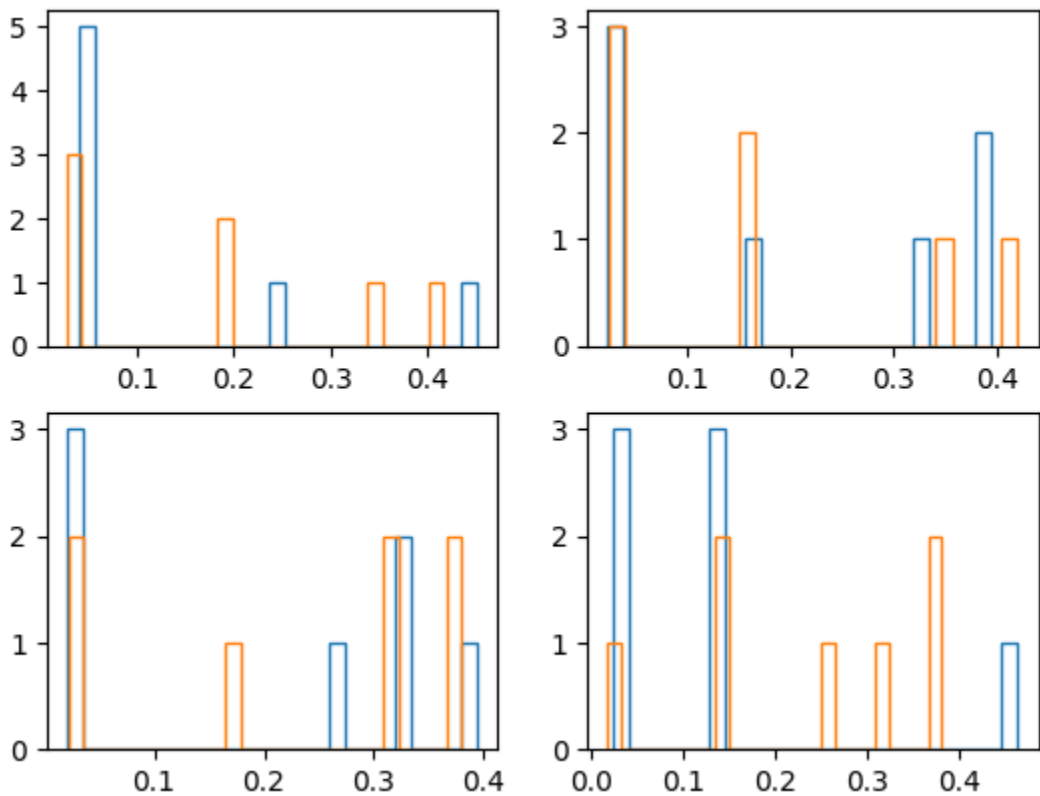


Fig 3. The normalization matrix.

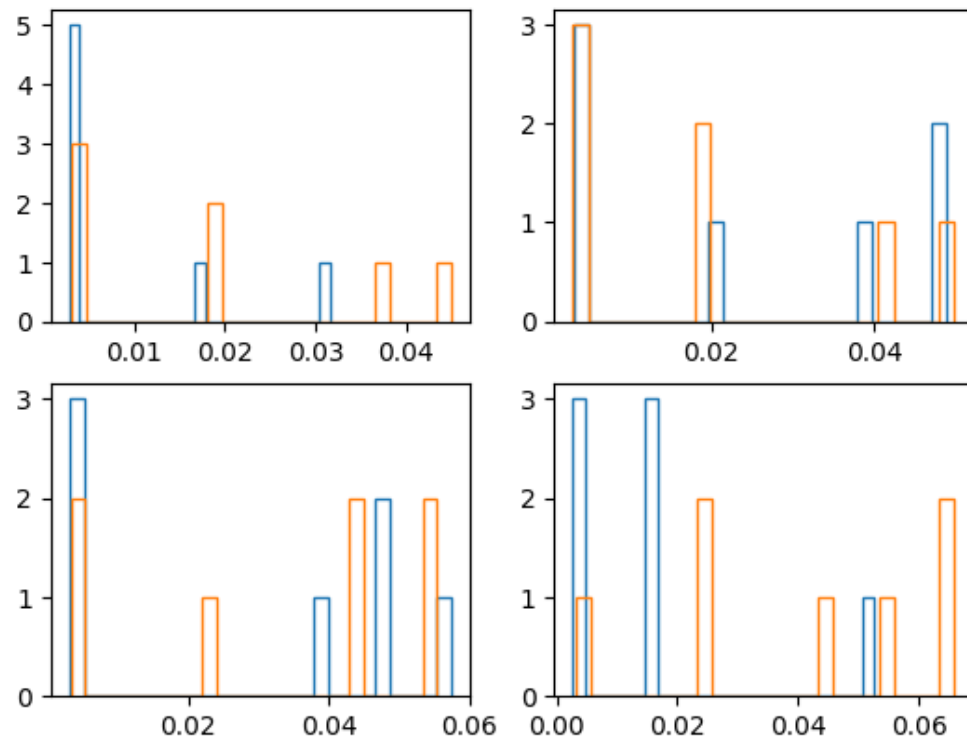


Fig 4. The weighted normalization matrix.

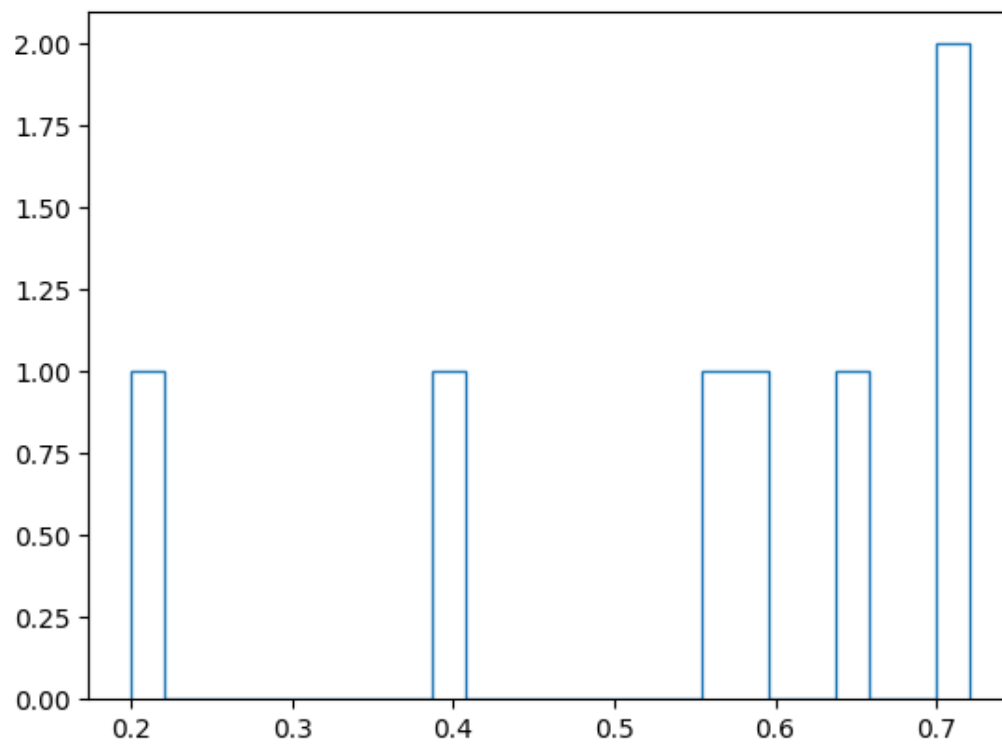


Fig 5. The R values.

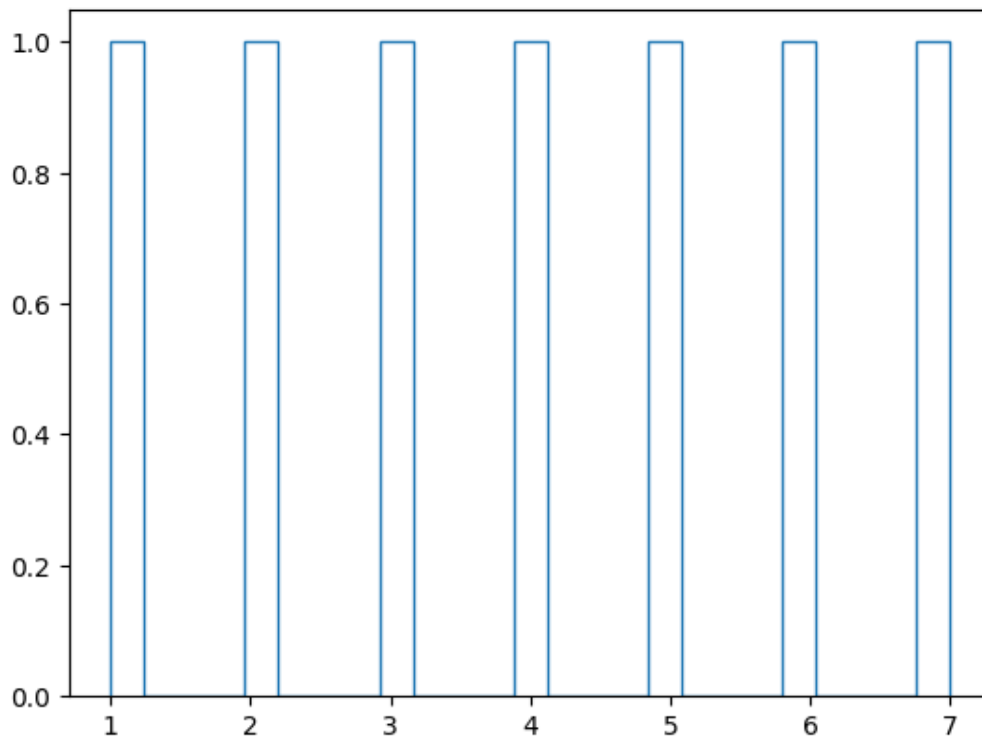


Fig 6. The ranks of alternatives.

5. Analysis

This section is divided into two parts. In the first part, we show the sensitivity analysis by changing the value of δ between 0 and 1 as in Eq. (11). Then we apply the proposed approach to show different R values as shown in Fig 7. Then we rank the alternatives as shown in Fig 8.

Fig 8 presents the rank positions of seven alternatives (A1 to A7) evaluated over ten scenarios (Case 1 to Case 10). The rankings are on a scale from 1 (best) to 7 (worst).

A1: Constant Rank: 6 in all cases

A1 is consistently ranked second-worst across all evaluations. This suggests it performs poorly relative to other alternatives with little to no variation.

A2: Mostly Rank: 3, Rank 4 in Case 10

A2 maintains a steady mid-range performance, showing moderate quality. The slight dip in Case 10 indicates a minor drop in perceived value or performance.

A3: Rank 5 in Case 1, then Rank 4 (mostly), Rank 3 in Case 10

A3 shows slight improvement across scenarios. It starts below average but edges closer to the top half, indicating growing acceptance or effectiveness.

A4: Constant Rank: 2 (except Case 10: Rank 1)

A4 is a consistently high performer, only beaten by one alternative (A5). Its rise to Rank 1 in Case 10 implies improvement or shift in evaluator preference.

A5: Constant Rank: 1 in all cases

A5 is the top-ranked alternative in every single case. This indicates it is consistently the best performer, likely across all evaluation criteria.

A6: Rank 4 in Case 1, then Rank 5 (mostly), Rank 6 in Case 10

A6 shows slightly declining performance, especially in later cases, pointing to potential quality degradation or weaker adaptation over time.

A7: Constant Rank: 7 in all cases except Case 10: Rank 4

A7 is the lowest ranked alternative in nearly all cases. The sudden jump to Rank 4 in Case 10 may suggest a situational improvement or reevaluation in that scenario

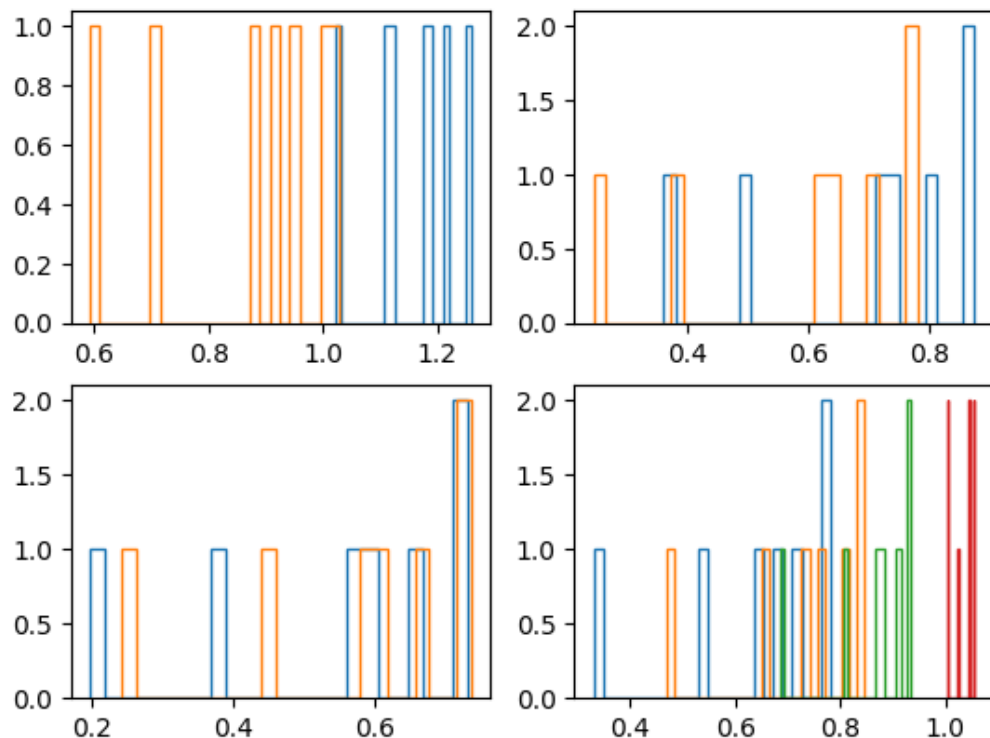


Fig 7. Th R value under different δ values.

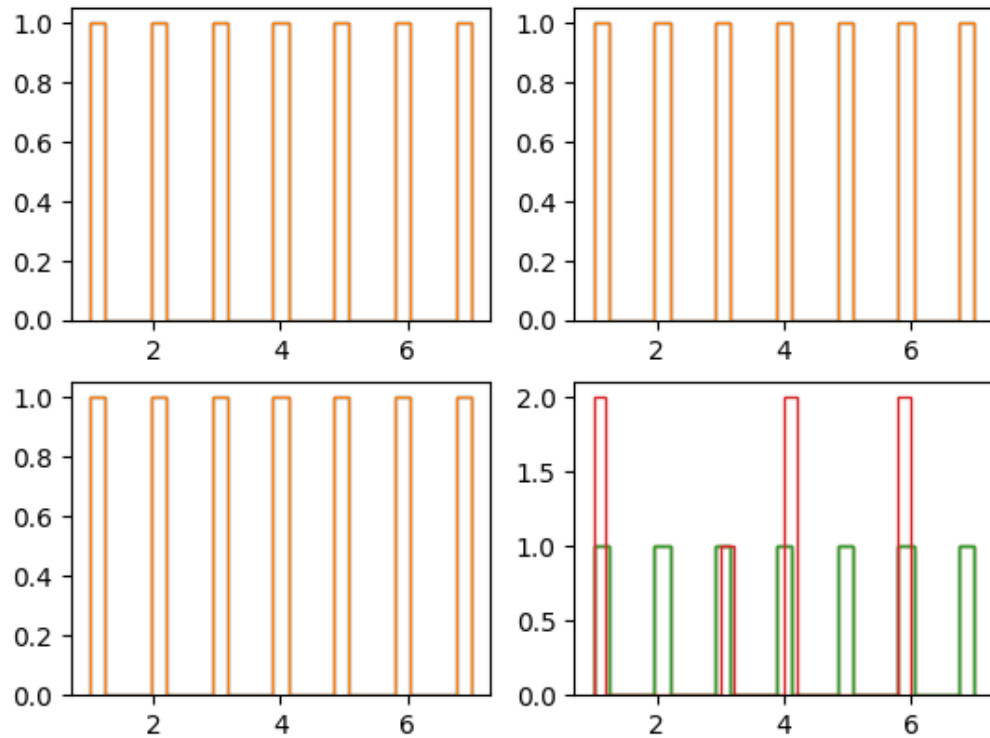
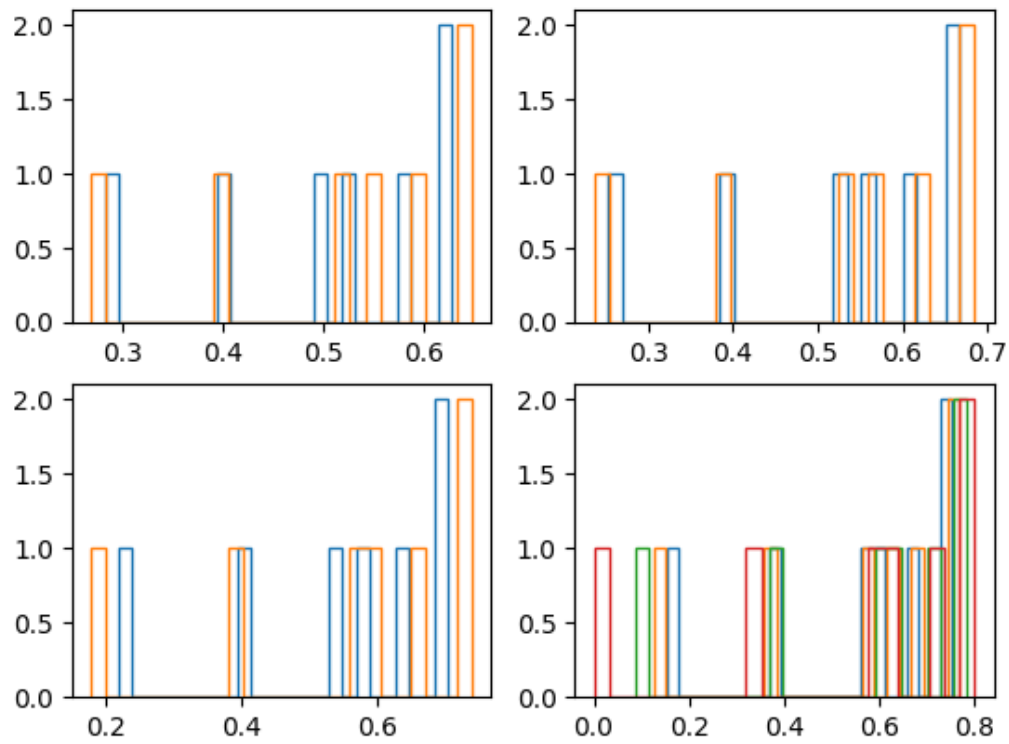
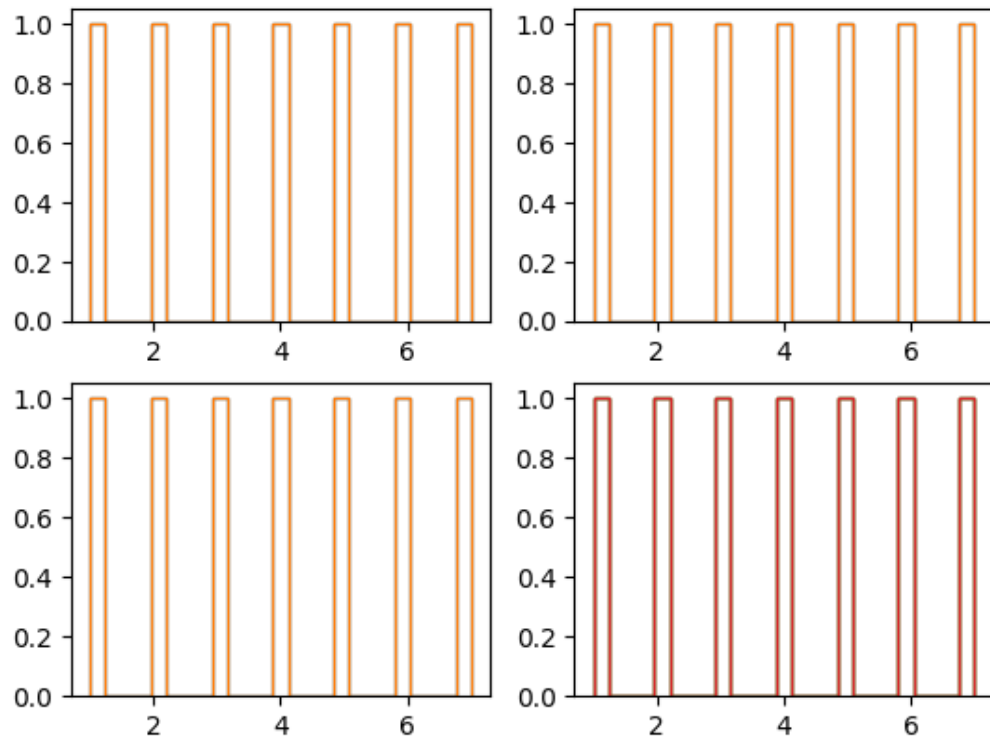


Fig 8. The ranks of alternatives under different δ values.

In the second part, we change the value of ω between 0 and 1. Then we apply the proposed approach. Fig 9 shows the different values of R. Then we rank the alternative as shown in Fig 10.

Fig 9. Th R value under different ω values.Fig 10. The ranks of alternatives under different ω values.

6. Conclusions

The increasing sophistication of AI painting tools necessitates a reliable framework for evaluating their outputs. By considering both computational and artistic aspects, this study provides a balanced, multidimensional approach to painting effects evaluation. The proposed criteria and alternatives reflect current advancements in the field, paving the way for informed development, creative exploration, and academic discourse. As AI continues to influence the arts, tools for its assessment will be vital in shaping a thoughtful and inclusive digital creative landscape. MCDM approach to rank the alternatives using the AROMAN method. The Triangular OverNorm is used in this study to deal with different decision matrices. The results of sensitivity analysis show the stability of the ranks under different cases.

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Evaluating Sustainability in the Protection of Intangible Cultural Heritage Through Sports Practices under the Triangular UnderConorm

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Abstract: The identity and customs of communities around the world depend heavily on the preservation of intangible cultural heritage (ICH). As a social and cultural activity, sports are increasingly being used as a means of preserving these intangible components. This study uses a multi-criteria decision-making (MCDM) framework to assess how sustainable sports practices are in preserving ICH. The study offers a formal foundation for evaluating and improving sports-related heritage efforts by defining important evaluation criteria and examining a collection of representative options. We use the MARCOS methodology as an MCDM methodology to rank the alternatives based on a set of criteria. This study uses four criteria and 15 alternatives. The Triangular UnderConorm is used to deal with the different decision matrix. The results are tested by sensitivity analysis. The results emphasize how important cultural authenticity, educational value, environmental alignment, and local involvement are to long-term preservation. Policymakers, cultural institutions, and community stakeholders are given recommendations.

Keywords: Triangular UnderConorm; Sustainability; Sports Practices; Protection of Intangible Cultural Heritage.

1. Introduction

A fundamental component of cultural identity, intangible cultural heritage (ICH) includes customs, representations, expressions, knowledge, and abilities that have been passed down

through the centuries. It is dynamic, living, and firmly anchored in local customs and values, in contrast to material legacy[1], [2].

Conventional and contemporary sports are among the less studied yet significant channels for spreading ICH. Around the world, a lot of sports are intricately woven with regional traditions, festivals, and rituals. Indigenous running ceremonies, traditional wrestling, and martial arts with philosophical and ceremonial connotations are a few examples[3], [4].

Sports can be used as a cultural repository as well as a means of staying active, especially as modernity and globalization put pressure on indigenous knowledge and cultural manifestations. These methods can aid in reviving endangered customs if they are in line with sustainability ideals[5], [6].

In this context, sustainability refers to the ability of communities to sustain these sporting practices while upholding cultural integrity, intergenerational transmission, and environmental stewardship. Therefore, social and ecological factors must be considered while assessing sustainability[7], [8].

To evaluate the sustainability of sports-based ICH efforts, this study suggests an evaluative framework that considers several cultural, social, and environmental factors. The assessment seeks to identify the best practices and direct upcoming projects[9].

Cultural ministries, non-profits, sports organizations, and community organizations are among the stakeholders engaged in the safeguarding of ICH, and they need a standardized yet flexible approach to rank initiatives that yield long-term benefits[10], [11].

This work closes the gap between applied sport-based interventions and heritage conservation theory by incorporating decision-making tools into cultural evaluation. It makes comparative study possible in a variety of cultural and geographic contexts[12], [13].

The result is a strategy that guides the creation of new, long-lasting programs for preserving intangible cultural traditions through sports in addition to rating ongoing projects. It adds to the growing conversation about cultural sustainability in unconventional fields.

To quantitatively Evaluating Sustainability in the Protection of Intangible Cultural Heritage Through Sports Practices, MCDM issue has implemented several multi-criteria decisions making (MCDM) methodologies. Evaluating Sustainability in the Protection of Intangible Cultural Heritage Through Sports Practices in an MCDM framework is carried out by a team of knowledgeable specialists[14], [15].

Their disparate areas of expertise, however, might cause them to clash, which would result in ineffective assessment results. As a result, one of the primary challenges for Evaluating Sustainability in the Protection of Intangible Cultural Heritage Through Sports Practices is increasing expert consensus.

Numerous MCDM techniques, including MARCOS, and average method, can be used to produce the assessment score. These strategies fall into two categories: outranking-based MCDM techniques and utility values-based MCDM techniques. The latter type can demonstrate the intricate relationship between sports in greater detail than the former type[16], [17].

One of the most important tasks for Evaluating Sustainability in the Protection of Intangible Cultural Heritage Through Sports Practices framework based on the MCDM method is to aggregate the opinions of experts to produce the collective analysis results. They are effective at aggregating information. Most of them assume that specialists make their own decisions. Researchers, shown the interfering effects of experts in the decision-making process through a series of behavioral experiments. Therefore, modeling such an effect is crucial to obtaining trustworthy analysis conclusions[18], [19].

Smarandache (2007) expanded the uncertain set to include OverSet (when some neutrosophic component is > 1) because he noted that, for instance, an employee who works overtime should have a degree of membership > 1 compared to an employee who only works regular full-time and whose degree of membership $= 1$; UnderSet (when some neutrosophic component is < 0) because, for instance, an employee who causes more harm than good to his company should have a degree of membership < 0 in comparison to an employee who benefits the company and has a degree of membership > 0 ; and OffSet (when some neutrosophic components are off the interval $[0, 1]$, i.e. some neutrosophic component > 1 and some neutrosophic component < 0) [20].

Subsequently, the concepts of logic, measure, probability, statistics, etc. have been expanded to include over-, under-, and off- logic, measure, probability, statistics, etc. [21].

To address the Evaluating Sustainability in the Protection of Intangible Cultural Heritage Through Sports Practices MCDM that integrates average method and MARCOS is presented. The following is an introduction to the suggested method's primary contributions:

The average method is used to compute the criteria weights between the criteria which affect the decision-making process.

The MARCOS method is used to rank the alternatives based on the different decision matrix between the criteria and alternatives.

The triangular UnderConorm is used to combine the decision matrix.

The sensitivity analysis is applied by changing the criteria weights by eight cases. The results show the ranks of alternatives are stable.

This study aims to Evaluating Sustainability in the Protection of Intangible Cultural Heritage Through Sports Practices by suing four criteria and 15 sports.

2. Triangular UnderConorm

We can define the triangular UnderConorm on the interval $[\delta, 1], \delta < 0$ [22].

$$T_{underconorm}: [\delta, 1] \times [\delta, 1] \rightarrow [\delta, 1] \text{ where } \theta > 0 \quad (1)$$

$$\text{And for any } (J, K) \in [\delta, 1] \times [\delta, 1] \quad (2)$$

$$T_{underconorm}(J, K) = T_{underconorm}(K, J), \text{ commutativity} \quad (3)$$

$$T_{underconorm}(K, T_{underconorm}(K, L)) = T_{underconorm}(T_{underconorm}(J, K), L) \text{ associativity} \quad (4)$$

$$\text{if } K \leq L, \text{ then } T_{underconorm}(J, K) \leq T_{underconorm}(J, L) \text{ monotonicity} \quad (5)$$

$$T_{underconorm}(J, \theta) = J, \text{ or the neutral elements is } \theta \quad (6)$$

$$T_{underconorm}(G, H) = \max(G, H) \quad (7)$$

The steps of the MARCOS method are shown below to rank the alternatives.

Create the decision matrix.

Four experts created the decision matrix using interval values. These matrices are combined using the $T_{underconorm}$. The decision matrix is used to obtain the weights of criteria by the average method to show the best and worst criterion effect on the ranks of alternatives.

Compute the normalized decision matrix for the positive and cost criteria.

$$d_{ij} = \frac{x_{ij}}{\max_i x_{ij}} \quad (8)$$

$$d_{ij} = \frac{x_{ij}}{\min_i x_{ij}} \quad (9)$$

Compute the weighted matrix.

$$u_{ij} = W_j d_{ij} \quad (10)$$

Calculate the utility degree numbers.

$$F_i^+ = \frac{u_i}{\max_i u_i} \quad (11)$$

$$F_i^- = \frac{u_i}{\min_i u_i} \quad (12)$$

$$F_i = \sum_{j=1}^n u_{ij} \quad (13)$$

Compute the utility function

$$S(F_i) = \frac{F_i^+ + F_i^-}{1 + \frac{1 - f(F_i^+)}{f(F_i^+)} + \frac{1 - f(F_i^-)}{f(F_i^-)}} \quad (14)$$

$$S(F_i^+) = \frac{F_i^-}{F_i^+ + F_i^-} \quad (15)$$

$$S(F_i^-) = \frac{F_i^+}{F_i^+ + F_i^-} \quad (16)$$

Rank the options.

3. Results and Discussion

This section shows the results of the proposed approach to select the best criterion and alternative from different alternatives. Three experts created the decision matrix as shown in Table 1. We use the $T_{underconorm}$ to combine the decision matrix. We use four criteria and 15 alternatives (sports) as: Cultural Integrity (C1) – The extent to which the sport practice retains its original meaning, symbolism, and community significance. Intergenerational Transmission (C2) – The ability of the practice to engage and educate youth, ensuring continuity. Environmental Compatibility (C3) – How well the practice aligns with principles of environmental sustainability (e.g., resource use, ecological footprint). Community Participation (C4) – The level of local engagement, ownership, and inclusiveness in sustaining the sport practice.

Table 1. The opinions of experts.

	C ₁	C ₂	C ₃	C ₄
A ₁	[-1,0.99]	[-2,0.85]	[-3,0.75]	[-4,0.65]
A ₂	[-1,0.99]	[-7,0.15]	[-6,0.35]	[-5,0.25]
A ₃	[-2,0.85]	[-3,0.75]	[-4,0.65]	[-5,0.25]
A ₄	[-4,0.65]	[-3,0.75]	[-2,0.85]	[-1,0.99]
A ₅	[-5,0.25]	[-4,0.65]	[-3,0.75]	[-2,0.85]
A ₆	[-5,0.25]	[-5,0.25]	[-4,0.65]	[-5,0.25]
A ₇	[-6,0.35]	[-6,0.35]	[-3,0.75]	[-6,0.35]
A ₈	[-5,0.25]	[-4,0.65]	[-6,0.35]	[-7,0.15]
A ₉	[-6,0.35]	[-3,0.75]	[-2,0.85]	[-1,0.99]
A ₁₀	[-7,0.15]	[-1,0.99]	[-2,0.85]	[-3,0.75]
A ₁₁	[-3,0.75]	[-4,0.65]	[-5,0.25]	[-6,0.35]
A ₁₂	[-1,0.99]	[-7,0.15]	[-6,0.35]	[-6,0.35]
A ₁₃	[-7,0.15]	[-6,0.35]	[-5,0.25]	[-7,0.15]
A ₁₄	[-4,0.65]	[-5,0.25]	[-6,0.35]	[-1,0.99]
A ₁₅	[-7,0.15]	[-6,0.35]	[-5,0.25]	[-4,0.65]
	C ₁	C ₂	C ₃	C ₄
A ₁	[-4,0.65]	[-2,0.85]	[-3,0.75]	[-4,0.65]
A ₂	[-3,0.75]	[-7,0.15]	[-6,0.35]	[-4,0.65]
A ₃	[-2,0.85]	[-3,0.75]	[-4,0.65]	[-3,0.75]
A ₄	[-1,0.99]	[-3,0.75]	[-2,0.85]	[-2,0.85]
A ₅	[-7,0.15]	[-4,0.65]	[-3,0.75]	[-1,0.99]
A ₆	[-4,0.65]	[-4,0.65]	[-4,0.65]	[-7,0.15]
A ₇	[-3,0.75]	[-3,0.75]	[-3,0.75]	[-6,0.35]
A ₈	[-5,0.25]	[-4,0.65]	[-4,0.65]	[-3,0.75]
A ₉	[-6,0.35]	[-3,0.75]	[-2,0.85]	[-1,0.99]
A ₁₀	[-7,0.15]	[-4,0.65]	[-3,0.75]	[-2,0.85]
A ₁₁	[-3,0.75]	[-4,0.65]	[-5,0.25]	[-6,0.35]
A ₁₂	[-2,0.85]	[-1,0.99]	[-4,0.65]	[-1,0.99]
A ₁₃	[-7,0.15]	[-4,0.65]	[-3,0.75]	[-2,0.85]
A ₁₄	[-1,0.99]	[-7,0.15]	[-6,0.35]	[-3,0.75]
A ₁₅	[-5,0.25]	[-6,0.35]	[-5,0.25]	[-4,0.65]

	C ₁	C ₂	C ₃	C ₄
A ₁	[-1,0.99]	[-2,0.85]	[-3,0.75]	[-4,0.65]
A ₂	[-7,0.15]	[-7,0.15]	[-6,0.35]	[-5,0.25]
A ₃	[-6,0.35]	[-1,0.99]	[-4,0.65]	[-5,0.25]
A ₄	[-5,0.25]	[-7,0.15]	[-1,0.99]	[-1,0.99]
A ₅	[-4,0.65]	[-6,0.35]	[-7,0.15]	[-1,0.99]
A ₆	[-3,0.75]	[-5,0.25]	[-6,0.35]	[-7,0.15]
A ₇	[-2,0.85]	[-4,0.65]	[-5,0.25]	[-6,0.35]
A ₈	[-5,0.25]	[-4,0.65]	[-6,0.35]	[-1,0.99]
A ₉	[-6,0.35]	[-3,0.75]	[-2,0.85]	[-1,0.99]
A ₁₀	[-7,0.15]	[-1,0.99]	[-7,0.15]	[-6,0.35]
A ₁₁	[-3,0.75]	[-4,0.65]	[-5,0.25]	[-6,0.35]
A ₁₂	[-7,0.15]	[-6,0.35]	[-5,0.25]	[-5,0.25]
A ₁₃	[-1,0.99]	[-7,0.15]	[-6,0.35]	[-6,0.35]
A ₁₄	[-5,0.25]	[-4,0.65]	[-3,0.75]	[-7,0.15]
A ₁₅	[-7,0.15]	[-6,0.35]	[-5,0.25]	[-1,0.99]
	C ₁	C ₂	C ₃	C ₄
A ₁	[-6,0.35]	[-7,0.15]	[-2,0.85]	[-7,0.15]
A ₂	[-7,0.15]	[-1,0.99]	[-1,0.99]	[-1,0.99]
A ₃	[-7,0.15]	[-2,0.85]	[-7,0.15]	[-2,0.85]
A ₄	[-2,0.85]	[-2,0.85]	[-2,0.85]	[-5,0.25]
A ₅	[-1,0.99]	[-1,0.99]	[-1,0.99]	[-1,0.99]
A ₆	[-7,0.15]	[-7,0.15]	[-7,0.15]	[-2,0.85]
A ₇	[-1,0.99]	[-3,0.75]	[-4,0.65]	[-5,0.25]
A ₈	[-5,0.25]	[-4,0.65]	[-3,0.75]	[-3,0.75]
A ₉	[-4,0.65]	[-3,0.75]	[-2,0.85]	[-2,0.85]
A ₁₀	[-7,0.15]	[-1,0.99]	[-2,0.85]	[-5,0.25]
A ₁₁	[-2,0.85]	[-7,0.15]	[-3,0.75]	[-5,0.25]
A ₁₂	[-2,0.85]	[-1,0.99]	[-4,0.65]	[-1,0.99]
A ₁₃	[-1,0.99]	[-7,0.15]	[-5,0.25]	[-2,0.85]
A ₁₄	[-2,0.85]	[-1,0.99]	[-2,0.85]	[-3,0.75]
A ₁₅	[-1,0.99]	[-2,0.85]	[-7,0.15]	[-4,0.65]

Then we compute the criteria weights by the average method. We show the descriptions for the criteria given weights in the context of evaluating sustainability in the protection of Intangible Cultural Heritage (ICH) through Sports Practices:

C1 (Weight: 0.2355), Cultural Integrity is a significant factor in preserving the authenticity and original values of a sport-related heritage practice. A weight of 0.2355 reflects its strong influence in the overall evaluation. Practices that preserve traditional rituals, symbols, and meanings score higher in this criterion.

C2 (Weight: 0.2566), This criterion assesses how effectively the sport practice is passed from older to younger generations. With a weight of 0.2566, it holds the highest importance, emphasizing the need for continuity and youth engagement to ensure long-term sustainability of the ICH.

C3 (Weight: 0.2455), Environmental Compatibility examines whether the sport is practiced in a way that respects ecological balance, uses local materials, and minimizes environmental harm. A weight of 0.2455 indicates its nearly equal importance to other social-cultural criteria in sustainable evaluations.

C4 (Weight: 0.2624), This criterion evaluates the level of local community involvement in organizing, preserving, and participating in the sport. With the highest weight of 0.2624, it reflects the central role of community ownership in sustaining ICH through active engagement and shared responsibility.

Eq. (8) is used to compute the normalized decision matrix for the positive and cost criteria as shown in Fig 1.

Eq. (10) is used to compute the weighted matrix as shown in Fig 2.

Eqs. (11-13) are used to calculate the utility degree numbers as shown in Fig 3.

Eqs. (14-16) are used Compute the utility function as shown in Fig 4 and Fig 5.

Rank the options as shown in Fig 6.

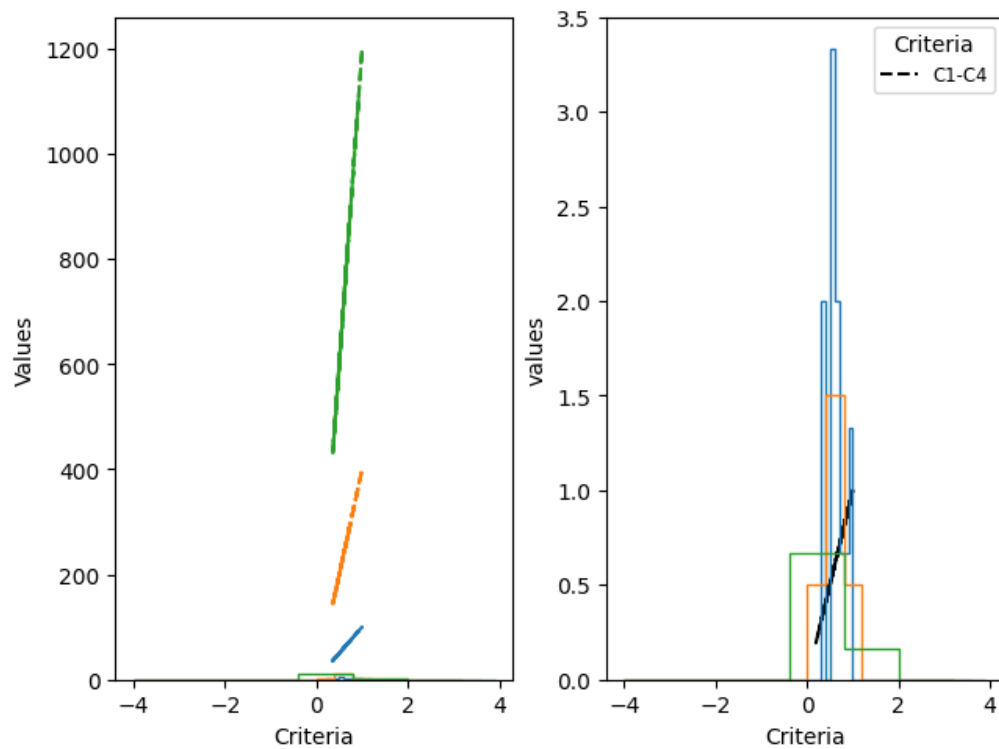


Fig 1. The normalized decision matrix.

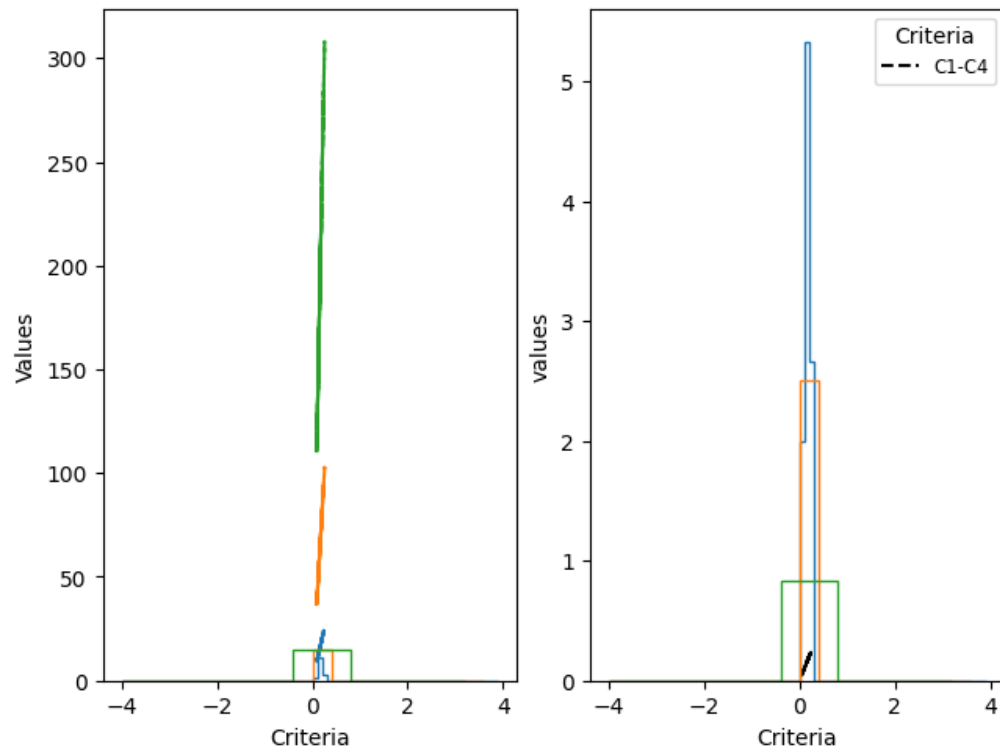
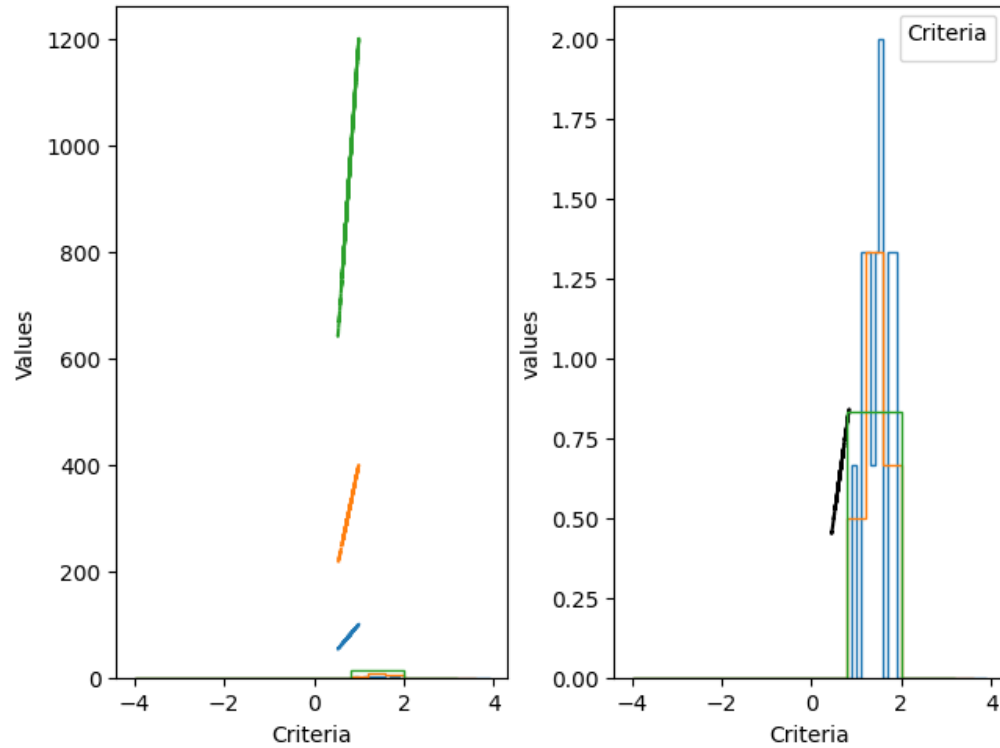


Fig 2. The weighted matrix.

Fig 3. The values of F_i^+ , F_i^- , and F_i .

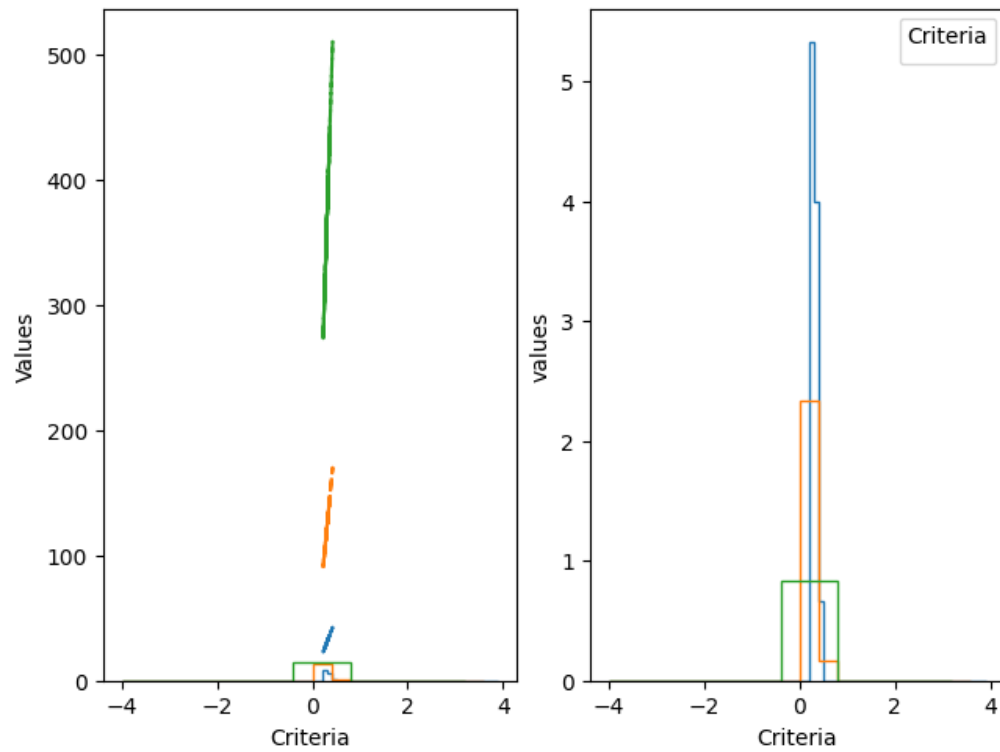


Fig 4. The values of $S(F_i^+)$ and $S(F_i^-)$.

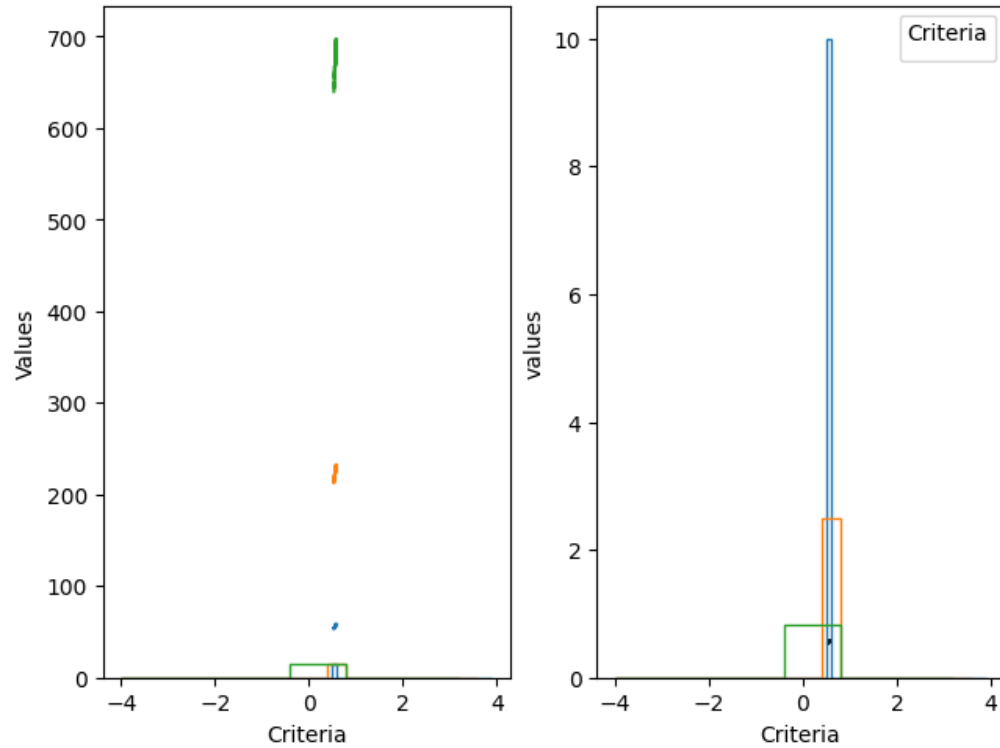


Fig 5. The values of $S(F_i)$.

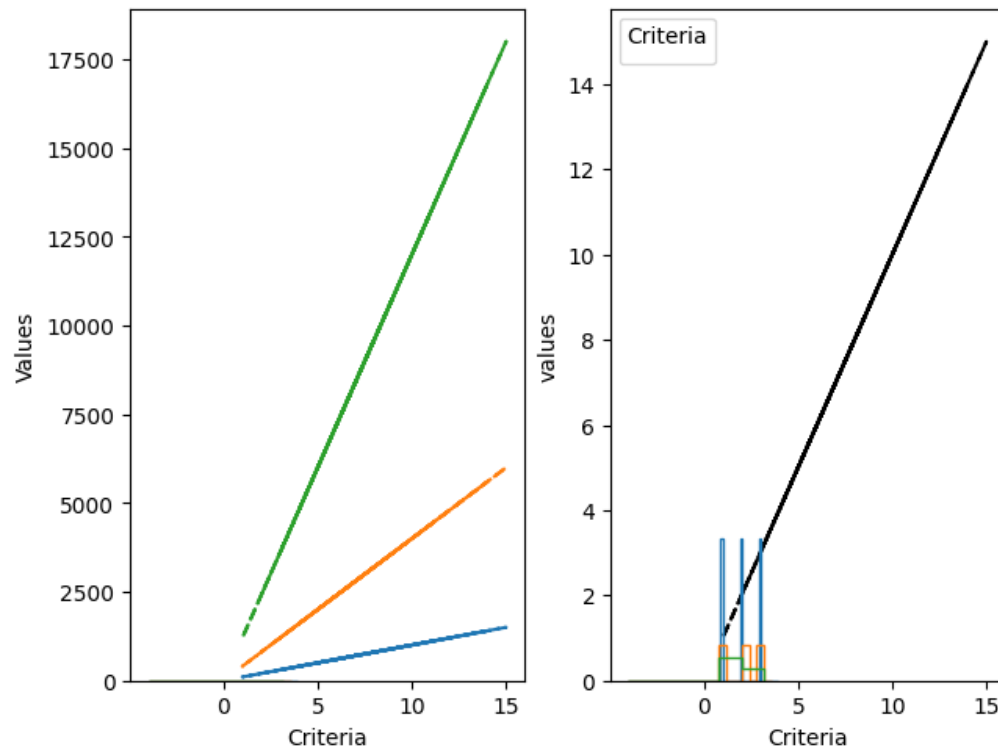


Fig 6. The ranks of alternatives.

4. Analysis

This part shows the sensitivity analysis by updating the weights of criteria by eight cases as shown in Fig 7. Then we apply the MARCOS method to rank the alternatives. Fig 8 shows the utility function. Fig 9 shows the ranks of the alternatives.

Case 1 The focus is evenly distributed, but (C4) has the highest weight (0.3329), highlighting the importance of local involvement in this context. (C1) is low (0.0299), suggesting it's less prioritized here.

Case 2, (C1) is highly emphasized (0.3062), whereas (C2) is minimal (0.0334), suggesting a current-centric approach rather than future-focused transmission.

Case 3 Emphasis shifts to (0.3294) and (0.3368), indicating a dual focus on both people-based sustainability and cultural continuity.

Case 4 Cultural Integrity again scores high (0.3083) along with Intergenerational Transmission (0.3360), while (C4) drops sharply (0.0344)—possibly a more institutional or heritage-preservation-focused case.

Case 5 A more balanced approach across all four criteria, with weights ranging from 0.0580 (C1) to 0.3233 (C4). (C3) and (C2) are strong, suggesting a holistic sustainable model.

Case 6 (C1) is again the lowest (0.2963), while Community Participation (0.3302) takes the lead. This case likely emphasizes collective ownership over individual or symbolic cultural elements.

Case 7 The lowest value is assigned to (C3 – 0.0611), possibly reflecting a community that prioritizes heritage over ecological concerns. High weight to (C4 – 0.3265) again emphasizes collective effort.

Case 8 This case balances the four criteria more equally than most, with all weights between ~0.29–0.33. It reflects a well-rounded evaluation model that does not overly prioritize one aspect.

(C4) consistently receives the highest weights in most cases, underscoring its centrality in sustainable heritage practices.

(C1) shows significant variation, suggesting it is valued differently depending on local or institutional contexts.

(C3) tends to be underweighted in a few cases, indicating a need to raise awareness of ecological aspects in some heritage practices.

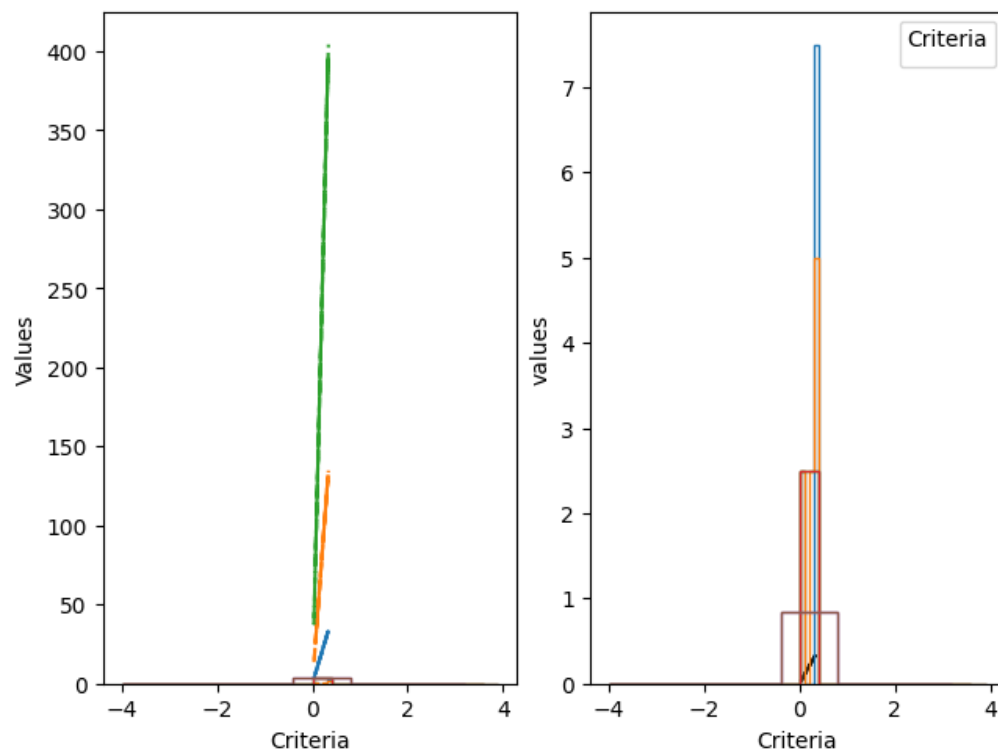


Fig 7. Eight cases in criteria weights.

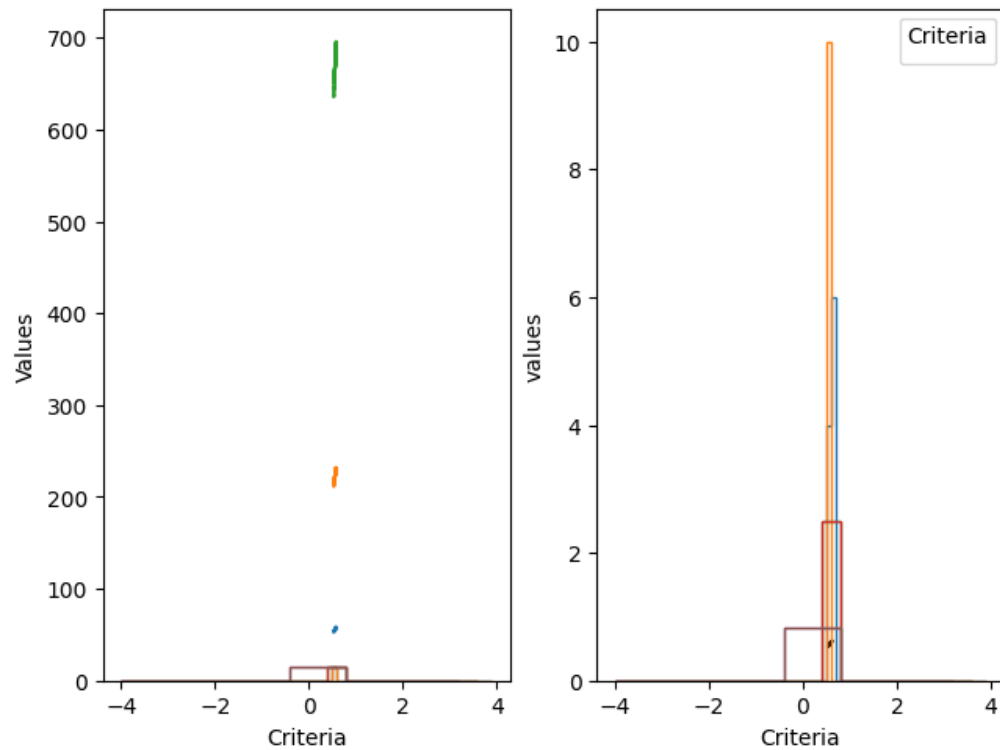


Fig 8. Eight cases in utility function.

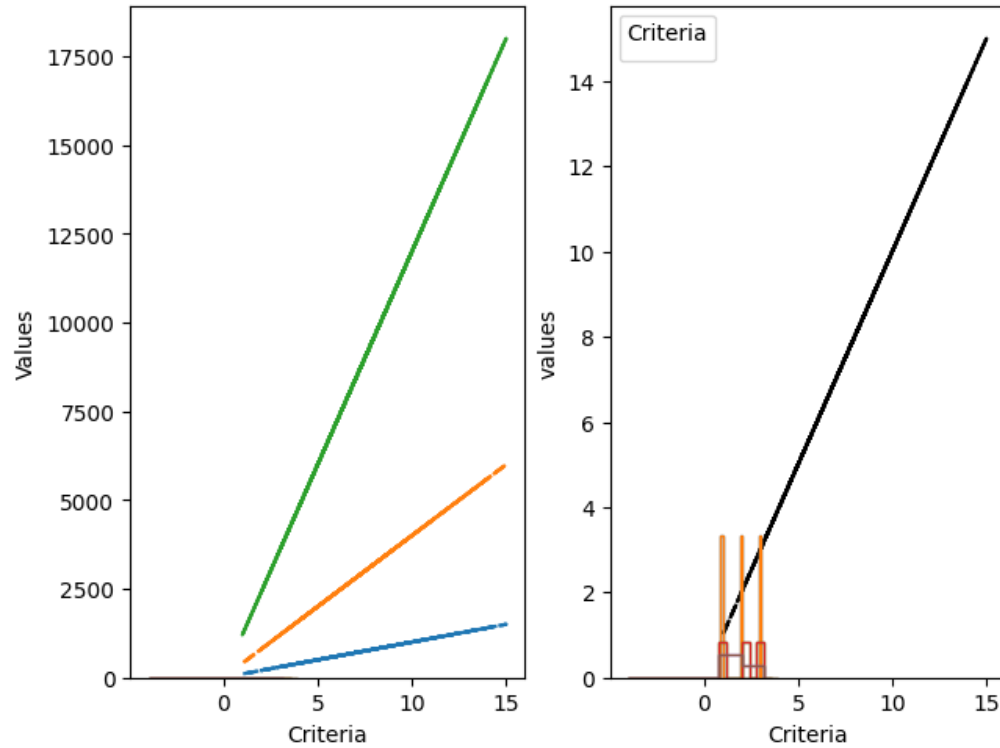


Fig 9. Eight cases in ranks of alternatives.

A1: Moderate to Low Rankings: Ranges from 11 to 14. Generally, underperforms compared to others. May lack comprehensive sustainability elements or strong cultural impact.

A2: Consistently High Performer: Ranks range between 2 and 7. Highly regarded across all scenarios—shows well-rounded sustainability, cultural relevance, and community involvement.

A3: Lower Mid Rankings: Ranks mostly 8 to 11. Seen as sustainable but may lack in certain criteria like environmental alignment or community depth.

A4: Very High Consistency but Low Performance: Always among the bottom three (ranks 13 to 15). Indicates weak alignment with evaluation criteria—possibly outdated methods or lacking participatory dimensions.

A5: Low Performer: Typically ranks 10 to 14. May have cultural value but struggles with practical sustainability or broader engagement.

A6: Top Performer: Frequently Rank 1, with only a couple Rank 2s. Indicates this alternative is the most effective and sustainable by all evaluation criteria.

A7: Middle Tier: Usually ranks 6 to 12. Decent performance but lacks consistency to compete with top-ranking alternatives like A6 or A2.

A8: Upper Mid Performer: Mostly ranks 4 to 9. It shows balanced strengths across different sustainability criteria but isn't top tier.

A9: Lowest Overall: Consistently ranks 13 to 15, usually 15. Perform poorly across all evaluation models. Possibly lacks innovation or relevance.

A10: Mixed Ratings: Ranks range from 3 to 12. Strong in some criteria (e.g., C2 and C6), but possibly inconsistent in delivery or outcomes.

A11: Strong Upper-Mid Performance: Often Rank 2 to 7. Shows clear potential and relevance, particularly in community-driven or intergenerational contexts.

A12: Lower Mid-Tier: Stays around 8 to 11. Might need development or innovation to stand out.

A13: High Performer: Usually between 2 and 5. Frequently ranks in the top 5; likely well-balanced in cultural, environmental, and community aspects.

A14: Mid to Lower Range: Ranks 7 to 11. Offers consistent but not standout value in sustainability efforts.

A15: Highly Competitive: Often Rank 1 or 2, never below 5. Indicates strong integration of sustainability and cultural preservation—among the most favored options.

5. Conclusions

Through sports, intangible cultural legacy can be preserved while providing a special platform for tradition, physical expression, and community building. It has been shown that assessing such practices according to community involvement, intergenerational transmission, environmental compatibility, and cultural integrity produces insightful results. This paradigm makes it possible to establish context-sensitive, culturally respectful policies and tailored actions. Communities can enhance their cultural identity and support global diversity and heritage preservation by supporting these sports in a sustainable way. This study developed an MCDM approach to evaluate 15 sports under four criteria. Four experts created the decision matrix between the criteria and alternatives. The Triangular UnderConorm is used to deal with different decision matrices. The sensitivity analysis is created with eight cases by changing in the weights of criteria. The results show that the ranks of alternatives are stable in different cases.

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On Data Envelopment Analysis Framework for Optimizing Two-Stage

Systems Under Neutrosophic Environment

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Abstract: Recently, DEA became one of the evaluation method and used widely by the researchers for the analysis. It's used to evaluate units under evaluation that produce multiple outputs by consuming multiple inputs This paper presents a neutrosophic set-based, two-stage DEA framework to surmount the weak linkages within the traditional DEA model when uncertainties and imprecise values exist. Neutrosophic sets capture the uncertainty by considering membership degrees of truth, indeterminacy, and falsity. Overall, this framework decomposes overall system efficiency into stage efficiencies and provides deep insights into the system performance. We developed a two-stage DEA optimization model with neutrosophic variables and constraints. This framework can apply to any organization in order to support decision-makers to make rational decisions and allocate resources optimally, especially in the modern competitive business environment. A case study introduced to show how the model could handle neutrosophic uncertainty, and the mathematical formulation presents succinctly for practical implementation.

Keywords: Data Envelopment Analysis, Mathematical model; Neutrosophic Sets, Two-Stage Model, Efficiency Assessment, Competitive Business Environment, Uncertainty

1. Introduction

In today's complex and interlinked world, several real-life systems work just like two-stage processes, with the output of one stage normally forming the input to the other. Examples include supply chain networks, healthcare delivery systems, and manufacturing processes. It is thus clear that for optimum resource allocation and performance improvement with a view to achieving strategic objectives, efficiency analysis of such systems becomes quite indispensable. Traditional DEA has been one of the popular methods for evaluating the efficiency of DMUs involving multiple inputs and outputs. However, conventional DEA models cannot handle uncertain, incomplete, and ambiguous data, which is a common characteristic with real-world systems. This limitation has spurred the development of advanced DEA approaches, such as fuzzy DEA and

neutrosophic DEA, with a view to embracing uncertainty and indeterminacy in efficiency evaluation [1]. Neutrosophic logic is the extension of fuzzy logic, in which the membership functions of truth, indeterminacy, and falsity define a very powerful tool in mathematics when dealing with uncertainty. It is therefore going to be more applicable for modeling those complex systems containing imprecise, inconsistent partial unknown data. In such context, including neutrosophic logic within the DEA will yield more realistic estimates regarding the efficiency of systems when the conditions of uncertainty arise. Some recent studies demonstrate the potential of neutrosophic DEA in various applications; however, applying it to a two-stage system is still scant. Two-stage systems are pretty special, considering that the interdependence between stages requires specialized models that can accurately capture the flow of intermediate inputs and outputs [2].

Decision science, operations management, and economics are the fields of research that need to explain the complexity of the systems. In the beginning of the 70s, the DEA as a method of efficiency evaluation was developed by Charnes, Cooper, and Rhodes [3], and by now has already been used as a powerful tool in the multidimensional efficiency measurement of the decision-making unit. But the main reason why the traditional DEA models, for the sake of DEA that can handle the uncertainty and ambiguity of real-life data, limit their application, is the assumption made by these models that the datasets used in the process are crisp. In this regard, over the last decades, fuzzy DEA was suggested to the scientific community in order to deal with data imprecision [4]. The fuzzy logic-based DEA has been experimented with in various fields [5, 6, 7, 8, 9, 10, 11, 12, 13]. Therefore, these studies emphasize uncertainty resolution and information representation effects on technology development and efficiency. Along with neutrosophic logic, another approach, called neutrosophic logic, is provided by Smarandache [14], who tries to capture the concept of indeterminacy in yet another dimension through this creative representation of complex systems that may contain uncertain or incomplete data. Meanwhile, newly originated studies were carried out in order to add neutrosophic logic in with DEA as a means to add analysis tools under conditions of (%) uncertainty. Additionally, various methods are available to tackle diverse challenges in a neutrosophic environment [15, 16, 17, 18, 19, 20, 21].

Thus, in a study by Abdel-Basset et al. [22], a neutrosophic DEA model was proposed to be used in the efficiency evaluation of a renewable energy system, which turns out to be more accurate while having the capacity of dealing with the indeterminacy that cannot be solved by traditional DEA. Similarly, Khan et al. [23] utilized neutrosophic DEA for the performance evaluation of financial institutions to direct resource allocation to these entities, a better practice in a condition of ambiguity. The results in these studies reveal that utilization of neutrosophic DEA technology can eliminate some problems with conventional and fuzzy DEA models, especially in a partial or inconsistent data defense. In a very beneficial way for both research and practice, El-Demerdash et al. [24] proposed neutrosophic input-oriented DEA model. As an extension to the work of El-Demerdash et al., Farnam et al. [25] proposed a neutrosophic DEA model in which input and output variables are considered neutrosophic intrinsic to handle uncertainties in the input and output data. An important step forward was given by Almutairi et al. [26], who proposed a general model which is able to: decide which variables have to be modeled as deterministic or neutrosophic, consider both input and output orientation, handle different types of returns to scale, such as constant and variable. Two-stage systems are of high interest where one stage's output constitutes the input of another stage. These systems have been widely used in various settings such as supply chain management, manufacturing, and service operations; therefore, they have attracted significant attention. The traditional DEA method has limited applications in the

evaluation of these systems due to the fact that the interdependencies amidst the stages are ignored and their inputs/outputs are not individually modeled. In contrast, a two-stage DEA has been proposed by some researchers with a view to providing a comprehensive solution for that problem. Tabatabaee et al., [27] proposed a new model for ranking efficient units by introducing an ideal unit and calculating the distance between each efficient unit and the ideal unit.

So, this paper proposes an optimization framework for the two-stage system using Neutrosophic Data Envelopment Analysis (NDEA). The present approach surmounts some of the limitations of classical DEA by embracing neutrosophic sets to model uncertain input-output data and also developing a two-stage model which takes care of interdependency between stages. This proposed approach will not only provide efficiency at each stage but will also provide status about overall system performance, thus helping decision-makers to identify an area of inefficiency and providing targeted improvements. This study connects neutrosophic logic with two-stage DEA, contributing to the developing body of efficiency analysis under uncertainty, hence offering the practical tool for optimization of complex real-world systems.

The rest of this paper is structured as follows. Section 2 presents the preliminaries. Section 3 presents the basic two-stages DEA model. Section 4 view the developing Neutrosophic Two-Stage DEA Model. Section 5 highlights case study. Section 6 shows the conclusion and future works.

2. Preliminaries

Neutrosophic sets were defined by Florentin Smarandache [14], thus providing a unified framework for dealing with the uncertainties, indeterminacy, and inconsistencies that arise in any kind of data. Neutrosophic sets, in contrast to classical fuzzy sets, which take into consideration only the degree of membership, and intuitionistic fuzzy sets, which consider both membership and nonmembership degrees, consider membership along with non-membership degrees and an extra dimension of indeterminacy. This indeterminate nature makes neutrosophic sets suitable for the representation of several real tasks filled with complex ambiguities and scenarios wherein the information available is incomplete or even imprecise.

Definition 1 [14]: A neutrosophic set A in a universe of discourse X is defined as:

$A = \{(x, T_A(x), I_A(x), F_A(x)) | x \in X\}$, where $T_A(x), I_A(x), F_A(x)$ represent truth-membership function, indeterminacy-membership function, and falsity-membership function respectively. Each of these functions are independent and maps elements of X to the interval $[0,1]$.

Definition 2 [14]: Neutrosophic sets generalize classical fuzzy sets and intuitionistic fuzzy sets.

When $I_A(x) = 0$, a neutrosophic set reduces to an intuitionistic fuzzy set, and when both $I_A(x) = 0$ and $F_A(x) = 1 - T_A(x)$, it reduces to a classical fuzzy set.

Definition 3 [14]: Neutrosophic sets support a variety of operations, including union, intersection, and complement, let A and B are neutrosophic set, $A = \{(x, T_A(x), I_A(x), F_A(x)) | x \in X\}$, $B = \{(x, T_B(x), I_B(x), F_B(x)) | x \in X\}$ which are defined as follows:

1. $A \cup B = \{(x, \max(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \min(F_A(x), F_B(x))) | x \in X\}$

$$2. \quad A \cap B = \{(x, \min(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \max(F_A(x), F_B(x))) | x \in X\}$$

$$3. \quad A^c = \{(x, F_A(x), 1 - I_A(x), T_A(x)) | x \in X\}$$

Definition 4 [14]: A Neutrosophic Set with Triangular Fuzzy Single-Valued Membership Functions (NSTFSV) is a specialized form of a neutrosophic set. Assume that $\tilde{Q}^{TN} = \langle (x, y, z); \beta_{\tilde{Q}^{TN}}, \gamma_{\tilde{Q}^{TN}}, \delta_{\tilde{Q}^{TN}} \rangle$ be NSTFSV, then the score function (SF) defined as follows:

$$SF(\tilde{Q}^{TN}) = \left(\frac{1}{4}(a + 2b + c) \right) \left(\frac{1}{3}(2 + \beta_{\tilde{Q}^{TN}} - \gamma_{\tilde{Q}^{TN}} - \delta_{\tilde{Q}^{TN}}) \right), \text{ where } a, b, c \in \mathbb{R} \text{ such that } a \leq b \leq c.$$

3. Basic Two-stage Data Envelopment Analysis Model

Two-stage DEA models investigate processes that are characterized by two distinct successive stages. Thus, in the first stage of a two-stage process, certain inputs lead to specific outputs. The two-stage DEA process that has ‘n’ DMUs working along the process. It has been formulated to evaluate the efficiency of organizations with a two-stage internal structure as shown in Figure 1. Hence, each DMU can have:

Inputs $\rightarrow$ first stage $\rightarrow$ intermediate outputs $\rightarrow$ second stage $\rightarrow$ final outputs.

It is a very useful approach in evaluating processes where outputs from the first stage serve as inputs for the second stage. It proceeds to two stages or termed (subprocess – 1 or first stage, and subprocess – 2 or second stage).

In it, weight λ_i is assigned to each DMU. In first stage, ‘m’ number of inputs, where x_{ij} is the amount of

input j consumed by DMU i ($j = 1$ to m), and ‘k’ number of outputs were produced, where z_{ik} is the amount of

output k generated by DMU i ($k = 1$ to s). The outputs of the first stage are regarded as intermediate measures

and input to second stage. For second stage takes ‘v’ independent inputs denoted as h_{it} , the amount of

independent input ‘t’ used by DMU i ($t = 1$ to v) and belches out ‘u’ outputs represented by y_{ir} , the amount of

output ‘r’ produced by DMU i ($r = 1$ to u). Let ϕ_p^1 be the efficiency of the p^{th} DMU in first stage, while the

efficiency of the p^{th} DMU second stage, would be ϕ_p^2 . Together the efficiency of the p^{th} DMU can be expressed

as ϕ_p . The basic two-stage output-oriented variable returns to scale (VRS) DEA model of the p^{th} DMU

considers model (M-1) for the first stage and model (M-2) for the second.

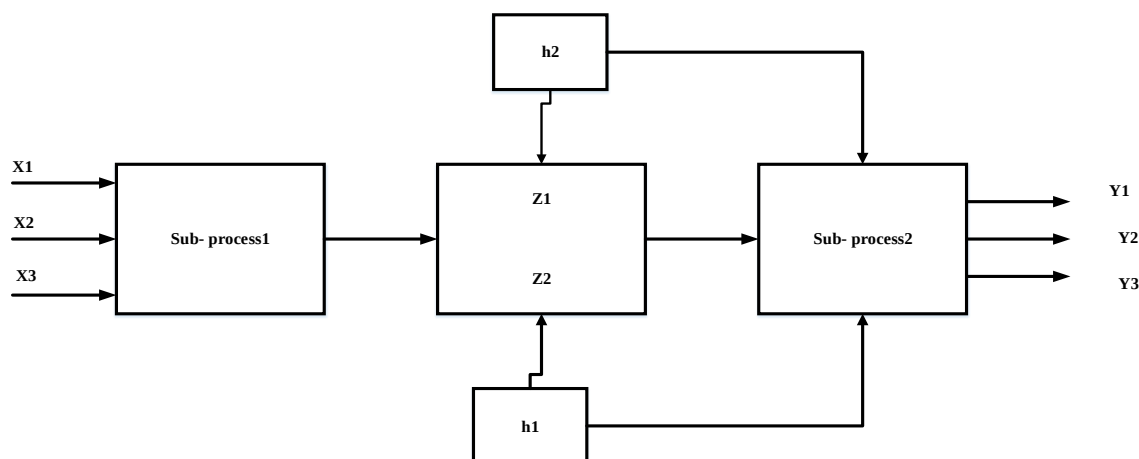


Figure 1. Two-stage DEA Model

- For 1st stage: A basic Variable Return to Scale – Output Oriented - DEA model

$$\text{Max } \phi_p^1$$

Subject to

$$\sum_{i=1}^n \lambda_i z_{ik} \geq \phi_p^1 z_{pk}, \forall k = 1 \dots s$$

$$\sum_{i=1}^n \lambda_i x_{ij} \leq x_{pj}, \forall j = 1 \dots m$$

$$\sum_{i=1}^n \lambda_i = 1$$

$$\lambda_i \geq 0, (i = 1, 2, \dots, n) \quad (M-1)$$

- For 2nd stage: A basic Variable Return to Scale – Output Oriented - DEA model

$$\text{Max } \phi_p^2$$

Subject to

$$\sum_{i=1}^n \lambda_i y_{ir} \geq \phi_p^2 y_{pr}, \forall r = 1 \dots u$$

$$\sum_{i=1}^n \lambda_i z_{ik} \leq z_{pk}, \forall k = 1 \dots s$$

$$\sum_{i=1}^n \lambda_i h_{it} \leq h_{pt}, \forall t = 1 \dots v$$

$$\sum_{i=1}^n \lambda_i = 1$$

$$\lambda_i \geq 0, (i = 1, 2, \dots, n) \quad (M-2)$$

- Overall Efficiency for p^{th} DMU (ϕ_p) = $\phi_p^1 * \phi_p^2$

4. Developing a Novel Neutrosophic Two-Stage DEA Model

These two decades witness complication in the decision-making process and real uncertainties in data around the world to yield even more and highly flexible approaches regarding analytical efficiencies. An integrated neutrosophic DEA Model is here presented, which considers an output-oriented approach with two stages, during uncertain conditions with respect to its existence and flowing between such stages. The proposed model in this paper, as opposed to traditional two-stage DEA models that use deterministic data, incorporates neutrosophic sets in order to reflect the inherent vagueness and indeterminacy present in the input and output variables. This thereby provides a reasonable assessment interface for real efficiency investigation dealing with complicated systems. The formulated model takes two-stage DEA concepts further, whereby the first stage produces some intermediate outputs entering the second stage as inputs. However, it extends those principles by incorporating neutrosophic logic into handling uncertain data and also incorporates a variable return to scale assumption to account for variations in operational scale.

This dual focus on uncertainty and scalability issues renders the model especially useful in diverse areas of application in supply chain management, healthcare, and financial services, where lots of uncertainty in data and scale heterogeneity are present. The following subsections will elaborate upon the mathematical formulation of the model, key features, and application in a theoretical framework. Thus, the development of this model constitutes a significant milestone in the efficiency analysis direction, with each efficiency analysis thereby equipped by a better tool to venture the modern decision-making settings. Three separate phases are formed within our model. First, the membership function assigned to the inputs and outputs of a neutrosophic kind is defined at the first phase. Then in the second phase, defined membership functions for the neutrosophic variables to calculate the score function for them. Then the third phase concerns the computation for each DMU level of relative efficiency. Finally, the VRS output-oriented neutrosophic DEA model developed to evaluate the relative efficiency performance of the p^{th} DMU presents in the mathematical model below:

$$\text{Max } \phi_p^1$$

Subject to

$$\sum_{i=1}^n \lambda_i z_{ik} \geq \phi_p^1 z_{pk}, \forall k \in K_D$$

$$\begin{aligned}
\sum_{i=1}^n \lambda_i \tilde{z}_{ik}^{TN} &\geq \tilde{z}_{pk}^{TN}, \forall k \in K_N \\
\sum_{i=1}^n \lambda_i x_{ij} &\leq x_{pj}, \forall j \in J_D \\
\sum_{i=1}^n \lambda_i \tilde{x}_{ij}^{TN} &\leq \tilde{x}_{pj}^{TN}, \forall j \in J_N \\
\sum_{i=1}^n \lambda_i &= 1 \\
\lambda_i &\geq 0, (i = 1, 2, \dots, n)
\end{aligned} \tag{M-3}$$

Where:

- $\tilde{z}_{ik}^{TN}$: output neutrosophic variable 'k' produced by DMU 'i' in first stage,
- $\tilde{x}_{ij}^{TN}$: input neutrosophic variable 'j' utilized by DMU 'i' in first stage,
- J_D : set of deterministic input variables for first stage,
- J_N : set of neutrosophic input variables for first stage,
- J : set of all input variables for first stage, where $J = J_D \cup J_N$
- K_D : set of deterministic output variables for first stage,
- K_N : set of neutrosophic output variables for first stage,
- K : set of all output variables for first stage, where $K = K_D \cup K_N$.

The membership functions of neutrosophic variables are assumed to be of triangular structure in this proposed model, following which a score function as defined in Definition 4 is applied to convert the VRS output-oriented triangular neutrosophic DEA model to a crisp DEA model. By this transformation, the computational function gets simplified, and the model-governing equations become easier to solve. More detail on this approach is given below.

$$\text{Max } \phi_p^1$$

Subject to

$$\begin{aligned}
\sum_{i=1}^n \lambda_i z_{ik} &\geq \phi_p^1 z_{pk}, \forall k \in K_D \\
\sum_{i=1}^n \lambda_i SF(\tilde{z}_{ik}^{TN}) &\geq \phi_p^1 SF(\tilde{z}_{pk}^{TN}), \forall k \in K_N \\
\sum_{i=1}^n \lambda_i x_{ij} &\leq x_{pj}, \forall j \in J_D \\
\sum_{i=1}^n \lambda_i SF(\tilde{x}_{ij}^{TN}) &\leq SF(\tilde{x}_{pj}^{TN}), \forall j \in J_N \\
\sum_{i=1}^n \lambda_i &= 1 \\
\lambda_i &\geq 0, (i = 1, 2, \dots, n)
\end{aligned} \tag{M-4}$$

Where:

- $SF(\tilde{z}_{ik}^{TN})$: score function for output neutrosophic variable 'k' produced by DMU 'i' in first stage,
- $SF(\tilde{x}_{ij}^{TN})$: score function for input neutrosophic variable 'j' utilized by DMU 'i' in first stage.

In the same way, the triangular output-oriented neutrosophic VRS DEA model has been converted into a crisp DEA model just as in the first stage, for the second stage. Since this conversion relieves the problem from the difficulties of having neutrosophic variables, applying crisp DEA techniques becomes much easier. The function that scores the input and output of the DMU should enable a model to work under uncertainty and will remove complexities from the computation process. The very simplified version made under this approach will allow for quick evaluation of DMUs without compromise on the accuracy. The following explains how the transformation is implemented in detail.

$$\text{Max } \phi_p^2$$

Subject to

$$\sum_{i=1}^n \lambda_i y_{ir} \geq \phi_p^2 y_{pr}, \forall r \in R_D$$

$$\begin{aligned}
\sum_{i=1}^n \lambda_i SF(\tilde{y}_{ir}^{TN}) &\geq \phi_p^2 SF(\tilde{y}_{pr}^{TN}) \quad , \forall r \in R_N \\
\sum_{i=1}^n \lambda_i z_{ik} &\leq z_{pk} \quad , \forall k \in K_D \\
\sum_{i=1}^n \lambda_i h_{it} &\leq h_{pt} \quad , \forall t \in T_D \\
\sum_{i=1}^n \lambda_i SF(\tilde{z}_{ik}^{TN}) &\leq SF(\tilde{z}_{pk}^{TN}) \quad , \forall k \in K_N \\
\sum_{i=1}^n \lambda_i SF(\tilde{h}_{it}^{TN}) &\leq SF(\tilde{h}_{pt}^{TN}) \quad , \forall t \in T_N \\
\sum_{i=1}^n \lambda_i &= 1 \\
\lambda_i &\geq 0, (i = 1, 2, \dots, n)
\end{aligned} \tag{M-5}$$

Where:

- $\tilde{h}_{it}^{TN}$: independent neutrosophic input variable ‘t’ utilized by DMU ‘i’ in second stage,
- $\tilde{y}_{ir}^{TN}$: final output neutrosophic variable ‘r’ produced by DMU ‘i’ in second stage,
- $SF(\tilde{h}_{it}^{TN})$: score function for independent neutrosophic input variable ‘t’ utilized by DMU ‘i’ in second stage
- $SF(\tilde{y}_{ir}^{TN})$: score function for final output neutrosophic variable ‘r’ produced by DMU ‘i’ in second stage
- R_D : set of deterministic final output variables for second stage,
- R_N : set of neutrosophic final output variables for second stage,
- R : set of all final output variables for second stage, where $R = R_D \cup R_N$,
- T_D : set of independent deterministic input variables for second stage,
- T_N : set of independent neutrosophic input variables for second stage,

- T : set of all input independent variables for second stage, where $T = T_D \cup T_N$.

5. Case Study

Performance evaluation of branches assumes great significance in the banking sector for decision makers. Traditional DEA could be seen almost everywhere, but in uncertain conditions due to lack of data, it is all the time being dependent on vague or imprecise data, no central tendency remains there. In this paper, we do NDEA to evaluate the efficiency of a two-stage banking system, where the first stage represents resource utilization, and the second stage includes the operations of customer service. Seven different bank branches will be analyzed using NDEA to assess efficiency by also incorporating the uncertainty regarding financial and operational aspects. The first stage consists of bank branches operating with three deterministic input variables (Number of employees, Operating expenses (in \$1000s), Branch area (square meters), which are required for the banking processes) and two neutrosophic output variables (Number of processed transactions (in 1000) and Total funds managed (in \$1000,00s)). In second stage, the bank branches continue to operate with two intermediate input variables (Output variables from 1st stage) as well as having an independent: deterministic input added to it (New accounts opened) and two neutrosophic final output variables (Customer satisfaction rate, and Total revenue generated (\$1000,00s)). The complete case study data is shown in Tables 1, and 2. It is a problem that tries to come to terms with the relative performance of bank branches using our two-stage NDEA model.

Table 1 - 1st stage (resource utilization process) data for the bank branches

Branch	Inputs			Intermediate Outputs	
	Number of employees	Operating expenses (in \$1000s)	Branch area (square meters)	Processed Transactions (in 1000)	Total Funds Managed (\$1000,00s)
B_1	15	80	160	$\langle(4.9,5.1,5.3); 0.9,0.4,0.05\rangle$	$\langle(20,22,24); 0.9,0.4,0.1\rangle$
B_2	12	75	150	$\langle(4.6,4.8,5); 0.9,0.15,0.1\rangle$	$\langle(18,20,22); 0.9,0.7,0.1\rangle$
B_3	18	95	170	$\langle(5.3,5.5,5.7); 0.9,0.1,0.04\rangle$	$\langle(23,25,27); 0.9,0.4,0.1\rangle$
B_4	10	65	140	$\langle(4.1,4.3,4.5); 0.8,0.5,0.12\rangle$	$\langle(16,18,20); 0.8,0.5,0.1\rangle$
B_5	14	85	155	$\langle(5.2,5.4); 0.8,0.2,0.05\rangle$	$\langle(21,23,25); 0.8,0.5,0.2\rangle$
B_6	9	60	135	$\langle(3.8,4.4,2); 0.9,0.5,0.15\rangle$	$\langle(14,16,18); 0.9,0.5,0.2\rangle$
B_7	16	90	165	$\langle(5.1,5.3,5.5); 0.9,0.4,0.03\rangle$	$\langle(22,24,26); 0.9,0.4,0.1\rangle$

Table 2 - 2nd stage (customer service operations) data for the bank branches

Branch	Independent Input	Final Output	
	Number of new accounts opened	Customer satisfaction	Total revenue generated (\$1000,00s)
B_1	210	$\langle(0.8,0.87,0.94); 1.0, 0.0,0.06\rangle$	$\langle(4.8,5.2,5.6); 1.0, 0.14,0.06\rangle$
B_2	190	$\langle(0.75,0.82,0.89); 1.0,0.0,0.08\rangle$	$\langle(4.5,4.9,5.3); 1.0,0.16,0.08\rangle$
B_3	230	$\langle(0.83,0.9,0.97); 1.0,0.0,0.05\rangle$	$\langle(5.6,6.6,6.4); 1.0,0.12,0.05\rangle$
B_4	170	$\langle(0.68,0.75,0.82); 1.0,0.0,0.1\rangle$	$\langle(4.1,4.5,4.9); 1.0,0.18,0.1\rangle$
B_5	220	$\langle(0.81,0.88,0.95); 1.0,0.0,0.06\rangle$	$\langle(5.4,5.8,6.2); 1.0,0.13,0.06\rangle$
B_6	150	$\langle(0.63,0.7,0.77); 1.0,0.0,0.15\rangle$	$\langle(3.8,4.2,4.6); 1.0,0.2,0.15\rangle$
B_7	225	$\langle(0.82,0.89,0.96); 1.0,0.0,0.05\rangle$	$\langle(5.5,5.9,6.3); 1.0,0.12,0.05\rangle$

Table 3 - Bank Branches Relative Efficiency Level

Branch	NDEA Model			Conventional DEA Model		
	ϕ_p^1	ϕ_p^2	ϕ_p	ϕ_p^1	ϕ_p^2	ϕ_p
B_1	1	0.82	0.82	0.49	0.58	0.28
B_2	0.65	1	0.65	0.4	1	0.40
B_3	1	1	1	1	1	1
B_4	1	1	1	1	1	1
B_5	0.48	0.75	0.36	0.37	0.86	0.32
B_6	0.29	0.89	0.26	0.21	1	0.21
B_7	1	0.88	0.88	0.40	0.50	0.20

Table 3 presents comparative analysis between NDEA model and the Conventional DEA model applied on the seven bank branches to measure their performance. The efficiency of each branch is denoted by: ϕ_p^1 : (First-stage efficiency-resource utilization); ϕ_p^2 : (Second-stage efficiency-customer service operations); and

ϕ_p (Overall efficiency). The traditional DEA models fail to account for uncertainties and imprecise data in banking operations. On the other hand, it is obvious that NDEA describes neutrosophic numbers, thus providing a more realistic and flexible view of efficiency evaluation. Both branches B₃ and B₄ scored a total efficiency level of 1 for both models in ϕ_p indicating full efficiency in terms of resource usages and customer services. The branches show more improved efficiencies on NDEA than conventional DEA, where their degrees of efficiencies were $\phi_{B_1}=0.82$ and $\phi_{B_7}=0.88$, compared to $\phi_{B_1}=0.28$ and $\phi_{B_7}=0.20$: this gives a better indication that uncertainties in transaction and customer satisfaction data were tended better within NDEA. Both branches B₅ and B₆ present low efficient scores in both models; but NDEA provides a closer mirror of the performance because it factors in uncertainty in all revenue and customers' satisfaction. Also, again compared First-State vs. Second-State Efficiency: Branches B₁, B₃, B₄, and B₇ received a score of 1 for the first stage (ϕ_p^1), singling them out as being highly efficient with regards to the branch's employees, operating expenses, and area maximization for transactions and funds managed. Branch B₂ was inefficient in the first stage ($\phi_p^1=0.65$) but fully efficient in the second stage ($\phi_p^2=1$), indicating a good service delivery and revenue generation capacity despite having a relatively lower resource utilization efficiency. Branches B₅ and B₆ were the least efficient in the first stage, at $\phi_p^1=0.48$ and then $\phi_p^1=0.29$ respectively, which points out the issue in resource allocation. Finally, to improve efficiency for inefficient branches, we can recommend Branches B₅ and B₆ should focus on improving resource utilization (optimizing staff productivity and branch expenses). Branch B₂ should enhance first-stage efficiency by increasing employee productivity and optimizing expenses. Branches B₁ and B₇ should work on enhancing customer service quality to convert operational efficiency into higher satisfaction scores and revenue.

6. Conclusions and future works

The optimization framework presented in this paper is for two-stage systems, making use of Neutrosophic Data Envelopment Analysis (NDEA) in meeting the limitations of the traditional DEA against uncertainties and imprecise data. The proposed model has made the evaluation of efficiency in very complex systems, such as the banking sector, fairly realistic because it accommodates neutrosophic numbers. The case study on bank branch performance demonstrated the effectiveness of NDEA in decomposing overall efficiency into resource utilization and customer service operations from which decision-makers could derive more profound insights. Furthermore, NDEA measured efficiency more suitably than a comparison between NDEA and ordinary DEA; particularly branches regarding which the volume of transaction, customer satisfaction level, and revenue generated are uncertain. Major results of the study include both branches B₃ and B₄ were fully efficient in both models, which proves that they optimized their resources and performed good customer

service. NDEA raised efficiency scores for branches such as B_1 and B_7 , undermeasured by standard DEA due to uncertainty in financial and operational data. Branches B_5 and B_6 had the least efficiency scores. Therefore, these branches have to better utilize their available resources and optimization strategy.

This study does not end at establishing the virtues of Neutrosophic Data Envelopment Analysis (NDEA) in improving the efficiency appraisals of two-stage systems but opens up a whole lot of areas yet to be covered. This becomes an opportunity for further research, which should try out a new two-stage NDEA framework once again in other industries beyond the banking sector such as healthcare, supply chain management, education, and manufacturing. Moreover, ascertaining whether it is adaptable across different industries forms the basis for validating the robustness of the methodology and improves decision-making in different environments. An attractive prospect would be combining NDEA with machine-learning techniques to add prediction capability to the NDEA. AI programs would identify patterns related to efficiency improvements, forecast future inefficiencies, and recommend automated optimization strategies for decision-makers.

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ForestSoft Set for Mechanical Automation Production Control Systems Analysis Based on an Intelligent Manufacturing Environment

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Abstract: Intelligent manufacturing has become a disruptive force in the global industrial landscape in the age of Industry 4.0. In mechanical automation, responsive and flexible control systems are essential as manufacturing lines become increasingly intricate. The purpose of this research is to assess production control systems for mechanical automation in the context of intelligent manufacturing settings. The evaluation of system effectiveness, flexibility, data integration, and real-time responsiveness is prioritized. The study investigates how intelligent settings affect system performance and decision-making quality by examining important operational and technological factors. We use the ForestSoft set to split each main criterion into a TreeSoft set and in each TreeSoft we compute the criteria weights and rank the alternatives. We use the WPM method as an MCDM to rank alternatives. The results highlight how crucial it is to include smart infrastructure and advanced analytics into production systems to improve flexibility and competitiveness in contemporary manufacturing.

Keywords: ForestSoft Set; Mechanical Automation Production; Control Systems; Intelligent Manufacturing Environment.

1. Introduction

Technological developments and growing demands for sustainability, quality, and personalization are causing a major upheaval in the manufacturing sector. Among these advancements, the emergence of intelligent manufacturing environments, made possible by AI, IoT, and cyber-physical systems, has completely changed conventional production paradigms[1], [2]. These settings enable autonomous decision-making, smooth machine-to-machine connection, and real-time data transmission.

Production control systems for mechanical automation are essential for organizing manufacturing procedures. They manage production schedules, streamline operational operations, and guarantee synchronization between different hardware components[3], [4]. However, when faced with the changing demands of contemporary manufacturing environments, classic control systems are limited in their capacity to adapt and respond.

Control systems that are not just automated but also intelligent—able to learn, adapt, and optimize operations through data-driven insights—are essential in intelligent industrial environments[4], [5]. These systems need to be strong enough to manage intricate decision-making situations and adaptable enough to consider regular modifications to delivery dates, batch sizes, and product designs.

Aligning performance measurements with smart factory aspirations is a major difficulty in assessing mechanical automation control systems. Newer metrics like system interoperability, data integration, predictive maintenance, and scalability must now be used in addition to more traditional ones like speed, accuracy, and uptime.

The emphasis has switched from solely mechanical performance to a more comprehensive view of system intelligence due to the integration of smart sensors, edge computing, and real-time analytics[6], [7]. These days, evaluating control systems includes determining how well they use data to detect errors, improving operational efficiency, and predictably adjusting to shifting production floor conditions.

Control systems that are in line with intelligent manufacturing principles offer competitive advantages from a strategic and managerial standpoint. These include improved product quality, lower operating expenses, and a quicker time to market[8]. Therefore, for manufacturers looking for long-term resilience, assessing and enhancing these systems is not only a technical issue but also a strategic requirement.

By creating a multifaceted assessment methodology specifically suited for intelligent manufacturing scenarios, our study tackles these problems. Technical, operational, and intelligent performance metrics are all included in the framework[9], [10]. The study assesses several automation control systems to determine their advantages, disadvantages, and areas for development using a mix of qualitative and quantitative data.

Finally, by highlighting the significance of intelligent system integration, this article adds to the larger conversation on digital transformation in manufacturing. For engineers, researchers, and decision-makers working on automated production control design or management in the context of Industry 4.0, it also offers practical insights.

Six additional categories of soft sets were added by Smarandache between 2018 and 2024: HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, SuperHyperSoft Set, TreeSoft Set, and ForestSoft Set[11], [12], [13]. These sets can deal with the criteria values to show the relations

between them. The ForestSoft set is used to divide each main criterion into a TreeSoft set and each TreeSoft set we can compute the criteria weights and ranking the alternatives.

1.1 Motivation Behind the Study

In recent years, manufacturing has been changing rapidly. The rise of smart technologies like artificial intelligence and the Internet of Things has transformed how production lines operate. Traditional systems often struggle to keep up with this fast pace. This study was motivated by the need to explore better ways to manage and control manufacturing processes in this new environment.

1.2 Problem Statement

Most existing mechanical control systems were not built with flexibility or intelligence in mind. They work well in stable settings but cannot quickly adapt when production conditions change. The main problem this research addresses is how to evaluate and improve these systems to meet the demands of modern smart factories.

1.3 Objectives of the Research

This study aims to provide a way to measure how well control systems perform in intelligent manufacturing environments. It focuses on key areas like how responsive the system is, how it integrates with data, and how flexible it can be. The goal is to help engineers choose or design systems that truly match today's manufacturing needs.

1.4 Literature Review

When it comes to handling attributes, Plithogenic-based decision-making techniques are more flexible and accommodating. Sathya et al. [14] created a Plithogenic-centered decision-making model using PFHS representations and presents the ideas of the Plithogenic Forest hypersoft set (PFHS).

The recently presented PFHS representations are combined with the Plithogenic method of making decisions based on contradictions to create a strong decision-making process that can handle attributes and sub-attribute values on a bigger scale. The decision-making dilemma of choosing a location for manufacturing plant establishment is addressed by the integrated approach put forward in this work. The key characteristics are determined, and using the potential sub-attributes, the corresponding Plithogenic forest hypersoft sets are created.

In this instance, the alternatives are exposed to each criterion to ascertain the best ranking according to the criteria, and each of the core characteristics itself creates a Plithogenic tree hypersoft set representations with many sub-attribute values. To provide a more thorough ranking, aggregate score values are also calculated. To develop new approaches to decision-making, the PFHS concept will be combined with other approaches.

It is well known that the Plithogenic Set generalizes ideas like Neutrosophic Sets and Fuzzy Sets. By providing a framework for approximating subsets using lower and upper boundaries determined by equivalence relations, rough sets help to capture uncertainty in data analysis and classification. Concepts like Hyper Plithogenic Sets, Super hyper Plithogenic Sets, Hyperrough Sets, and Superhyperrough Sets are extensions of these fundamental ideas. By presenting the Forest HyperPlithogenic Set, the Forest SuperHyperPlithogenic Set, the Forest HyperRough Set, and the Forest SuperHyperRough Set, Fujita [15] expanded upon these ideas. These frameworks are broad extensions of established paradigms in set theory.

1.4.1 Evolution of Intelligent Manufacturing Systems

Manufacturing has evolved from simple automation to highly intelligent systems. These smart systems use data, sensors, and learning algorithms to make decisions on their own. Over time, researchers have developed many models to describe and improve this process.

1.4.2 Comparative Analysis of Existing Control Frameworks

Various approaches like fuzzy sets, neutrosophic sets, and plithogenic models have been proposed to improve decision-making in manufacturing. Each has its strengths and weaknesses. This research builds on these ideas and suggests that ForestSoft Sets may offer a more detailed and organized way to evaluate system performance.

1.4.3 Gaps in Current Research

Despite many advancements, most existing studies do not fully consider the complex structure of smart factories. They often miss how different performance factors interact with each other. There's a clear need for a more complete method that can handle detailed sub-criteria and link them together meaningfully.

2. ForestSoft Set

To create a single function whose domain is the union of all tree-nodes' power sets and whose values in $P(H)$ combine the data provided by the separate TreeSoft Sets, a ForestSoft Set is created by "gluing" (uniting) a collection of TreeSoft Sets together[16].

Let U be a universal of discourse and $H \subseteq U$ be a non-empty subset, and $P(H)$ is the power set of H . We have a set of TreeSoft sets such as:

$$\left\{ F_t: P\left(Tree\left(A^{(t)} \right) \rightarrow P(H) \right) \right\}_{t \in T} \quad (1)$$

We can define the forest by union of all trees

$$Forest\left(\left\{ A^{(t)} \right\}_{t \in T}\right) = \bigcup_{t \in T} A^{(t)} \quad (2)$$

$$F: P \left(Forest \left(A^{(t)} \right) \right) \rightarrow P(H) \quad (3)$$

The evaluation process followed the WPM method to rank the alternatives based on expert input. First, a decision matrix was developed by comparing each alternative against a set of defined criteria. Experts provided their assessments using a normalized scale from 0 to 1, reflecting the relative performance of each alternative under each criterion.

Once all expert inputs were collected, the individual matrices were merged into a single, unified decision matrix. From this, the relative importance of each criterion was calculated using a simple averaging method. These weights were then applied to create a weighted decision matrix, which enabled a more balanced and meaningful comparison across the alternatives.

$$F_{ij} = (H_{ij})^{w_j} \quad (4)$$

Where H_{ij} refers to the decision matrix.

Find the final score

$$k_i = \prod_{j=1}^n F_{ij} \quad (5)$$

Rank the alternatives.

2.1 Structure and Composition of the ForestSoft Framework

ForestSoft Sets are made by combining several TreeSoft Sets. Each TreeSoft Set represents a group of related performance factors. When these are connected, they form a larger structure, the ForestSoft Set which gives a clearer picture of the overall system performance.

2.2 Justification for Using the WPM Method

The Weighted Product Method (WPM) was chosen because it is simple yet powerful. It allows us to compare alternatives by multiplying weighted scores, which makes it easier to see which system performs better overall. It also handles different types of values smoothly.

2.3 Mapping of Criteria to TreeSoft Nodes

Each major performance area is broken into smaller parts and mapped into TreeSoft nodes. For example, "System Intelligence" includes things like real-time decision-making and fault diagnosis. This detailed breakdown helps experts focus on specific system features during evaluation.

3. Detailed Analysis and Ranking of System Alternatives

This section presents the outcomes of the proposed evaluation approach applied to seven alternative control systems using a structured, multi-level decision framework. The analysis was

organized around four main performance criteria, each designed to reflect a key aspect of intelligent manufacturing systems. Each criterion was modeled as a TreeSoft set to allow a hierarchical breakdown into detailed sub-criteria, capturing both technical and operational characteristics.

The first criterion, System Intelligence and Adaptability, focused on a system's ability to respond to changes, process data in real-time, and improve over time through self-learning. This included evaluating decision-making speed, accuracy of learning models, and how quickly and effectively the system can detect and recover from faults.

The second criterion, System Integration and Compatibility, assessed how well the system connects and communicates with other hardware and software components within the manufacturing environment. Sub-criteria included sensor network quality, compatibility with existing software interfaces, and the ability to support legacy systems.

The third criterion, Production Performance and Efficiency, measured the system's contribution to optimizing manufacturing processes. This included task execution times, reduction of idle periods, throughput rates, and energy consumption efficiency—factors directly linked to cost savings and operational effectiveness.

The fourth criterion, Maintainability and Scalability, examined how easily the system can be updated, expanded, and maintained. It covered aspects such as modular system design, cost of maintenance, firmware flexibility, and diagnostic capabilities.

For each of these criteria, a separate TreeSoft set was created. Expert evaluations were collected for all sub-criteria across the seven alternatives. Three domain experts provided structured assessments, which were consolidated into a single decision matrix for each criterion.

First TreeSoft Set (System Intelligence and Adaptability)

The first decision matrix was constructed based on expert input, as shown in Figure 1. From this, the relative importance of each sub-criterion was calculated, with the resulting weights shown in Figure 2. These weights were applied to generate a weighted decision matrix (Figure 3), and the overall performance scores for each alternative were then computed (Figure 4). Based on these scores, the alternatives were ranked in order of performance (Figure 5).

Second TreeSoft Set (System Integration and Compatibility)

In the second TreeSoft set, the process was repeated following the same methodology. Expert evaluations were merged (Figure 6), and the weights of the sub-criteria were calculated accordingly (Figure 7). The weighted decision matrix was then generated (Figure 8), and final scores for each system were derived (Figure 9). The ranking based on these scores is shown in Figure 10.

Third TreeSoft Set (Production Performance and Efficiency)

For the third set, expert judgments were gathered and compiled into a single matrix (Figure 11). After determining the sub-criteria weights (Figure 12), the weighted matrix was built (Figure 13),

and the overall performance scores were calculated (Figure 14). These scores were used to determine the final rankings (Figure 15).

Fourth TreeSoft Set (Maintainability and Scalability)

Finally, the fourth TreeSoft set was developed using the same structured process. Experts completed the evaluation matrix (Figure 16), and the importance of each sub-criterion was calculated (Figure 17). A weighted matrix followed (Figure 18), leading to the final performance scores (Figure 19), and the ranking of alternatives (Figure 20).

Each TreeSoft set offered insights into how well the alternatives performed in specific functional areas. The results showed a noticeable variation in performance across the alternatives. Some systems performed well in adaptability but fell short in maintainability, while others demonstrated strong energy efficiency but lacked integration flexibility.

By using this multi-layered approach, the evaluation captured both the strengths and weaknesses of each alternative in a structured and transparent way. The detailed breakdown helped reveal not just which system was best overall, but why it achieved that ranking—providing a valuable decision-making tool for selecting or improving intelligent control systems in smart manufacturing environments.

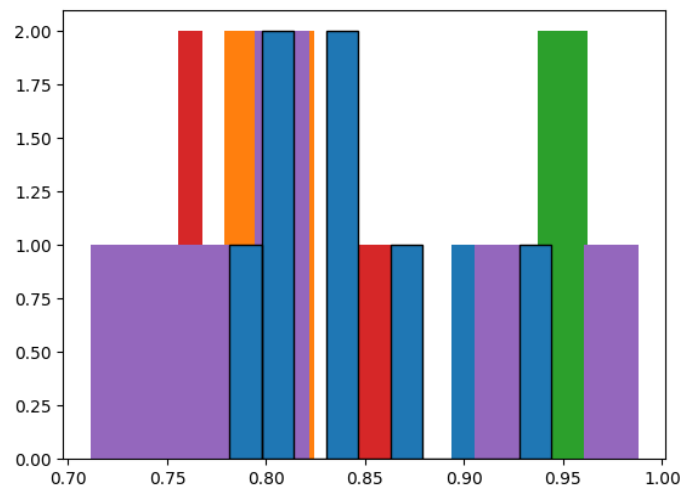


Fig 1. Combined decision matrix.

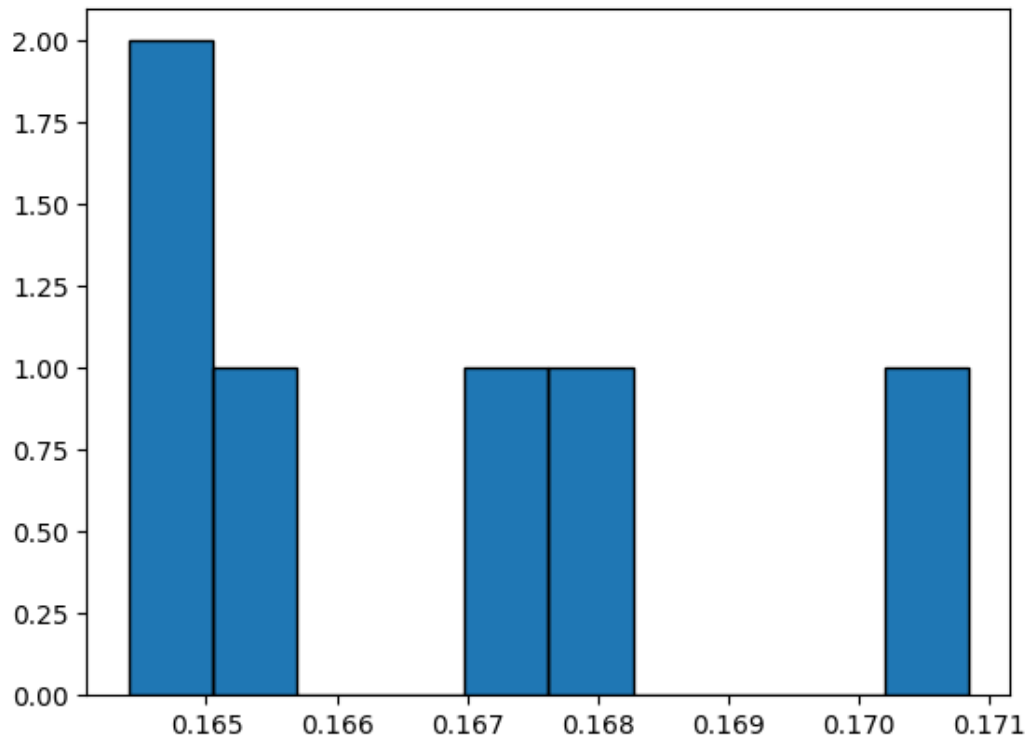


Fig 2. The criteria weights.

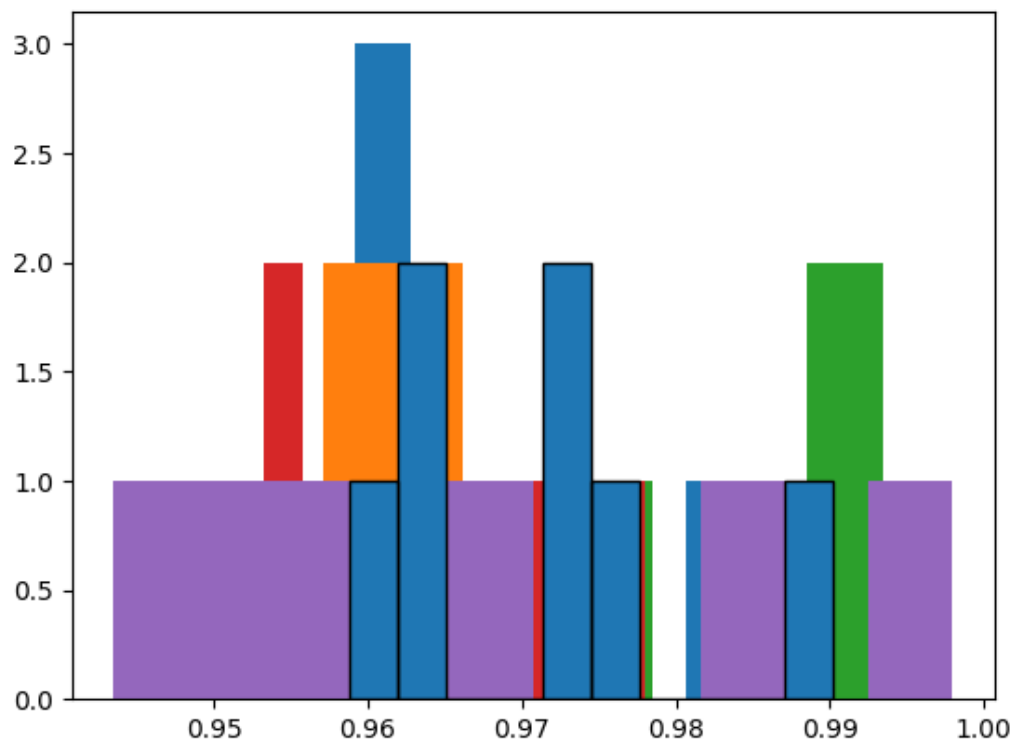


Fig 3. The weighted decision matrix.

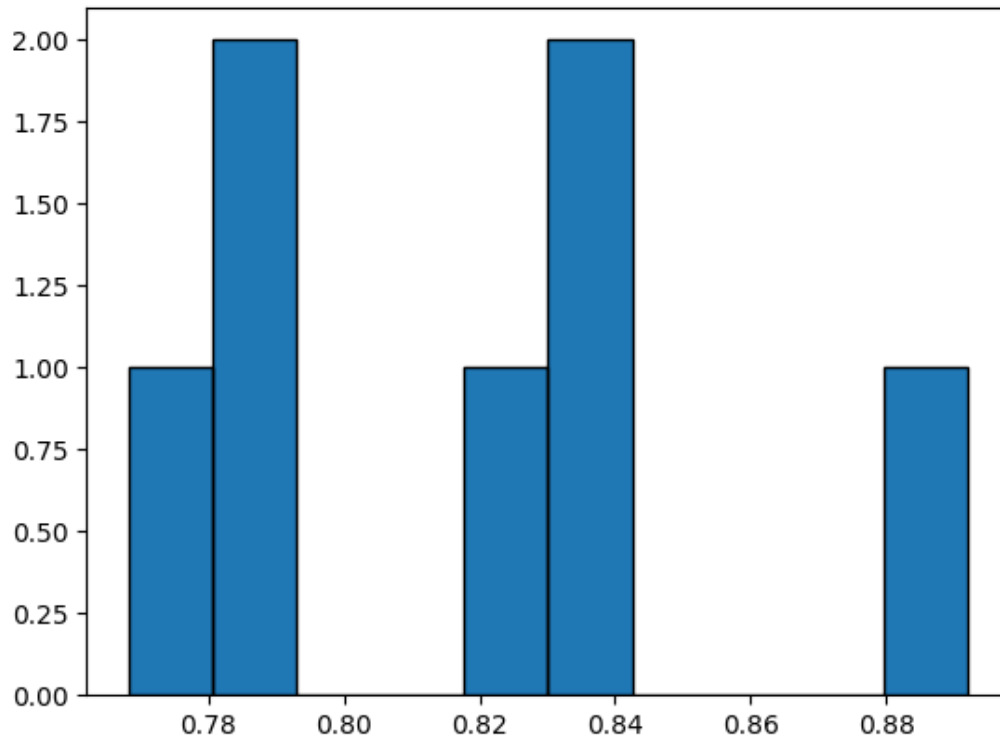


Fig 4. The final score of each alternative.

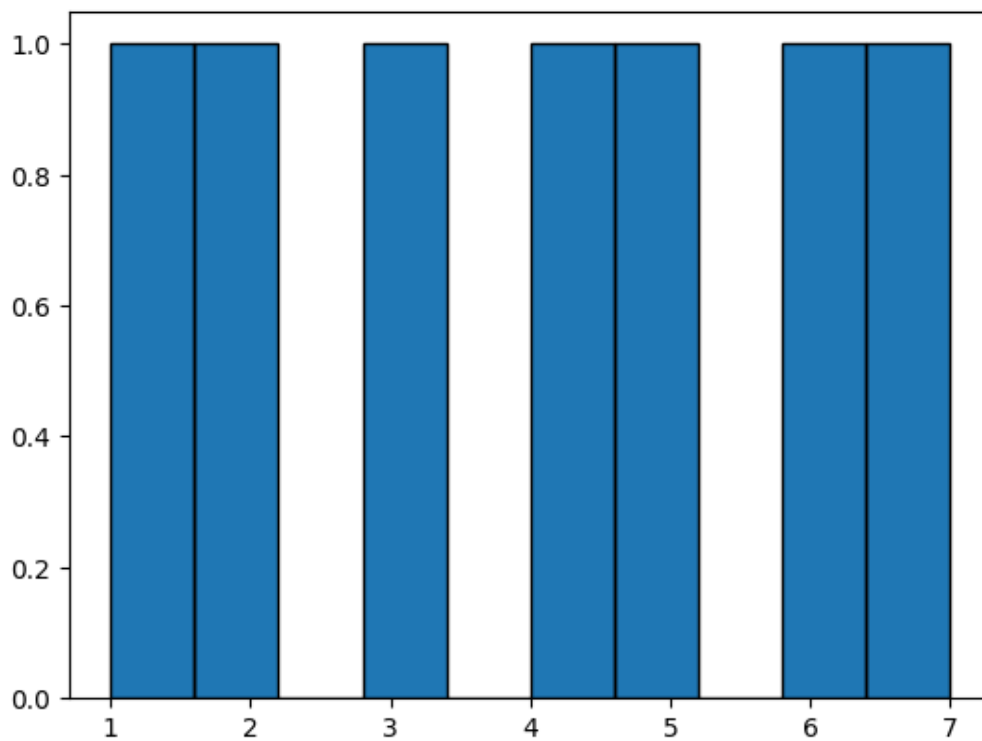


Fig 5. The rank of alternatives.

In the second TreeSoft set, the evaluation process followed a similar structured approach. Three experts assessed how each alternative performed in relation to the defined criteria. Their evaluations were combined with forming a single decision matrix that reflected a shared perspective, as illustrated in Figure 6. After collecting the data, the next step was to determine the weight of each criterion. These weights represent how important each factor is in the decision-making process and were calculated by averaging the experts' inputs. The resulting values are shown in Figure 7. Using these weights, a weighted decision matrix was developed. This helped to adjust the influence of each criterion based on its importance, providing a more accurate comparison between the alternatives. The weighted values are displayed in Figure 8. From this matrix, a final score for each alternative was calculated using the given formula. These scores, shown in Figure 9, offer a clear view of how each system performed overall. Finally, based on these results, the alternatives were ranked from highest to lowest performance in Figure 10.

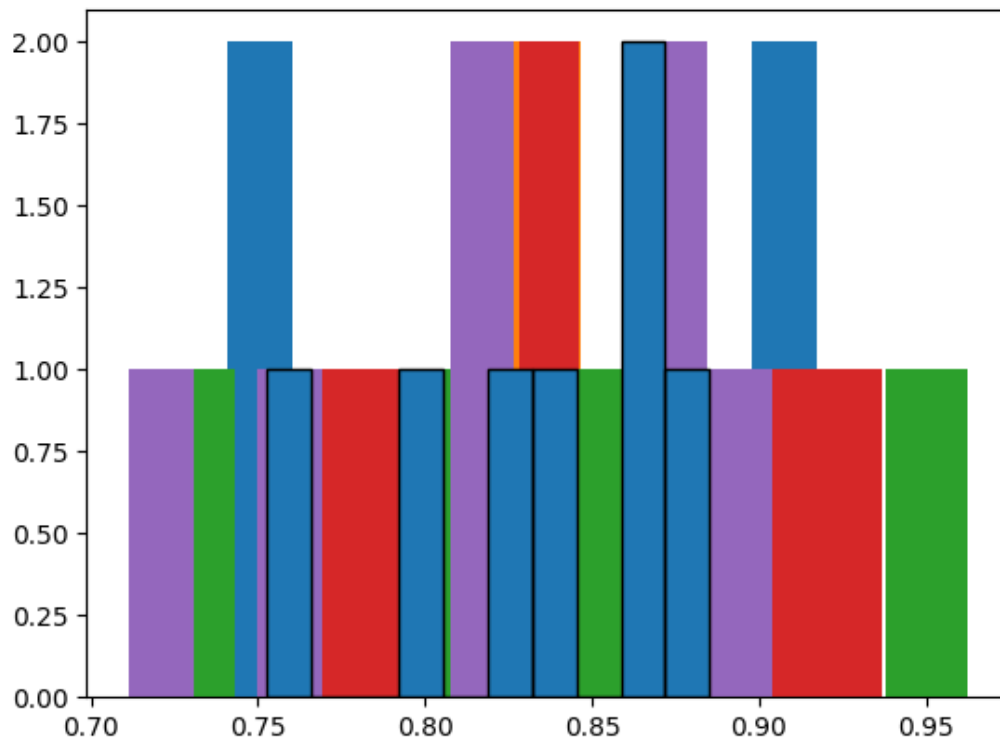


Fig 6. Combined decision matrix.

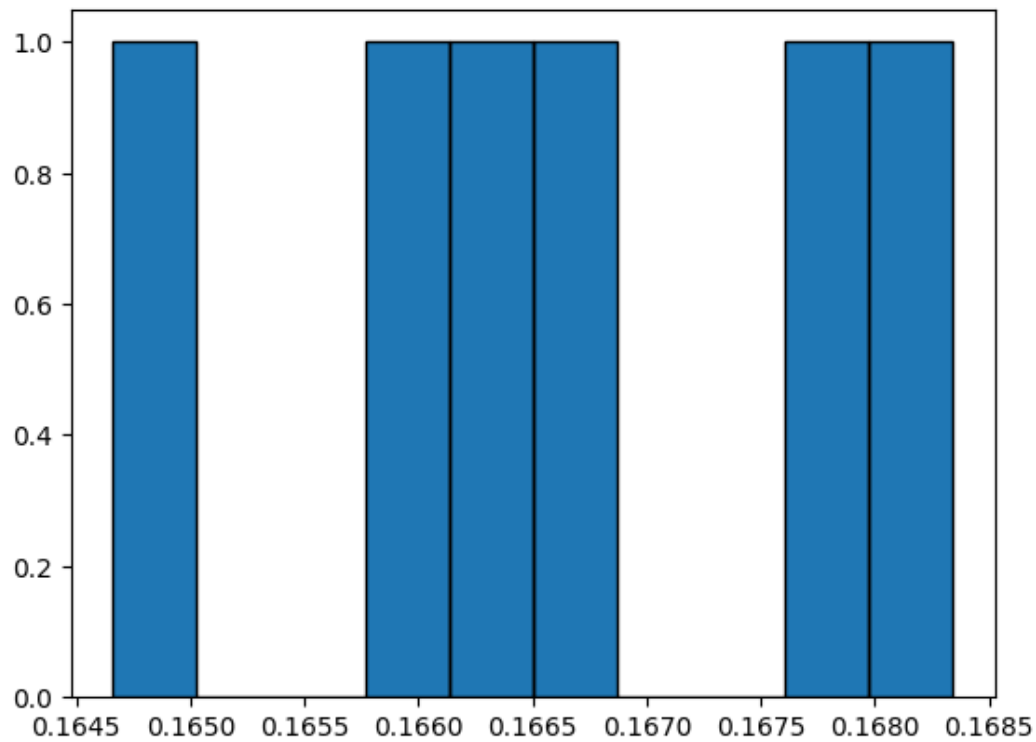


Fig 7. The criteria weights.

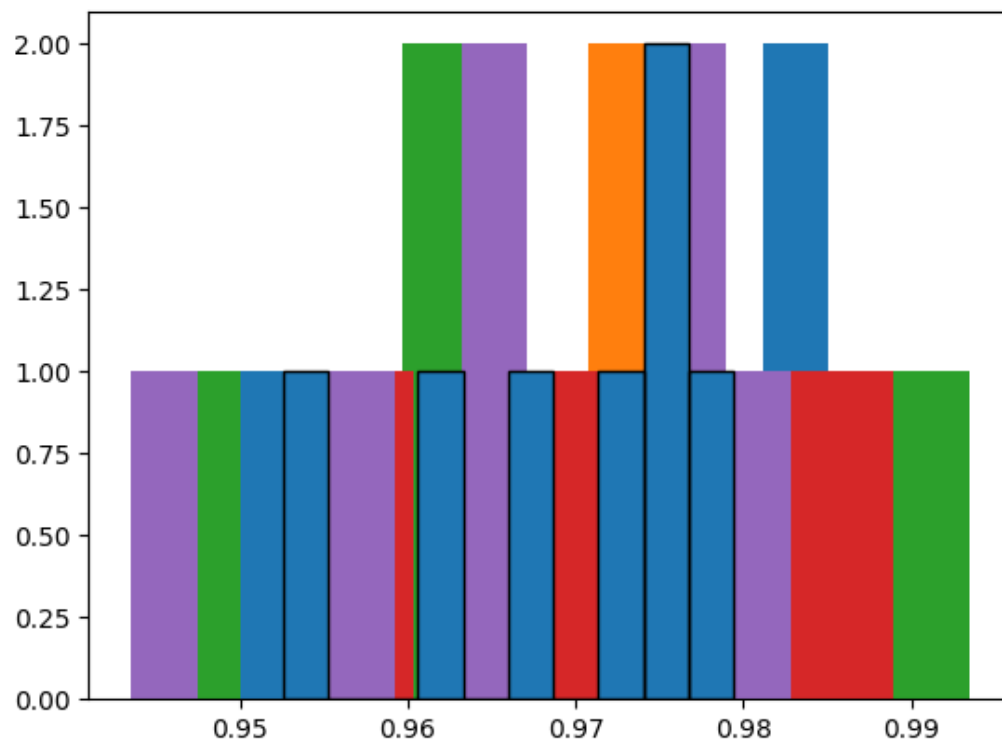


Fig 8. The weighted decision matrix.

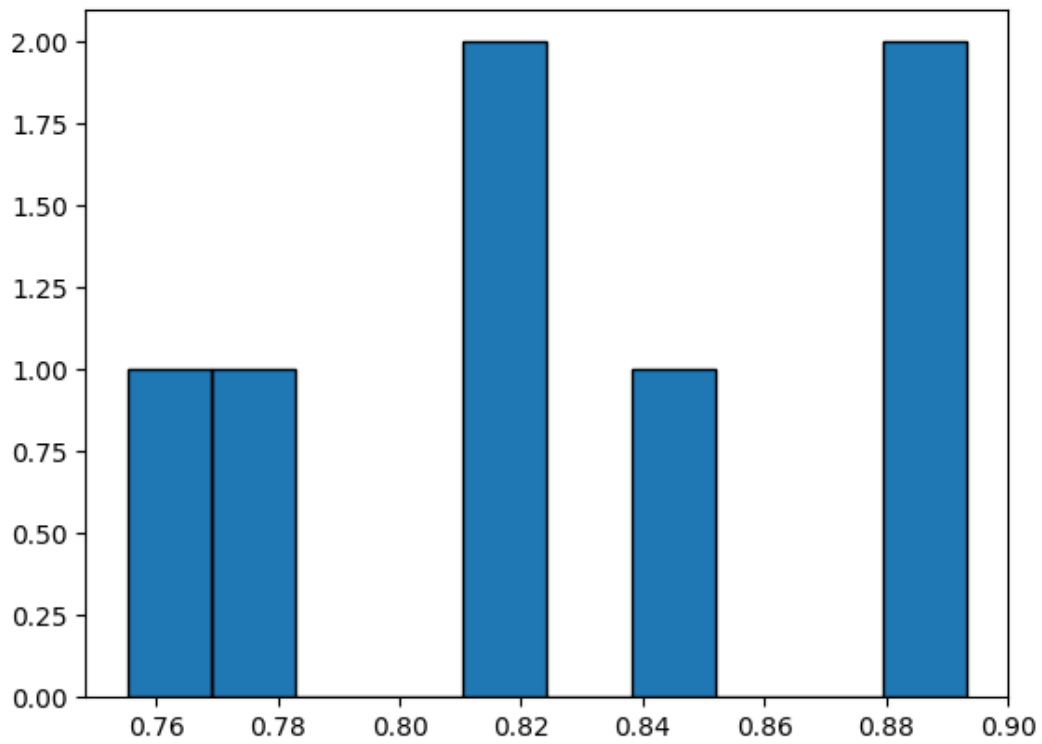


Fig 9. The final score of each alternative.

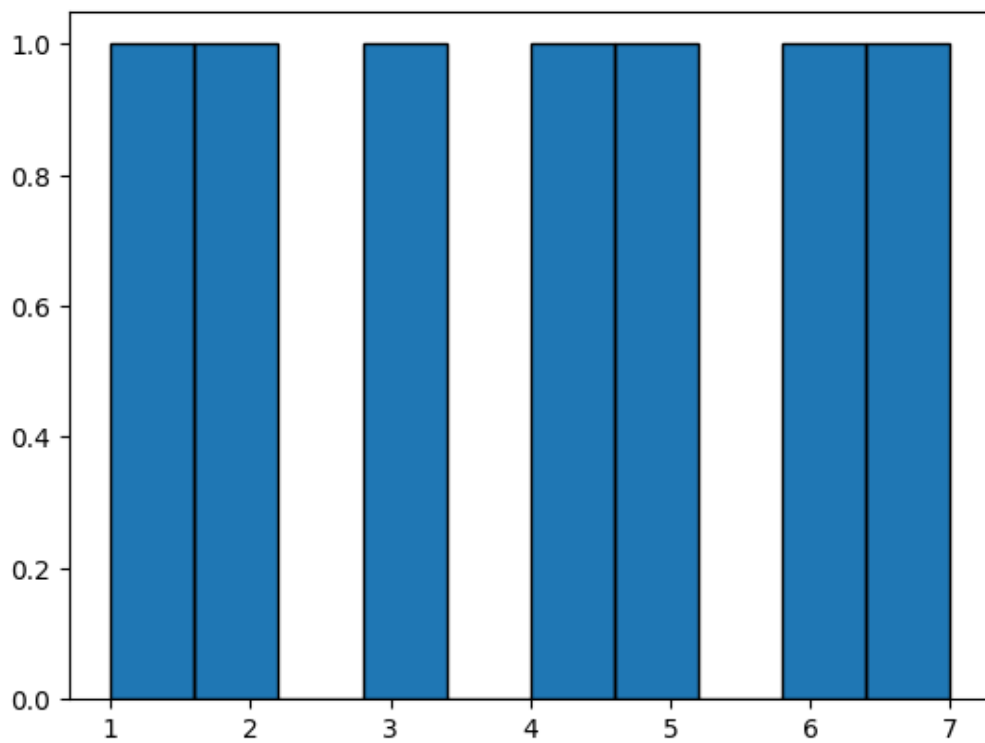


Fig 10. The rank of alternatives.

In the third TreeSoft set, the evaluation began with three experts providing their judgments on how each alternative performed according to the selected criteria. Their individual assessments were carefully reviewed and then merged into one unified decision matrix, as presented in Figure 11. Once the combined matrix was ready, the importance of each criterion was calculated based on the experts' input. These weights reflect which criteria had the most influence on the final decision and are shown in Figure 12.

With the criteria weights in place, a weighted decision matrix was created. This matrix allowed the influence of each criterion to be factored into how each alternative was scored, making the comparison more balanced and realistic. The weighted values are displayed in Figure 13.

The next step was to calculate a final score for each alternative, summarizing their performance across all weighted criteria. These scores, shown in Figure 14, helped identify which systems performed best overall. Based on these results, the alternatives were ranked accordingly, as shown in Figure 15.

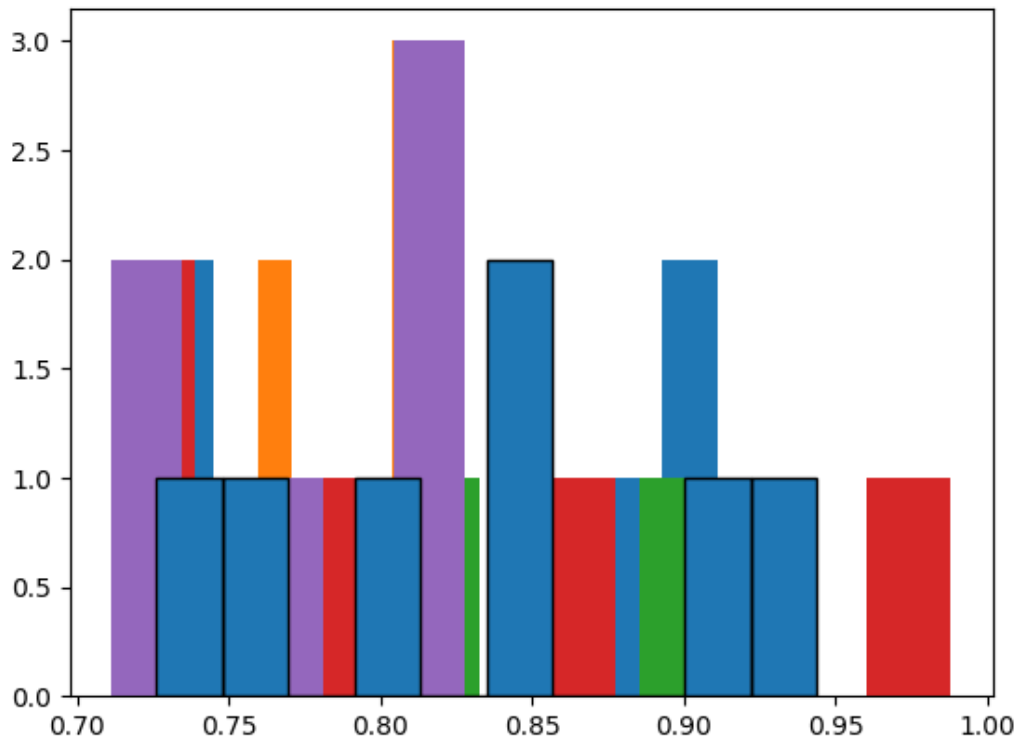


Fig 11. Combined decision matrix.

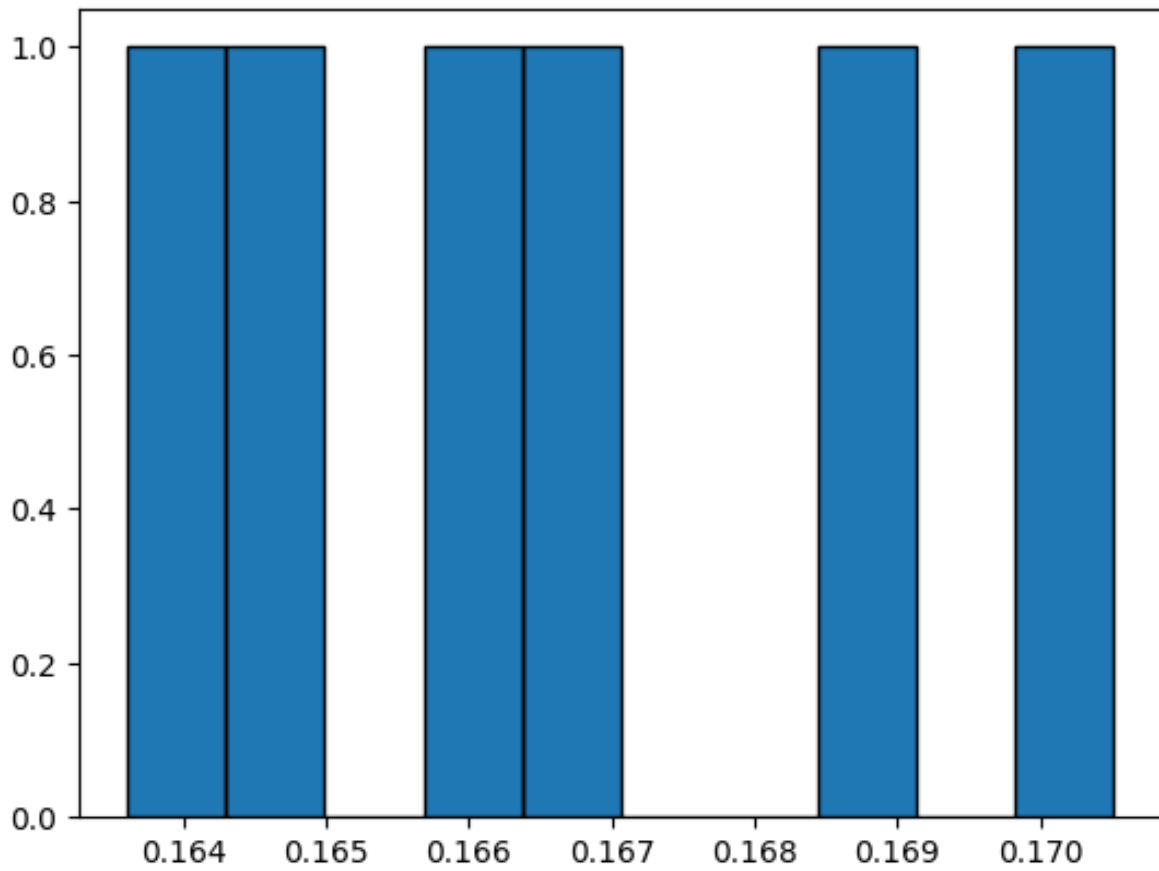


Fig 12. The criteria weights.

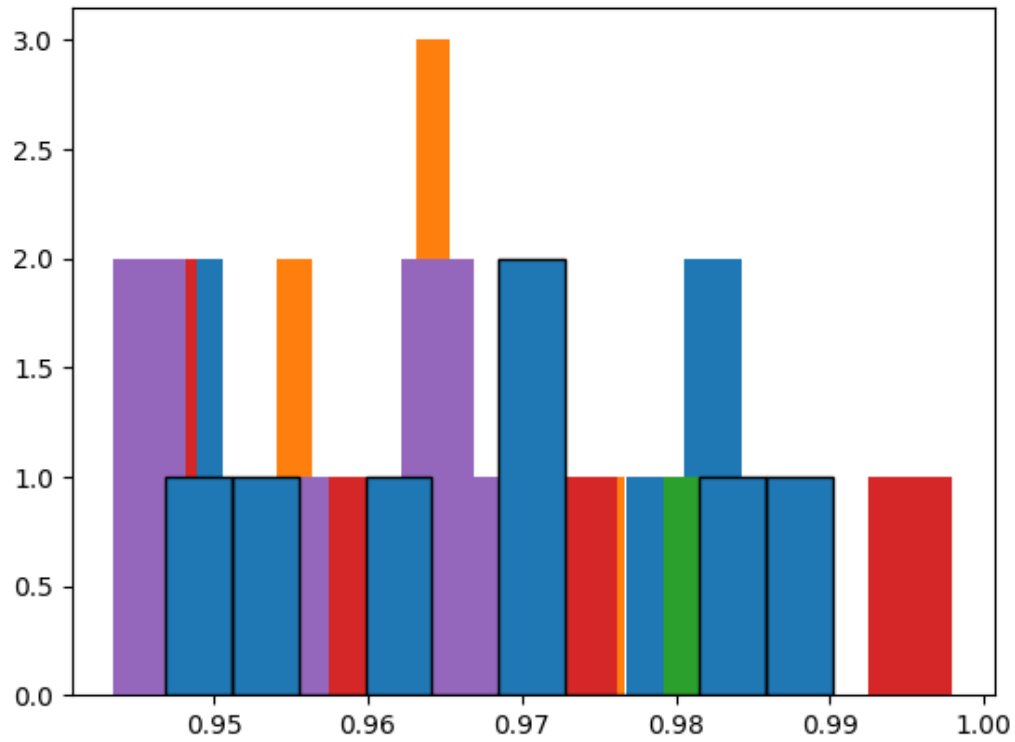


Fig 13. The weighted decision matrix.

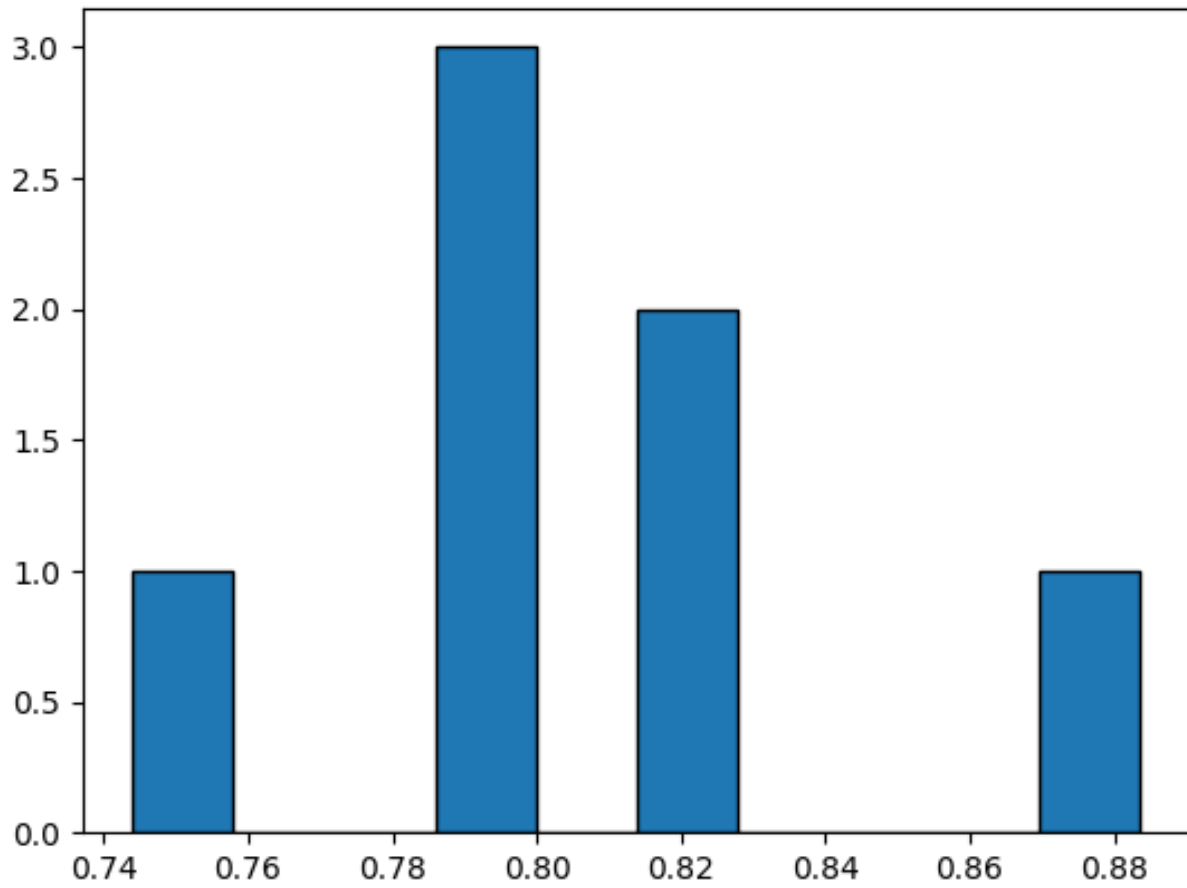


Fig 14. The final score of each alternative.

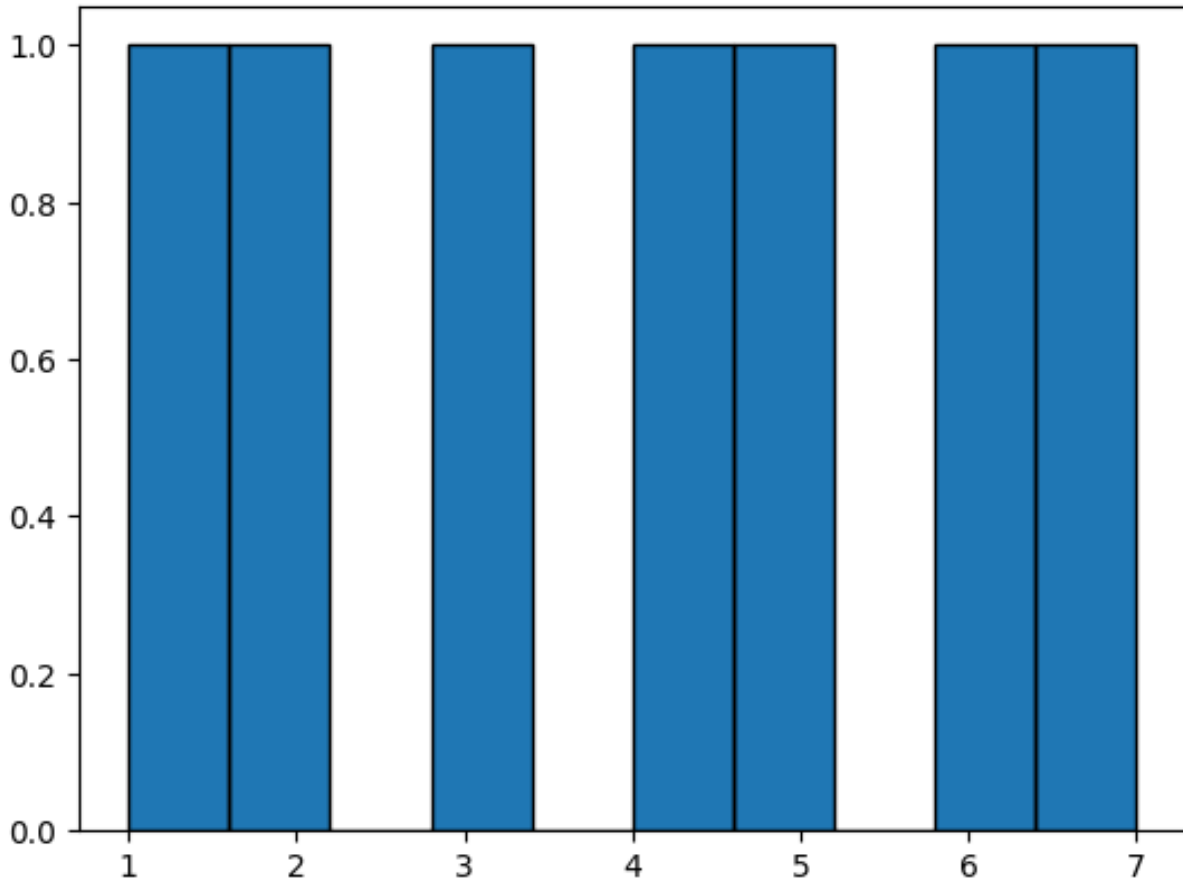


Fig 15. The rank of alternatives.

In the fourth TreeSoft set, three experts were involved in evaluating the alternatives against the set of defined criteria. Each expert provided their assessments separately, and their inputs were then merged into a single, combined matrix to represent a shared view, as illustrated in Figure 16. Once the matrix was prepared, the next step was to calculate the weights for each criterion, reflecting their importance in the decision process. These weights were derived from the average of expert opinions and are shown in Figure 17.

Using the assigned weights, a weighted decision matrix was constructed. This matrix helped to highlight how each alternative performed after considering the importance of each criterion. The weighted matrix is presented in Figure 18.

Afterward, the overall score for each alternative was calculated using the formula provided earlier. These final scores, shown in Figure 19, represent the combined performance of each system across all the criteria. Based on these scores, the alternatives were ranked from best to worst in Figure 20, completing the evaluation for this TreeSoft set.

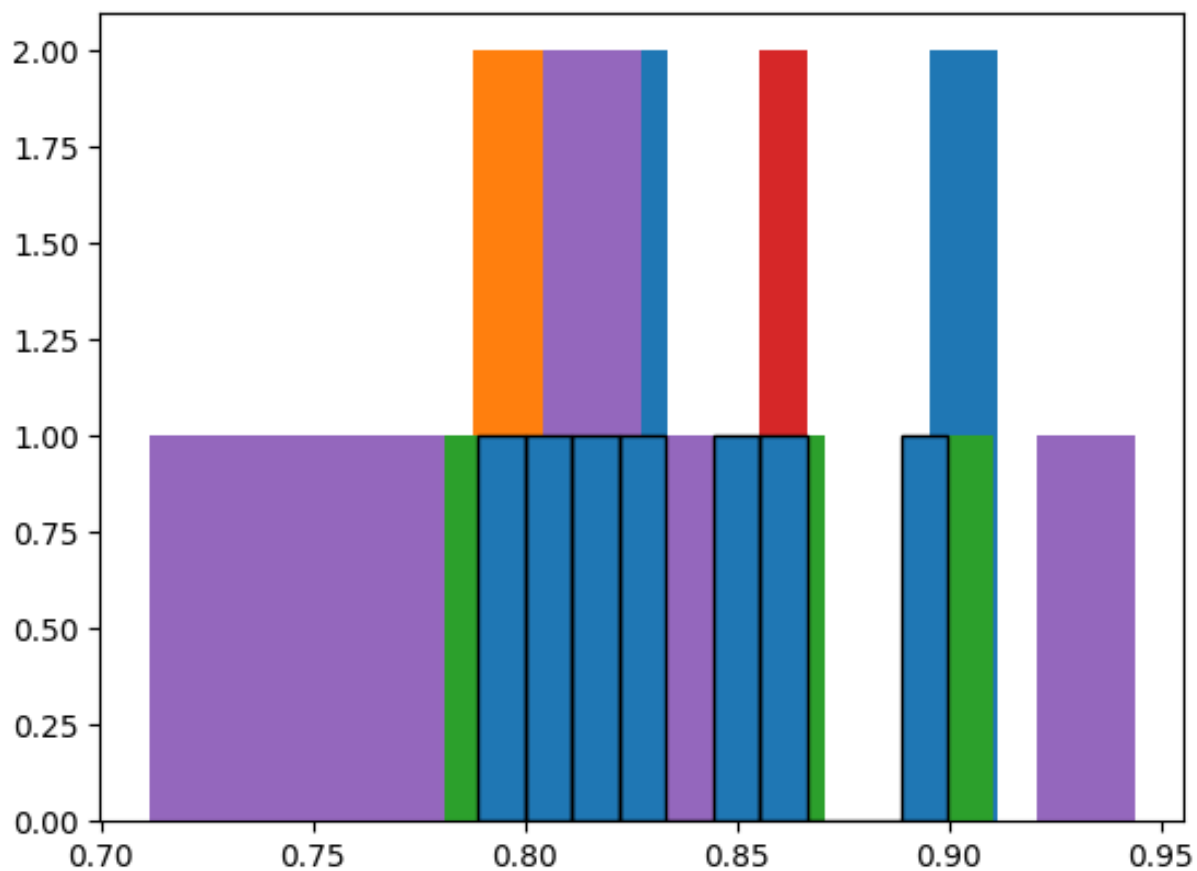


Fig 16. Combined decision matrix.

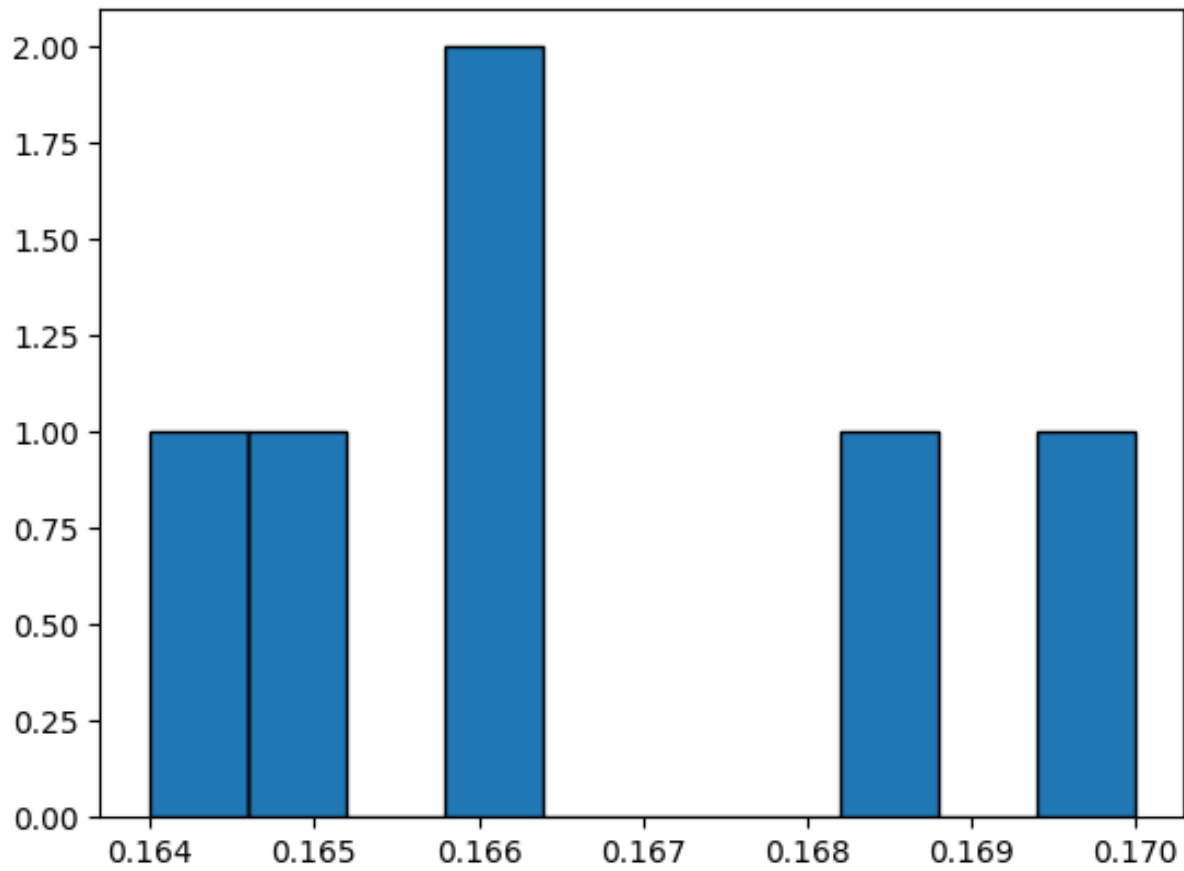


Fig 17. The criteria weights.

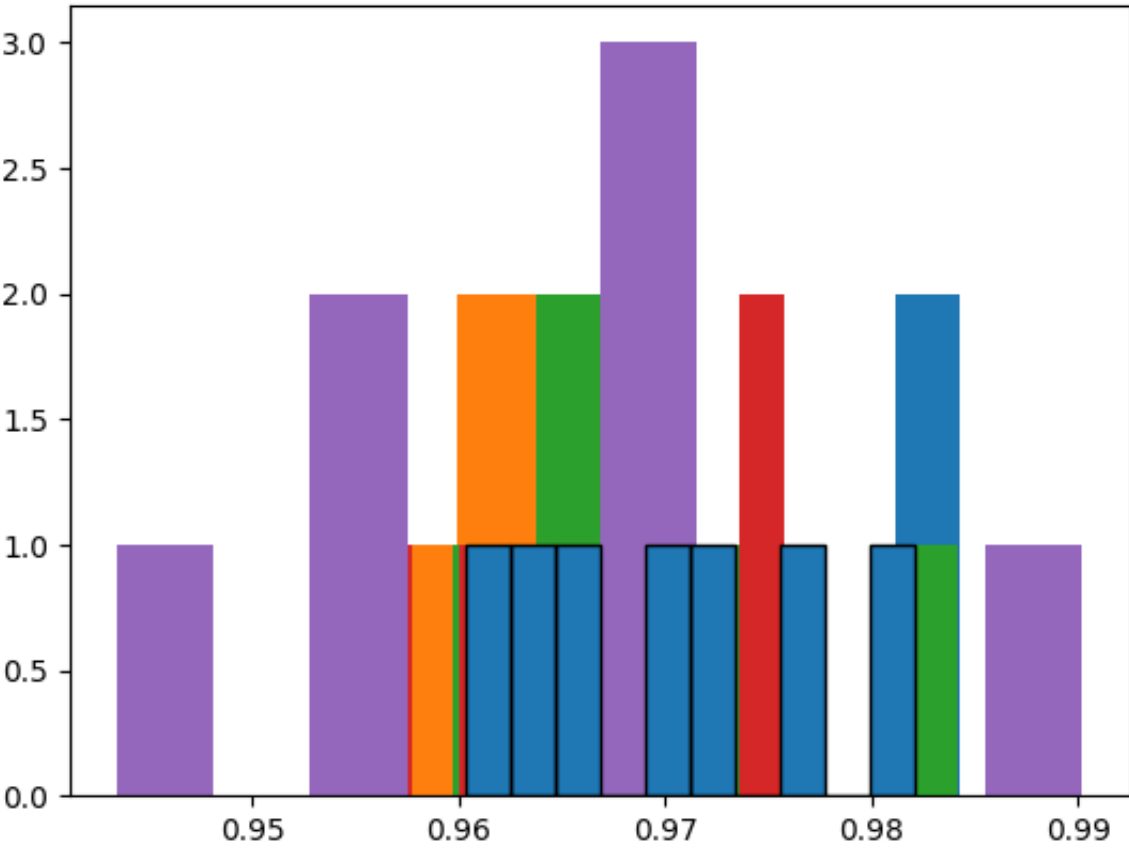


Fig 18. The weighted decision matrix.

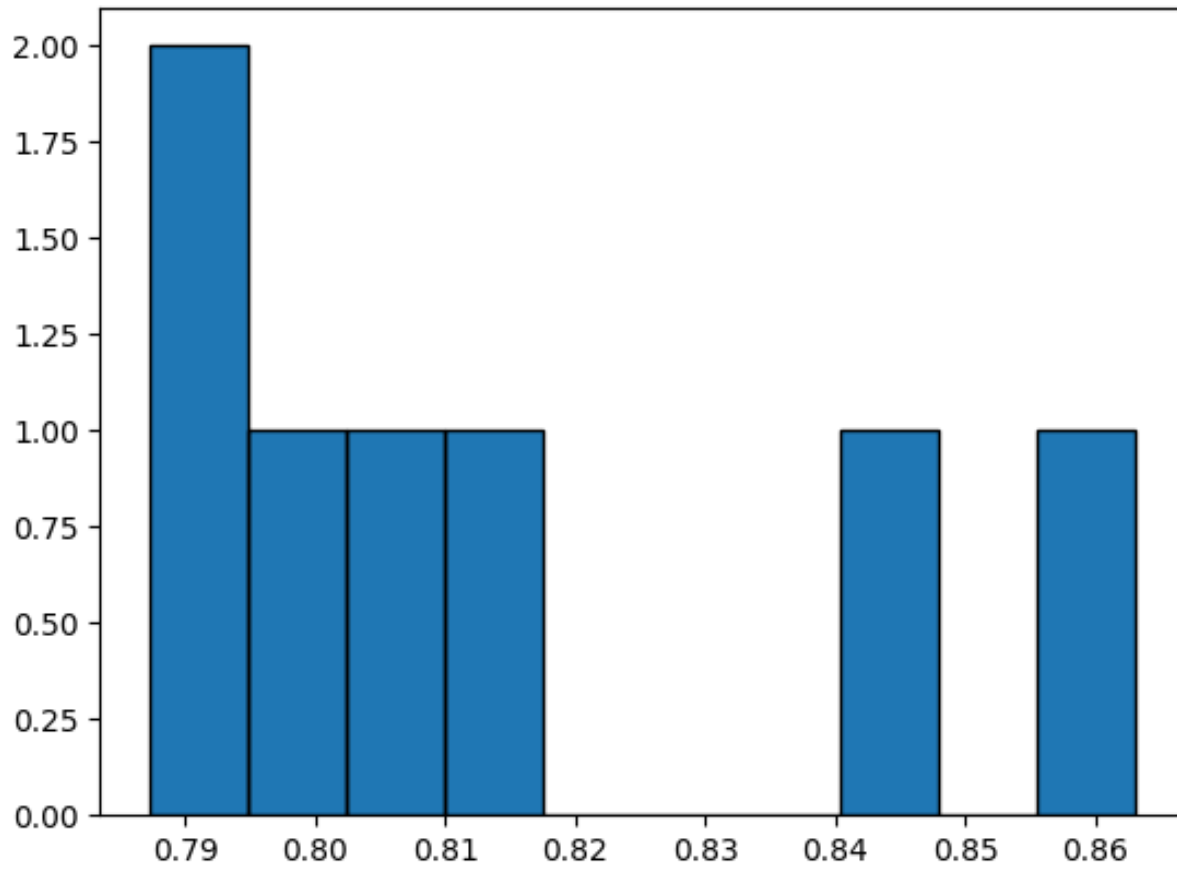


Fig 19. The final score of each alternative.

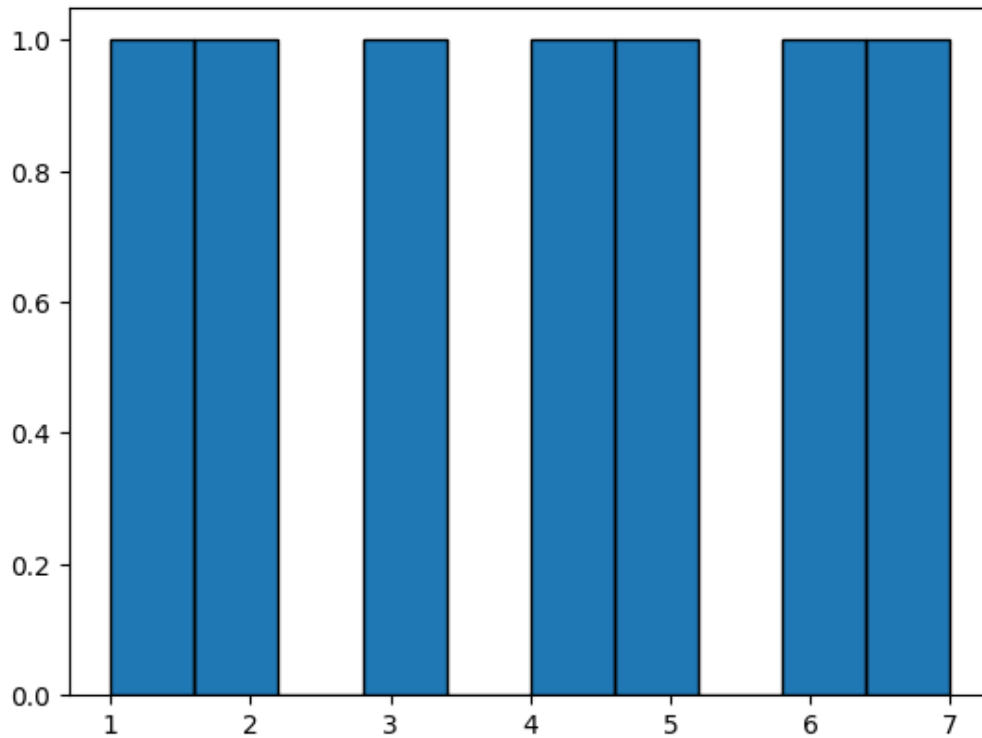


Fig 20. The rank of alternatives.

After evaluating all four TreeSoft sets, the performance of each alternative revealed interesting patterns. Alternative A1 showed a steady performance overall. While it didn't reach the top in any category, it performed consistently, especially in the third tree where it came in second place. This suggests that A1 is a balanced option, offering reliability without sharp ups or downs.

A2, on the other hand, performed slightly below average across the board. It struggled more in the first and second trees but held a consistent fourth-place rank in the last two. This reflects a level of stability, though it doesn't stand out as a strong performer.

A3 kept a middle-ground position throughout the evaluation. It ranked third in the first two trees and dropped a bit in the last two. Its results didn't shift dramatically, which makes it appear dependable, though it didn't show any signs of exceptional strength.

Alternative A4 started off strong with the best and second-best rankings in the first two trees. However, its performance dropped sharply in the later evaluations, ending up at the bottom. This sharp decline may mean that while A4 handles certain criteria very well, it struggles when the focus shifts to other areas.

A5 had a more mixed journey. It came in second and first in the early trees but fell to sixth in the third. Interestingly, it bounced back again in the fourth tree, showing that it has strong potential but lacks consistent performance across all areas.

A6 had one of the most notable shifts. It started low, ranking sixth and fourth in the first two trees, but then rose to first place in both the third and fourth trees. This major improvement

highlights its strength in areas related to later evaluation criteria, possibly adaptability or maintainability.

Finally, A7 ranked the lowest or nearly the lowest among most trees. It only saw a slight improvement in the third tree, which wasn't enough to change the overall picture. Its performance suggests it may not be suitable in its current form for environments with diverse and dynamic requirements.

4.1 Summary of Individual TreeSoft Set Evaluations

Each of the four main criteria was analyzed separately using its own TreeSoft Set. The results showed how each alternative system performed in each specific area. Some systems did well in intelligence but not in maintainability, for instance.

4.2 Cross-Comparison Among Alternatives

By comparing the rankings across all criteria, we can see which systems are strong all around and which ones have more specialized strengths. For example, one system may be excellent in integration but only average in energy efficiency.

4.3 Discussion of Ranking Trends

The ranking results revealed some interesting trends. Some alternatives started strong but dropped in later evaluations. Others, like A6, started with low scores but improved over time. These patterns give clues about the systems' consistency and long-term potential.

6. Validation of the Evaluation Method

The method used in this study depends on expert ratings and a ranking process that combines many factors using the ForestSoft and WPM approaches. To make sure the results are trustworthy, it's important to check if the method gives similar outcomes when tested under different conditions.

One way to test this is by changing the weights of the criteria just a little and seeing if the rankings change a lot. If the rankings stay mostly the same, the method is considered stable. Another check is comparing how different experts rated the systems. If their answers are very close, this supports the accuracy of the method.

Even though the evaluation process in this study was based on expert input, these checks help confirm that the results are not random or biased. They also show that the method can be used again in other cases and give useful results.

7. Practical Implications and Use in Real Factories

This study gives a structured way to evaluate control systems in smart manufacturing, but it's also important to think about how this can be used in real factories. Managers and engineers can apply this model when they are choosing or upgrading systems.

For example, before buying a new control system, a team can use the same steps—defining criteria, collecting expert ratings, and calculating rankings—to compare options. This saves time and reduces the risk of choosing a system that doesn't fit the factory's needs.

Also, factories can use this model to review their current systems. If a system scores low in flexibility or energy use, that tells the team where improvements are needed. Over time, this can help improve efficiency, reduce downtime, and make smarter decisions about upgrades.

The evaluation process doesn't need special tools. With a basic setup and expert input, factories can apply the same method to their own environment. This makes the model practical and useful—not just a research idea, but a tool that can be used on the ground.

8. Involving Stakeholders in System Evaluation

Control systems are used by many people in a factory, not just designers or analysts. The people who operate, maintain, or manage these systems see things from different angles. What works well for a developer might not be easy for a technician on the factory floor.

To make the evaluation better, future models should include opinions from different users. For example, criteria like user-friendliness, support tools, or how quickly errors can be fixed could be added. This makes the ranking of systems more realistic.

When people feel involved in the choice of new systems, they are more likely to use them effectively. This improves the chance of success after implementation. So, the evaluation process should not just be about numbers—it should also include the experience of the people who use the system every day.

9. Conclusions

A notable change in assessment paradigms is revealed by the examination of production control systems for mechanical automation in intelligent manufacturing settings. Mechanical efficiency alone is no longer sufficient to gauge system performance; instead, systems must exhibit intelligence, adaptability, and data reactivity. The need for sophisticated frameworks that represent the intricacy and interdependencies of smart production environments is highlighted by this study. It also emphasizes how technological innovation, human skills, and strategic vision must all come together for implementation to be successful. Integrating intelligent control systems will be crucial to maintaining operational excellence and maintaining global competitiveness as manufacturing continues to change. We used the MCDM methodology for computing the criteria weights and ranking the alternatives. We used the WPM methodology to rank the alternatives. We use the ForestSoft set to dela with different criteria values. This study used four main criteria, six sub criteria and seven alternatives.

9.1 Key Takeaways for Manufacturing Practitioners

For people working in smart manufacturing, this research provides clear guidelines on what to look for when choosing or improving a control system. It shows the importance of adaptability, data usage, and system flexibility.

9.2 Implications for Future Research

This study opens the door for more advanced research. Future studies might combine ForestSoft Sets with machine learning techniques or apply this method in specific industries like automotive or electronics for deeper insights.

9.3 Limitations of the Current Study

Like any research, this one has its limits. It relied on a small number of expert evaluations and used a fixed number of alternatives. More diverse expert input or applying the method in real-world factory settings could make the results even stronger.

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Neutrosophic Reliability Assessment of Hydroelectric Power Plant using Universal Generating Function Technique

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Abstract: The main objective of present study is to assess the reliability measures of a hydroelectric power plant under neutrosophic environment. There are several reliability evaluations like Markovian approach, Semi-Markovian approach, etc. However, these techniques fail to deal with the fuzziness of the data due to impreciseness. For this purpose, here universal generating function technique is used to evaluate the reliability and mean time to failure of hydroelectric power plant. The hydroelectric power plant is a complex structure having six subsystems having configured in series structure. The subsystem, namely turbine configured according to parallel redundancy. Each neutrosophic variable is illustrated by a triangular neutrosophic number and follows the Weibull distribution. Numerical results of reliability and MTTF are derived for understanding the system's behaviour using triangular neutrosophic number. To highlight the importance of MTTF results are also represented graphically.

Keywords: Neutrosophic reliability, triangular neutrosophic number, mean time to failure, sustainable manufacturing system, universal generating function.

1. Introduction

Reliability in systems plays a major role, as it ensures constant performance, reduces failures, and increases user trust. There are several methods, such as the Markov technique, fault tree analysis, semi-Markovian technique and reliability block diagram, which are used for the reliability evaluation of process industries. However, these techniques fail to deal with the fuzziness and uncertainty of the data due to imprecision. Uncertainty theory is a branch of mathematics that deals with modeling and analyzing uncertain behavior, like fuzziness and randomness. The fuzzy set theory was first introduced by Zadeh [1], and since then, the various properties of fuzzy uncertainty theory have evolved in a new direction. Fuzzy set theory can handle uncertainties arising from vagueness or partial belonging, but it cannot deal with all types of uncertainties, especially in real-life problems having incomplete information. Atanassov [2] modified the fuzzy set theory to create intuitionistic fuzzy set theory. In this modified intuitionistic fuzzy theory, the concept of both membership function and non-membership function was introduced. These membership functions and non-membership functions can measure the degree of belonging and non-belongingness in a set. Liu and Yuan [3] applied the concept of triangular fuzzy numbers and an intuitionistic fuzzy set theory to introduce a triangular intuitionistic fuzzy set. Anand and Bharatraj [4] conducted the number theory analysis of triangular fuzzy numbers. Seikh et al. [5] discussed triangular fuzzy numbers from an intuitionistic fuzzy perspective. In this study, some basic rules and non-linear operations of triangular intuitionistic fuzzy numbers (TIFN) are developed to establish the relation between two TIFNs. The average ranking index was introduced, and a method was explained to approximate a TIFN to a closely estimated interval number. As the traditional set theory and fuzzy set theory failed to handle many situations involving uncertain information, the neutrosophic set theory was introduced to address these uncertainties more effectively. While fuzzy set theory only deals with truth values but,

neutrosophic set theory overcomes these limitations by dealing with three components-truth, indeterminacy, and falsity. Smarandache [6,7,8,9] established the concept of a neutrosophic set and considered the truth membership function, indeterminacy function, and falsity function. It can manage uncertainty and imprecision in data more distinctly. This theory can fulfill all the limitations of traditional fuzzy set theory, as it can solve real-world problems more effectively and be used in decision-making. Several researchers proposed extensions of the neutrosophic set to handle different types of problems [10,11,12]. Though very few works reported on the reliability evaluation of industrial systems under neutrosophic environment. Mahapatra and Roy [13] analyzed fuzzy system reliability by considering the reliability of each component as a triangular intuitionistic fuzzy number and described the expressions for fuzzy reliability of series and parallel systems. An electric network model of a dark room is considered and the reliability of each component of this system is measured as triangular intuitionistic fuzzy numbers. Garg et al. [14] analyzed the reliability of industrial systems. The lambda-tau methodology was used, and the uncertain and imprecise data was analyzed through the intuitionistic fuzzy set (IFS) theory and the results were compared with existing traditional crisp and fuzzy methodologies results. Guilani et al. [15] studied the reliability analysis of non-repairable three-state systems by introducing a new technique based on the Markov process and compared it to traditional methods like universal generation function (UGF) and recursive algorithms. Li et al. [16] used traditional universal generating function (UGF) to improve the reliability and availability of multi-state system and random fuzzy variables were used by an extension hybrid UGF (HUGF). Gao et al. [17] reflected the factors like load, strength, operational states, and probabilities as a fuzzy factor and design the reliability models for multi-state systems. To design the model components and systems, the universal generating function method was used. Chakraborty et al. [18] explored neutrosophic numbers, including linear and non-linear generalized triangular neutrosophic numbers. The concept of de-eutrophication was also presented and used these concepts to imprecise project evaluation and route selection problems. Ebeling [19] described reliability as the probability that a system or component will perform its required functions under stated conditions for a specified period of time. Measurement of reliability is quite challenging due to circumstances like system behavior change, human error, and the lack of proper data availability (insufficient data). For the success of any product/system, reliability plays a very important role. To enhance the design of any system, traditional reliability methods are commonly used by researchers. The involvement of complexity, human behaviour, and natural phenomena's inconsistency/uncertainties arises in the failure data of systems. In such situations traditional probability-based models face difficulties in dealing with uncertainties in data. The uncertainties in failure probabilities cannot be clearly measured by these traditional methods. To deal with this limitation, various fuzzy sets, intuitionistic fuzzy sets, Fermatean fuzzy sets, Pythagorean fuzzy sets, etc., were used to handle uncertainty in reliability analysis. Alhasan and Smarandache [20] focused on neutrosophic Weibull distribution and its family. Various variations of this distribution are also explained such as Neutrosophic Inverse Weibull, Neutrosophic Rayleigh, Neutrosophic three-parameter Weibull, Neutrosophic Beta Weibull, Neutrosophic five Weibull, Neutrosophic six Weibull distributions and help to deal with problems having uncertain, vague, and imprecise. Saini et al. [21] presented the simplified version of a single-valued neutrosophic number (single-valued trapezoidal neutrosophic number). It uses the process called de-neutrophication for practical applications which convert neutrosophic number into a crisp number. Mondal et al. [22] studied a mining safety model by using the neutrosophic sets, triangular single-valued neutrosophic numbers, trapezoidal single-valued neutrosophic numbers, and neutrosophic differential equations. Alamin et al. [23] emphasizes the value of neutrosophic set theory in discrete systems. The homogeneous linear differential equations were solved under neutrosophic environment, and the initial conditions and coefficients are used as neutrosophic numbers. Saini and Ahirwar [24] applied interval-valued trapezoidal neutrosophic numbers for solving neutrosophic transportation problems (NTP) during uncertain and inconsistent situations. Kamal et al. [25] suggested a decision-making model to enhance the reliability of complex systems. The components are repaired or replaced during

maintenance to make the system more efficient. The best results were found by using a combination of four-valued neutrosophic technique and fuzzy logic. Ram et al. [26] evaluates the reliability of solar panel k-out-of-n systems using universal generating function. Kumar et al. [27] studied a gas turbine system and triangular neutrosophic sets and its applications were used to control uncertainties and enhance its reliability. Mondal et al. [28] focuses on the safety of nuclear power plants by handling the uncertainties using single-valued triangular neutrosophic numbers. Chachra et al. [29] used hesitant and dual hesitant fuzzy sets to examine the reliability of the summer air-conditioner system. The system's reliability and mean time to failure were calculated by using the universal generating function method where fuzzy variables follow the Weibull distribution. Yang [30] suggested a technique to develop the safety risk management in fabricated residential construction. The safety decisions become more accurate because this new technique helps to deal with uncertainty, indeterminacy, and inconsistency and the success of this technique was proved by a real-life case study. Zhang [31] examined the gent Garbage Sorting Cart using a decision-making approach and deals with the uncertainty in waste sorting. For this purpose, the Quadripartitioned Single-Valued Neutrosophic Z-Numbers (QSVNZN), CRITIC, and ARAS are combined. Zhao [32] evaluated the university dance teaching quality and established the triangular fuzzy neutrosophic number QUALIFLEX (TFNN-QUALIFLEX) to solve the attribute decision-making (MADM).

Renewable energy plants are very important as they generate electricity by using natural resources such as sunlight, wind, and water, and reduce their dependence on fossil fuels. Therefore, the reliability of renewable energy plants is also very important as it ensures regular power generation, reduces downtime, reduces maintenance costs, etc. But the traditional reliability assessment methods like Markovian and Semi-Markovian approaches do not properly deal with the uncertainty and imprecision present in the real-world failure data. The neutrosophic environment is used in the present study because it can deal with uncertain data more accurately. For the reliability analysis, the neutrosophic number can handle the uncertainty, imprecision, and indeterminacy more accurately. The accuracy of reliability was also improved by neutrosophic environment because it leads to better maintenance planning, risk management, and system optimization.

In this study, a model of a hydroelectric power plant was used to evaluate reliability and MTTF in a neutrosophic environment. From the introduction of the neutrosophic set and previous research, it was clearly found that the neutrosophic set can easily handle imprecise data where uncertainty is naturally combined into the set. By using universal generating function techniques and triangular neutrosophic numbers, this study can help to improve reliability and decision-making for hydroelectric power plant maintenance and operational efficiency. From the previous studies, it was noticed that no study of the hydroelectric power plant has been conducted in a neutrosophic environment. Thus, a mathematical model of a hydroelectric power plant was formed and analyzed its reliability and MTTF in the neutrosophic environment.

With knowledge of the above studies, we have introduced a new approach with some additional efforts, which include:

- The reliability of the hydroelectric power plant was first analyzed in the neutrosophic environment by using a universal generating function.
- Conventional reliability studies use probabilistic approaches, but this study deals with Weibull distribution in the neutrosophic environment.
- The study provides a graphical representation of triangular neutrosophic numbers and MTTF making results more explainable.

There are 6 sections in this paper. Section 1 consists of all the related previous studies with the novelty and motivation of our study. Section 2 contains preliminary concepts. The description of hydroelectric power plant was explained in Section 3. Section 4 shows the methodologies used for the study. Numerical results and limitations are considered in Section 5 and consequently, conclusions are considered in Section 6.

2. Preliminaries

2.1 Fuzzy Set

For a given non-empty set H , the fuzzy set $\tilde{R}$ is defined as:

$$\tilde{R} = \{(\tilde{h}, \mu_{\tilde{R}}(\tilde{h})) / \tilde{h} \in H\} \quad (1)$$

Where $\mu_{\tilde{R}}(\tilde{h})$ is the membership function such that $\mu_{\tilde{R}}(\tilde{h}): H \rightarrow [0,1]$.

2.2 Fuzzy Number

For the universal set of real numbers R , a fuzzy set $\tilde{R}$ is called fuzzy number if the membership function $\mu_{\tilde{R}}$ of $\tilde{R}$ follow the following conditions:

- i. $\mu_{\tilde{R}}(\tilde{h}): H \rightarrow [0,1]$ is continuous
- ii. $\mu_{\tilde{R}}(\tilde{h}) = 0$, for all $\tilde{h} \in (-\infty, a] \cup [d, \infty)$
- iii. $\mu_{\tilde{R}}(\tilde{h})$ is strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$
- iv. $\mu_{\tilde{R}}(\tilde{h}) = 1$, for all $\tilde{h} \in [b, c]$, where $a \leq b \leq c \leq d$.

2.3 Triangular Fuzzy Number

For a triangular fuzzy number (TFN), $\tilde{R}$ is represented as $\tilde{R} = (p, q, r)$ with the membership function $\mu_{\tilde{R}}(\tilde{h})$

$$\mu_{\tilde{R}}(\tilde{h}) = \begin{cases} 0 & \text{if } \tilde{h} < p \\ \frac{\tilde{h}-p}{q-p} & \text{if } p \leq \tilde{h} \leq q \\ \frac{r-\tilde{h}}{r-q} & \text{if } q \leq \tilde{h} \leq r \\ 0 & \text{if } \tilde{h} > r \end{cases} \quad (2)$$

Where p : lower bound

q : peak point

r : upper bound and $p \leq q \leq r$.

2.4 Intuitionistic Fuzzy Set

For a given set H , an intuitionistic fuzzy set (IFS) is defined as:

$$\tilde{R}^I = \{(\tilde{h}, \mu_{\tilde{R}^I}(\tilde{h}), \nu_{\tilde{R}^I}(\tilde{h})) / \tilde{h} \in H\} \quad (3)$$

Where $\mu_{\tilde{R}^I}(\tilde{h})$ is the membership function, $\nu_{\tilde{R}^I}(\tilde{h})$ is the non-membership function, $\mu_{\tilde{R}^I}(\tilde{h}): H \rightarrow [0,1]$, $\nu_{\tilde{R}^I}(\tilde{h}): H \rightarrow [0,1]$, and $0 \leq \mu_{\tilde{R}^I}(\tilde{h}) + \nu_{\tilde{R}^I}(\tilde{h}) \leq 1$.

For every IFS $\tilde{R}^I$ in H ,

$\pi_{\tilde{R}^I}(\tilde{h}) = 1 - \mu_{\tilde{R}^I}(\tilde{h}) - \nu_{\tilde{R}^I}(\tilde{h})$ is the intuitionistic fuzzy index of $\tilde{h}$ in H .

Where $\pi_{\tilde{R}^I}(\tilde{h})$ is the hesitancy degree.

2.5 Triangular Intuitionistic Fuzzy Number

A triangular intuitionistic fuzzy number (TIFN) is represented as:

$\tilde{\mathbb{R}}^I_{TIFN} = (a_1, a_2, a_3, a'_1, a'_2, a'_3)$, where triplet (a_1, a_2, a_3) shows membership function and (a'_1, a'_2, a'_3) shows non-membership function.

$$\text{Membership function: } \mu_{\tilde{\mathbb{R}}^I}(\tilde{h}) = \begin{cases} \frac{\tilde{h}-a_1}{a_2-a_1} & \text{for } a_1 \leq \tilde{h} \leq a_2 \\ \frac{a_3-\tilde{h}}{a_3-a_2} & \text{for } a_2 \leq \tilde{h} \leq a_3 \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

$$\text{Non-membership function: } \nu_{\tilde{\mathbb{R}}^I}(\tilde{h}) = \begin{cases} \frac{a_2-x}{a_2-a'_1} & \text{for } a'_1 \leq x \leq a_2 \\ \frac{x-a_2}{a'_3-a_2} & \text{for } a_2 \leq x \leq a'_3 \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

Where, $a'_1 \leq a_1 \leq a_2 \leq a_3 \leq a'_3$ and $\mu_{\tilde{\mathbb{R}}^I}(x) + \nu_{\tilde{\mathbb{R}}^I}(x) \leq 1$

2.6 Neutrosophic Set

For a given universal set H , the neutrosophic set $\tilde{\mathbb{R}}^{NS}$ is defined as:

$$\tilde{\mathbb{R}}^{NS} = \langle (\tilde{h}; T_{\tilde{\mathbb{R}}^{NS}}(\tilde{h}), I_{\tilde{\mathbb{R}}^{NS}}(\tilde{h}), F_{\tilde{\mathbb{R}}^{NS}}(\tilde{h})) : \tilde{h} \in H \rangle \quad (6)$$

Where, $T_{\tilde{\mathbb{R}}^{NS}}(\tilde{h}), I_{\tilde{\mathbb{R}}^{NS}}(\tilde{h}), F_{\tilde{\mathbb{R}}^{NS}}(\tilde{h})$ are truth membership, indeterminacy membership, falsity membership grade of $\tilde{h}$ in $\tilde{\mathbb{R}}^{NS}$ which are real standard or non-standard subsets of $]0^-, 1^+[$ and $\mu_T(\tilde{h}) + \nu_I(\tilde{h}) + \sigma_F(\tilde{h}) \leq 3^+$.

2.7 Single Valued Linear Neutrosophic Number (SVTNN) (Chakraborty et al. [18])

A triangular single valued neutrosophic number is represented as: $\tilde{\mathbb{R}}^I_{SVTNN} = (\varepsilon_1, \varepsilon_2, \varepsilon_3; \varphi_1, \varphi_2, \varphi_3; \omega_1, \omega_2, \omega_3)$, where the quantity of the truth, indeterminacy, falsity are not dependent. The truth membership, indeterminacy membership, and falsity membership functions defined as follows:

$$T_{\tilde{\mathbb{R}}^{SVTNN}}(\tilde{h}) = \begin{cases} \frac{\tilde{h}-\varepsilon_1}{\varepsilon_2-\varepsilon_1} & \text{if } \varepsilon_1 \leq \tilde{h} < \varepsilon_2 \\ 1 & \text{if } \tilde{h} = \varepsilon_2 \\ \frac{\varepsilon_3-\tilde{h}}{\varepsilon_3-\varepsilon_2} & \text{if } \varepsilon_2 < \tilde{h} \leq \varepsilon_3 \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

$$I_{\mathbb{R}SVTNN}(\tilde{h}) = \begin{cases} \frac{\varphi_2 - \tilde{h}}{\varphi_2 - \varphi_1} & \text{if } \varphi_1 \leq \tilde{h} < \varphi_2 \\ 0 & \text{if } \tilde{h} = \varphi_2 \\ \frac{\tilde{h} - \varphi_2}{\varphi_3 - \varphi_2} & \text{if } \varphi_2 < \tilde{h} \leq \varphi_3 \\ 1 & \text{otherwise} \end{cases} \quad (8)$$

$$F_{\mathbb{R}SVTNN}(\tilde{h}) = \begin{cases} \frac{\omega_2 - \tilde{h}}{\omega_2 - \omega_1} & \text{if } \omega_1 \leq \tilde{h} < \omega_2 \\ 0 & \text{if } \tilde{h} = \omega_2 \\ \frac{\tilde{h} - \omega_2}{\omega_3 - \omega_2} & \text{if } \omega_2 < \tilde{h} \leq \omega_3 \\ 1 & \text{otherwise} \end{cases} \quad (9)$$

Where $0 \leq T_{\mathbb{R}NS}(\tilde{h}) + I_{\mathbb{R}NS}(\tilde{h}) + F_{\mathbb{R}NS}(\tilde{h}) \leq 3$, $\tilde{h} \in \mathbb{R}^{NS}$

2.8 (α, β, γ) -Cuts or (α, β, γ) -Level Intervals

The (α, β, γ) -cut set is defined as the crisp set of elements $\tilde{h}$ satisfying the following conditions:

- The degree of truth membership $T_{\mathbb{R}NS}(\tilde{h})$ is at least α : $T_{\mathbb{R}NS}(\tilde{h}) \geq \alpha$.
- The degree of indeterminacy membership $I_{\mathbb{R}NS}(\tilde{h})$ is at most β : $I_{\mathbb{R}NS}(\tilde{h}) \leq \beta$.
- The degree of falsity membership $F_{\mathbb{R}NS}(\tilde{h})$ is at most γ : $F_{\mathbb{R}NS}(\tilde{h}) \leq \gamma$.

$$\mathbb{R}^{NS}_{\alpha, \beta, \gamma} = \left\{ \left(\tilde{h}, T_{\mathbb{R}TNN}(\tilde{h}), I_{\mathbb{R}TNN}(\tilde{h}), F_{\mathbb{R}TNN}(\tilde{h}) \right) : \tilde{h} \in H, \right. \\ \left. T_{\mathbb{R}TNN}(\tilde{h}) \leq \alpha, I_{\mathbb{R}TNN}(\tilde{h}) \leq \beta, F_{\mathbb{R}TNN}(\tilde{h}) \leq \gamma, \alpha, \beta, \gamma \in [0, 1] \right\} \quad (10)$$

Where $\alpha, \beta, \gamma \in [0, 1]$ and $\alpha + \beta + \gamma \leq 1$.

2.9 Weibull Distribution (Chachra et al. [29])

The Weibull Distribution is one of the most used distributions in reliability analysis. Weibull's probability density function is given as:

$$f(t) = \begin{cases} \frac{\delta}{\theta} \left(\frac{t}{\theta} \right)^{\delta-1} \exp \left\{ - \left(\frac{t}{\theta} \right)^{\delta} \right\}, & x > 0, \theta > 0, \delta > 0 \\ 0, & \text{otherwise} \end{cases} \quad (11)$$

Where θ is a scale parameter and δ is a shape parameter.

The hazard function is defined as,

$$h(t) = \lim_{\Delta t \rightarrow 0} \frac{P(t < x < t + \Delta t)}{\Delta t} = \frac{f(t)}{R(t)}$$

The fuzzy hazard function is given by,

$$h(t) = \frac{\delta}{\theta} \left(\frac{t}{\theta} \right)^{\delta-1} \quad (12)$$

The reliability function can be formed as:

$$\begin{aligned}
 R(t) &= \int_t^{\infty} f(x) dx \\
 R(t) &= \int_t^{\infty} \frac{\delta}{\theta} \left(\frac{t}{\theta}\right)^{\delta-1} \exp\left\{-\left(\frac{t}{\theta}\right)^{\delta}\right\} dx. \\
 R(t) &= \exp\left\{-\left(\frac{t}{\theta}\right)^{\delta}\right\}
 \end{aligned} \tag{13}$$

2.10 Neutrosophic Weibull Distribution (Alhasan and Smarandache [20])

According to Weibull distribution of 3 parameters, the probability density function is given as:

$$f(t; T_{\theta}, I_{\theta}, F_{\theta}, T_{\delta}, I_{\delta}, F_{\delta}, T_t, I_t, F_t) = \begin{cases} \frac{T_{\delta}}{T_{\theta}} \left(\frac{T_t}{T_{\theta}}\right)^{T_{\delta}-1} \exp\left(-\left(\frac{T_t}{T_{\theta}}\right)^{T_{\delta}}\right), & t > 0, \theta > 0, \delta > 0 \\ 0, & \text{otherwise} \end{cases} \tag{14}$$

Where

- i. $T_{\theta}, I_{\theta}, F_{\theta}$ denotes the truth, indeterminacy, and falsity components of the scale parameter θ .
- ii. $T_{\delta}, I_{\delta}, F_{\delta}$ denotes the truth, indeterminacy, and falsity components of the shape parameter δ .
- iii. T_t, I_t, F_t denotes the truth, indeterminacy, and falsity components of the random variable t .

The hazard function is defined as the ratio of the probability density function $f(t; T_{\theta}, I_{\theta}, F_{\theta}, T_{\delta}, I_{\delta}, F_{\delta}, T_t, I_t, F_t)$ to the survival function $R(t; T_{\theta}, I_{\theta}, F_{\theta}, T_{\delta}, I_{\delta}, F_{\delta}, T_t, I_t, F_t)$

$$h(t; T_{\theta}, I_{\theta}, F_{\theta}, T_{\delta}, I_{\delta}, F_{\delta}, T_t, I_t, F_t) = \frac{f(t; T_{\theta}, I_{\theta}, F_{\theta}, T_{\delta}, I_{\delta}, F_{\delta}, T_t, I_t, F_t)}{R(t; T_{\theta}, I_{\theta}, F_{\theta}, T_{\delta}, I_{\delta}, F_{\delta}, T_t, I_t, F_t)}$$

The neutrosophic hazard function is given by,

$$h(t; T_{\theta}, I_{\theta}, F_{\theta}, T_{\delta}, I_{\delta}, F_{\delta}, T_t, I_t, F_t) = \frac{T_{\delta}}{T_{\theta}} \left(\frac{T_t}{T_{\theta}}\right)^{T_{\delta}-1} \tag{15}$$

where,

- i. T_{θ} denotes the truth membership of the scale parameter θ .
- ii. T_{δ} denotes the indeterminacy membership of the shape parameter δ .
- iii. T_t denotes the falsity membership of the random variable t .

2.11 Reliability Function

The neutrosophic reliability function of system is formed as follows:

$$\begin{aligned}
 R(t) &= \int_t^{\infty} f(x) dx \\
 R(t) &= \int_t^{\infty} \frac{T_{\delta}}{T_{\theta}} \left(\frac{T_t}{T_{\theta}}\right)^{T_{\delta}-1} \exp\left(-\left(\frac{T_t}{T_{\theta}}\right)^{T_{\delta}}\right) dx. \\
 R(t) &= \exp\left(-\left(\frac{T_t}{T_{\theta}}\right)^{T_{\delta}}\right)
 \end{aligned} \tag{16}$$

2.12 Neutrosophic Mean Time to Failure (Jamkhaneh [33])

Neutrosophic mean time to failure (NMTTF) is the estimated time to failure. Basically, it is an extension of fuzzy mean time to failure (FMTTF), including truth, indeterminacy, and falsity

components to handle uncertainty, imprecision, and inconsistency in failure time estimation. It is defined as:

$$NMTTF = \int_0^{\infty} R(t) dt : \theta \in \tilde{\theta}[\alpha, \beta, \gamma] \quad (17)$$

Where $R(t)$ is the reliability function of the hydroelectric power plant, $\tilde{\theta}[\alpha, \beta, \gamma]$ represents the neutrosophic parameters with α, β and γ as the truth, indeterminacy, and falsity membership functions.

2.13 Universal Generating Function (Chachra et al. [29])

The universal generating function was introduced by Ushakov, I.A. [34] in the context of mathematical combinatorics and formal power series. The UGF gives more flexibility in modeling multi-parameter or multi-dimensional situations by representing the sequences and functions in a more general form rather than traditional generating functions. It is a mathematical tool used to study complex systems dealing with probability distributions and reliability functions. The UGF uses recursive methods to solve complicated algorithms. In this study, the UGF is used to analyze the reliability of hydroelectric power plants.

Mathematical representation of UGF:

The UGF of independent discrete random variable is represented as $u(y) = \sum_{n=1}^N p_n y^{x_n}$, where N is the total number of possible values of random variable X , p_n is the probability related with each value x_n of random variable X , x_n denotes the value taken by the random variable in the n -th state, and y is a complex number used in the generating function.

UGF for series configuration

Let $u_h(y)$ and $u_k(y)$ are two u-functions of two components h and k connected in series configuration. The UGF is represented as:

$$u_h(y) \otimes_{ser} u_k(y) = \sum_{n=1}^{N_h} p_{hn} y^{x_{hn}} \otimes_{ser} \sum_{i=1}^{N_k} p_{ik} y^{x_{ik}} = \sum_{n=1}^{N_h} \sum_{i=1}^{N_k} p_{hn} p_{ik} y^{\min\{x_{hn}, x_{ik}\}} \quad (18)$$

UGF for parallel configuration

Let $u_h(y)$ and $u_k(y)$ are two u-functions of two components h and k connected in series configuration. The UGF is represented as:

$$u_h(y) \otimes_{par} u_k(y) = \sum_{n=1}^{N_h} p_{hn} y^{x_{hn}} \otimes_{par} \sum_{i=1}^{N_k} p_{ik} y^{x_{ik}} = \sum_{n=1}^{N_h} \sum_{i=1}^{N_k} p_{hn} p_{ik} y^{\max\{x_{hn}, x_{ik}\}} \quad (19)$$

3. System Description

In this section, a brief description of a hydroelectric power plant is given. A hydroelectric power plant consists of six subsystems: governor system, turbine, generator system, generator converter, excitation system, and main unit transformer. All the subsystems are arranged in a series configuration. The visual representation of subsystems is shown in Figure 1. The working of different subsystems of the hydroelectric power plant is described below:

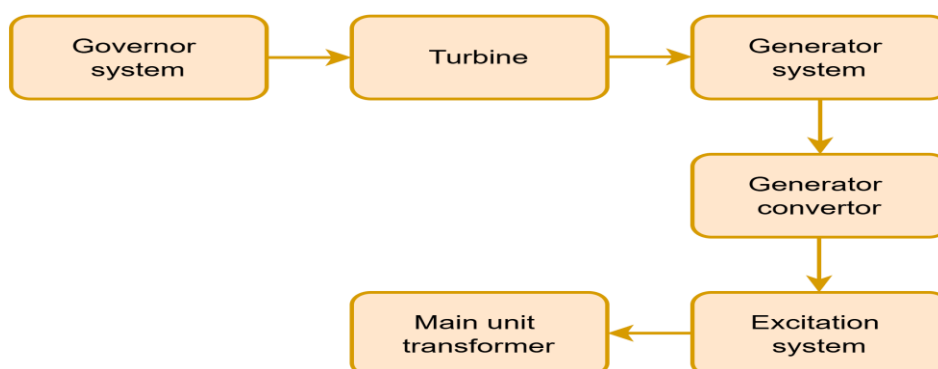


Figure 1: Flow diagram of hydroelectric power plant

- i. **Governor system:** It limits the turbine speed and output power by controlling the flow of water to the turbine. To maintain the speed and frequency, the governor system senses the speed deviations in the turbine and adjusts the wicket gates. It consists of a single unit and connects in series with the turbine. Failure of this unit results in system failure.
- ii. **Turbine:** It converts the kinetic and potential energy of water into mechanical energy by spinning its blades. The turbine blades spin due to the flow of water over them. It consists of two units connected in parallel configuration with each other and in a series configuration with generator system.
- iii. **Generator system:** It converts the mechanical energy produced in a turbine to electrical energy. The rotor of the generator is driven by the rotating shaft connected to the turbine. It consists of a single unit and connects in series with a generator converter. Failure of this unit causes system failure.
- iv. **Generator converter:** To meet the grid requirements, it transforms the properties of electrical output like frequency and voltage. It consists of a single unit and connects in series with an excitation system. Failure of this unit causes system failure.
- v. **Excitation system:** To maintain a steady magnetic field, it supplies the required field current to the generator's rotor. It adjusts the DC supply to the rotor. It consists of a single unit and connects in series with the main unit transformer. Failure of this unit causes system failure.
- vi. **Main unit transformer:** It increases the generated voltage for long-distance transmission. It transforms the low electric output from the generator to high voltage for transmission. It consists of a single unit. Failure of this unit causes system failure.

4. Methodology

4.1 Reliability function using UGF method

The reliability function of the hydroelectric power plant is formed by UGF method. The failed and working states are represented by 0 and 1 respectively. Figure 2 shows the block diagram of hydroelectric power plant. A denotes governor system, B and C are two turbines, D denotes generator system, E is the generator converter, F is the excitation system, and G denotes main unit transformer.

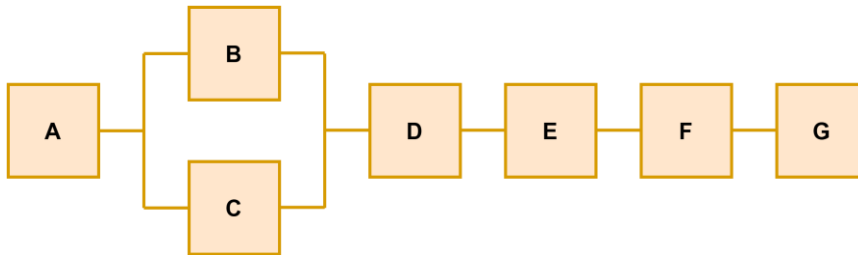


Figure 2: Block diagram of hydroelectric power plant

The u-functions for all six subsystems are represented as:

$$A : u_1(y) = P_1y^1 + (1 - P_1)y^0$$

$$B : u_2(y) = P_2y^1 + (1 - P_2)y^0$$

$$C : u_3(y) = P_3y^1 + (1 - P_3)y^0$$

$$D : u_4(y) = P_4y^1 + (1 - P_4)y^0$$

$$E : u_5(y) = P_5y^1 + (1 - P_5)y^0$$

$$F : u_6(y) = P_6y^1 + (1 - P_6)y^0$$

$$G : u_7(y) = P_7y^1 + (1 - P_7)y^0$$

Where $P_i, i = 1, 2, \dots, 6$ is the probability of the i^{th} working component and $(1 - P_i), i = 1, 2, \dots, 6$ is the probability of the failed component.

Using Eq. (18) and (19), the UGF is formed as follows:

$$u_8(y) = u_{2,3}(y) = u_2(y) \otimes_{\max} u_3(y)$$

$$u_{2,3}(y) = P_2P_3y^1 + P_2(1 - P_3)y^1 + P_3(1 - P_2)y^1 + (1 - P_2)(1 - P_3)y^0 \quad (20)$$

The u-function of the system is:

$$u(y) = u_1(y) \otimes_{\min} u_8(y) \otimes_{\min} u_4(y) \otimes_{\min} u_5(y) \otimes_{\min} u_6(y) \otimes_{\min} u_7(y) \quad (21)$$

The reliability is given by $u'(y)$ at $y = 1$. Hence, the reliability function of the hydroelectric power system is:

$$R = u'(1) = P_1P_2P_4P_5P_6P_7 + P_1P_2P_4P_5P_6P_7(1 - P_3) + P_1P_3P_4P_5P_6P_7(1 - P_2) \quad (22)$$

Since the subsystems are supposed to be independent and identically distributed, that is,

$$P_1 = P_2 = P_3 = P_4 = P_5 = P_6 = P_7 = r, \text{ then },$$

$$R = 2r^6 - r^7 \quad (23)$$

4.2 (α, β, γ) -Cuts or (α, β, γ) -Level Intervals

The triangular single valued neutrosophic number (SVTNN) used for truth, indeterminacy, and falsity membership is $\psi = (\varepsilon_1, \varepsilon_2, \varepsilon_3; \varphi_1, \varphi_2, \varphi_3; \omega_1, \omega_2, \omega_3) = (10, 15, 20; 14, 16, 22; 12, 15, 19)$. Based on this triangular input data, all the truth, indeterminacy, and falsity membership functions are attained on different α, β , and γ -cuts. By using equation (10), the α, β , and γ -cuts for triangular neutrosophic numbers are represented as:

$$T_{\mathbb{R}SVTNN}(\alpha - \text{cut}) = [\varepsilon_1 + \alpha_1(\varepsilon_2 - \varepsilon_1), \varepsilon_3 - \alpha_2(\varepsilon_3 - \varepsilon_2)]$$

$$T_{\mathbb{R}SVTNN}(\alpha - \text{cut}) = [10 + 5\alpha_1, 20 - 5\alpha_2] \quad (24)$$

$$I_{\mathbb{R}SVTNN}(\beta - \text{cut}) = [\varphi_2 - \beta_1(\varphi_2 - \varphi_1), \varphi_2 + \beta_2(\varphi_3 - \varphi_2)]$$

$$I_{\mathbb{R}SVTNN}(\beta - \text{cut}) = [16 - 2\beta_1, 16 + 6\beta_2] \quad (25)$$

$$F_{\mathbb{R}SVTNN}(\gamma - \text{cut}) = [\omega_2 + \gamma_1(\omega_2 - \omega_1), \omega_2 - \gamma_2(\omega_3 - \omega_2)]$$

$$F_{\mathbb{R}SVTNN}(\gamma - \text{cut}) = [15 - 3\gamma_1, 15 + 4\gamma_2] \quad (26)$$

The values of $T_{\mathbb{R}TNN}(\alpha_1), T_{\mathbb{R}TNN}(\alpha_2), I_{\mathbb{R}TNN}(\beta_1), I_{\mathbb{R}TNN}(\beta_2), F_{\mathbb{R}TNN}(\gamma_1), F_{\mathbb{R}TNN}(\gamma_2)$ and graphical representation of triangular neutrosophic number (TNN) are shown in Table 1 and Figure 3.

Table 1: Values of $T_{\mathbb{R}TNN}(\alpha_1), T_{\mathbb{R}TNN}(\alpha_2), I_{\mathbb{R}TNN}(\beta_1), I_{\mathbb{R}TNN}(\beta_2), F_{\mathbb{R}TNN}(\gamma_1), F_{\mathbb{R}TNN}(\gamma_2)$.

α, β, γ	$T_{\mathbb{R}SVTNN}(\alpha_1)$	$T_{\mathbb{R}SVTNN}(\alpha_2)$	$I_{\mathbb{R}SVTNN}(\beta_1)$	$I_{\mathbb{R}SVTNN}(\beta_2)$	$F_{\mathbb{R}SVTNN}(\gamma_1)$	$F_{\mathbb{R}SVTNN}(\gamma_2)$
0	10	20	16	16	15	15
0.1	10.5	19.5	15.8	16.6	14.7	15.4
0.2	11	19	15.6	17.2	14.4	15.8
0.3	11.5	18.5	15.4	17.8	14.1	16.2
0.4	12	18	15.2	18.4	13.8	16.6
0.5	12.5	17.5	15	19	13.5	17
0.6	13	17	14.8	19.6	13.2	17.4
0.7	13.5	16.5	14.6	20.2	12.9	17.8
0.8	14	16	14.4	20.8	12.6	18.2
0.9	14.5	15.5	14.2	21.4	12.3	18.6
1	15	15	14	22	12	19

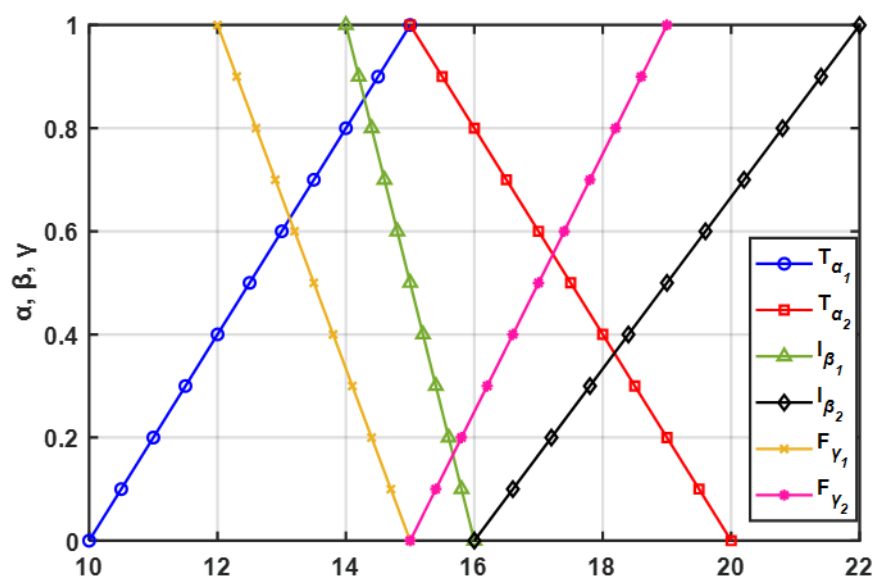


Figure 3: Graphical representation of triangular neutrosophic number $\psi = (10, 15, 20; 14, 16, 22; 12, 15, 19)$

4.3 Reliability Function

For triangular neutrosophic number, $\tilde{\mathbb{R}}^I_{SVTNN} = (\varepsilon_1, \varepsilon_2, \varepsilon_3; \varphi_1, \varphi_2, \varphi_3; \omega_1, \omega_2, \omega_3) = (10, 15, 20; 14, 16, 22; 12, 15, 19)$, the reliability functions for α, β , and γ cuts using equation (16) are represented as follows:

$$R_{SVTNN}(\alpha_1) = 2e^{-\left[\left(\frac{t}{10+5\alpha_1}\right)^\delta\right]^6} - e^{-\left[\left(\frac{t}{10+5\alpha_1}\right)^\delta\right]^7} \quad (27)$$

$$R_{SVTNN}(\alpha_2) = 2e^{-\left[\left(\frac{t}{20-5\alpha_2}\right)^\delta\right]^6} - e^{-\left[\left(\frac{t}{20-5\alpha_2}\right)^\delta\right]^7} \quad (28)$$

$$R_{SVTNN}(\beta_1) = 2e^{-\left[\left(\frac{t}{16-2\beta_1}\right)^\delta\right]^6} - e^{-\left[\left(\frac{t}{16-2\beta_1}\right)^\delta\right]^7} \quad (29)$$

$$R_{SVTNN}(\beta_2) = 2e^{-\left[\left(\frac{t}{16+6\beta_2}\right)^\delta\right]^6} - e^{-\left[\left(\frac{t}{16+6\beta_2}\right)^\delta\right]^7} \quad (30)$$

$$R_{SVTNN}(\gamma_1) = 2e^{-\left[\left(\frac{t}{15-3\gamma_1}\right)^\delta\right]^6} - e^{-\left[\left(\frac{t}{15-3\gamma_1}\right)^\delta\right]^7} \quad (31)$$

$$R_{SVTNN}(\gamma_2) = 2e^{-\left[\left(\frac{t}{15+4\gamma_2}\right)^\delta\right]^6} - e^{-\left[\left(\frac{t}{15+4\gamma_2}\right)^\delta\right]^7} \quad (32)$$

4.4 MTTF using TNN [Kumar and Singh (2015)]

When the lifetime follows neutrosophic Weibull distribution, then using equation (17), the neutrosophic mean time to failure cuts are represented as:

$$NMTTF = \{\theta\Gamma(1 + \delta^{-1}) | \theta \in \tilde{\theta}[\alpha, \beta, \gamma]\} \quad (33)$$

$$\begin{aligned} NMTTF(\alpha_1, \alpha_2) &= ((\varepsilon_1 + \alpha_1(\varepsilon_2 - \varepsilon_1))\Gamma(1 + \delta^{-1}), (\varepsilon_3 - \alpha_2(\varepsilon_3 - \varepsilon_2))\Gamma(1 + \delta^{-1})) \\ &= ((10 + 5\alpha_1)\Gamma(1 + \delta^{-1}), (20 - 5\alpha_2)\Gamma(1 + \delta^{-1})) \end{aligned} \quad (34)$$

$$\begin{aligned} NMTTF(\beta_1, \beta_2) &= ((\varphi_2 - \beta_1(\varphi_2 - \varphi_1))\Gamma(1 + \delta^{-1}), (\varphi_2 + \beta_2(\varphi_3 - \varphi_2))\Gamma(1 + \delta^{-1})) \\ &= ((16 - 2\beta_1)\Gamma(1 + \delta^{-1}), (16 + 6\beta_2)\Gamma(1 + \delta^{-1})) \end{aligned} \quad (35)$$

$$\begin{aligned} NMTTF(\gamma_1, \gamma_2) &= ((\omega_2 + \gamma_1(\omega_2 - \omega_1))\Gamma(1 + \delta^{-1}), (\omega_2 - \gamma_2(\omega_3 - \omega_2))\Gamma(1 + \delta^{-1})) \\ &= ((15 - 3\gamma_1)\Gamma(1 + \delta^{-1}), (15 + 4\gamma_2)\Gamma(1 + \delta^{-1})) \end{aligned} \quad (36)$$

5. Results

Neutrosophic fuzzy sets have a unique ability to handle uncertainty, imprecision, and indeterminacy in real-world situations over traditional fuzzy sets. A neutrosophic fuzzy set is an extension of traditional fuzzy sets and introduces three components truth, indeterminacy, and falsity, making it more flexible than traditional fuzzy sets which only work on truth value. It handles the uncertainty in a better way and describes complex systems more clearly. In an uncertain environment, the indeterminacy and falsity components provide more reliable and strong decision-making.

5.1 Single Valued Triangular Neutrosophic Reliability

In this study, the reliability function of hydroelectric power plant was formed by using universal generating method and reliability was analyzed over three specific time points $t=0$, $t=5$, and $t=10$, and

three value of shape parameter $\delta=0.8, \delta=0.9, \delta=1$ with the increment of 0.1 in the values of α, β , and γ from the interval $[0,1]$.

Table 2: Neutrosophic reliability of hydroelectric plants using TNN at $\delta=0.8$ and $t=0,5,10$.

$\delta=0.8$	$t=0$			$t=5$			$t=10$		
α, β, γ	R ($\alpha 1, \alpha 2$)	R ($\beta 1, \beta 2$)	R ($\gamma 1, \gamma 2$)	R ($\alpha 1, \alpha 2$)	R ($\beta 1, \beta 2$)	R ($\gamma 1, \gamma 2$)	R ($\alpha 1, \alpha 2$)	R ($\beta 1, \beta 2$)	R ($\gamma 1, \gamma 2$)
0	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.739468, 0.986338)	(0.962796, 0.962796)	(0.950563, 0.950563)	(2.62E-07, 0.536486)	(0.215943, 0.215943)	(0.133235, 0.133235)
0.1	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.782842, 0.984676)	(0.960667, 0.968406)	(0.945998, 0.955957)	(7.16E-06, 0.501425)	(0.198905, 0.267542)	(0.110383, 0.16542)
0.2	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.818583, 0.982764)	(0.958391, 0.973035)	(0.940927, 0.960667)	(8.77E-05, 0.464516)	(0.182041, 0.318951)	(0.089027, 0.1989)
0.3	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.847965, 0.98056)	(0.955957, 0.976876)	(0.935284, 0.964789)	(0.000595, 0.425892)	(0.165422, 0.369303)	(0.069546, 0.2331)
0.4	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.872115, 0.978008)	(0.953352, 0.980079)	(0.928996, 0.968406)	(0.002587, 0.385739)	(0.149124, 0.417977)	(0.052301, 0.26754)
0.5	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.891994, 0.975044)	(0.950563, 0.982764)	(0.921979, 0.971588)	(0.008016, 0.344302)	(0.133235, 0.464516)	(0.037596, 0.3019)
0.6	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.908398, 0.971588)	(0.947573, 0.985027)	(0.914137, 0.974395)	(0.019161, 0.301895)	(0.117852, 0.508588)	(0.025616, 0.33589)
0.7	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.921979, 0.967545)	(0.944367, 0.986941)	(0.905362, 0.976876)	(0.037596, 0.258927)	(0.103079, 0.549972)	(0.016381, 0.3693)
0.8	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.933264, 0.962796)	(0.940927, 0.988569)	(0.89553, 0.979074)	(0.063532, 0.215943)	(0.089027, 0.588546)	(0.009721, 0.40197)

0.9	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.942678, 0.957195)	(0.937233, 0.989958)	(0.8845, 0.981026)	(0.095956, 0.173696)	(0.075808, 0.624272)	(0.005282, 0.43375)
1	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.950563, 0.950563)	(0.933264, 0.99115)	(0.872115, 0.982764)	(0.133235, 0.133235)	(0.063532, 0.657185)	(0.00259, 0.46452)

The reliability of hydroelectric power plant for SVTNN with three specific time points $t=0$, $t=5$, and $t=10$, and three values of shape parameter $\delta=0.8, \delta=0.9, \delta=1$ was shown in Table 2, 3, and 4. The neutrosophic reliability for $\delta=0.8$ is (1.0,1.0) at $\alpha, \beta, \gamma=1$, and $t=0$, (0.950563, 0.950563) at $\alpha=1$, and $t=5$, (0.962796, 0.962796) at $\beta=0$ and $t=5$, (0.950563, 0.950563) at $\gamma=0$ and $t=5$, (0.133235, 0.133235) at $\alpha=1$, and $t=10$, (0.215943, 0.215943) at $\beta=0$ and $t=10$, (0.133235, 0.133235) at $\gamma=0$ and $t=10$.

Table 3: Neutrosophic reliability of hydroelectric plants using TNN at $\delta=0.9$ and $t=0, 5, 10$.

$\delta=0.9$	$t=0$			$t=5$			$t=10$		
α, β, γ	R ($\alpha 1$, $\alpha 2$)	R ($\beta 1$, $\beta 2$)	R ($\gamma 1$, $\gamma 2$)	R ($\alpha 1$, $\alpha 2$)	R ($\beta 1$, $\beta 2$)	R ($\gamma 1$, $\gamma 2$)	R ($\alpha 1$, $\alpha 2$)	R ($\beta 1$, $\beta 2$)	R ($\gamma 1$, $\gamma 2$)
0	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.91778, 0.99755)	(0.99221, 0.99221)	(0.98915, 0.98915)	(0.03067, 0.88167)	(0.68868, 0.68868)	(0.60334, 0.60334)
0.1	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.93504, 0.9972)	(0.99169, 0.99356)	(0.98797, 0.99052)	(0.06878, 0.86697)	(0.6730, 0.73133)	(0.57408, 0.6397)
0.2	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.94825, 0.9968)	(0.99113, 0.99464)	(0.98663, 0.99169)	(0.12122, 0.85022)	(0.65676, 0.76818)	(0.54314, 0.67308)
0.3	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.95845, 0.99632)	(0.99052, 0.99551)	(0.98512, 0.9927)	(0.18252, 0.83112)	(0.6397, 0.79985)	(0.51056, 0.70358)
0.4	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.96639, 0.99576)	(0.98986, 0.99622)	(0.9834, 0.99356)	(0.24782, 0.80937)	(0.6219, 0.827)	(0.47641, 0.73133)
0.5	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.97261, 0.9951)	(0.98915, 0.9968)	(0.98143, 0.9943)	(0.31388, 0.78462)	(0.60334, 0.85022)	(0.44081, 0.7565)
0.6	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.97752, 0.9943)	(0.98838, 0.99728)	(0.97919, 0.99495)	(0.37867, 0.7565)	(0.58402, 0.87006)	(0.4039, 0.77928)
0.7	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.98143, 0.99335)	(0.98754, 0.99767)	(0.97663, 0.99551)	(0.44081, 0.72464)	(0.56396, 0.88704)	(0.36589, 0.79985)

0.8	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.98457, 0.99221)	(0.98663, 0.998)	(0.97369, 0.996)	(0.49935, 0.68868)	(0.54314, 0.90156)	(0.32699 ,0.81841)
0.9	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.9871, 0.99083)	(0.98564, 0.99828)	(0.9703, 0.99642)	(0.55364, 0.64832)	(0.5216, 0.91401)	(0.28752 ,0.83514)
1	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.98915, 0.98915)	(0.98457, 0.99851)	(0.96639, 0.9968)	(0.60334, 0.60334)	(0.49935, 0.9247)	(0.24782 ,0.85022)

For $\delta = 0.9$, the neutrosophic reliability is (1.0,1.0) at $\alpha, \beta, \gamma=1$, and $t=0$, (0.98915,0.98915) at $\alpha=1$ and $t=5$, (0.99221,0.99221) at $\beta=0$ and $t=5$, (0.98915,0.98915) at $\gamma=0$ and $t=5$, (0.60334,0.60334) at $\alpha=1$ and $t=10$, (0.68868,0.68868) at $\beta=0$ and $t=10$, (0.98915,0.98915) at $\gamma=0$ and $t=10$.

Table 4: Neutrosophic reliability of hydroelectric plants using TNN at $\delta = 2$ and $t=0,5,10$.

$\delta = 1$	$t=0$			$t=5$			$t=10$		
α, β, γ	R ($\alpha 1$, $\alpha 2$)	R ($\beta 1$, $\beta 2$)	R ($\gamma 1$, $\gamma 2$)	R ($\alpha 1$, $\alpha 2$)	R ($\beta 1$, $\beta 2$)	R ($\gamma 1$, $\gamma 2$)	R ($\alpha 1$, $\alpha 2$)	R ($\beta 1$, $\beta 2$)	R ($\gamma 1$, $\gamma 2$)
0	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.97677, 0.99957)	(0.99842, 0.99842)	(0.99771, 0.99771)	(0.36787, 0.97677)	(0.92084, 0.92084)	(0.8887, 0.88875)
0.1	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.98235, 0.9995)	(0.99831, 0.99873)	(0.99743, 0.99803)	(0.45701, 0.97323)	(0.91535, 0.93504)	(0.87649, 0.90306)
0.2	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.98643, 0.99942)	(0.99818, 0.99896)	(0.9971, 0.99831)	(0.53871, 0.96906)	(0.90944, 0.94643)	(0.86273, 0.91535)
0.3	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.98946, 0.99932)	(0.99803, 0.99915)	(0.99673, 0.99853)	(0.61134, 0.96412)	(0.90306, 0.9556)	(0.84731, 0.92593)
0.4	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.99173, 0.9992)	(0.99788, 0.9993)	(0.99629, 0.99873)	(0.67434, 0.95825)	(0.89617, 0.96302)	(0.83001, 0.93504)
0.5	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.99346, 0.99906)	(0.99771, 0.99942)	(0.99579, 0.99889)	(0.72798, 0.95126)	(0.88875, 0.96906)	(0.81063, 0.94291)
0.6	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.99478, 0.99889)	(0.99753, 0.99951)	(0.99521, 0.99903)	(0.77306, 0.94291)	(0.88073, 0.97399)	(0.78893, 0.94971)
0.7	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.99579, 0.99868)	(0.99732, 0.99959)	(0.99454, 0.99915)	(0.81063, 0.93289)	(0.87208, 0.97803)	(0.76468, 0.9556)

0.8	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.99659, 0.99842)	(0.9971, 0.99966)	(0.99375, 0.99925)	(0.84176, 0.92084)	(0.86273, 0.98137)	(0.73765, 0.96072)
0.9	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.99722, 0.99811)	(0.99686, 0.99971)	(0.99282, 0.99934)	(0.86749, 0.90631)	(0.85265, 0.98413)	(0.7076, 0.96517)
1	(1.0,1.0)	(1.0,1.0)	(1.0,1.0)	(0.99771, 0.99771)	(0.99659, 0.99975)	(0.99173, 0.99942)	(0.88875, 0.88875)	(0.84176, 0.98643)	(0.67434, 0.96906)

For $\delta=1$, the neutrosophic reliability is (1.0,1.0) at $\alpha, \beta, \gamma=1$, and $t=0$, (0.99771,0.99771) at $\alpha=1$ and $t=5$, (0.99842,0.99842) at $\beta=0$ and $t=5$, (0.99771,0.99771) at $\gamma=0$ and $t=5$, (0.88875,0.88875) at $\alpha=1$ and $t=10$, (0.92084,0.92084) at $\beta=0$ and $t=10$, (0.88875,0.88875) at $\gamma=0$ and $t=10$.

5.2 MTTF using Triangular Neutrosophic Set

The neutrosophic MTTF for hydroelectric power plant is achieved by using equation (MTTF) and (cut sets). The MTTF for $\delta=0.8$ is (16.995, 16.995) at $\alpha=1$, (18.128,18.128) at $\beta=0$, and (16.995,16.995) at $\gamma=0$. The neutrosophic MTTF is obtained at $\delta=0.8$ and it is shown in Table 5 and Figure 4.

Table 5: Neutrosophic MTTF using TNN

α, β, γ	MTTF ($\alpha 1, \alpha 2$)	MTTF ($\beta 1, \beta 2$)	MTTF ($\gamma 1, \gamma 2$)
0	(11.33, 22.6601)	(18.128,18.128)	(16.995,16.995)
0.1	(11.8965, 22.0936)	(17.9014,18.8079)	(16.6551,17.4482)
0.2	(12.463, 21.5271)	(17.6748,19.4877)	(16.3152,17.9014)
0.3	(13.0295, 20.9606)	(17.4482,20.1675)	(15.9753,18.3547)
0.4	(13.596, 20.3941)	(17.2216,20.8473)	(15.6354,18.8079)
0.5	(14.1625, 19.8276)	(16.995,21.5271)	(15.2955,19.2611)
0.6	(14.729, 19.2611)	(16.7684,22.2069)	(14.9556,19.7143)
0.7	(15.2955, 18.6946)	(16.5418,22.8867)	(14.6157,20.1675)
0.8	(15.862, 18.128)	(16.3152,23.5665)	(14.2758,20.6207)
0.9	(16.4285, 17.5615)	(16.0886,24.2463)	(13.9359,21.0739)
1	(16.995, 16.995)	(15.862,24.9261)	(13.596,21.5271)

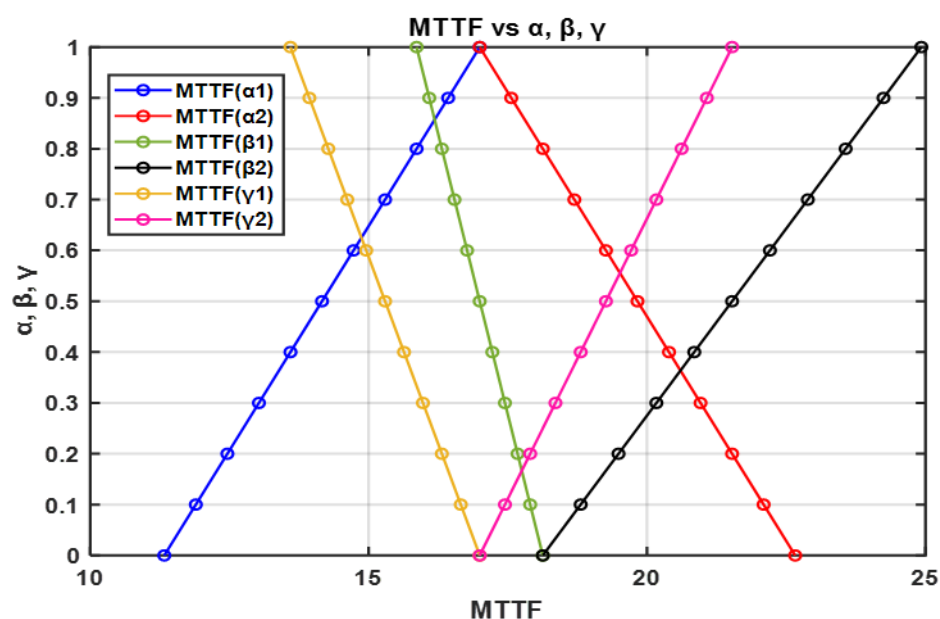


Figure 3: Graphical representation of MTTF using SVTNN

6. Conclusion

Reliability of any industrial system is a very important factor in performance evaluation and theory of uncertainty plays a key role in system modeling. For the situations having high uncertainty, fuzzy reliability can play a significant role. In this present study, we define the concept of triangular neutrosophic number and implement it on a hydroelectric power plant to estimate the reliability and MTTF where each neutrosophic variable follows Weibull distribution which was not defined earlier. The reliability function is formed by using a universal generating function. Table 1 to 5 demonstrates all the outcome results for reliability and MTTF. At $\delta=1$ and $t=5$, the reliability reaches its maximum value that is 0.99975 for $R(\alpha_1, \alpha_2)$ and MTTF attains its maximum value 24.9261. Numerical, as well as graphical methods, are used to represent triangular neutrosophic numbers and MTTF. The neutrosophic fuzzy methodology was used because it can handle uncertainty, imprecision, and indeterminacy easily and provide more accurate and flexible results for reliability and MTTF. As compared to the traditional reliability methods, the neutrosophic approach used in this study can deal with high level of uncertainties. Similarly, as compared to crisp or fuzzy reliability approaches, this neutrosophic fuzzy approach has more flexible and precise calculations of the system behavior. The above study is helpful in many fields and for future studies, but it has also contained some limitations like Weibull distribution used in this study may not be always optimal for real-world failure patterns. This study only considers the reliability and MTTF factors, but there are also many parameters like availability, maintainability, cost-related factors etc., which are also useful for reliability evaluation. The methodology used in the study is not validated for other industries like healthcare, aerospace etc. The methodologies used in this study and the results can be used in future for finding the behaviour of the system. These techniques can be used in other sectors like aerospace, healthcare, etc., as well as other neutrosophic numbers like trapezoidal neutrosophic numbers can also be used in the future. To improve real-time fault detection, the combination of neutrosophic

fuzzy reliability and machine learning can be used. Therefore, this study is very useful for future researchers who deal with system modeling with uncertainty.

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Fuzzy OverProbability for Competitiveness Evaluation of the Power Electronics Technology Industry with Multi-Criteria Decision-Making Method

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Abstract: The power electronics technology sector is essential to the advancement of industrial modernization and sustainable development as the demand for electrification and energy efficiency rises globally. A thorough competitiveness assessment model for the power electronics technology sector is presented in this study. Ten crucial criteria are developed using a multi-criteria decision-making (MCDM) framework, which considers the economic, technological, and innovation-based elements influencing industrial success. These criteria are used to evaluate ten major options that represent well-known businesses or areas. To improve industrial competitiveness in this high-impact industry, the study intends to assist academics, investors, and policymakers in identifying key strengths, weaknesses, and strategic orientations. We use the MULTIMOORA methodology to rank the alternatives. We use Fuzzy OverProbability to obtain the show the final probability between different interval values.

Keywords: Fuzzy OverProbability; Multi-Criteria Decision-Making; Power Electronics Technology Industry.

1. Introduction

Power electronics plays a key role in the transformation of electrical energy systems by facilitating effective power conversion and control in commercial, residential, and industrial settings. The industry includes embedded software, control systems, and semiconductor devices that work together to provide energy-saving solutions. To remain competitive, businesses in the power electronics industry need to quickly adjust to the rapidly changing technical landscape and expanding environmental laws. Assessing competitiveness becomes crucial for evaluating performance against international norms as well as for strategic decision-making[1], [2].

The multifaceted character of competitiveness in this industry is frequently overlooked by traditional evaluation methodologies. Therefore, a strong, criteria-based strategy is necessary. Innovation capacity, production efficiency, market performance, and other factors must all be taken into consideration in such a strategy. This study suggests a quantitative assessment method based on ten well-chosen criteria. These represent the most important elements influencing the development and prosperity of businesses in the power electronics technology sector[3], [4].

Industry studies, expert discussions, and literature evaluations serve as the foundation for the criteria selection process. To represent its relative significance in promoting competitiveness, each criterion is weighted and standardized. As evaluation alternatives, ten top businesses or local industrial clusters are selected. These options offer a fair foundation for comparison because they differ in terms of their operational structures, innovation focus, and market reach[5], [6].

To evaluate performance and produce useful rankings, the evaluation makes use of multi-criteria decision-making techniques. These techniques facilitate the integration of qualitative and quantitative data, guaranteeing thorough analysis. The results shed light on the criteria that have the biggest effects on competitiveness. The assessment also reveals differences between businesses, which could be the result of legal frameworks, investment trends, or strategy decisions[7], [8].

The study provides a forward-looking approach that considers the demands of modern industry by incorporating sustainability, technological innovation, and financial stability into the testing process. In the end, this paradigm supports scholarly discussion, policy creation, and strategic planning. It encourages long-term development and innovation in the field of power electronics technology by enabling stakeholders to make data-driven decisions[9], [10].

Compared to the recently developed intelligent knowledge-based systems, such as multi-criteria decision making (MCDM) for Competitiveness Evaluation of the Power Electronics Technology Industry, the scope of employing conventional methods and systems for Competitiveness Evaluation of the Power Electronics Technology Industry is comparatively limited. Nevertheless, there aren't many studies that have thoroughly examined MCDM in the power electronics industry.

One of the causes is that the standards used to choose power electronics vary slightly depending on the industry. For instance, the aerospace sector considers aspects like financial, production, technological, and environmental effect, while the cycling industry considers elements like supply chain.

The degree of flexibility in MCDM applications is another difficult component, since different sectors have different power electronics requirements, except for the choice of sustainable power electronics[11], [12]. The adoption and application of MCDM have been hampered by researchers. It is crucial to identify the power electronics used in the industry because the MCDM approaches

support informed decisions in power electronics selection. These methods also consider environmental factors in addition to digital transformation factors[13], [14].

Through power electronics selection from a variety of MCDM accessible, MCDM approaches would assist power electronics scientists, architects, engineers, and construction managers in accelerating sustainability. To provide a smooth power electronics selection process, this study aims to satisfy the needs and aspirations of the stakeholders in the power electronics industry, especially power electronics engineers, and sustainability scientists[15], [16].

The report offers a comprehensive analysis of the status of disruptive approaches in a range of industries, including their advantages, disadvantages, and prospects. The study also concentrates on the fundamental methods of MCDM. To provide a thorough analysis and practical suggestions on the tried-and-true MCDM techniques of major importance, this study has been meticulously planned using a methodical methodology[17], [18].

Smarandache (2007) expanded the uncertain set to include uncertain OverSet (when some component is > 1) because he noted that, for instance, an employee who works overtime should have a degree of membership > 1 , whereas an employee who works regular full-time and has a degree of membership $= 1$ [19], [20]; uncertain UnderSet (when some neutrosophic component is < 0), because, for instance, an employee who causes more harm than good to his company should have a degree of membership < 0 , whereas an employee who benefits the company and has a degree of membership > 0 ; and uncertain OffSet (when some neutrosophic components are off the interval $[0, 1]$, i.e. some neutrosophic component > 1 and some neutrosophic component < 0)[21], [22], [23].

This was followed by the extension of the uncertain logic, measure, probability, statistics, etc. to the corresponding uncertain over-, under-, and off-logic, measure, probability, statistics, etc. All fuzzy and fuzzy-extensions (intuitionistic fuzzy, neutrosophic, spherical fuzzy, plithogenic, etc.) are considered "uncertain".

2. Fuzzy OverProbability

What is student's OverProbability of passing the test (including receiving an A+)? One more question: what is the OverProbability of an A+ (degree > 1) for a student?[20]

We can compute the Fuzzy OverProbability between $[0.5, 1.1]$ and $[0, 1.1]$

$$F_{\text{OverProbability}} = \frac{1.1-0.5}{1.1-0} = 0.545 \quad (1)$$

We can compute the Fuzzy OverProbability between $[0.25, 1.1]$ and $[0, 1.1]$

$$F_{\text{OverProbability}} = \frac{1.1-0.25}{1.1-0} = 0.77272 \quad (2)$$

We can compute the Fuzzy OverProbability between $[0.3, 1.1]$ and $[0, 1.1]$

$$F_{\text{OverProbability}} = \frac{1.1-0.3}{1.1-0} = 0.7272 \quad (3)$$

We can compute the Fuzzy OverProbability between [0.6,1.1] and [0,1.1]

$$F_{\text{OverProbability}} = \frac{1.1-0.6}{1.1-0} = 0.4545 \quad (4)$$

We can compute the Fuzzy OverProbability between [0.7,1.1] and [0,1.1]

$$F_{\text{OverProbability}} = \frac{1.1-0.7}{1.1-0} = 0.3636 \quad (5)$$

We can compute the Fuzzy OverProbability between [0.8,1.1] and [0,1.1]

$$F_{\text{OverProbability}} = \frac{1.1-0.8}{1.1-0} = 0.2727 \quad (6)$$

We can compute the Fuzzy OverProbability between [0.1,1.1] and [0,1.1]

$$F_{\text{OverProbability}} = \frac{1.1-0.1}{1.1-0} = 0.9090 \quad (7)$$

We show the steps of the MULTIMOORA to rank the alternatives.

Create the decision matrix by using the opinions of experts and decision makers. We use the fuzzy OverProbability to obtain crisp values. We combine the decision matrices into one.

Calculate the weights of factors.

The weights of factors are calculated using the average method.

Compute the normalization values.

$$Q_{ij}^* = \frac{x_{ij}}{(\sum_{i=1}^m x_{ij}^2)^{0.5}} \quad (8)$$

Compute the ratio system

$$R_i = \sum_{j=1}^g w_j Q_{ij}^* - \sum_{j=g+1}^n w_j Q_{ij}^* \quad (9)$$

Calculate the reference point

$$P_i = \max_j |w_j \max Q_j^* - w_j Q_{ij}^*| \quad (10)$$

Calculate the full multiplicative form

$$F_i = \frac{\prod_{j=1}^g (Q_{ij}^*)^{w_j}}{\prod_{j=g+1}^n (Q_{ij}^*)^{w_j}} \quad (11)$$

Rank the alternatives based on ratio system, reference point, and full multiplicative form. We apply the dominance theory to show the final ranks of the alternatives.

3. Application

We show results of this study by showing the ranking of the alternatives and criteria weights. We use ten criteria and ten alternatives. Three experts created the decision matrix. We use the fuzzy OverProbability to obtain crisp values.

Technological Innovation Capacity (C1) – Measures R&D investment, patent filings, and new product development frequency.

Market Share and Penetration (C2) – Represents the company's presence in domestic and global markets.

Production Efficiency (C3) – Reflects unit cost, automation, and throughput.

Supply Chain Integration (C4) – Evaluates resilience, sourcing strategy, and supplier relationships.

Product Quality and Reliability (C5) – Indicates conformance to standards and failure rates.

Energy Efficiency and Environmental Compliance (C6) – Assesses adherence to green standards and eco-certifications.

Financial Performance (C7) – Based on revenue growth, profitability, and capital turnover.

Human Capital and Technical Expertise (C8) – Reflects workforce skills, training investments, and retention.

Global Strategic Partnerships (C9) – Evaluates alliances, joint ventures, and research collaborations.

Digital Transformation and Smart Manufacturing (C10) – Examines use of AI, IoT, and data analytics in operations.

A1: Siemens Power Electronics Division, A2: Infineon Technologies, A3: ABB Power Converters and Inverters, A4: Mitsubishi Electric Power Devices, A5: GE Power Electronics, A6: Hitachi Energy Power Electronics Systems, A7: Huawei Digital Power, A8: Texas Instruments (Power Management Division), A9: Toshiba Electronic Devices & Storage, A10: Chinese National Power Electronics Industrial Park.

Table 1. The decision matrix.

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈	C ₉	C ₁₀
A ₁	([0.25,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.10,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.8,1.1],[0,1.1])
A ₂	([0.10,1.1],[0,1.1])	([0.5,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.7,1.1],[0,1.1])
A ₃	([0.8,1.1],[0,1.1])	([0.5,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.5,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.6,1.1],[0,1.1])
A ₄	([0.7,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.5,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.3,1.1],[0,1.1])
A ₅	([0.6,1.1],[0,1.1])	([0.5,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.10,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.10,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.5,1.1],[0,1.1])	([0.10,1.1],[0,1.1])

A ₆	([0.3,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.10,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.8,1.1],[0,1.1])
A ₇	([0.10,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.5,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.10,1.1],[0,1.1])	([0.10,1.1],[0,1.1])	([0.5,1.1],[0,1.1])	([0.7,1.1],[0,1.1])
A ₈	([0.5,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.5,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.5,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.8,1.1],[0,1.1])
A ₉	([0.25,1.1],[0,1.1])	([0.10,1.1],[0,1.1])	([0.10,1.1],[0,1.1])	([0.10,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.6,1.1],[0,1.1])
A ₁₀	([0.8,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.10,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.5,1.1],[0,1.1])
	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈	C ₉	C ₁₀
A ₁	([0.5,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.5,1.1],[0,1.1])	([0.10,1.1],[0,1.1])	([0.8,1.1],[0,1.1])
A ₂	([0.25,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.5,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.7,1.1],[0,1.1])
A ₃	([0.3,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.5,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.7,1.1],[0,1.1])
A ₄	([0.6,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.10,1.1],[0,1.1])	([0.5,1.1],[0,1.1])	([0.10,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.3,1.1],[0,1.1])
A ₅	([0.7,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.5,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.10,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.5,1.1],[0,1.1])	([0.25,1.1],[0,1.1])
A ₆	([0.3,1.1],[0,1.1])	([0.10,1.1],[0,1.1])	([0.5,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.8,1.1],[0,1.1])
A ₇	([0.10,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.5,1.1],[0,1.1])	([0.5,1.1],[0,1.1])	([0.8,1.1],[0,1.1])
A ₈	([0.5,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.7,1.1],[0,1.1])
A ₉	([0.25,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.6,1.1],[0,1.1])
A ₁₀	([0.8,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.25,1.1],[0,1.1])
	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈	C ₉	C ₁₀
A ₁	([0.10,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.5,1.1],[0,1.1])
A ₂	([0.10,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.7,1.1],[0,1.1])
A ₃	([0.5,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.6,1.1],[0,1.1])
A ₄	([0.7,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.3,1.1],[0,1.1])
A ₅	([0.6,1.1],[0,1.1])	([0.10,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.25,1.1],[0,1.1])
A ₆	([0.3,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.8,1.1],[0,1.1])
A ₇	([0.25,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.7,1.1],[0,1.1])
A ₈	([0.8,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.10,1.1],[0,1.1])	([0.10,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.10,1.1],[0,1.1])	([0.8,1.1],[0,1.1])
A ₉	([0.25,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.6,1.1],[0,1.1])
A ₁₀	([0.5,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.3,1.1],[0,1.1])	([0.25,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.7,1.1],[0,1.1])	([0.8,1.1],[0,1.1])	([0.6,1.1],[0,1.1])	([0.3,1.1],[0,1.1])

We compute the criteria weights by the average method.

C1 (0.1107): This criterion receives the highest weight, emphasizing the strategic importance of R&D, patent portfolios, and innovative product development in maintaining a competitive edge in a technology-driven industry.

C2 (0.1022): Slightly lower in weight, this reflects the value of expansive market presence and customer reach, essential for economies of scale and brand strength.

C3 (0.1003): A nearly equal weight underlines the role of cost-effective, lean manufacturing practices that improve throughput and lower waste without compromising quality.

C4 (0.1082): High importance is placed on the ability to coordinate sourcing, logistics, and supplier collaboration—key for resilience and timely delivery in competitive markets.

C5 (0.1008): This weight indicates the necessity of consistent product performance, low failure rates, and adherence to international quality standards, especially in mission-critical applications.

C6 (0.0970): Reflects growing pressures from regulators and consumers to adopt green practices, ensuring long-term operational and reputational sustainability.

C7 (0.0926): Though slightly lower in weight, this criterion measures the company's liquidity, growth potential, and ability to fund expansion or R&D without compromising fiscal responsibility.

C8 (0.0962): Highlights the contribution of skilled engineers, training programs, and talent retention to ongoing innovation and operational excellence.

C9 (0.1063): Emphasizes the value of collaborations with universities, research labs, or other corporations for joint ventures, technological exchange, and global market access.

C10 (0.0858): Although the lowest in weight, it captures the transformative role of Industry 4.0 tools (e.g., AI, IoT, robotics) in enhancing responsiveness, customization, and productivity.

We compute normalization values using eq. (8) as shown in Fig 1. Fig 2 shows the weighted decision matrix.

We compute the ratio system using eq. (9) as shown in Fig 3.

We calculate the reference point using eq. (10) as shown in Fig 4. Fig 5 shows the reference point values.

We calculate the full multiplicative form using eq. (11) as shown in Fig 6. Fig 7 shows the full multiplicative form values.

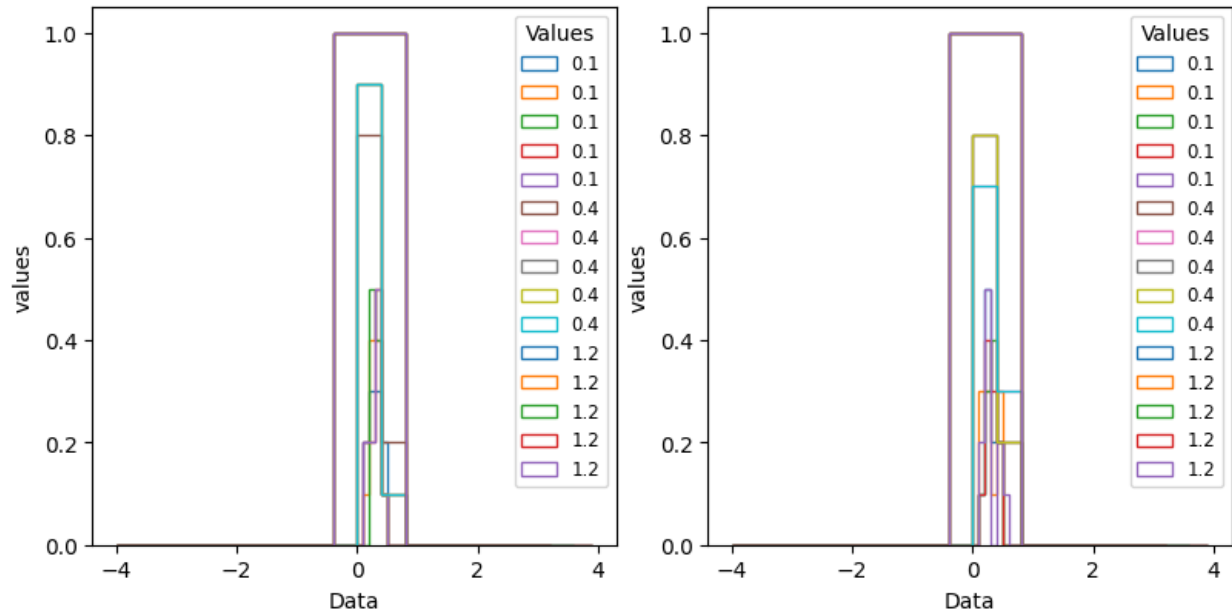


Fig 1. The normalization numbers.

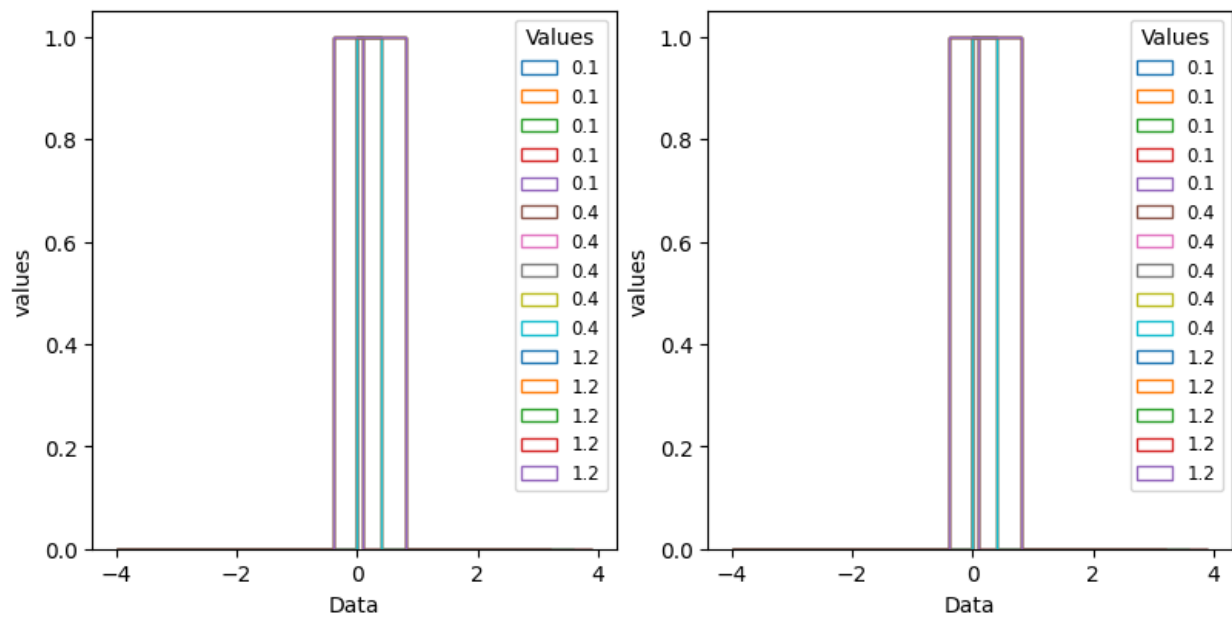


Fig 2. The weighted matrix.

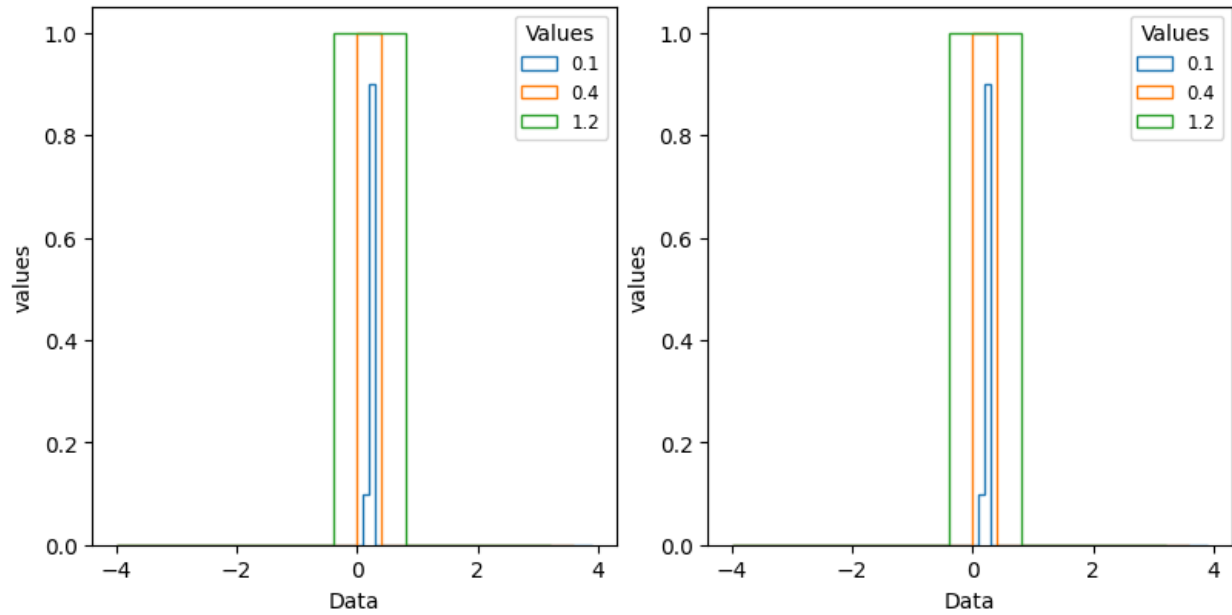


Fig 3. The ratio system values.

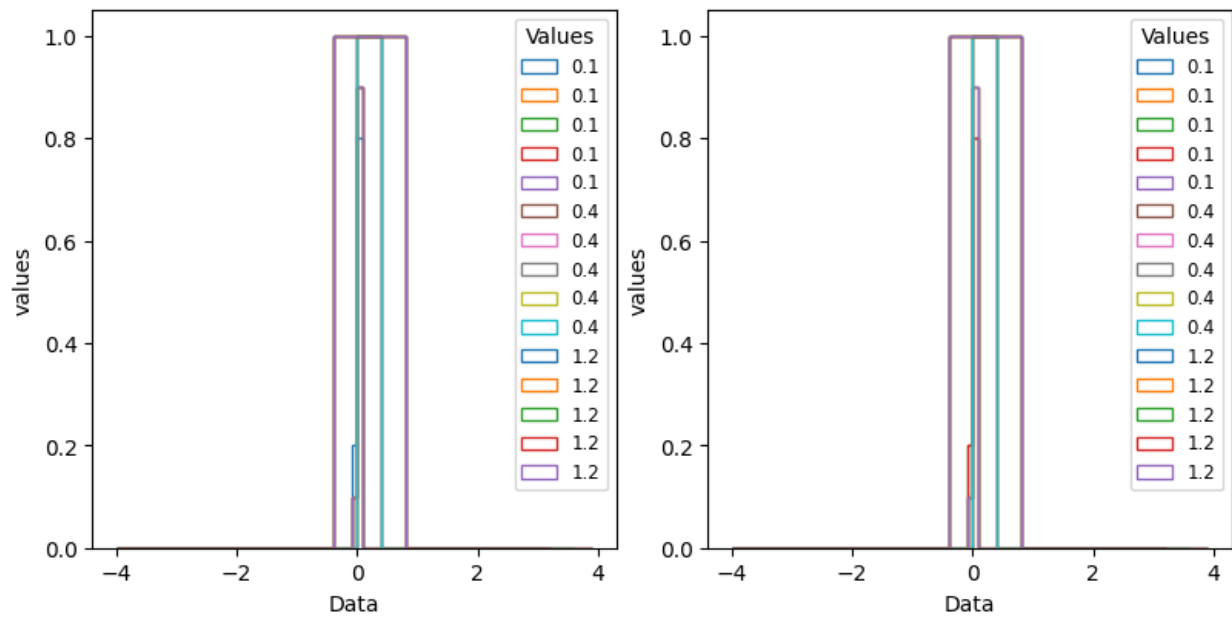


Fig 4. The reference point matrix.

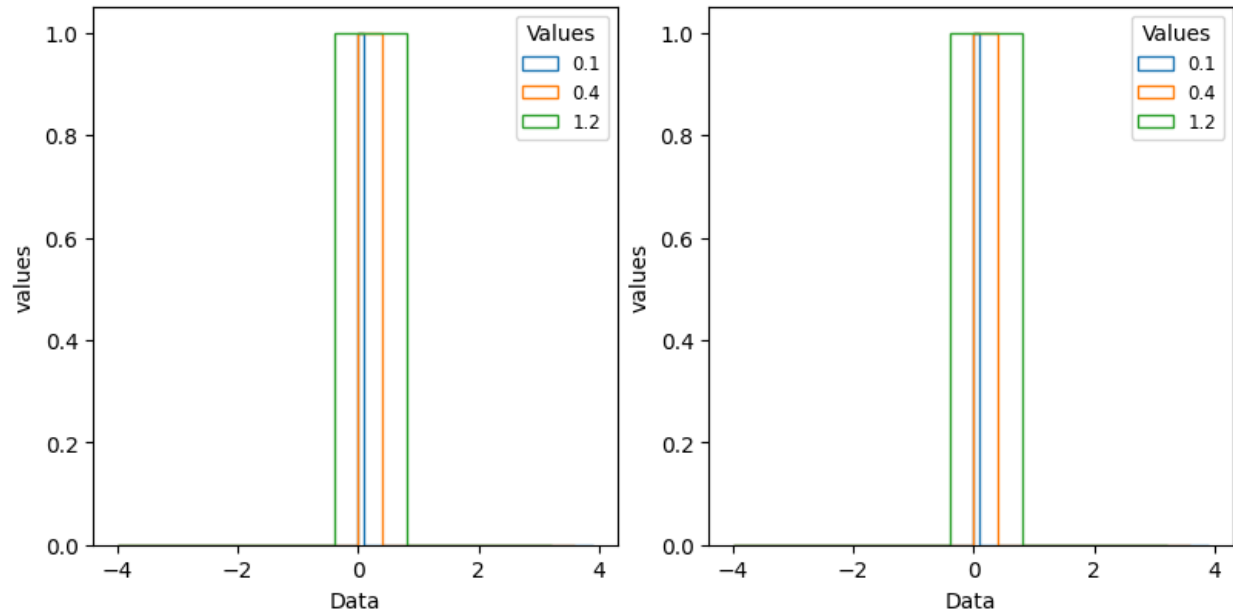


Fig 5. The reference point values.

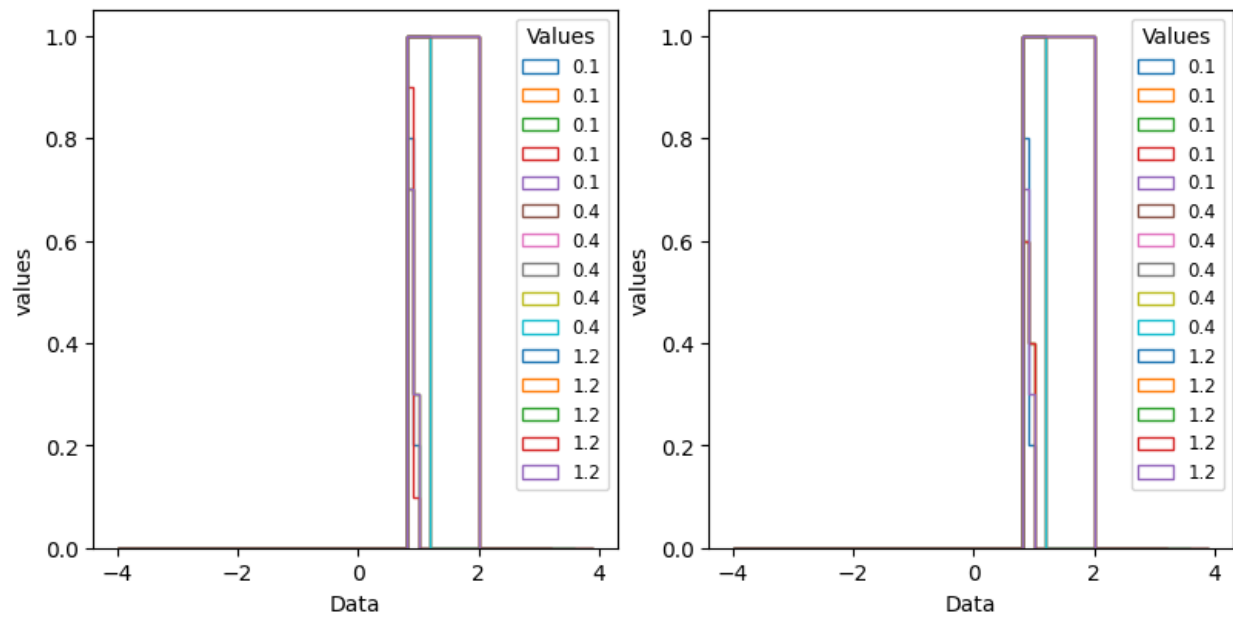


Fig 6. The multiplicative form matrix.

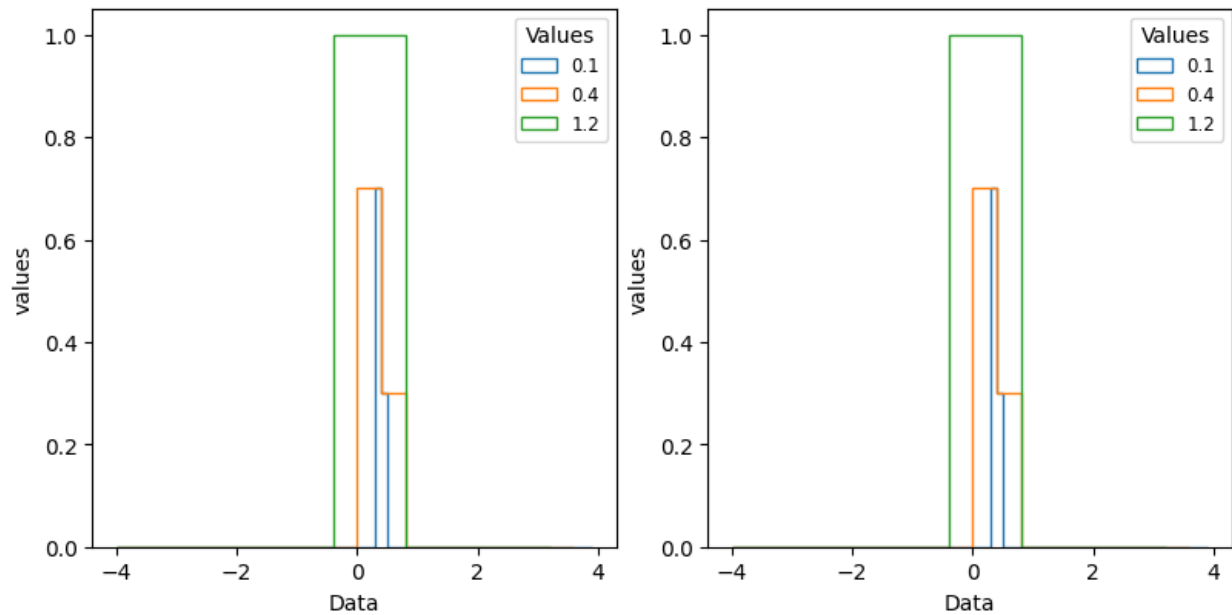


Fig 7. The multiplicative form values.

4. Analysis

This section shows the ranks of each three strategies such as ratio system, reference point, and full multiplicative form. Then we show the final ranks by the dominance strategy as shown in Fig 9.

Ratio System Method

A1 – Rank 4: A1 performs well, showing stability in core metrics, though it doesn't lead in any specific domain.

A2 – Rank 3: A strong candidate with a balanced competitive edge, outperforming most but just shy of leadership.

A3 – Rank 1: A top-ranked performer in efficiency and overall value, setting the benchmark for others.

A4 – Rank 10: The lowest in this method, A4 likely faces challenges in productivity or strategic alignment.

A5 – Rank 9: Struggles to gain a foothold competitively; possibly lagging in innovation or market adaptability.

A6 – Rank 5: Holds the middle ground, reflecting moderate strengths with potential for further development.

A7 – Rank 8: Below average, perhaps hindered by inefficiencies or lower operational scalability.

A8 – Rank 7: A standard outcome, indicating reliability but limited advancement.

A9 – Rank 6: Close to the median, suggesting steady but unexceptional performance.

A10 – Rank 2: Near the top; demonstrates impressive consistency and sound strategy.

Reference Point Method

A1 – Rank 1: Excels when benchmarked against ideal values, showing minimal deviation from target metrics.

A2 – Rank 5: A solid performance with a few weak areas, staying competitive overall.

A3 – Rank 8: Unexpectedly low in this method, possibly due to sensitivity to certain variable targets.

A4 – Rank 7: Below average, potentially affected by poor alignment with optimal benchmarks.

A5 – Rank 10: The furthest from ideal values; may need significant recalibration of strategic focus.

A6 – Rank 1: Matches A1 in hitting precise targets, reinforcing its operational accuracy.

A7 – Rank 6: Adequate performance with slight inefficiencies in key comparison areas.

A8 – Rank 4: Performs better than average, with some proximity to ideal target points.

A9 – Rank 9: Falls short in several benchmark criteria, highlighting areas for improvement.

A10 – Rank 3: Aligns well with optimal standards, showing solid strategic execution.

Full Multiplicative Form Method

A1 – Rank 3: Maintains robust performance through cumulative factor strength, indicating well-rounded capability.

A2 – Rank 4: Slightly behind A1, suggesting dependable execution but less synergy across metrics.

A3 – Rank 1: Dominates this method due to high combined effectiveness of contributing variables.

A6 – Rank 5: Reasonably competitive, with areas of strength and a few minor limitations.

A7 – Rank 10: Weakest cumulative performance; factors may be misaligned or under-optimized.

A8 – Rank 7: Average output; likely lacking in a few critical areas despite overall consistency.

A9 – Rank 6: Modestly effective, though not standing out in high-impact contributions.

A10 – Rank 2: Shows nearly top-tier synergy across elements, just behind the leader.

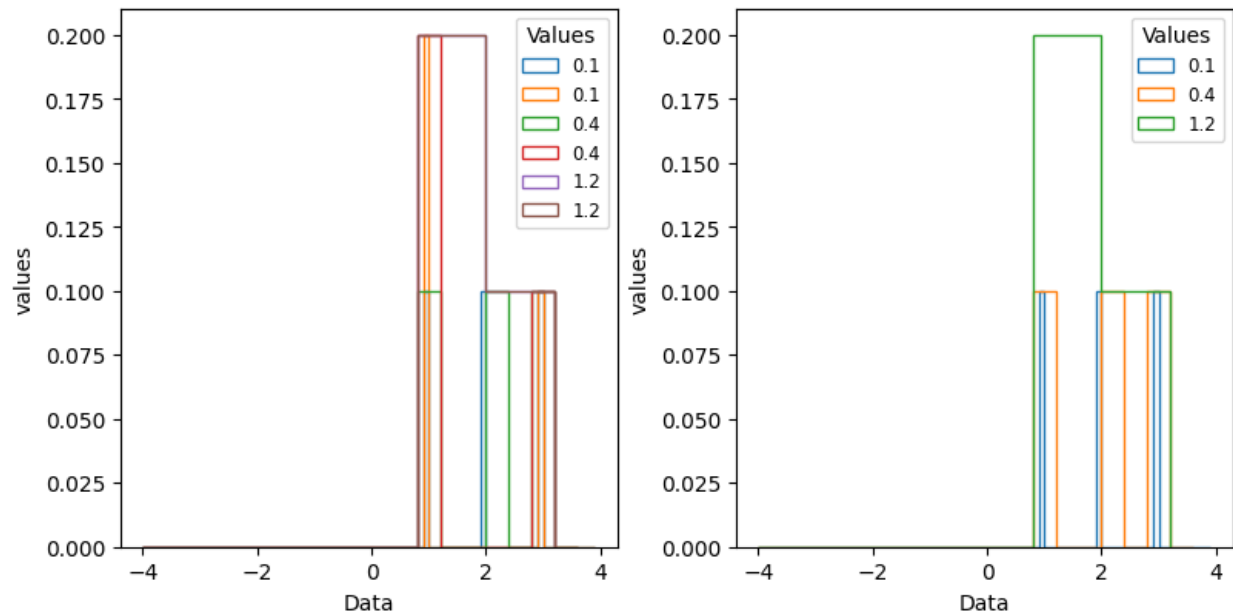


Fig 8. The ranks of three strategies.

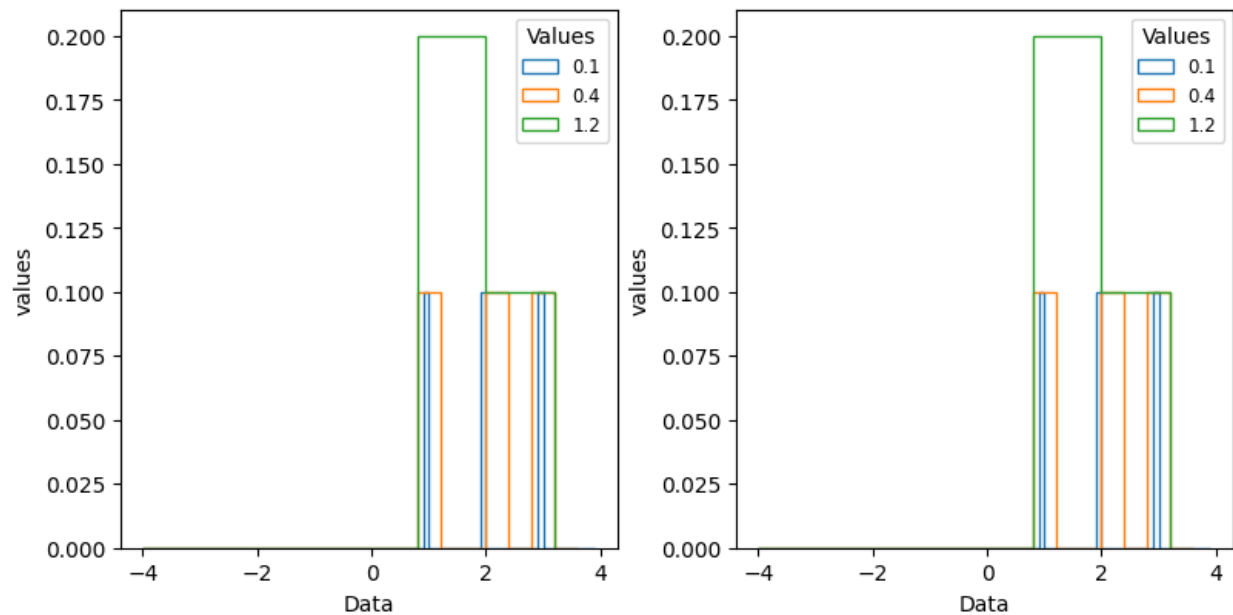


Fig 9. The final ranks.

5. Conclusions

The thorough instrument for evaluating and comparing competitiveness in the power electronics technology sector is this assessment methodology. The model emphasizes the importance of technological innovation, sustainability, financial performance, and digitization in determining industry success by using multi-dimensional factors. By using this paradigm, legislators can create legislation that will help, guide future investments, and encourage strategic improvements.

We used the MULTIMOORA methodology to rank the alternatives. The MCDM approach is used under Fuzzy OverProbability to obtain crisp values. Ten criteria and ten alternatives are used to show the application of the proposed approach. The alternatives are ranked based on three factors on the MULTIMOORA method. In the end, the ability to innovate sustainably while preserving operational excellence in a world that is becoming more digitally and globally integrated determines competitiveness in this sector.

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HyperSoft Set and MCDM Methodology for Impact of a Volleyball Program on Undergraduate Students' Physical Fitness and Health Elements

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Abstract: Using a multi-criteria decision-making framework, this study assesses how an organized volleyball program affects undergraduate students' physical fitness and health-related factors. The study evaluates 10 alternatives according to their involvement and program development, and it establishes eight important characteristics that represent advances in physical and health. To guarantee thorough assessment, the system combines quantitative performance measurements with qualitative observations. We use eight criteria and ten alternatives. We use the REGIME method to rank the alternatives. We use the HyperSoft set to deal with different criteria values. The findings show that participants' levels of improvement varied, underscoring the volleyball program's potential as a useful, entertaining tool for improving student health. Results back up the use of team sports in higher education institutions' health promotion initiatives.

Keywords: HyperSoft Set; MCDM Methodology; Volleyball Program; Physical Fitness and Health Elements; Students

1. Introduction

Major institutions have greatly increased volleyball's popularity, and college students are particularly interested in playing the sport. As a result, volleyball's growth in colleges and universities follows a specific scale. Furthermore, pertinent courses are always getting better. In

contrast to other sports, volleyball has specific demands on college students' athletic prowess as well as skill level needs[1], [2].

Consequently, playing volleyball greatly contributes to raising college students' general caliber. Additionally, playing volleyball mostly involves aerobic exercise, and this activity clearly improves college students' cardiopulmonary endurance. The goals of volleyball programs at professional sports colleges and colleges and universities varies significantly. However, they all serve to enhance college students' general physical well-being. Daily volleyball practice can significantly enhance college students' mental health, social skills, and quality of life while also reducing their negative emotions[3], [4].

We can assist in improving the college volleyball course by researching how the course affects the physical attributes of college students. It can serve as an adequate reference for college students' training and is essential to the growth of collegiate athletics[5], [6].

The physical education (PE) program that they offer their students must be assessed by physical educators. This is significant because physical education can help pupils get the recommended quantity of physical activity. This can then help to counteract the rise in obesity by lowering the prevalence of physical inactivity among students. Health-related fitness evaluations are one method of assessing a physical education curriculum[7], [8].

Schoolchildren's fitness levels have increased because of PE activities. However, university students rarely do such studies. Furthermore, no research has been done to ascertain how volleyball training affects university students who are not athletes. This is noteworthy because volleyball is a competitive sport that primarily uses explosive strength and stamina to execute skills like smashing and blocking. The purpose of this study was to ascertain how a volleyball course affected the male university students' health-related fitness[9], [10].

A basic human activity, decision-making takes place in many facets of daily life. It is a mental process that entails weighing several possibilities and choosing the one that best suits the decision-makers' objectives or requirements[11], [12]. Deterministic approaches, which assumed certainty in the process, were the focus of early decision-making research. However, ambiguity frequently occurs in decision-making because of our limitations in fully comprehending the objective reality. Accurately comprehending and characterizing this uncertainty has proven to be difficult for scholars[13], [14].

Making decisions frequently entails weighing several options according to several criteria, which can have a big impact on results in different situations. A methodical technique for assessing and choosing options based on several criteria is called multi-criteria decision-making, or MCDM. By carefully examining trade-offs between factors like cost, quality, performance, and risk, it helps decision-makers evaluate a variety of possibilities. MCDM is frequently used to assist with complicated decision-making issues in domains such as business, engineering, medicine, and disaster management[15], [16].

Instead, then choosing the best option, most of these decision-making situations require ranking all the available options. One of the significant MCDM difficulties, for example, is the assessment of the Impact of a Volleyball Program on Undergraduate Students' Physical Fitness and Health Elements, which ranks the alternatives based on their criteria by considering various characteristics[17], [18].

The aim of this study is organized as follows:

HyperSoft Set is used in this study to deal with different criteria values in the decision-making process.

Eight criteria and ten alternatives are used to evaluate the Impact of a Volleyball Program on Undergraduate Students' Physical Fitness and Health Elements.

The average method is used to compute the criteria weights in the decision matrix between the criteria and alternatives.

The REGIME method is a MCDM is used to rank the alternatives based on a set of criteria.

2. HyperSoft Set with Decision Making Method

This section shows the definitions of HyperSoft set and the steps of the MCDM approach to rank the alternatives.

By linking qualities (or parameters) to subsets of a universal set, a Soft Set provides a simple method for parameterized decision modeling that successfully addresses uncertainty in a structured way. To address various facets of uncertainty, several related mathematical frameworks have been created, including fuzzy sets, neutrosophic sets, and rough sets. The HyperSoft Set expands on this framework by adding multi-attribute decision modeling to Soft Sets. A HyperSoft Set enhances its ability to handle complicated decision-making scenarios by mapping combinations of numerous qualities to subsets of the universal set rather than assigning a single parameter to a subset[19], [20].

Let U be a universal set and set of criteria such as $C_1, C_2, \dots, C_m$. The cartesian product of these criteria can be obtained as: $C = C_1 \times C_2 \times \dots \times C_m$. The HyperSoft set over U is a pair (A, C) and $A: C \rightarrow P(U)$. We can define the HyperSoft Set as:

$$(A, C) = \{(y, A(y)) | y \in C, A(y) \in P(U)\} \quad (1)$$

$$y = (y_1, y_2, \dots, y_m) \in C; y_i \in C_i; i = 1, 2, \dots, m, A(y) \text{ shows the subset of } U \text{ of criteria values } y_1, y_2, \dots, y_m \quad (2)$$

We show the steps of the REGIME with the HyperSoft Set to rank the alternatives.

Create the decision matrix using the scale between 0 and 1. We combine these numbers into a single matrix. Create the weights of criteria by the average method.

Calculate the superiority index A_{fl} .

Calculate the superiority identifier.

$$A_{fl} = \sum_{j \in E_{fl}} w_j \quad (3)$$

Create the impact matrix.

Create the REGIME matrix among the options.

$$A_{fl,j} = \begin{cases} -1 & \text{if } u_{fj} < u_{lj} \\ 0 & \text{if } u_{fj} = u_{lj} \\ +1 & \text{if } u_{fj} > u_{lj} \end{cases} \quad (4)$$

Calafate the guide index.

$$B_{fl} = \sum_{j=1}^n A_{fl} w_j \quad (5)$$

3. Results

This section shows the results of the proposed approach. We use eight criteria with values for HyperSoft Set and ten alternatives such as:

- C1. Cardiovascular Endurance Low, Moderate, High
- C2. Muscular Strength Weak, Average, Strong
- C3. Flexibility Poor, Fair, Good
- C4. Body Composition Overweight, Normal, Lean
- C5. Agility Slow, Moderate, Fast
- C6. Coordination Below Average, Average, Excellent
- C7. Mental Wellbeing Low Motivation, Stable Mood, Highly Engaged
- C8. Participation Consistency Irregular, Regular, Highly Consistent
- Cardiovascular Endurance (C1): Measures improvements in stamina and aerobic capacity through timed running and heart rate recovery tests.
- Muscular Strength (C2): Assesses upper and lower body strength using metrics such as bench press and squat repetitions.
- Flexibility (C3): Evaluated through sit-and-reach tests to determine the extent of flexibility gains, especially in the lower back and hamstrings.
- Body Composition (C4): Tracks changes in fat mass and lean muscle ratio, based on BMI and body fat percentage analysis.
- Agility (C5): Measures quick directional changes and reaction time using shuttle run and cone drills.
- Coordination (C6): Observed through volleyball-specific drills like passing accuracy and ball control tasks.
- Mental Wellbeing (C7): Surveys and interviews used to evaluate reductions in stress and improvements in mood due to regular participation.

- Attendance and Participation Consistency (C8): Quantifies commitment and motivation through attendance records and active engagement levels.

Three experts created the decision matrix between the criteria and alternatives. We combined the decision matrix and computed the criteria weights using the average method.

C1 – (0.1221) This criterion assesses how effectively the volleyball program improves the ability of the heart and lungs to supply oxygen during sustained physical activity. With a moderate weight, it reflects a vital but balanced component of overall health.

C2 – (0.1247) This measures the increase in the students' ability to exert force. The volleyball program contributes to developing upper and lower body strength through repeated jumping, hitting, and blocking.

C3 – (0.1320) This criterion evaluates how long students can perform physical activity without fatigue. With a relatively higher weight, it shows the volleyball program's strong influence on building sustained physical performance.

C4 – (0.1284) Flexibility considers the range of motion in joints and muscles, often improved through volleyball's dynamic movements. This criterion has notable importance due to its role in injury prevention and functional movement.

C5 – (0.1209) This relates to the balance of fat and lean mass in the body. The volleyball program helps in managing weight and improving body composition through aerobic and anaerobic exertion.

C6 – (0.1261) This assesses how well participants control body movement during complex actions. Volleyball's rapid shifts and multidirectional play help improve these motor skills, making it a significant aspect of the evaluation.

C7 – (0.1103) This criterion captures the psychological and emotional benefits of participation, such as reduced stress and improved mood. Though slightly lower in weight, it remains a key holistic health component.

C8 – (0.1356) The highest-weighted criterion, emphasizing the volleyball program's role in fostering social bonds, communication, and collaboration among students. This aligns with educational goals promoting interpersonal development alongside physical fitness.

We select the best values such as:

- C1. High
- C2. Strong
- C3. Good
- C4. Normal
- C5. Fast
- C6. Excellent

- C7. Highly Engaged
- C8. Highly Consistent

We calculate the superiority index A_{fl} and calculate the superiority identifier using eq. (3). We create the impact matrix.

We create the REGIME matrix among the options using eq. (4) as shown in Fig 1.

We compute the guide index using eq. (5) as shown in Fig 2. Fig 3 shows the values of guide index. We rank the alternatives as shown in Fig 4.

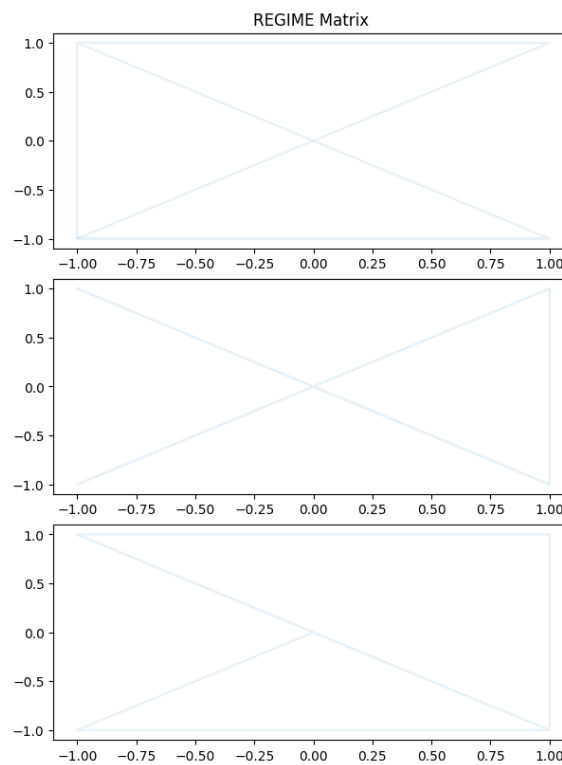
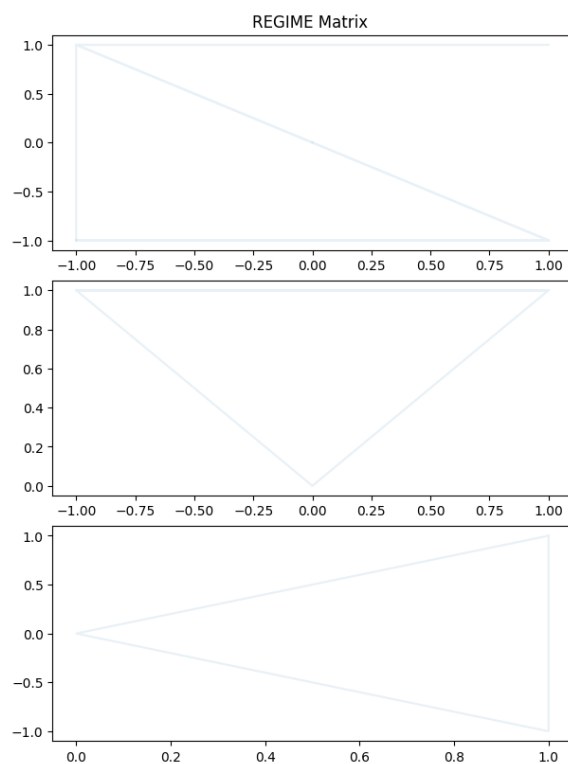
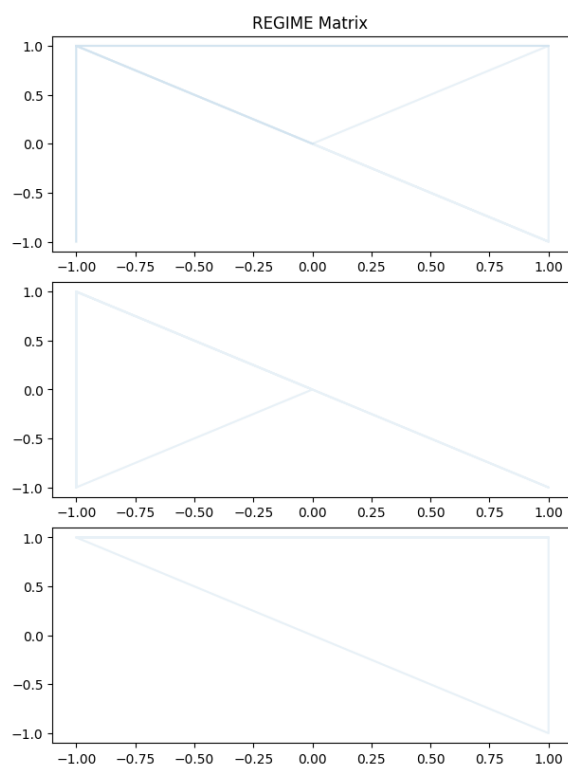
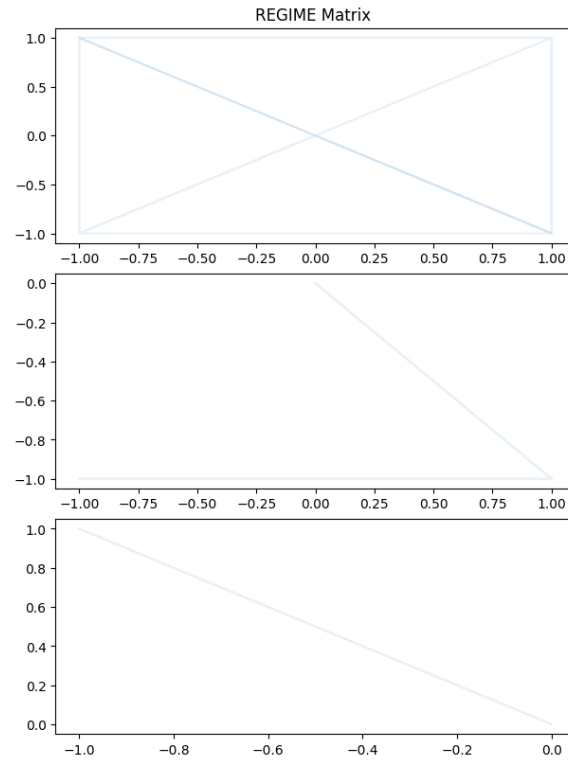
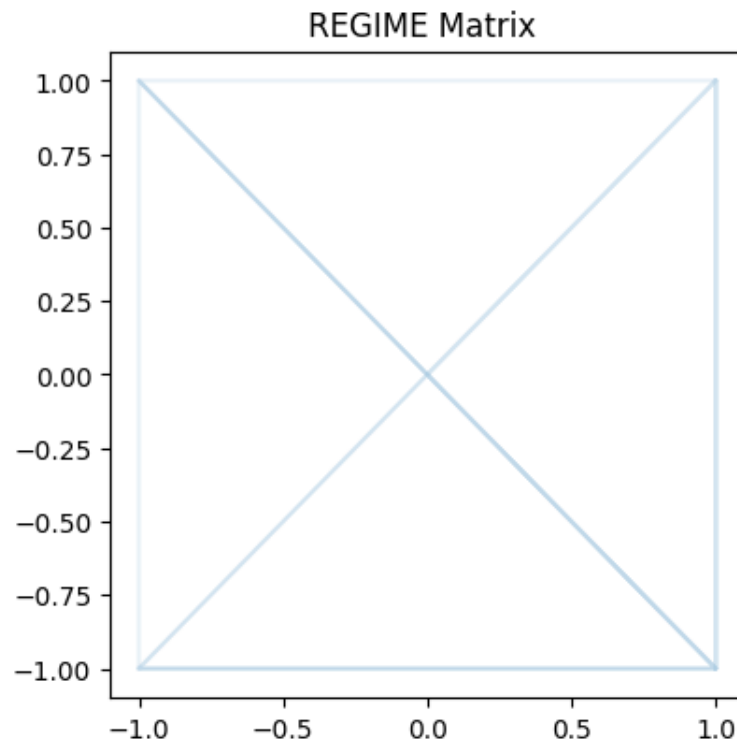
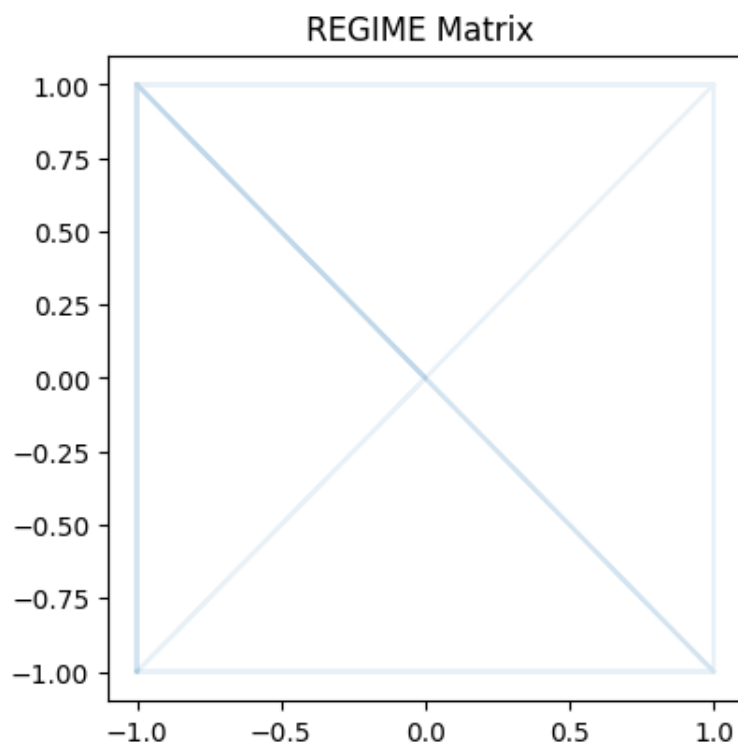
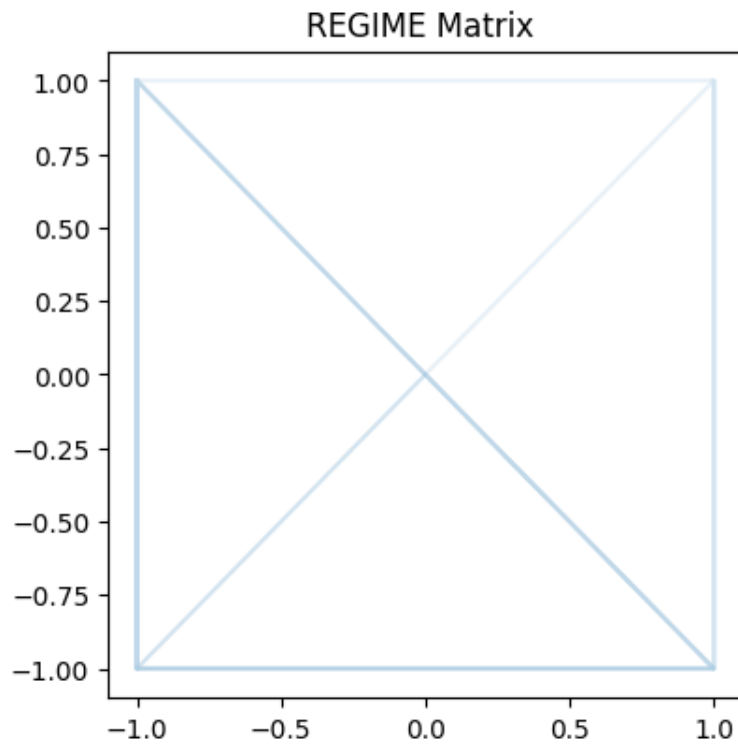
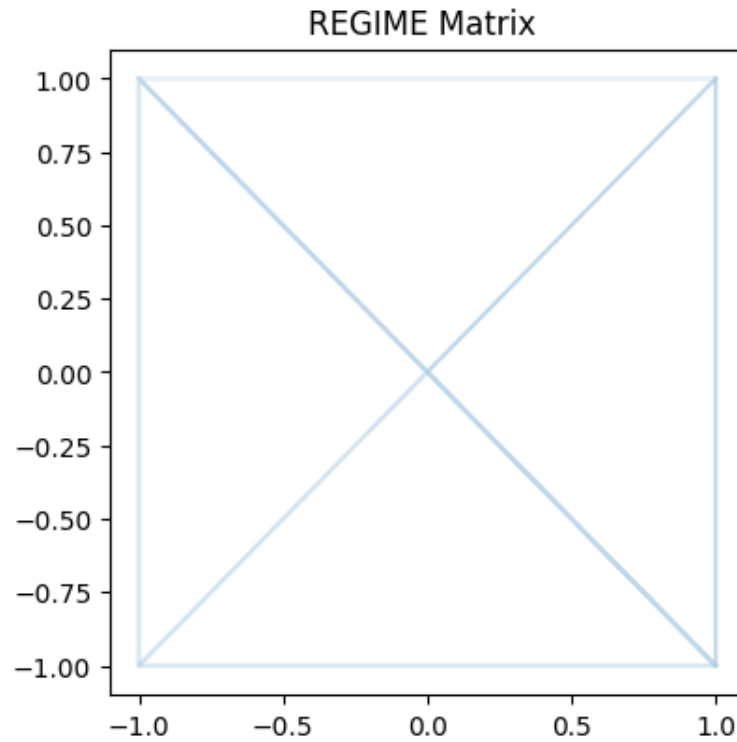
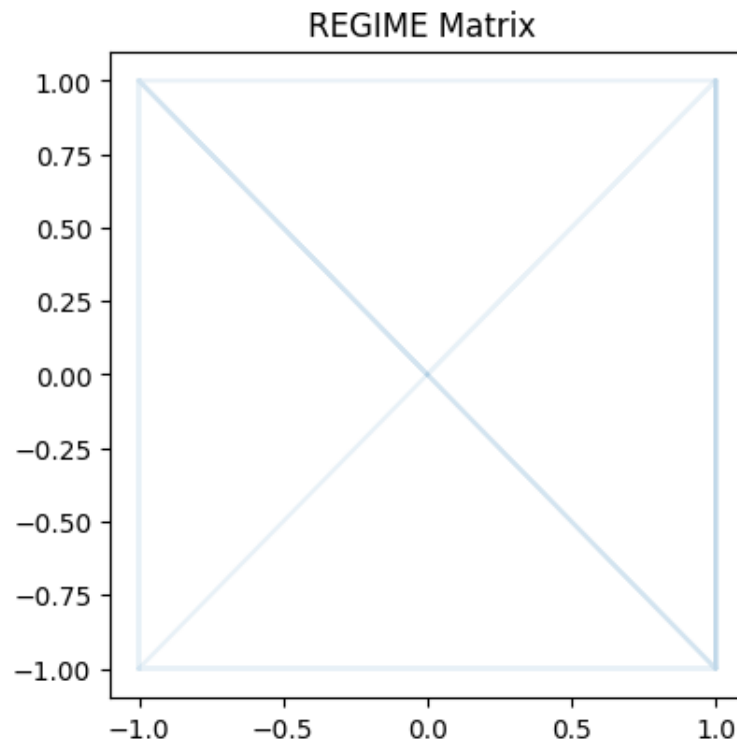


Fig 1-a. The REGIME matrix for A_1 .

Fig 1-b. The REGIME matrix for A_2 .Fig 1-c. The REGIME matrix for A_3 .

Fig 1-d. The REGIME matrix for A_4 .Fig 1-e. The REGIME matrix for A_5 .

Fig 1-f. The REGIME matrix for A_6 .Fig 1-g. The REGIME matrix for A_7 .

Fig 1-h. The REGIME matrix for A_8 .Fig 1-i. The REGIME matrix for A_9 .

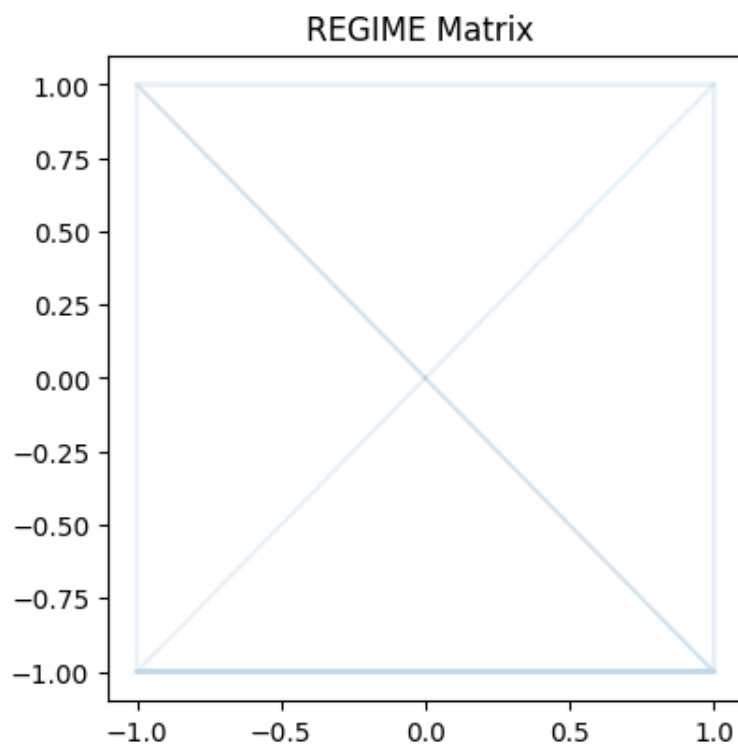


Fig 1-j. The REGIME matrix for A_{10} .

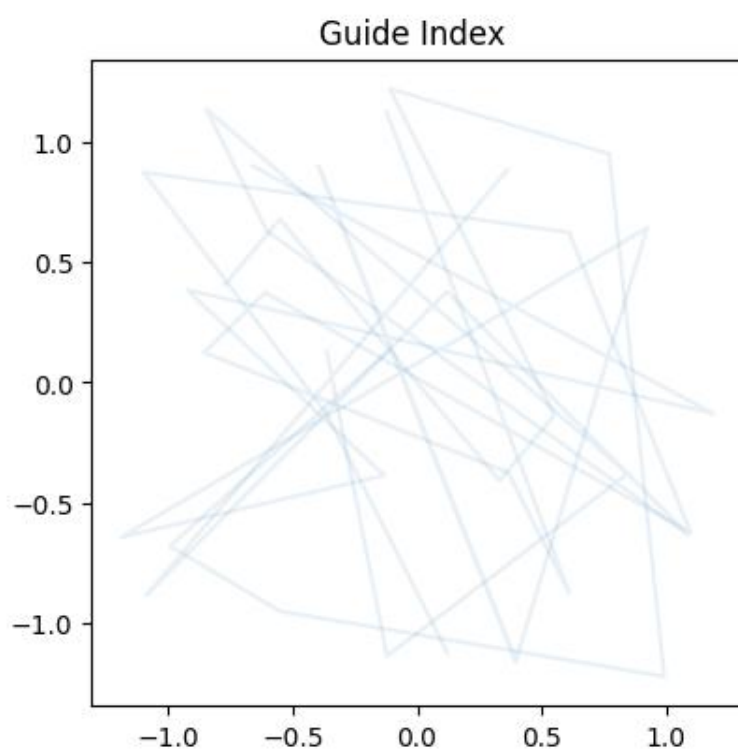


Fig 2. The weighted guide index.

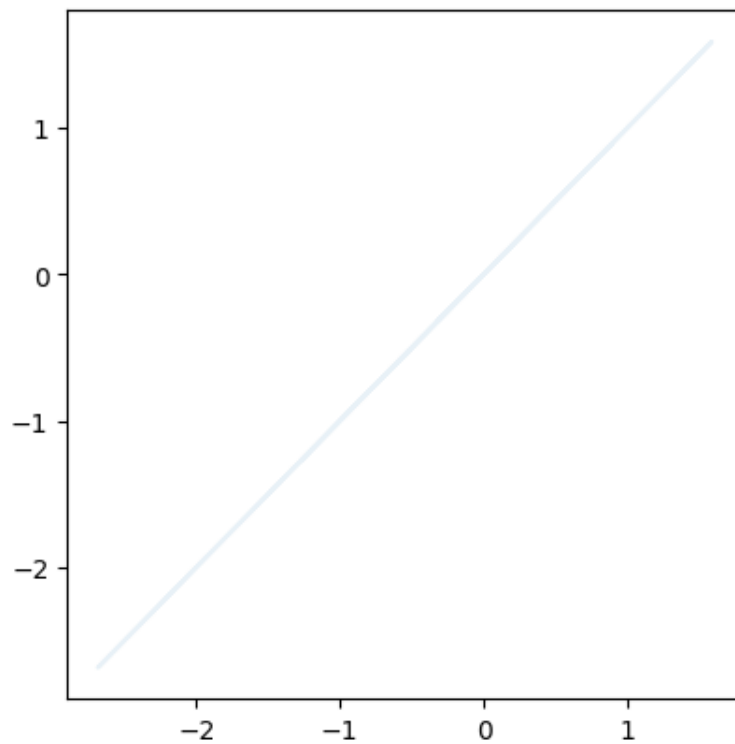


Fig 3. Values of guide index.

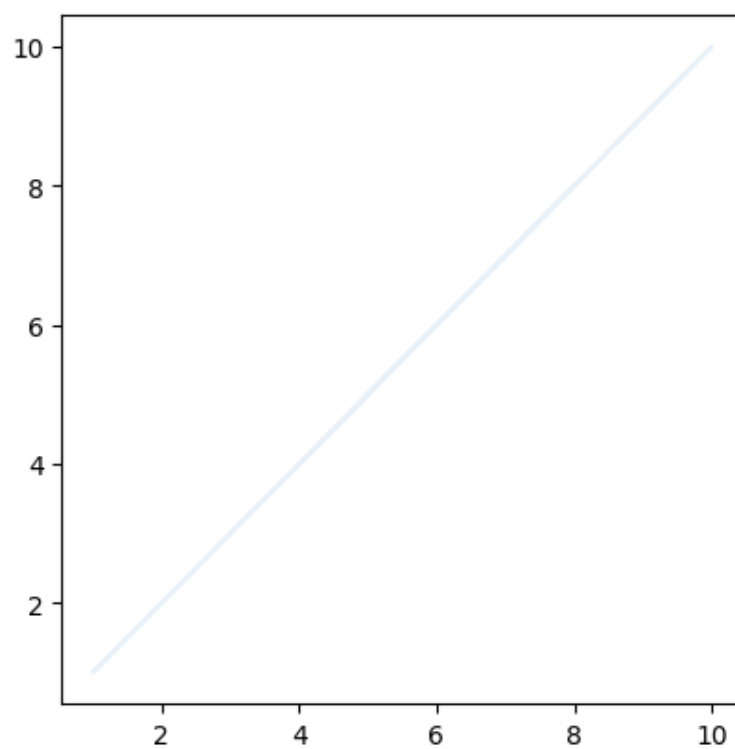


Fig 4. The ranks of alternatives.

A1 – Rank 5

This alternative performed moderately well across the criteria. It shows a balanced improvement in fitness and health elements, placing it in the upper mid-tier among all options.

A2 – Rank 1 (Lowest)

This alternative had the least impact across the evaluated criteria. It might indicate minimal effectiveness of the volleyball program in improving physical fitness and health for this group.

A3 – Rank 2

Slightly better than A2, this alternative still falls in the lower-performing range. It likely showed modest improvements in some areas but lacked consistency across all criteria.

A4 – Rank 8

This is among the stronger-performing alternatives. It demonstrated significant benefits, especially in areas like cardiovascular endurance or social interaction, though not at the very top.

A5 – Rank 4

This alternative showed fairly strong outcomes. It might reflect effective implementation of the volleyball program with notable gains in several key health and fitness criteria.

A6 – Rank 9

A high-performing alternative, indicating substantial benefits from the volleyball program. Likely to have excelled in a combination of physical and psychosocial elements.

A7 – Rank 10 (Highest)

This is the top-performing alternative, showing outstanding results across nearly all criteria. It suggests excellent impact of the volleyball program on students' physical fitness and health.

A8 – Rank 7

On the other hand, this alternative achieved good results, possibly excelling in one or two areas like flexibility or teamwork, but not as comprehensively as A7 or A6.

A9 – Rank 3

A solid alternative that outperformed most others except the top two. It indicates effective engagement and measurable improvements in health metrics.

A10 – Rank 6

Sitting at the center, this alternative showed consistent but not standout results. A generally effective outcome with room for improvement.

5. Conclusions

The study confirms the positive effects of a volleyball program on various dimensions of students' physical health and mental wellbeing. The eight evaluation criteria provided a well-rounded framework to capture improvements beyond physical fitness, incorporating psychological and behavioral aspects. While some participants showed exceptional physical development, others benefitted more in terms of mental health, highlighting the multifaceted benefits of team sports in educational settings. We use the HyperSoft Set to deal with criteria values. The MCDM method is used to select the best alternative. The REGIME method is used to rank the alternatives. Based on the findings, institutions should consider integrating volleyball and similar programs into the curriculum as part of a comprehensive health promotion strategy.

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An Approach to Neutrosophic Hyper Soft Rough Matrix and its Application

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Abstract. This paper introduces a novel of Neutrosophic Hyper Soft Rough Matrix (NHSRM). We discuss various operators on NHSRM such as addition, multiplication, scalar multiplication and complement. Additionally, we develop several properties, including associativity, distributivity and De Morgan's laws, utilizing operators such as union, intersection, complement, arithmetic mean, weighted arithmetic mean, geometric mean, weighted geometric mean, harmonic mean and weighted harmonic mean. Further we propose a score function as part of this work. To demonstrate the application of the proposed work, a multi-attribute decision-making problem is considered. The score matrix obtained from three different operators are compared, and the results are illustrated graphically.

Keywords: Neutrosophic hyper soft matrix; Neutrosophic hyper soft rough matrix; Weighted arithmetic mean; Weighted geometric mean; Weighted harmonic mean; Score matrix; Multi-attribute decision-making.

1. Introduction

In a traditional matrix, each element is well-defined and unambiguous, and these matrices are widely used in various mathematical fields, including linear transformations in vector spaces, computer graphics, signal processing, and data analysis. Fuzzy matrices have been widely studied for their utility in representing uncertainty and imprecision. In a fuzzy matrix [22], each element is either a fuzzy number or a degree of membership, typically within the interval $[0, 1]$, indicating levels of truth, membership, or uncertainty. These matrices are particularly valuable in addressing situations involving vague or imprecise information. Their

applications span various domains, including artificial intelligence, control systems, and decision support systems, where traditional matrices often lack the flexibility required to handle such complex scenarios.

The soft matrix, rooted in soft set theory, provides an advanced framework by allowing multiple and context-specific parameterizations to describe uncertainty, offering greater flexibility in modeling and analysis. This approach extends the capabilities of traditional methods by incorporating parameter dependency, reducing ambiguity, broadening applicability, and enhancing versatility. Soft matrices prove particularly useful in various fields where traditional crisp values are insufficient to capture the complexities of real-world situations. The Dynamism of fuzzy soft matrices lies in their ability to incorporate parameter dependency, reduce ambiguity, and broaden applicability across diverse fields. Applications have been explored in artificial intelligence, data analysis, decision support systems, and pattern recognition, where traditional frameworks often fail to capture the complexities and subtleties of real-world scenarios. As a result, fuzzy soft matrices have emerged as a powerful tool for tackling challenges involving high levels of uncertainty and variability. Cagman and Enginoglu [5], Borah et al. [3] developed fuzzy soft matrices, which serve as a matrix representation of fuzzy soft sets, and established operations on these matrices that facilitate theoretical studies in fuzzy soft set theory.

Fuzzy Soft Rough Matrix extends the fuzzy soft matrix by integrating rough set theory, which provides mechanisms to approximate sets using lower and upper approximations. This allows the matrix to handle situations where there is incomplete or indeterminate information, distinguishing between lower approximation and upper approximations. It is widely applied in decision support systems, pattern recognition, data mining, medical diagnostics, environmental modeling, control systems, supply chain management, and risk assessment. Muthukumar and Krishnan [14] initiated generalized fuzzy soft rough matrices and their operations, which play a crucial role in advancing theoretical studies in fuzzy soft rough sets. Their work provides a foundation for applying these matrices to model and analyze complex systems.

The Fuzzy Soft Matrix is extended to the Intuitionistic Fuzzy Soft Matrix (IFSM) by incorporating the concept of intuitionistic fuzzy sets, which includes both membership and non-membership degrees along with a degree of hesitation. The IFSM provides a framework for analyzing problems with multiple sources of uncertainty, capturing more detailed and nuanced information than standard fuzzy soft matrices. Chetia et al. [6] established five distinct types of product for intuitionistic fuzzy soft matrices and explore their theoretical properties. These operations and definitions make IFSMs a functional tool for advancing studies in intuitionistic fuzzy soft set theory, with potential applications in fields that require nuanced

decision-making in engineering. Neutrosophic Fuzzy Soft Matrices (NFSMs) enhance the representation of uncertainty by explicitly incorporating indeterminacy as a distinct parameter alongside truth and falsehood, providing a more detailed and versatile framework compared to the hesitation-based representation in Intuitionistic Fuzzy Soft Matrices (IFSs). NFSMs is designed for more complex and dynamic environments, such as big data analytics, where explicit modeling of indeterminate states is essential. Sumathi and Arockiarani [20] introduced new operations on fuzzy neutrosophic soft matrices, expanding the potential applications of these matrices in dealing with uncertainty and imprecision in complex systems. Their research highlights innovative matrix operations, such as addition and multiplication, which enhance the versatility of neutrosophic models in decision-making and other computational applications. These contributions provide a solid foundation for understanding the practical implementation of fuzzy neutrosophic soft matrices in various real-world contexts. Uma et al. [21] explored Fuzzy Neutrosophic Soft Matrices of Type I and Type II, offering significant advancements in the application of neutrosophic principles to matrix operations. Their work presents novel methods for handling uncertainty and imprecision in complex systems, particularly in decision-making environments. These contributions broaden the scope of fuzzy neutrosophic models and provide a more robust framework for analyzing real-world problems involving multi-criteria decision analysis and other computational tasks.

Smarandache [16] extended the concept of soft sets to hyper soft sets by transforming the function into a multi-attribute function. This development allowed for a more comprehensive representation of data involving multiple parameters. A hyper soft matrices lies in the latter's ability to represent and analyze multi-attribute data, enabling more flexible and comprehensive modeling of complex, interdependent systems compared to the single-parameter framework of soft matrices. These matrices are particularly useful in scenarios where multiple interrelated attributes must be considered, such as in complex system modeling, risk assessment, and multi-objective optimization. They also enhance the handling of unpredictability in medical decision support, environmental modeling, and financial forecasting. In contrast, hyper soft rough matrices go a step further by introducing the lower and upper approximations of rough set theory, which allow for distinguishing between known and unknown information, particularly when dealing with higher degree of uncertainty. It is offering a more advanced mechanism for classification and decision-making in systems where data may be ambiguous or partial.

An Intuitionistic Hyper Soft Rough Matrix (IHSRM) was introduced to offer a more comprehensive and flexible framework that can handle uncertainty at different levels through membership, non-membership, and indeterminacy. This makes it particularly suitable for complex systems, multi-criteria decision-making, and fields like data mining, machine learning, and information retrieval, where precise modeling and handling of diverse uncertainties

are essential. Lei Zhou et al. [12] proposed a comprehensive framework for studying relation-based intuitionistic fuzzy rough approximation operators. This framework incorporates both constructive and axiomatic approaches and establishes several fundamental properties of intuitionistic fuzzy rough approximation operators. Jafar and Saeed [8] expanded the concept of Neutrosophic hypersoft sets to Neutrosophic Hyper Soft Matrices (NHSM), providing a detailed study of these matrices along with relevant examples. Jayasudha and Raghavi [10] explored various operations on Neutrosophic Hypersoft Matrices and demonstrated their practical applications in complex decision-making scenarios. Their work provided a comprehensive framework for applying neutrosophic principles to matrix operations, expanding the scope of these models in real-world applications. This study offers valuable insights that complement the methodologies applied in the current research, particularly in addressing the specific aspects of uncertainty in data modeling and multi-criteria decision analysis.

In this paper, we extend the concept of NHSM to NHSRM. The NHSRM is a novel mathematical model designed to address these challenges by integrating several advanced concepts from neutrosophic logic, soft set theory, hyper-soft sets, and rough set theory. This hybrid model offers a powerful and flexible tool for managing complex decision-making scenarios where traditional methods fall short. This enables NHSRM to handle situations where data is not only uncertain but also partially known or ambiguous, which is particularly useful in fields like pattern recognition, decision-making under uncertainty, and machine learning, where such approximations improve decision boundaries and classification accuracy. NHSM focuses on the representation of uncertainty without incorporating the rough set-based approximations, making it suitable for less complex scenarios. In contrast, NHSRM provides a more comprehensive framework by integrating rough set-based approximations, which makes it better suited for handling more complex and nuanced problems. The scope of this work is to develop some notions and operations with various properties of NHSRM along with a suitable decision-making problem.

In this paper, the following section of the proposed work organizes as follows:

- Section 2 presents the foundational key preliminaries, establishing the essential concepts and terminologies necessary for understanding the theoretical framework.
- Section 3 defines the key concepts, operators, and various results related to NHSRM. This section also discusses the mathematical operations used in NHSRM, such as addition, multiplication, and composition, and examines the properties and results that emerge from applying these operations.
- The proposed work illustrates the method through its application to decision-making problems, comparing the weighted arithmetic mean, weighted geometric mean, and weighted harmonic mean operators with the use of a score function in section 4. This

section effectively highlights how these operators can be applied within the NHSRM framework to tackle complex decision-making tasks.

- In section 5, an in-depth evaluation of the performance, accuracy, or efficiency of the proposed method, model, or technique is typically conducted in comparison to existing methods. In the context of this work, the NHSRM can be compared with traditional decision-making models like weighted arithmetic mean, weighted geometric mean, and weighted harmonic mean operators. These operators, when applied within NHSRM, can be assessed in terms of their effectiveness in solving decision-making problems that involve multi-attribute data and uncertainty.
- Conclusion and result are discussed in section 6.

The Neutrosophic Hyper Soft Rough Matrix (NHSRM) model is constrained by the requirement for accurate parameter adjustments, challenges in interpreting intricate outcomes, dependence on expert input for weight determination, and limited flexibility in handling dynamic data changes.

2. Preliminaries

Definition 1 (Neutrosophic set) [17] Let U be universe set, the neutrosophic set N on the universe set U is defined as $N = [T(x), I(x), F(x)]$, where the characteristic functions $T, I, F : U \rightarrow [0, 1]$ and $0 \leq T(x) + I(x) + F(x) \leq 3$, T, I, F are neutrosophic components which defines the degree of truth, the degree of indeterminacy and the degree of falsity respectively.

Definition 2 (Soft rough set) [14] Let $R = (\varphi, A)$ be a soft set over U the pair $S = (U, R)$ is called a soft approximation space. Based on the soft approximation space S , define the following two operations:

$$\underline{apr}_S(X) = \{u \in U; \exists a \in A, u \in \varphi(a) \subseteq X\}$$

$$\overline{apr}_S(X) = \{u \in U; \exists a \in A, u \in \varphi(a), \varphi(a) \cap X \neq \emptyset\}$$

Assigning to every subset $X \subseteq U$ two sets $\underline{apr}_S(X)$ and $\overline{apr}_S(X)$, which are called the soft S lower approximation and the soft S upper approximation of X respectively. In general, $\underline{apr}_S(X)$ and $\overline{apr}_S(X)$ as soft rough approximation of X with respect to S . Moreover the sets $Pos_S(X) = \underline{apr}_S(X)$

$$Neg_S(X) = U - \overline{apr}_S(X)$$

$Bnd_S(X) = \overline{apr}_S(X) - \underline{apr}_S(X)$ are called the soft S - positive region, the soft S - negative region and the soft S - boundary region of X respectively.

If $\overline{apr}_S(X) = \underline{apr}_S(X)$, X is said to be S - definable, otherwise X is called a soft S - rough set.

Definition 3 (Hyper Soft Set) [9] Let U be the universe set and $P(U)$ be the power set of U . Suppose $a_1, a_2, a_3, \dots, a_n$ where $n \geq 1$ be n distinct attributes whose corresponding attribute

values respectively the set $A_1, A_2, \dots, A_n$ with $A_i \cap A_j = \phi, i \neq j$ and $i, j \in \{1, 2, \dots, n\}$ then the pair $(\varphi, A_1 \times A_2 \times \dots \times A_n)$ is called a hyper soft set over U , where $\varphi : A_1 \times A_2 \times \dots \times A_n \rightarrow P(U)$.

Definition 4 (Hyper soft rough set) [11] Let U be the universe set and $P(U)$ be the power set of U . Suppose $a_1, a_2, a_3, \dots, a_n$ where $n \geq 1$ be n distinct attributes whose corresponding attribute values $A_1, A_2, \dots, A_n$ respectively. Let $S_j \subseteq A_j, j \in \{1, 2, \dots, n\}$ then $\prod_{j=1}^n S_j \subseteq \prod_{j=1}^n A_j$. The pair $(\varphi, \prod_{j=1}^n S_j) = P(U)$ where φ is a mapping defined by $\varphi : \prod_{j=1}^n S_j \rightarrow P(U)$ is called hyper soft rough set. The lower and upper neutrosophic hyper soft approximation spaces of $X \in P(U)$ with respect to $(\varphi, \prod_{j=1}^n S_j^k)$ are denoted by $\underline{apr}(X)$ and $\overline{apr}(X)$ respectively, defined by

$$\underline{apr}_S(X) = \{u \in U; \exists a_1, a_2, \dots, a_n \in A, u \in \varphi(a_1, a_2, \dots, a_n) \subseteq X\}$$

$$\overline{apr}_S(X) = \{u \in U; \exists a_1, a_2, \dots, a_n \in A, u \in \varphi(a_1, a_2, \dots, a_n), \varphi(a_1, a_2, \dots, a_n) \cap X \neq \phi\}$$

If $\overline{apr}(X) \neq \underline{apr}(X)$, then X is hyper soft rough set, otherwise it is called as hyper soft rough definable set.

Definition 5 (Neutrosophic Hyper Soft Matrix) [8] Let $U = \{u_1, u_2, \dots, u_\alpha\}$ and $P(U)$ be the universal set and the power set, respectively. Consider $A_1, A_2, \dots, A_\beta$, for $\beta \geq 1$, where β is well-defined attributes, the corresponding attributive values are $A_1^{q_1}, A_2^{q_2}, \dots, A_\beta^{q_n}$ and their relation $A_1^{q_1} \times A_2^{q_2} \times \dots \times A_\beta^{q_n}$, where $q_1, q_2, q_3, \dots, q_n = 1, 2, \dots, n$; then the pair $(\varphi, A_1^{q_1} \times A_2^{q_2} \times \dots \times A_\beta^{q_n})$ is said to be a neutrosophic hyper soft set over U , where $\varphi : (A_1^{q_1} \times A_2^{q_2} \times \dots \times A_\beta^{q_n}) \rightarrow P(U)$ is defined as

$$\varphi(A_1^{q_1} \times A_2^{q_2} \times \dots \times A_\beta^{q_n}) = \{\langle u, T_\lambda(u), I_\lambda(u), F_\lambda(u) \rangle \in U, \lambda \in (A_1^{q_1} \times A_2^{q_2} \times \dots \times A_\beta^{q_n})\}$$

If $B_{ij} = X(u_i, A_j^k)$, where $i = 1, 2, 3, \dots, \alpha, j = 1, 2, 3, \dots, \beta$ and $k = q_1, q_2, q_3, \dots, q_n$ then a

matrix is defined as $[B_{ij}]_{\alpha \times \beta} = \begin{bmatrix} B_{11} & B_{12} & \dots & B_{1\beta} \\ B_{21} & B_{22} & \dots & B_{2\beta} \\ \vdots & \vdots & \ddots & \vdots \\ B_{\alpha 1} & B_{\alpha 2} & \dots & B_{\alpha \beta} \end{bmatrix}$

where $B_{ij} = (T_{A_j^k}(u_i), I_{A_j^k}(u_i), F_{A_j^k}(u_i), u_i \in U, A_j^k \in (A_1^{q_1} \times A_2^{q_2} \times \dots \times A_\beta^{q_n})) = (T_{ij}^B, I_{ij}^B, F_{ij}^B)$

Definition 6 (Neutrosophic Hyper Soft Rough Set) [19] Let U be a non-empty universe set and $P(U)$ be the power set of U . Let E be the set of parameters, $E = \{A_1, A_2, \dots, A_n\}$, where $A_i \cap A_j = \emptyset$ for $i \neq j$. Let $S_j \subseteq A_j, j \in \{1, 2, \dots, n\}$ then $\prod_{j=1}^n S_j^k \subseteq \prod_{j=1}^n A_j^k$. The pair $(\varphi, \prod_{j=1}^n S_j^k) = P(U)$ where φ is a mapping defined by $\varphi : \prod_{j=1}^n S_j^k \rightarrow P(U)$ is called neutrosophic hyper soft rough set. The triplet $(U, \varphi, \prod_{j=1}^n S_j^k)$ is called neutrosophic hyper soft approximation space. The lower and upper neutrosophic hyper soft approximation spaces of $X \in P(U)$ with respect to $(U, \varphi, \prod_{j=1}^n S_j^k)$ are denoted by $\overline{apr}(X)$ and $\underline{apr}(X)$ respectively, defined by

$$\underline{apr}_N(X) = \{(\prod_{j=1}^n S_j, \{\langle \frac{u}{\mu_A(u), v_A(u), w_A(u)} \rangle\}, \forall u \in U\} \text{ and } \overline{apr}_N(X) = N(U) - \underline{apr}_N(X^c)$$

If $\overline{apr}_N(X) \neq \underline{apr}_N(X)$, then X is neutrosophic hyper soft rough set, otherwise it is called as neutrosophic hyper soft rough definable set.

3. Neutrosophic Hyper Soft Rough Matrix (NHSRM)

In this section, the definition, basic operations and various properties of NHSRM are presented.

Definition 7 (NHSRM) Let $U = \{u_1, u_2, \dots, u_\alpha\}$ be an non-empty universe set and $P(U)$ be the power set of U . Let E be the set of parameters, $E = \{A_1, A_2, \dots, A_n\}$, where $A_i \cap A_j = \emptyset$ for $i \neq j$. Let $S_j \subseteq A_j, j \in \{1, 2, \dots, n\}$ then $\prod_{j=1}^n S_j^k \subseteq \prod_{j=1}^n A_j^k$. The pair $(\varphi, \prod_{j=1}^n S_j^k) = P(U)$, where φ is a mapping defined by $\varphi : \prod_{j=1}^n S_j^k \rightarrow P(U)$ is called neutrosophic hyper soft rough set. Each element $u \in U$ is associated with the values determined by the hyper soft set, where each parameter can take multiple values. For each element $u \in U$ related with a parameter A_j is represented by the triplet (T_{ij}, I_{ij}, F_{ij}) where T_{ij} is the truth membership function, I_{ij} is the indeterminacy membership function and F_{ij} is the falsity membership function, $T_{ij}, I_{ij}, F_{ij} \in [0, 1]$. If $P_{ij} = \gamma(u_i, A_j^k)$, where $i = 1, 2, 3, \dots, m, j = 1, 2, 3, \dots, n$ and $k = q_1, q_2, q_3, \dots, q_n$ then a NHSRM is defined as

$$P = [P_{ij}] = \begin{bmatrix} \langle \underline{P}_{11}; \bar{P}_{11} \rangle & \langle \underline{P}_{12}; \bar{P}_{12} \rangle & \dots & \langle \underline{P}_{1n}; \bar{P}_{1n} \rangle \\ \langle \underline{P}_{21}; \bar{P}_{21} \rangle & \langle \underline{P}_{22}; \bar{P}_{22} \rangle & \dots & \langle \underline{P}_{2n}; \bar{P}_{2n} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle \underline{P}_{m1}; \bar{P}_{m1} \rangle & \langle \underline{P}_{m2}; \bar{P}_{m2} \rangle & \dots & \langle \underline{P}_{mn}; \bar{P}_{mn} \rangle \end{bmatrix}$$

Lower Approximation matrix is denoted by $\underline{P}_{ij} = (\underline{T}_{ij}^P, \underline{I}_{ij}^P, \underline{F}_{ij}^P)$, $0 \leq \underline{T}_{ij}^P + \underline{I}_{ij}^P + \underline{F}_{ij}^P \leq 3$ and Upper Approximation matrix is denoted by $\bar{P}_{ij} = (\bar{T}_{ij}^P, \bar{I}_{ij}^P, \bar{F}_{ij}^P)$, $0 \leq \bar{T}_{ij}^P + \bar{I}_{ij}^P + \bar{F}_{ij}^P \leq 3$. Thus, we can represent any neutrosophic Hyper Soft Rough Set in term of neutrosophic Fuzzy Hyper Soft Rough Matrix.

Example 1

The NHSRM P of order 4×3 is written as

$$P = \begin{bmatrix} \langle (0.8, 0.3, 0.2); (0.6, 0.5, 0.1) \rangle & \langle (0.4, 0.2, 0.3); (0.9, 0.2, 0.1) \rangle & \langle (0.5, 0.4, 0.2); (0.7, 0.5, 0.1) \rangle \\ \langle (0.3, 0.4, 0.0); (0.5, 0.8, 0.7) \rangle & \langle (0.8, 0.7, 0.5); (0.9, 0.5, 0.4) \rangle & \langle (1.0, 0.5, 0.2); (0.5, 0.6, 0.0) \rangle \\ \langle (0.9, 0.1, 0.2); (0.5, 0.6, 0.0) \rangle & \langle (0.7, 0.2, 0.1); (0.6, 0.5, 0.0) \rangle & \langle (0.8, 0.2, 0.3); (0.3, 0.2, 0.5) \rangle \\ \langle (1.0, 0.5, 0.4); (0.2, 0.3, 0.5) \rangle & \langle (0.9, 0.5, 0.1); (0.7, 0.8, 1.0) \rangle & \langle (0.6, 0.4, 0.2); (0.5, 0.2, 0.5) \rangle \end{bmatrix}$$

Definition 8 (Square NHSRM) A NHSRM of order $m \times n$ is said to be square NHSRM, if $m = n$ i.e., the number of rows and numbers of columns are equal. This "square" structure signifies that the matrix represents an equal balance between the objects in the universe of discourse and the hierarchical parameters.

Example 2

The square NHSRM P of order 3×3 is written as

$$P = \begin{bmatrix} \langle(0.8, 0.3, 0.2); (0.6, 0.5, 0.1)\rangle & \langle(0.4, 0.2, 0.3); (0.9, 0.2, 0.1)\rangle & \langle(0.5, 0.4, 0.2); (0.7, 0.5, 0.1)\rangle \\ \langle(0.3, 0.4, 0.0); (0.5, 0.8, 0.7)\rangle & \langle(0.8, 0.7, 0.5); (0.9, 0.5, 0.4)\rangle & \langle(1.0, 0.5, 0.2); (0.5, 0.6, 0.0)\rangle \\ \langle(0.9, 0.1, 0.2); (0.5, 0.6, 0.0)\rangle & \langle(0.7, 0.2, 0.1); (0.6, 0.5, 0.0)\rangle & \langle(0.8, 0.2, 0.3); (0.3, 0.2, 0.5)\rangle \end{bmatrix}$$

Definition 9 (Diagonal NHSRM) A square NHSRM of order $n \times n$ is said to be a diagonal NHSRM, if all of its non-diagonal elements are $(0, 0, 1)$ and the main diagonal elements are considered as non-zero.

Example 3

The diagonal NHSRM P of order 3×3 is written as

$$P = \begin{bmatrix} \langle(0.8, 0.3, 0.2); (0.6, 0.5, 0.1)\rangle & \langle(0.0, 0.0, 1.0); (0.0, 0.0, 1.0)\rangle & \langle(0.0, 0.0, 1.0); (0.0, 0.0, 1.0)\rangle \\ \langle(0.0, 0.0, 1.0); (0.0, 0.0, 1.0)\rangle & \langle(1.0, 0.3, 0.5); (0.4, 0.2, 1.0)\rangle & \langle(0.0, 0.0, 1.0); (0.0, 0.0, 1.0)\rangle \\ \langle(0.0, 0.0, 1.0); (0.0, 0.0, 1.0)\rangle & \langle(0.0, 0.0, 1.0); (0.0, 0.0, 1.0)\rangle & \langle(0.8, 0.2, 0.3); (0.3, 0.2, 0.5)\rangle \end{bmatrix}$$

Definition 10 (Symmetric NHSRM) A square NHSRM of order $n \times n$ is said to be a symmetric NHSRM, if its transpose be equal to itself. This means that for every element in the matrix, the entry at position (i, j) is identical to the entry at the position (j, i) . i.e. $P = P^T$ (or) $[P_{ij}] = [P_{ji}] \forall i, j$

Example 4

The symmetric NHSRM P of order 3×3 is written as

$$P = \begin{bmatrix} \langle(0.8, 0.3, 0.2); (0.6, 0.5, 0.1)\rangle & \langle(0.3, 0.4, 0.0); (0.5, 0.8, 0.7)\rangle & \langle(0.5, 0.4, 0.2); (0.7, 0.5, 0.1)\rangle \\ \langle(0.3, 0.4, 0.0); (0.5, 0.8, 0.7)\rangle & \langle(0.8, 0.7, 0.5); (0.9, 0.5, 0.4)\rangle & \langle(0.7, 0.2, 0.1); (0.6, 0.5, 0.0)\rangle \\ \langle(0.5, 0.4, 0.2); (0.7, 0.5, 0.1)\rangle & \langle(0.7, 0.2, 0.1); (0.6, 0.5, 0.0)\rangle & \langle(0.8, 0.2, 0.3); (0.3, 0.2, 0.5)\rangle \end{bmatrix}$$

Definition 11 (Transpose) The transpose of a square NHSRM of order $n \times n$ is obtained by switching its rows and columns, similar to the transpose operation in standard matrices. This operation allows us to examine relationships from a different perspective, effectively interchanging the roles of elements and hierarchical parameters.

$$[P_{ij}]^T = \left[\left(\underline{T}_{ji}^P, \underline{I}_{ji}^P, \underline{F}_{ji}^P \right); \left(\bar{T}_{ji}^P, \bar{I}_{ji}^P, \bar{F}_{ji}^P \right) \right]$$

Example 5

Consider Example 2, the transpose of P is given by

$$P^T = \begin{bmatrix} \langle(0.8, 0.3, 0.2); (0.6, 0.5, 0.1)\rangle & \langle(0.3, 0.4, 0.0); (0.5, 0.8, 0.7)\rangle & \langle(0.9, 0.1, 0.2); (0.5, 0.6, 0.0)\rangle \\ \langle(0.4, 0.2, 0.3); (0.9, 0.2, 0.1)\rangle & \langle(0.8, 0.7, 0.5); (0.9, 0.5, 0.4)\rangle & \langle(0.7, 0.2, 0.1); (0.6, 0.5, 0.0)\rangle \\ \langle(0.5, 0.4, 0.2); (0.7, 0.5, 0.1)\rangle & \langle(1.0, 0.5, 0.2); (0.5, 0.6, 0.0)\rangle & \langle(0.8, 0.2, 0.3); (0.3, 0.2, 0.5)\rangle \end{bmatrix}$$

Definition 12 (Addition) Let $P = [P_{ij}]$ and $Q = [Q_{ij}]$ be two NHSRMs of same order, where $[P_{ij}] = \left[\left(\underline{T}_{ij}^P, \underline{I}_{ij}^P, \underline{F}_{ij}^P \right); \left(\bar{T}_{ij}^P, \bar{I}_{ij}^P, \bar{F}_{ij}^P \right) \right]$ and $[Q_{ij}] = \left[\left(\underline{T}_{ij}^Q, \underline{I}_{ij}^Q, \underline{F}_{ij}^Q \right); \left(\bar{T}_{ij}^Q, \bar{I}_{ij}^Q, \bar{F}_{ij}^Q \right) \right]$ we define the addition in NHSRM as follows:

$$P + Q = \left[\left(\max(\underline{T}_{ij}^P, \underline{T}_{ij}^Q), \max(\underline{I}_{ij}^P, \underline{I}_{ij}^Q), \min(\underline{F}_{ij}^P, \underline{F}_{ij}^Q) \right); \left(\max(\bar{T}_{ij}^P, \bar{T}_{ij}^Q), \max(\bar{I}_{ij}^P, \bar{I}_{ij}^Q), \min(\bar{F}_{ij}^P, \bar{F}_{ij}^Q) \right) \right]$$

Example 6

Let P and Q be two NHSRM of order 3×3 ,

$$P = \begin{bmatrix} \langle (0.8, 0.3, 0.2); (0.6, 0.5, 0.1) \rangle & \langle (0.4, 0.2, 0.3); (0.9, 0.2, 0.1) \rangle & \langle (0.5, 0.4, 0.2); (0.7, 0.5, 0.1) \rangle \\ \langle (0.3, 0.4, 0.0); (0.5, 0.8, 0.7) \rangle & \langle (0.8, 0.7, 0.5); (0.9, 0.5, 0.4) \rangle & \langle (1.0, 0.5, 0.2); (0.5, 0.6, 0.0) \rangle \\ \langle (0.9, 0.1, 0.2); (0.5, 0.6, 0.0) \rangle & \langle (0.7, 0.2, 0.1); (0.6, 0.5, 0.0) \rangle & \langle (0.8, 0.2, 0.3); (0.3, 0.2, 0.5) \rangle \end{bmatrix}$$

and

$$Q = \begin{bmatrix} \langle (0.5, 0.2, 0.0); (0.4, 0.3, 0.1) \rangle & \langle (0.6, 0.8, 0.2); (0.7, 0.3, 0.2) \rangle & \langle (0.7, 0.6, 0.0); (1.0, 0.4, 0.1) \rangle \\ \langle (0.6, 0.7, 0.1); (0.8, 0.9, 0.3) \rangle & \langle (0.4, 1.0, 0.1); (0.9, 0.2, 0.3) \rangle & \langle (1.0, 0.6, 0.0); (0.7, 0.1, 0.3) \rangle \\ \langle (0.4, 0.3, 0.2); (0.7, 0.6, 0.2) \rangle & \langle (1.0, 0.5, 0.7); (0.8, 0.4, 1.0) \rangle & \langle (0.3, 0.2, 0.4); (0.5, 0.7, 0.6) \rangle \end{bmatrix}$$

then

$$P+Q = \begin{bmatrix} \langle (0.8, 0.3, 0.0); (0.6, 0.5, 0.1) \rangle & \langle (0.6, 0.8, 0.2); (0.9, 0.3, 0.1) \rangle & \langle (0.7, 0.6, 0.0); (1.0, 0.5, 0.1) \rangle \\ \langle (0.6, 0.7, 0.0); (0.8, 0.9, 0.3) \rangle & \langle (0.8, 1.0, 0.1); (0.9, 0.5, 0.3) \rangle & \langle (1.0, 0.6, 0.0); (0.7, 0.6, 0.0) \rangle \\ \langle (0.9, 0.3, 0.2); (0.7, 0.6, 0.0) \rangle & \langle (1.0, 0.5, 0.1); (0.8, 0.5, 0.0) \rangle & \langle (0.8, 0.2, 0.3); (0.5, 0.7, 0.5) \rangle \end{bmatrix}$$

Definition 13 (Multiplication) Let $P = [P_{ij}]$ and $Q = [Q_{ij}]$ be two NHSRMs of same order, where $[P_{ij}] = \left[\left(\underline{T}_{ij}^P, \underline{I}_{ij}^P, \underline{F}_{ij}^P \right); \left(\bar{T}_{ij}^P, \bar{I}_{ij}^P, \bar{F}_{ij}^P \right) \right]$ and $[Q_{ij}] = \left[\left(\underline{T}_{ij}^Q, \underline{I}_{ij}^Q, \underline{F}_{ij}^Q \right); \left(\bar{T}_{ij}^Q, \bar{I}_{ij}^Q, \bar{F}_{ij}^Q \right) \right]$ we define the multiplication in NHSRM as follows:

$$P.Q = \left[\left(\min(\underline{T}_{ij}^P, \underline{T}_{ij}^Q), \min(\underline{I}_{ij}^P, \underline{I}_{ij}^Q), \max(\underline{F}_{ij}^P, \underline{F}_{ij}^Q) \right); \left(\min(\bar{T}_{ij}^P, \bar{T}_{ij}^Q), \min(\bar{I}_{ij}^P, \bar{I}_{ij}^Q), \max(\bar{F}_{ij}^P, \bar{F}_{ij}^Q) \right) \right]$$

Example 7

In Example 6, the result is derived by applying Definition 13.

$$P.Q = \begin{bmatrix} \langle (0.5, 0.2, 0.2); (0.4, 0.3, 0.1) \rangle & \langle (0.4, 0.2, 0.3); (0.7, 0.2, 0.2) \rangle & \langle (0.5, 0.4, 0.2); (0.7, 0.4, 0.1) \rangle \\ \langle (0.3, 0.4, 0.1); (0.5, 0.8, 0.7) \rangle & \langle (0.4, 0.7, 0.5); (0.9, 0.2, 0.4) \rangle & \langle (1.0, 0.5, 0.2); (0.5, 0.1, 0.3) \rangle \\ \langle (0.4, 0.1, 0.2); (0.5, 0.6, 0.2) \rangle & \langle (0.7, 0.2, 0.7); (0.6, 0.4, 1.0) \rangle & \langle (0.3, 0.2, 0.4); (0.3, 0.2, 0.6) \rangle \end{bmatrix}$$

Definition 14 (Scalar Multiplication) Let $P = [P_{ij}]$ be the NHSRM of order $m \times n$, where $[P_{ij}] = \left[\left(\underline{T}_{ij}^P, \underline{I}_{ij}^P, \underline{F}_{ij}^P \right); \left(\bar{T}_{ij}^P, \bar{I}_{ij}^P, \bar{F}_{ij}^P \right) \right]$ and $\alpha \in [0, 1]$ is the scalar then scalar multiplication of NHSRM P is given by

$$\alpha P = [\alpha P_{ij}] = \left[\left(\alpha \underline{T}_{ij}^P, \alpha \underline{I}_{ij}^P, \alpha \underline{F}_{ij}^P \right); \left(\alpha \bar{T}_{ij}^P, \alpha \bar{I}_{ij}^P, \alpha \bar{F}_{ij}^P \right) \right]$$

Example 8

By using definition 14 in Example 2, if $\alpha = 0.1$ then the result is as follows:

$$(0.1)P = \begin{bmatrix} \left\langle (0.08, 0.03, 0.02); \right\rangle & \left\langle (0.04, 0.02, 0.03); \right\rangle & \left\langle (0.05, 0.04, 0.02); \right\rangle \\ \left\langle (0.06, 0.05, 0.01) \right\rangle & \left\langle (0.09, 0.02, 0.01) \right\rangle & \left\langle (0.07, 0.05, 0.01) \right\rangle \\ \left\langle (0.03, 0.04, 0.00); \right\rangle & \left\langle (0.08, 0.07, 0.05); \right\rangle & \left\langle (0.10, 0.05, 0.02); \right\rangle \\ \left\langle (0.05, 0.08, 0.07) \right\rangle & \left\langle (0.09, 0.05, 0.04) \right\rangle & \left\langle (0.05, 0.06, 0.00) \right\rangle \\ \left\langle (0.09, 0.01, 0.02); \right\rangle & \left\langle (0.07, 0.02, 0.01); \right\rangle & \left\langle (0.08, 0.02, 0.03); \right\rangle \\ \left\langle (0.05, 0.06, 0.00) \right\rangle & \left\langle (0.06, 0.05, 0.00) \right\rangle & \left\langle (0.03, 0.02, 0.05) \right\rangle \end{bmatrix}$$

Definition 15 (Complement) Let $P = [P_{ij}]$ be the NHSRM of order $m \times n$, where $[P_{ij}] = \left[\left(\underline{T}_{ij}^P, \underline{I}_{ij}^P, \underline{F}_{ij}^P \right); \left(\bar{T}_{ij}^P, \bar{I}_{ij}^P, \bar{F}_{ij}^P \right) \right]$ then $P^c = \left[\left(\underline{F}_{ij}^P, 1 - \underline{I}_{ij}^P, \underline{T}_{ij}^P \right); \left(\bar{F}_{ij}^P, 1 - \bar{I}_{ij}^P, \bar{T}_{ij}^P \right) \right]$ is the complement of P.

Example 9

The complement of NHSRM P in Example 2 is formulated as

$$P^c = \begin{bmatrix} \langle (0.2, 0.7, 0.8); (0.1, 0.5, 0.6) \rangle & \langle (0.3, 0.8, 0.4); (0.5, 0.8, 0.7) \rangle & \langle (0.2, 0.6, 0.5); (0.1, 0.5, 0.7) \rangle \\ \langle (0.0, 0.6, 0.3); (0.7, 0.2, 0.5) \rangle & \langle (0.5, 0.3, 0.8); (0.4, 0.5, 0.9) \rangle & \langle (0.2, 0.5, 1.0); (0.0, 0.4, 0.5) \rangle \\ \langle (0.2, 0.9, 0.9); (0.0, 0.4, 0.5) \rangle & \langle (0.1, 0.8, 0.7); (0.0, 0.5, 0.6) \rangle & \langle (0.3, 0.8, 0.8); (0.5, 0.8, 0.3) \rangle \end{bmatrix}$$

Definition 16 (Score Function) Let $P = [P_{ij}]$ be the NHSRM of order $m \times n$ where $[P_{ij}] = \left[\left(\underline{T}_{ij}^P, \underline{I}_{ij}^P, \underline{F}_{ij}^P \right); \left(\bar{T}_{ij}^P, \bar{I}_{ij}^P, \bar{F}_{ij}^P \right) \right]$ then the score function is defined as

$$S(TP_n) = \frac{1}{3} \left[\left(2 + \left(\frac{\underline{T}_{ij}^P + \bar{T}_{ij}^P}{2} \right) - \left(\frac{\underline{I}_{ij}^P + \bar{I}_{ij}^P}{2} \right) - \left(\frac{\underline{F}_{ij}^P + \bar{F}_{ij}^P}{2} \right) \right) \right]$$

Proposition 1

Let $P = [P_{ij}]$ and $Q = [Q_{ij}]$ be two NHRMs of same order, where $[P_{ij}] = \left[\left(\underline{T}_{ij}^P, \underline{I}_{ij}^P, \underline{F}_{ij}^P \right); \left(\bar{T}_{ij}^P, \bar{I}_{ij}^P, \bar{F}_{ij}^P \right) \right]$ and $[Q_{ij}] = \left[\left(\underline{T}_{ij}^Q, \underline{I}_{ij}^Q, \underline{F}_{ij}^Q \right); \left(\bar{T}_{ij}^Q, \bar{I}_{ij}^Q, \bar{F}_{ij}^Q \right) \right]$ for two scalars $\alpha, \beta \in [0, 1]$, then

- (i) $\alpha(P + Q) = \alpha P + \alpha Q$
- (ii) $\alpha(\beta P) = (\alpha\beta)P$

Proof

$$\begin{aligned} & \text{(i) } \alpha(P + Q) \\ &= \alpha \left(\left[\left(\underline{T}_{ij}^P, \underline{I}_{ij}^P, \underline{F}_{ij}^P \right); \left(\bar{T}_{ij}^P, \bar{I}_{ij}^P, \bar{F}_{ij}^P \right) \right] + \left[\left(\underline{T}_{ij}^Q, \underline{I}_{ij}^Q, \underline{F}_{ij}^Q \right); \left(\bar{T}_{ij}^Q, \bar{I}_{ij}^Q, \bar{F}_{ij}^Q \right) \right] \right) \\ &= \left[\left(\alpha \underline{T}_{ij}^P, \alpha \underline{I}_{ij}^P, \alpha \underline{F}_{ij}^P \right); \left(\alpha \bar{T}_{ij}^P, \alpha \bar{I}_{ij}^P, \alpha \bar{F}_{ij}^P \right) \right] + \left[\left(\alpha \underline{T}_{ij}^Q, \alpha \underline{I}_{ij}^Q, \alpha \underline{F}_{ij}^Q \right); \left(\alpha \bar{T}_{ij}^Q, \alpha \bar{I}_{ij}^Q, \alpha \bar{F}_{ij}^Q \right) \right] \\ &= \alpha \left[\left(\underline{T}_{ij}^P, \underline{I}_{ij}^P, \underline{F}_{ij}^P \right); \left(\bar{T}_{ij}^P, \bar{I}_{ij}^P, \bar{F}_{ij}^P \right) \right] + \alpha \left[\left(\underline{T}_{ij}^Q, \underline{I}_{ij}^Q, \underline{F}_{ij}^Q \right); \left(\bar{T}_{ij}^Q, \bar{I}_{ij}^Q, \bar{F}_{ij}^Q \right) \right] \\ &= \alpha P + \alpha Q \end{aligned}$$

$$\begin{aligned} & \text{(ii) } \alpha(\beta P) \\ &= \alpha \left[\left(\beta \underline{T}_{ij}^P, \beta \underline{I}_{ij}^P, \beta \underline{F}_{ij}^P \right); \left(\beta \bar{T}_{ij}^P, \beta \bar{I}_{ij}^P, \beta \bar{F}_{ij}^P \right) \right] \\ &= \left[\left(\alpha \beta \underline{T}_{ij}^P, \alpha \beta \underline{I}_{ij}^P, \alpha \beta \underline{F}_{ij}^P \right); \left(\alpha \beta \bar{T}_{ij}^P, \alpha \beta \bar{I}_{ij}^P, \alpha \beta \bar{F}_{ij}^P \right) \right] \\ &= (\alpha\beta) \left[\left(\underline{T}_{ij}^P, \underline{I}_{ij}^P, \underline{F}_{ij}^P \right); \left(\bar{T}_{ij}^P, \bar{I}_{ij}^P, \bar{F}_{ij}^P \right) \right] \\ &= (\alpha\beta)P \end{aligned}$$

3.1. Operations on NHSRM

In this section, we discuss various operators such as union, intersection, arithmetic mean, weighted arithmetic mean, geometric mean, weighted geometric mean, harmonic mean and weighted harmonic mean on NHSRM.

Let $P = [P_{ij}]$ and $Q = [Q_{ij}]$ be two NHSRMs of same order, where $[P_{ij}] = \left[\left(\underline{T}_{ij}^P, \underline{I}_{ij}^P, \underline{F}_{ij}^P \right); \left(\bar{T}_{ij}^P, \bar{I}_{ij}^P, \bar{F}_{ij}^P \right) \right]$ and $[Q_{ij}] = \left[\left(\underline{T}_{ij}^Q, \underline{I}_{ij}^Q, \underline{F}_{ij}^Q \right); \left(\bar{T}_{ij}^Q, \bar{I}_{ij}^Q, \bar{F}_{ij}^Q \right) \right]$. Then

(i) Union

$$P \cup Q = \left[\left(\underline{T}_{ij}^R, \underline{I}_{ij}^R, \underline{F}_{ij}^R \right); \left(\bar{T}_{ij}^R, \bar{I}_{ij}^R, \bar{F}_{ij}^R \right) \right],$$

where

$$T_{ij}^R = \max(T_{ij}^P, T_{ij}^Q),$$

$$I_{ij}^R = \frac{I_{ij}^P + I_{ij}^Q}{2},$$

$$F_{ij}^R = \min(F_{ij}^P, F_{ij}^Q)$$

(ii) Intersection

$$P \cap Q = \left[\left(\underline{T}_{ij}^R, \underline{I}_{ij}^R, \underline{F}_{ij}^R \right); \left(\bar{T}_{ij}^R, \bar{I}_{ij}^R, \bar{F}_{ij}^R \right) \right],$$

where

$$T_{ij}^R = \min(T_{ij}^P, T_{ij}^Q),$$

$$I_{ij}^R = \frac{I_{ij}^P + I_{ij}^Q}{2},$$

$$F_{ij}^R = \max(F_{ij}^P, F_{ij}^Q)$$

(iii) Arithmetic Mean

$$P \odot_{AM} Q = \left[\left(\underline{T}_{ij}^R, \underline{I}_{ij}^R, \underline{F}_{ij}^R \right); \left(\bar{T}_{ij}^R, \bar{I}_{ij}^R, \bar{F}_{ij}^R \right) \right],$$

where

$$T_{ij}^R = \frac{T_{ij}^P + T_{ij}^Q}{2},$$

$$I_{ij}^R = \frac{I_{ij}^P + I_{ij}^Q}{2},$$

$$F_{ij}^R = \frac{F_{ij}^P + F_{ij}^Q}{2}$$

(iv) Weighted Arithmetic Mean

$$P \odot_{WAM} Q = \left[\left(\underline{T}_{ij}^R, \underline{I}_{ij}^R, \underline{F}_{ij}^R \right); \left(\bar{T}_{ij}^R, \bar{I}_{ij}^R, \bar{F}_{ij}^R \right) \right],$$

where

$$T_{ij}^R = \frac{w_1 T_{ij}^P + w_2 T_{ij}^Q}{w_1 + w_2},$$

$$I_{ij}^R = \frac{w_1 I_{ij}^P + w_2 I_{ij}^Q}{w_1 + w_2},$$

$$F_{ij}^R = \frac{w_1 F_{ij}^P + w_2 F_{ij}^Q}{w_1 + w_2}, 0 < w_1 + w_2 \leq 1$$

(v) Geometric Mean

$$P \odot_{GM} Q = \left[\left(\underline{T}_{ij}^R, \underline{I}_{ij}^R, \underline{F}_{ij}^R \right); \left(\bar{T}_{ij}^R, \bar{I}_{ij}^R, \bar{F}_{ij}^R \right) \right],$$

where

$$T_{ij}^R = \sqrt{T_{ij}^P \cdot T_{ij}^Q},$$

$$I_{ij}^R = \sqrt{I_{ij}^P \cdot I_{ij}^Q},$$

$$F_{ij}^R = \sqrt{F_{ij}^P \cdot F_{ij}^Q}$$

(vi) Weighted Geometric Mean

$$P \odot_{WGM} Q = \left[\left(\underline{T}_{ij}^R, \underline{I}_{ij}^R, \underline{F}_{ij}^R \right); \left(\bar{T}_{ij}^R, \bar{I}_{ij}^R, \bar{F}_{ij}^R \right) \right],$$

where

$$T_{ij}^R = \sqrt[w_1+w_2]{\left(T_{ij}^P\right)^{w_1} \cdot \left(T_{ij}^Q\right)^{w_2}},$$

$$I_{ij}^R = \sqrt[w_1+w_2]{\left(I_{ij}^P\right)^{w_1} \cdot \left(I_{ij}^Q\right)^{w_2}},$$

$$F_{ij}^R = \sqrt[w_1+w_2]{\left(F_{ij}^P\right)^{w_1} \cdot \left(F_{ij}^Q\right)^{w_2}}, 0 < w_1 + w_2 \leq 1$$

(vii) Harmonic Mean

$$P \odot_{GM} Q = \left[\left(\underline{T}_{ij}^R, \underline{I}_{ij}^R, \underline{F}_{ij}^R \right); \left(\bar{T}_{ij}^R, \bar{I}_{ij}^R, \bar{F}_{ij}^R \right) \right],$$

where

$$T_{ij}^R = \frac{2T_{ij}^P T_{ij}^Q}{T_{ij}^P + T_{ij}^Q},$$

$$I_{ij}^R = \frac{2I_{ij}^P I_{ij}^Q}{I_{ij}^P + I_{ij}^Q},$$

$$F_{ij}^R = \frac{2F_{ij}^P F_{ij}^Q}{F_{ij}^P + F_{ij}^Q}$$

(viii) Weighted Harmonic Mean

$$P \odot_{WHM} Q = \left[\left(\underline{T}_{ij}^R, \underline{I}_{ij}^R, \underline{F}_{ij}^R \right); \left(\bar{T}_{ij}^R, \bar{I}_{ij}^R, \bar{F}_{ij}^R \right) \right],$$

where

$$T_{ij}^R = \frac{w_1 + w_2}{\left(w_1/T_{ij}^P\right) + \left(w_2/T_{ij}^Q\right)},$$

$$I_{ij}^R = \frac{w_1 + w_2}{\left(w_1/I_{ij}^P\right) + \left(w_2/I_{ij}^Q\right)},$$

$$F_{ij}^R = \frac{w_1 + w_2}{\left(w_1/F_{ij}^P\right) + \left(w_2/F_{ij}^Q\right)}, 0 < w_1 + w_2 \leq 1$$

3.2. Algebraic properties of NHSRM

In this section, we present the algebraic properties of NHSRM, which are transpose property, commutative property, associative property and distributive property.

Theorem 1 Let $P = [P_{ij}]$ and $Q = [Q_{ij}]$ be two NHSRMs with same order, where $[P_{ij}] = \left[\left(\underline{T}_{ij}^P, \underline{I}_{ij}^P, \underline{F}_{ij}^P \right); \left(\bar{T}_{ij}^P, \bar{I}_{ij}^P, \bar{F}_{ij}^P \right) \right]$ and $[Q_{ij}] = \left[\left(\underline{T}_{ij}^Q, \underline{I}_{ij}^Q, \underline{F}_{ij}^Q \right); \left(\bar{T}_{ij}^Q, \bar{I}_{ij}^Q, \bar{F}_{ij}^Q \right) \right]$ then

- (i) $(P \cup Q)^T = P^T \cup Q^T$
- (ii) $(P \cap Q)^T = P^T \cap Q^T$
- (iii) $(P \odot_{AM} Q)^T = P^T \odot_{AM} Q^T$
- (iv) $(P \odot_{WAM} Q)^T = P^T \odot_{WAM} Q^T$
- (v) $(P \odot_{GM} Q)^T = P^T \odot_{GM} Q^T$
- (vi) $(P \odot_{WGM} Q)^T = P^T \odot_{WGM} Q^T$
- (vii) $(P \odot_{HM} Q)^T = P^T \odot_{HM} P^T$
- (viii) $(P \odot_{WHM} Q)^T = P^T \odot_{WHM} Q^T$

Proof(i) $(P \cup Q)^T$

$$\begin{aligned}
&= \left[\left(\max \left(\underline{T}_{ij}^P, \underline{T}_{ij}^Q \right), \frac{\underline{I}_{ij}^P + \underline{I}_{ij}^Q}{2}, \min \left(\underline{F}_{ij}^P, \underline{F}_{ij}^Q \right) \right); \left(\max \left(\bar{T}_{ij}^P, \bar{T}_{ij}^Q \right), \frac{\bar{I}_{ij}^P + \bar{I}_{ij}^Q}{2}, \min \left(\bar{F}_{ij}^P, \bar{F}_{ij}^Q \right) \right) \right]^T \\
&= \left[\left(\max \left(\underline{T}_{ji}^P, \underline{T}_{ji}^Q \right), \frac{\underline{I}_{ji}^P + \underline{I}_{ji}^Q}{2}, \min \left(\underline{F}_{ji}^P, \underline{F}_{ji}^Q \right) \right); \left(\max \left(\bar{T}_{ji}^P, \bar{T}_{ji}^Q \right), \frac{\bar{I}_{ji}^P + \bar{I}_{ji}^Q}{2}, \min \left(\bar{F}_{ji}^P, \bar{F}_{ji}^Q \right) \right) \right] \\
&= \left[\left(\underline{T}_{ji}^P, \underline{I}_{ji}^P, \underline{F}_{ji}^P \right); \left(\bar{T}_{ji}^P, \bar{I}_{ji}^P, \bar{F}_{ji}^P \right) \right] \cup \left[\left(\underline{T}_{ji}^Q, \underline{I}_{ji}^Q, \underline{F}_{ji}^Q \right); \left(\bar{T}_{ji}^Q, \bar{I}_{ji}^Q, \bar{F}_{ji}^Q \right) \right] \\
&= \left[\left(\underline{T}_{ij}^P, \underline{I}_{ij}^P, \underline{F}_{ij}^P \right); \left(\bar{T}_{ij}^P, \bar{I}_{ij}^P, \bar{F}_{ij}^P \right) \right]^T \cup \left[\left(\underline{T}_{ij}^Q, \underline{I}_{ij}^Q, \underline{F}_{ij}^Q \right); \left(\bar{T}_{ij}^Q, \bar{I}_{ij}^Q, \bar{F}_{ij}^Q \right) \right]^T \\
&= P^T \cup Q^T
\end{aligned}$$

Similarly the result (ii) can be proved.

(iii) $(P \odot_{AM} Q)^T$

$$\begin{aligned}
&= \left[\left(\frac{\underline{T}_{ij}^P + \underline{T}_{ij}^Q}{2}, \frac{\underline{I}_{ij}^P + \underline{I}_{ij}^Q}{2}, \frac{\underline{F}_{ij}^P + \underline{F}_{ij}^Q}{2} \right); \left(\frac{\bar{T}_{ij}^P + \bar{T}_{ij}^Q}{2}, \frac{\bar{I}_{ij}^P + \bar{I}_{ij}^Q}{2}, \frac{\bar{F}_{ij}^P + \bar{F}_{ij}^Q}{2} \right) \right]^T \\
&= \left[\left(\frac{\underline{T}_{ji}^P + \underline{T}_{ji}^Q}{2}, \frac{\underline{I}_{ji}^P + \underline{I}_{ji}^Q}{2}, \frac{\underline{F}_{ji}^P + \underline{F}_{ji}^Q}{2} \right); \left(\frac{\bar{T}_{ji}^P + \bar{T}_{ji}^Q}{2}, \frac{\bar{I}_{ji}^P + \bar{I}_{ji}^Q}{2}, \frac{\bar{F}_{ji}^P + \bar{F}_{ji}^Q}{2} \right) \right] \\
&= \left[\left(\underline{T}_{ji}^P, \underline{I}_{ji}^P, \underline{F}_{ji}^P \right); \left(\bar{T}_{ji}^P, \bar{I}_{ji}^P, \bar{F}_{ji}^P \right) \right] \odot_{AM} \left[\left(\underline{T}_{ji}^Q, \underline{I}_{ji}^Q, \underline{F}_{ji}^Q \right); \left(\bar{T}_{ji}^Q, \bar{I}_{ji}^Q, \bar{F}_{ji}^Q \right) \right] \\
&= \left[\left(\underline{T}_{ij}^P, \underline{I}_{ij}^P, \underline{F}_{ij}^P \right); \left(\bar{T}_{ij}^P, \bar{I}_{ij}^P, \bar{F}_{ij}^P \right) \right]^T \odot_{AM} \left[\left(\underline{T}_{ij}^Q, \underline{I}_{ij}^Q, \underline{F}_{ij}^Q \right); \left(\bar{T}_{ij}^Q, \bar{I}_{ij}^Q, \bar{F}_{ij}^Q \right) \right]^T \\
&= P^T \odot_{AM} Q^T
\end{aligned}$$

(iv) $(P \odot_{WAM} Q)^T$

$$\begin{aligned}
&= \left[\left(\frac{w_1 \underline{T}_{ij}^P + w_2 \underline{T}_{ij}^Q}{w_1 + w_2}, \frac{w_1 \underline{I}_{ij}^P + w_2 \underline{I}_{ij}^Q}{w_1 + w_2}, \frac{w_1 \underline{F}_{ij}^P + w_2 \underline{F}_{ij}^Q}{w_1 + w_2} \right); \right. \\
&\quad \left. \left(\frac{w_1 \bar{T}_{ij}^P + w_2 \bar{T}_{ij}^Q}{w_1 + w_2}, \frac{w_1 \bar{I}_{ij}^P + w_2 \bar{I}_{ij}^Q}{w_1 + w_2}, \frac{w_1 \bar{F}_{ij}^P + w_2 \bar{F}_{ij}^Q}{w_1 + w_2} \right) \right]^T
\end{aligned}$$

$$\begin{aligned}
&= \left[\left(\frac{w_1 \underline{T}_{ji}^P + w_2 \underline{T}_{ji}^Q}{w_1 + w_2}, \frac{w_1 \underline{I}_{ji}^P + w_2 \underline{I}_{ji}^Q}{w_1 + w_2}, \frac{w_1 \underline{F}_{ji}^P + w_2 \underline{F}_{ji}^Q}{w_1 + w_2} \right) \right. \\
&\quad \left. ; \left(\frac{w_1 \bar{T}_{ji}^P + w_2 \bar{T}_{ji}^Q}{w_1 + w_2}, \frac{w_1 \bar{I}_{ji}^P + w_2 \bar{I}_{ji}^Q}{w_1 + w_2}, \frac{w_1 \bar{F}_{ji}^P + w_2 \bar{F}_{ji}^Q}{w_1 + w_2} \right) \right] \\
&= \left[\left(\underline{T}_{ji}^P, \underline{I}_{ji}^P, \underline{F}_{ji}^P \right); \left(\bar{T}_{ji}^P, \bar{I}_{ji}^P, \bar{F}_{ji}^P \right) \right] \odot_{WAM} \left[\left(\underline{T}_{ji}^Q, \underline{I}_{ji}^Q, \underline{F}_{ji}^Q \right); \left(\bar{T}_{ji}^Q, \bar{I}_{ji}^Q, \bar{F}_{ji}^Q \right) \right] \\
&= \left[\left(\underline{T}_{ij}^P, \underline{I}_{ij}^P, \underline{F}_{ij}^P \right); \left(\bar{T}_{ij}^P, \bar{I}_{ij}^P, \bar{F}_{ij}^P \right) \right]^T \odot_{WAM} \left[\left(\underline{T}_{ij}^Q, \underline{I}_{ij}^Q, \underline{F}_{ij}^Q \right); \left(\bar{T}_{ij}^Q, \bar{I}_{ij}^Q, \bar{F}_{ij}^Q \right) \right]^T \\
&= P^T \odot_{WAM} Q^T
\end{aligned}$$

Similarly the results (v) – (viii) can be proved.

Theorem 2 Let $P = [P_{ij}]$ and $Q = [Q_{ij}]$ be two NHSRMs with same order, where $[P_{ij}] = \left[\left(\underline{T}_{ij}^P, \underline{I}_{ij}^P, \underline{F}_{ij}^P \right); \left(\bar{T}_{ij}^P, \bar{I}_{ij}^P, \bar{F}_{ij}^P \right) \right]$ and $[Q_{ij}] = \left[\left(\underline{T}_{ij}^Q, \underline{I}_{ij}^Q, \underline{F}_{ij}^Q \right); \left(\bar{T}_{ij}^Q, \bar{I}_{ij}^Q, \bar{F}_{ij}^Q \right) \right]$ then

- (i) $(P \cup Q)^c = P^c \cap Q^c$
- (ii) $(P \cap Q)^c = P^c \cup Q^c$
- (iii) $(P \odot_{AM} Q)^c = P^c \odot_{AM} Q^c$
- (iv) $(P \odot_{WAM} Q)^c = P^c \odot_{WAM} Q^c$
- (v) $(P \odot_{GM} Q)^c = P^c \odot_{GM} Q^c$
- (vi) $(P \odot_{WGM} Q)^c = P^c \odot_{WGM} Q^c$
- (vii) $(P \odot_{HM} Q)^c = P^c \odot_{HM} Q^c$
- (viii) $(P \odot_{WHM} P)^c = P^c \odot_{WHM} Q^c$

Proof

$$\begin{aligned}
&(i) (P \cup Q)^c \\
&= \left[\left(\max \left(\underline{T}_{ij}^P, \underline{T}_{ij}^Q \right), \frac{\underline{I}_{ij}^P + \underline{I}_{ij}^Q}{2}, \min \left(\underline{F}_{ij}^P, \underline{F}_{ij}^Q \right) \right); \left(\max \left(\bar{T}_{ij}^P, \bar{T}_{ij}^Q \right), \frac{\bar{I}_{ij}^P + \bar{I}_{ij}^Q}{2}, \min \left(\bar{F}_{ij}^P, \bar{F}_{ij}^Q \right) \right) \right]^c \\
&= \left[\left(\min \left(\underline{F}_{ij}^P, \underline{F}_{ij}^Q \right), 1 - \frac{\underline{I}_{ij}^P + \underline{I}_{ij}^Q}{2}, \max \left(\underline{T}_{ij}^P, \underline{T}_{ij}^Q \right) \right); \right. \\
&\quad \left. \left(\min \left(\bar{F}_{ij}^P, \bar{F}_{ij}^Q \right), 1 - \frac{\bar{I}_{ij}^P + \bar{I}_{ij}^Q}{2}, \max \left(\bar{T}_{ij}^P, \bar{T}_{ij}^Q \right) \right) \right] \\
&= \left[\left(\underline{F}_{ij}^P, 1 - \underline{I}_{ij}^P, \underline{T}_{ij}^P \right); \left(\bar{F}_{ij}^Q, 1 - \bar{I}_{ij}^Q, \bar{T}_{ij}^Q \right) \right] \cap \left[\left(\underline{F}_{ij}^P, 1 - \underline{I}_{ij}^P, \underline{T}_{ij}^P \right); \left(\bar{F}_{ij}^Q, 1 - \bar{I}_{ij}^Q, \bar{T}_{ij}^Q \right) \right] \\
&= \left[\left(\underline{T}_{ij}^P, \underline{I}_{ij}^P, \underline{F}_{ij}^P \right); \left(\bar{T}_{ij}^Q, \bar{I}_{ij}^Q, \bar{F}_{ij}^Q \right) \right]^c \cap \left[\left(\underline{T}_{ij}^P, \underline{I}_{ij}^P, \underline{F}_{ij}^P \right); \left(\bar{T}_{ij}^Q, \bar{I}_{ij}^Q, \bar{F}_{ij}^Q \right) \right]^c \\
&= P^c \cap Q^c
\end{aligned}$$

In similar manner, the result (ii) can be proved.

$$(iii) (P \odot_{AM} Q)^c$$

$$\begin{aligned}
&= \left[\left(\frac{T_{ij}^P + T_{ij}^Q}{2}, \frac{I_{ij}^P + I_{ij}^Q}{2}, \frac{F_{ij}^P + F_{ij}^Q}{2} \right); \left(\frac{\bar{T}_{ij}^P + \bar{T}_{ij}^Q}{2}, \frac{\bar{I}_{ij}^P + \bar{I}_{ij}^Q}{2}, \frac{\bar{F}_{ij}^P + \bar{F}_{ij}^Q}{2} \right) \right]^c \\
&= \left[\left(\frac{F_{ij}^P + F_{ij}^Q}{2}, 1 - \frac{I_{ij}^P + I_{ij}^Q}{2}, \frac{T_{ij}^P + T_{ij}^Q}{2} \right); \left(\frac{\bar{F}_{ij}^P + \bar{F}_{ij}^Q}{2}, 1 - \frac{\bar{I}_{ij}^P + \bar{I}_{ij}^Q}{2}, \frac{\bar{T}_{ij}^P + \bar{T}_{ij}^Q}{2} \right) \right] \\
&= \left[\left(F_{ij}^P, 1 - I_{ij}^P, T_{ij}^P \right); \left(\bar{F}_{ij}^Q, 1 - \bar{I}_{ij}^Q, \bar{T}_{ij}^Q \right) \right] \odot_{AM} \left[\left(F_{ij}^P, 1 - I_{ij}^P, T_{ij}^P \right); \left(\bar{F}_{ij}^Q, 1 - \bar{I}_{ij}^Q, \bar{T}_{ij}^Q \right) \right] \\
&= \left[\left(T_{ij}^P, I_{ij}^P, F_{ij}^P \right); \left(\bar{T}_{ij}^P, \bar{I}_{ij}^P, \bar{F}_{ij}^P \right) \right]^c \odot_{AM} \left[\left(T_{ij}^Q, I_{ij}^Q, F_{ij}^Q \right); \left(\bar{T}_{ij}^Q, \bar{I}_{ij}^Q, \bar{F}_{ij}^Q \right) \right]^c = P^c \odot_{AM} Q^c
\end{aligned}$$

Similarly the results (iv) – (viii) can be proved.

Theorem 3 Let $P = [P_{ij}]$ and $Q = [Q_{ij}]$ be two NHRMs with same order, where $[P_{ij}] = \left[\left(T_{ij}^P, I_{ij}^P, F_{ij}^P \right); \left(\bar{T}_{ij}^P, \bar{I}_{ij}^P, \bar{F}_{ij}^P \right) \right]$ and $[Q_{ij}] = \left[\left(T_{ij}^Q, I_{ij}^Q, F_{ij}^Q \right); \left(\bar{T}_{ij}^Q, \bar{I}_{ij}^Q, \bar{F}_{ij}^Q \right) \right]$ then

- (i) $P \cup Q = Q \cup P$
- (ii) $P \cap Q = Q \cap P$
- (iii) $P \odot_{AM} Q = Q \odot_{AM} P$
- (iv) $P \odot_{WAM} Q = Q \odot_{WAM} P$
- (v) $P \odot_{GM} Q = Q \odot_{GM} P$
- (vi) $P \odot_{WGM} Q = Q \odot_{WGM} P$
- (vii) $P \odot_{HM} Q = Q \odot_{HM} P$
- (viii) $P \odot_{WHM} Q = Q \odot_{WHM} P$

Proof

$$\begin{aligned}
&(i) P \cup Q \\
&= \left[\left(\max \left(T_{ij}^P, T_{ij}^Q \right), \frac{I_{ij}^P + I_{ij}^Q}{2}, \min \left(F_{ij}^P, F_{ij}^Q \right) \right); \left(\max \left(\bar{T}_{ij}^P, \bar{T}_{ij}^Q \right), \frac{\bar{I}_{ij}^P + \bar{I}_{ij}^Q}{2}, \min \left(\bar{F}_{ij}^P, \bar{F}_{ij}^Q \right) \right) \right] \\
&= \left[\left(\max \left(T_{ij}^Q, T_{ij}^P \right), \frac{I_{ij}^Q + I_{ij}^P}{2}, \min \left(F_{ij}^Q, F_{ij}^P \right) \right); \left(\max \left(\bar{T}_{ij}^Q, \bar{T}_{ij}^P \right), \frac{\bar{I}_{ij}^Q + \bar{I}_{ij}^P}{2}, \min \left(\bar{F}_{ij}^Q, \bar{F}_{ij}^P \right) \right) \right] \\
&= \left[\left(T_{ij}^Q, I_{ij}^Q, F_{ij}^Q \right); \left(\bar{T}_{ij}^Q, \bar{I}_{ij}^Q, \bar{F}_{ij}^Q \right) \right] \cup \left[\left(T_{ij}^P, I_{ij}^P, F_{ij}^P \right); \left(\bar{T}_{ij}^P, \bar{I}_{ij}^P, \bar{F}_{ij}^P \right) \right] = Q \cup P
\end{aligned}$$

In similar manner, the results (ii) – (iv) can be proved.

$$\begin{aligned}
&(v) P \odot_{GM} Q \\
&= \left[\left(\sqrt{T_{ij}^P \cdot T_{ij}^Q}, \sqrt{I_{ij}^P \cdot I_{ij}^Q}, \sqrt{F_{ij}^P \cdot F_{ij}^Q} \right); \left(\sqrt{\bar{T}_{ij}^P \cdot \bar{T}_{ij}^Q}, \sqrt{\bar{I}_{ij}^P \cdot \bar{I}_{ij}^Q}, \sqrt{\bar{F}_{ij}^P \cdot \bar{F}_{ij}^Q} \right) \right] \\
&= \left[\left(\sqrt{T_{ij}^Q \cdot T_{ij}^P}, \sqrt{I_{ij}^Q \cdot I_{ij}^P}, \sqrt{F_{ij}^Q \cdot F_{ij}^P} \right); \left(\sqrt{\bar{T}_{ij}^Q \cdot \bar{T}_{ij}^P}, \sqrt{\bar{I}_{ij}^Q \cdot \bar{I}_{ij}^P}, \sqrt{\bar{F}_{ij}^Q \cdot \bar{F}_{ij}^P} \right) \right] \\
&= \left[\left(T_{ij}^Q, I_{ij}^Q, F_{ij}^Q \right); \left(\bar{T}_{ij}^Q, \bar{I}_{ij}^Q, \bar{F}_{ij}^Q \right) \right] \odot_{GM} \left[\left(T_{ij}^P, I_{ij}^P, F_{ij}^P \right); \left(\bar{T}_{ij}^P, \bar{I}_{ij}^P, \bar{F}_{ij}^P \right) \right] = Q \odot_{GM} P
\end{aligned}$$

In similar manner, the results (vi) – (viii) can be proved.

Theorem 4 Let $P = [P_{ij}]$, $Q = [Q_{ij}]$ and $R = [R_{ij}]$ be three NHRMs with same order,

where $[P_{ij}] = \left[\left(\underline{T}_{ij}^P, \underline{I}_{ij}^P, \underline{F}_{ij}^P \right); \left(\bar{T}_{ij}^P, \bar{I}_{ij}^P, \bar{F}_{ij}^P \right) \right]$, $[Q_{ij}] = \left[\left(\underline{T}_{ij}^Q, \underline{I}_{ij}^Q, \underline{F}_{ij}^Q \right); \left(\bar{T}_{ij}^Q, \bar{I}_{ij}^Q, \bar{F}_{ij}^Q \right) \right]$ and $[R_{ij}] = \left[\left(\underline{T}_{ij}^R, \underline{I}_{ij}^R, \underline{F}_{ij}^R \right); \left(\bar{T}_{ij}^R, \bar{I}_{ij}^R, \bar{F}_{ij}^R \right) \right]$ then

- (i) $(P \cup Q) \cup R = P \cup (Q \cup R)$
- (ii) $(P \cap Q) \cap R = P \cap (Q \cap R)$
- (iii) $(P \odot_{AM} Q) \odot_{AM} R \neq P \odot_{AM} (Q \odot_{AM} R)$
- (iv) $(P \odot_{GM} Q) \odot_{GM} R \neq P \odot_{GM} (Q \odot_{GM} R)$
- (v) $(P \odot_{HM} Q) \odot_{HM} R \neq P \odot_{HM} (Q \odot_{HM} R)$

Proof

$$\begin{aligned}
 & \text{(i) } (P \cup Q) \cup R \\
 &= \left[\left(\max \left(\underline{T}_{ij}^P, \underline{T}_{ij}^Q \right), \frac{\underline{I}_{ij}^P + \underline{I}_{ij}^Q}{2}, \min \left(\underline{F}_{ij}^P, \underline{F}_{ij}^Q \right) \right); \left(\max \left(\bar{T}_{ij}^P, \bar{T}_{ij}^Q \right), \frac{\bar{I}_{ij}^P + \bar{I}_{ij}^Q}{2}, \min \left(\bar{F}_{ij}^P, \bar{F}_{ij}^Q \right) \right) \right] \\
 & \cup \left[\left(\underline{T}_{ij}^R, \underline{I}_{ij}^R, \underline{F}_{ij}^R \right); \left(\bar{T}_{ij}^R, \bar{I}_{ij}^R, \bar{F}_{ij}^R \right) \right] \\
 &= \left[\left(\max \left(\underline{T}_{ij}^P, \underline{T}_{ij}^Q, \underline{T}_{ij}^R \right), \frac{\underline{I}_{ij}^P + \underline{I}_{ij}^Q + \underline{I}_{ij}^R}{3}, \min \left(\underline{F}_{ij}^P, \underline{F}_{ij}^Q, \underline{F}_{ij}^R \right) \right); \right. \\
 & \quad \left. \left(\max \left(\bar{T}_{ij}^P, \bar{T}_{ij}^Q, \bar{T}_{ij}^R \right), \frac{\bar{I}_{ij}^P + \bar{I}_{ij}^Q + \bar{I}_{ij}^R}{3}, \min \left(\bar{F}_{ij}^P, \bar{F}_{ij}^Q, \bar{F}_{ij}^R \right) \right) \right] \\
 &= \left[\left(\underline{T}_{ij}^P, \underline{I}_{ij}^P, \underline{F}_{ij}^P \right); \left(\bar{T}_{ij}^P, \bar{I}_{ij}^P, \bar{F}_{ij}^P \right) \right] \cup \left[\left(\max \left(\underline{T}_{ij}^Q, \underline{T}_{ij}^R \right), \frac{\underline{I}_{ij}^Q + \underline{I}_{ij}^R}{2}, \min \left(\underline{F}_{ij}^Q, \underline{F}_{ij}^R \right) \right); \right. \\
 & \quad \left. \left(\max \left(\bar{T}_{ij}^Q, \bar{T}_{ij}^R \right), \frac{\bar{I}_{ij}^Q + \bar{I}_{ij}^R}{2}, \min \left(\bar{F}_{ij}^Q, \bar{F}_{ij}^R \right) \right) \right] \\
 &= P \cup (Q \cup R)
 \end{aligned}$$

In similar manner, the result (ii) can be proved.

(iii) Consider LHS,

$$\begin{aligned}
 (P \odot_{AM} Q) \odot_{AM} R &= \left[\left(\frac{\underline{T}_{ij}^P + \underline{T}_{ij}^Q}{2}, \frac{\underline{I}_{ij}^P + \underline{I}_{ij}^Q}{2}, \frac{\underline{F}_{ij}^P + \underline{F}_{ij}^Q}{2} \right); \left(\frac{\bar{T}_{ij}^P + \bar{T}_{ij}^Q}{2}, \frac{\bar{I}_{ij}^P + \bar{I}_{ij}^Q}{2}, \frac{\bar{F}_{ij}^P + \bar{F}_{ij}^Q}{2} \right) \right] \\
 & \odot_{AM} \left[\left(\underline{T}_{ij}^R, \underline{I}_{ij}^R, \underline{F}_{ij}^R \right); \left(\bar{T}_{ij}^R, \bar{I}_{ij}^R, \bar{F}_{ij}^R \right) \right] \\
 &= \left[\left(\frac{\left(\underline{T}_{ij}^P + \underline{T}_{ij}^Q \right) / 2 + \underline{T}_{ij}^R}{2}, \frac{\left(\underline{I}_{ij}^P + \underline{I}_{ij}^Q \right) / 2 + \underline{I}_{ij}^R}{2}, \frac{\left(\underline{F}_{ij}^P + \underline{F}_{ij}^Q \right) / 2 + \underline{F}_{ij}^R}{2} \right); \right. \\
 & \quad \left. \left(\frac{\left(\bar{T}_{ij}^P + \bar{T}_{ij}^Q \right) / 2 + \bar{T}_{ij}^R}{2}, \frac{\left(\bar{I}_{ij}^P + \bar{I}_{ij}^Q \right) / 2 + \bar{I}_{ij}^R}{2}, \frac{\left(\bar{F}_{ij}^P + \bar{F}_{ij}^Q \right) / 2 + \bar{F}_{ij}^R}{2} \right) \right] \quad (1)
 \end{aligned}$$

Consider RHS of (iii)

$$\begin{aligned}
 P \odot_{AM} (Q \odot_{AM} R) &= \left[\left(\underline{T}_{ij}^P, \underline{I}_{ij}^P, \underline{F}_{ij}^P \right); \left(\bar{T}_{ij}^P, \bar{I}_{ij}^P, \bar{F}_{ij}^P \right) \right] \odot_{AM} \left[\left(\frac{\underline{T}_{ij}^Q + \underline{T}_{ij}^R}{2}, \frac{\underline{I}_{ij}^Q + \underline{I}_{ij}^R}{2}, \frac{\underline{F}_{ij}^Q + \underline{F}_{ij}^R}{2} \right); \right. \\
 &\quad \left. \left(\frac{\bar{T}_{ij}^Q + \bar{T}_{ij}^R}{2}, \frac{\bar{I}_{ij}^Q + \bar{I}_{ij}^R}{2}, \frac{\bar{F}_{ij}^Q + \bar{F}_{ij}^R}{2} \right) \right] \\
 &= \left[\left(\frac{\underline{T}_{ij}^P + \left(\frac{\underline{T}_{ij}^Q + \underline{T}_{ij}^R}{2} \right) / 2}{2}, \frac{\underline{I}_{ij}^P + \left(\frac{\underline{I}_{ij}^Q + \underline{I}_{ij}^R}{2} \right) / 2}{2}, \frac{\underline{F}_{ij}^P + \left(\frac{\underline{F}_{ij}^Q + \underline{F}_{ij}^R}{2} \right) / 2}{2} \right); \right. \\
 &\quad \left. \left(\frac{\bar{T}_{ij}^P + \left(\frac{\bar{T}_{ij}^Q + \bar{T}_{ij}^R}{2} \right) / 2}{2}, \frac{\bar{I}_{ij}^P + \left(\frac{\bar{I}_{ij}^Q + \bar{I}_{ij}^R}{2} \right) / 2}{2}, \frac{\bar{F}_{ij}^P + \left(\frac{\bar{F}_{ij}^Q + \bar{F}_{ij}^R}{2} \right) / 2}{2} \right) \right] \quad (2)
 \end{aligned}$$

From Equation (1) & Equation (2), $(P \odot_{AM} Q) \odot_{AM} R \neq P \odot_{AM} (Q \odot_{AM} R)$

In similar manner, the results (iv) and (v) can be proved.

Theorem 5 Let $P = [P_{ij}]$, $Q = [Q_{ij}]$ and $R = [R_{ij}]$ be three NHSRMs with same order, where $[P_{ij}] = \left[\left(\underline{T}_{ij}^P, \underline{I}_{ij}^P, \underline{F}_{ij}^P \right); \left(\bar{T}_{ij}^P, \bar{I}_{ij}^P, \bar{F}_{ij}^P \right) \right]$, $[Q_{ij}] = \left[\left(\underline{T}_{ij}^Q, \underline{I}_{ij}^Q, \underline{F}_{ij}^Q \right); \left(\bar{T}_{ij}^Q, \bar{I}_{ij}^Q, \bar{F}_{ij}^Q \right) \right]$ and $[R_{ij}] = \left[\left(\underline{T}_{ij}^R, \underline{I}_{ij}^R, \underline{F}_{ij}^R \right); \left(\bar{T}_{ij}^R, \bar{I}_{ij}^R, \bar{F}_{ij}^R \right) \right]$ then

- (i) $P \cup (Q \odot_{AM} R) = (P \cup Q) \odot_{AM} (P \cup R)$
- (ii) $P \cap (Q \odot_{AM} R) = (P \cap Q) \odot_{AM} (P \cap R)$
- (iii) $(P \odot_{AM} Q) \cup R = (P \cup R) \odot_{AM} (Q \cup R)$
- (iv) $(P \odot_{AM} Q) \cap R = (P \cap R) \odot_{AM} (Q \cap R)$

Proof

(i) $P \cup (Q \odot_{AM} R)$

$$\begin{aligned}
 &= \left[\left(\underline{T}_{ij}^P, \underline{I}_{ij}^P, \underline{F}_{ij}^P \right); \left(\bar{T}_{ij}^P, \bar{I}_{ij}^P, \bar{F}_{ij}^P \right) \right] \cup \left[\left(\left(\frac{\underline{T}_{ij}^Q + \underline{T}_{ij}^R}{2} \right), \left(\frac{\underline{I}_{ij}^Q + \underline{I}_{ij}^R}{2} \right), \left(\frac{\underline{F}_{ij}^Q + \underline{F}_{ij}^R}{2} \right) \right); \right. \\
 &\quad \left. \left(\left(\frac{\bar{T}_{ij}^Q + \bar{T}_{ij}^R}{2} \right), \left(\frac{\bar{I}_{ij}^Q + \bar{I}_{ij}^R}{2} \right), \left(\frac{\bar{F}_{ij}^Q + \bar{F}_{ij}^R}{2} \right) \right) \right] \\
 &= \left[\left(\max \left(\underline{T}_{ij}^P, \frac{\underline{T}_{ij}^Q + \underline{T}_{ij}^R}{2} \right), \frac{\underline{I}_{ij}^P + (\underline{I}_{ij}^Q + \underline{I}_{ij}^R)/2}{2}, \min \left(\underline{F}_{ij}^P, \frac{\underline{F}_{ij}^Q + \underline{F}_{ij}^R}{2} \right) \right); \right. \\
 &\quad \left. \left(\max \left(\bar{T}_{ij}^P, \frac{\bar{T}_{ij}^Q + \bar{T}_{ij}^R}{2} \right), \frac{\bar{I}_{ij}^P + (\bar{I}_{ij}^Q + \bar{I}_{ij}^R)/2}{2}, \min \left(\bar{F}_{ij}^P, \frac{\bar{F}_{ij}^Q + \bar{F}_{ij}^R}{2} \right) \right) \right]
 \end{aligned}$$

$$\begin{aligned}
&= \left[\left(\max \left(\frac{\underline{T}_{ij}^P + \underline{T}_{ij}^Q}{2}, \frac{\underline{T}_{ij}^P + \underline{T}_{ij}^R}{2} \right), \left(\frac{(\underline{I}_{ij}^P + \underline{I}_{ij}^Q)/2 + (\underline{I}_{ij}^P + \underline{I}_{ij}^R)/2}{2} \right), \min \left(\frac{\underline{F}_{ij}^P + \underline{F}_{ij}^Q}{2}, \frac{\underline{F}_{ij}^P + \underline{F}_{ij}^R}{2} \right) \right) \right. \\
&\quad \left. \left(\max \left(\frac{\bar{T}_{ij}^P + \bar{T}_{ij}^Q}{2}, \frac{\bar{T}_{ij}^P + \bar{T}_{ij}^R}{2} \right), \left(\frac{(\bar{I}_{ij}^P + \bar{I}_{ij}^Q)/2 + (\bar{I}_{ij}^P + \bar{I}_{ij}^R)/2}{2} \right), \min \left(\frac{\bar{F}_{ij}^P + \bar{F}_{ij}^Q}{2}, \frac{\bar{F}_{ij}^P + \bar{F}_{ij}^R}{2} \right) \right) \right] \\
&= \left[\left(\left(\max \left(\underline{T}_{ij}^P, \underline{T}_{ij}^Q \right), \frac{\underline{I}_{ij}^P + \underline{I}_{ij}^Q}{2}, \min \left(\underline{F}_{ij}^P, \underline{F}_{ij}^Q \right) \right); \left(\max \left(\bar{T}_{ij}^P, \bar{T}_{ij}^Q \right), \frac{\bar{I}_{ij}^P + \bar{I}_{ij}^Q}{2}, \min \left(\bar{F}_{ij}^P, \bar{F}_{ij}^Q \right) \right) \right) \right. \\
&\quad \left. \odot_{AM} \left(\left(\max \left(\underline{T}_{ij}^P, \underline{T}_{ij}^R \right), \frac{\underline{I}_{ij}^P + \underline{I}_{ij}^R}{2}, \min \left(\underline{F}_{ij}^P, \underline{F}_{ij}^R \right) \right); \left(\max \left(\bar{T}_{ij}^P, \bar{T}_{ij}^R \right), \frac{\bar{I}_{ij}^P + \bar{I}_{ij}^R}{2}, \min \left(\bar{F}_{ij}^P, \bar{F}_{ij}^R \right) \right) \right) \right] \\
&= \left[\left(\left(\underline{T}_{ji}^P, \underline{I}_{ji}^P, \underline{F}_{ji}^P \right); (\bar{T}_{ji}^P, \bar{I}_{ji}^P, \bar{F}_{ji}^P) \right) \cup \left(\left(\underline{T}_{ji}^Q, \underline{I}_{ji}^Q, \underline{F}_{ji}^Q \right); (\bar{T}_{ji}^Q, \bar{I}_{ji}^Q, \bar{F}_{ji}^Q) \right) \right. \\
&\quad \left. \odot_{AM} \left(\left(\underline{T}_{ji}^P, \underline{I}_{ji}^P, \underline{F}_{ji}^P \right); (\bar{T}_{ji}^P, \bar{I}_{ji}^P, \bar{F}_{ji}^P) \right) \cup \left(\left(\underline{T}_{ji}^R, \underline{I}_{ji}^R, \underline{F}_{ji}^R \right); (\bar{T}_{ji}^R, \bar{I}_{ji}^R, \bar{F}_{ji}^R) \right) \right] \\
&= (P \cup Q) \odot_{AM} (P \cup R)
\end{aligned}$$

In similar manner, the results (ii) – (iv) can be proved.

4. NHSRM in Multi-attribute Decision-making using Score Function

In this section, we present the application of NHSRM in decision-making using score function. This proposed work involves evaluating alternatives based on multi-attribute decision-making problem. The approach focuses on determining the most suitable alternative by analyzing and ranking them according to multiple attributes. This is achieved by integrating the NHSRM framework with a score function to ensure a systematic and reliable decision-making process.

Let us consider a multi-attribute decision-making problem with m alternatives $(Z_1, Z_2, \dots, Z_m)$ and n attribute $(N_1, N_2, \dots, N_n)$. The goal is to evaluate the best score of m alternatives based on the performance of n attributes, using a structured decision-making approach. Each alternatives $Z_i, i=1,2,\dots,m$ combined with each attribute $N_j, j=1,2,\dots,n$, forms a relation represented by NHSRM. The corresponding to these NHSRM values are portrayed in the order $m \times n$. Further, the value matrix is calculated using one of three operators such as weighted arithmetic mean, weighted geometric mean and weighted harmonic mean. The value matrix is a real matrix that adheres to all the properties of NHSRM. The score matrix is then derived from the value matrix using the score function. This function simplifies complex data, such as those represented in neutrosophic sets, into a scalar value for straightforward comparison. By applying the score function within the NHSRM framework, the decision-making process becomes more efficient and transparent. The function aggregates the values of the attribute for each alternative, accounting for both the uncertainty and the partial truth-values

inherent in neutrosophic sets. This enables a clearer comparison of alternatives, even when the data is incomplete or imprecise.

4.1. Algorithm

- (1) Construct NHSRMs as defined in section 3.
- (2) Apply weighted arithmetic mean, weighted geometric mean and weighted harmonic mean mentioned in section 3.2.
- (3) Calculate the value matrix $V[P, Q]$ by using any one of the three operators in step 2.
- (4) Determine the score matrix with the help of the value matrix, by the definition 16.
- (5) Compute the aggregate scores matrix across criteria by using $S_{Total}(Z_m) = \sum_{i=1}^m S(Z_i)$
- (6) Find the Rank alternative from the aggregate score.

Example 10

To illustrate the working of the decision-making problem, two physician P and Q needs to select the best treatment plan (Z_1, Z_2, Z_3) for an asthma patient, considering the patient's symptoms, response to past treatments, and uncertainty in diagnosis arising from the variable factors like environmental triggers and patient adherence.

Alternatives (Treatment plan)

- (1) Z_1 : Inhaled corticosteroids
- (2) Z_2 : Combination therapy (Inhaled corticosteroids+ long-acting beta agonists)
- (3) Z_3 : Biologics for severe asthma

Attribute for decision-making

- (1) N_1 : Effectiveness – Symptom Control and Frequency of Exacerbations
- (2) N_2 : Side effects – Short term and Long term
- (3) N_3 : Cost – High and Low

The two physicians give their valuable opinion about the asthma patient in the form of NHSRM as

$$P = \begin{matrix} & \begin{matrix} N_1 & N_2 & N_3 \end{matrix} \\ \begin{matrix} Z_1 \\ Z_2 \\ Z_3 \end{matrix} & \left[\begin{array}{ccc} \langle (0.9, 0.5, 0.2); (0.8, 0.3, 0.1) \rangle & \langle (0.8, 0.4, 0.3); (0.6, 0.2, 0.1) \rangle & \langle (0.7, 0.4, 0.1); (0.6, 0.3, 0.2) \rangle \\ \langle (0.8, 0.3, 0.1); (0.7, 0.1, 0.1) \rangle & \langle (0.7, 0.2, 0.1); (0.8, 0.3, 0.1) \rangle & \langle (0.9, 0.4, 0.3); (0.6, 0.3, 0.1) \rangle \\ \langle (0.7, 0.4, 0.2); (0.8, 0.2, 0.1) \rangle & \langle (0.7, 0.3, 0.2); (0.9, 0.4, 0.2) \rangle & \langle (0.8, 0.3, 0.1); (0.7, 0.3, 0.1) \rangle \end{array} \right] \end{matrix}$$

$$Q = \begin{matrix} & \begin{matrix} N_1 & N_2 & N_3 \end{matrix} \\ \begin{matrix} Z_1 \\ Z_2 \\ Z_3 \end{matrix} & \left[\begin{array}{ccc} \langle (0.5, 0.2, 0.1); (0.4, 0.3, 0.1) \rangle & \langle (0.8, 0.6, 0.2); (0.7, 0.3, 0.2) \rangle & \langle (0.7, 0.6, 0.3); (1.0, 0.4, 0.2) \rangle \\ \langle (0.7, 0.6, 0.1); (0.8, 0.9, 0.3) \rangle & \langle (1.0, 0.4, 0.1); (0.9, 0.2, 0.3) \rangle & \langle (0.8, 0.6, 0.3); (0.7, 0.5, 0.1) \rangle \\ \langle (0.4, 0.3, 0.2); (0.7, 0.6, 0.2) \rangle & \langle (0.7, 0.5, 0.2); (0.8, 0.4, 0.1) \rangle & \langle (0.9, 0.3, 0.2); (0.5, 0.3, 0.2) \rangle \end{array} \right] \end{matrix}$$

Weighted Arithmetic Mean

Using section 3.2 (iv), we obtained the value matrix

$$V[P, Q] = \begin{matrix} & N_1 & N_2 & N_3 \\ \begin{matrix} Z_1 \\ Z_2 \\ Z_3 \end{matrix} & \left[\begin{array}{ccc} \left\langle \begin{array}{l} (0.7, 0.35, 0.1); \\ (0.6, 0.3, 0.1) \end{array} \right\rangle & \left\langle \begin{array}{l} (0.8, 0.5, 0.25); \\ (0.65, 0.25, 0.15) \end{array} \right\rangle & \left\langle \begin{array}{l} (0.7, 0.5, 0.05); \\ (0.8, 0.35, 0.2) \end{array} \right\rangle \\ \left\langle \begin{array}{l} (0.75, 0.45, 0.1); \\ (0.75, 0.5, 0.2) \end{array} \right\rangle & \left\langle \begin{array}{l} (0.85, 0.3, 0.1); \\ (0.85, 0.25, 0.2) \end{array} \right\rangle & \left\langle \begin{array}{l} (0.85, 0.5, 0.3); \\ (0.65, 0.4, 0.1) \end{array} \right\rangle \\ \left\langle \begin{array}{l} (0.55, 0.35, 0.2); \\ (0.75, 0.4, 0.15) \end{array} \right\rangle & \left\langle \begin{array}{l} (0.7, 0.4, 0.2); \\ (0.85, 0.4, 0.15) \end{array} \right\rangle & \left\langle \begin{array}{l} (0.85, 0.3, 0.15); \\ (0.6, 0.3, 0.15) \end{array} \right\rangle \end{array} \right] \end{matrix}$$

By using Definition 16, the score matrix is derived from the above value matrix

$$S(Z_m) = \begin{matrix} & N_1 & N_2 & N_3 \\ \begin{matrix} Z_1 \\ Z_2 \\ Z_3 \end{matrix} & \left[\begin{array}{ccc} 0.74 & 0.72 & 0.73 \\ 0.71 & 0.81 & 0.70 \\ 0.70 & 0.73 & 0.76 \end{array} \right] \end{matrix}$$

Now compute the aggregate score matrix by adding the entires of the score matrix

$$Total\ Score = \begin{bmatrix} 2.19 \\ 2.22 \\ 2.19 \end{bmatrix}$$

Weighted Geometric Mean

Using section 3.2 (vi), we obtained the value matrix

$$V[P, Q] = \begin{matrix} & N_1 & N_2 & N_3 \\ \begin{matrix} Z_1 \\ Z_2 \\ Z_3 \end{matrix} & \left[\begin{array}{ccc} \left\langle \begin{array}{l} (0.82, 0.56, 0.38); \\ (0.75, 0.55, 0.32) \end{array} \right\rangle & \left\langle \begin{array}{l} (0.89, 0.7, 0.49); \\ (0.81, 0.49, 0.38) \end{array} \right\rangle & \left\langle \begin{array}{l} (0.84, 0.7, 0.42); \\ (0.88, 0.59, 0.45) \end{array} \right\rangle \\ \left\langle \begin{array}{l} (0.87, 0.65, 0.32); \\ (0.87, 0.55, 0.42) \end{array} \right\rangle & \left\langle \begin{array}{l} (0.91, 0.53, 0.32); \\ (0.92, 0.49, 0.42) \end{array} \right\rangle & \left\langle \begin{array}{l} (0.92, 0.7, 0.55); \\ (0.81, 0.62, 0.32) \end{array} \right\rangle \\ \left\langle \begin{array}{l} (0.73, 0.59, 0.45); \\ (0.87, 0.59, 0.38) \end{array} \right\rangle & \left\langle \begin{array}{l} (0.84, 0.62, 0.45); \\ (0.92, 0.63, 0.38) \end{array} \right\rangle & \left\langle \begin{array}{l} (0.92, 0.55, 0.38); \\ (0.77, 0.55, 0.38) \end{array} \right\rangle \end{array} \right] \end{matrix}$$

The score matrix is constructed from the above Value matrix, as per Definition 16,

$$S(Z_m) = \begin{matrix} & N_1 & N_2 & N_3 \\ \begin{matrix} Z_1 \\ Z_2 \\ Z_3 \end{matrix} & \left[\begin{array}{ccc} 0.63 & 0.61 & 0.59 \\ 0.63 & 0.68 & 0.59 \\ 0.60 & 0.61 & 0.64 \end{array} \right] \end{matrix}$$

Now compute the aggregate score matrix by adding the entires of the score matrix

$$Total\ Score = \begin{bmatrix} 1.83 \\ 1.90 \\ 1.85 \end{bmatrix}$$

Weighted Harmonic Mean

Using section 3.2 (viii), we obtained the value matrix

$$V[P, Q] = \begin{matrix} & N_1 & N_2 & N_3 \\ \begin{matrix} Z_1 \\ Z_2 \\ Z_3 \end{matrix} & \begin{bmatrix} \left\langle (0.64, 0.29, 0.13); (0.53, 0.3, 0.1) \right\rangle \\ \left\langle (0.75, 0.4, 0.1); (0.75, 0.18, 0.15) \right\rangle \\ \left\langle (0.51, 0.35, 0.2); (0.75, 0.3, 0.13) \right\rangle \end{bmatrix} & \begin{bmatrix} \left\langle (0.8, 0.48, 0.24); (0.65, 0.25, 0.13) \right\rangle \\ \left\langle (0.82, 0.27, 0.1); (0.85, 0.24, 0.15) \right\rangle \\ \left\langle (0.7, 0.38, 0.2); (0.85, 0.4, 0.13) \right\rangle \end{bmatrix} & \begin{bmatrix} \left\langle (0.7, 0.48, 0.15); (0.75, 0.34, 0.2) \right\rangle \\ \left\langle (0.85, 0.48, 0.3); (0.65, 0.38, 0.1) \right\rangle \\ \left\langle (0.85, 0.3, 0.13); (0.58, 0.3, 0.13) \right\rangle \end{bmatrix} \end{matrix}$$

By using Definition 16, the score matrix is obtained from the above value matrix

$$S(Z_m) = \begin{matrix} & N_1 & N_2 & N_3 \\ \begin{matrix} Z_1 \\ Z_2 \\ Z_3 \end{matrix} & \begin{bmatrix} 0.73 & 0.73 & 0.71 \\ 0.78 & 0.82 & 0.71 \\ 0.71 & 0.74 & 0.76 \end{bmatrix} \end{matrix}$$

Now compute the aggregate score matrix by adding the entires of the score matrix

$$Total\ Score = \begin{bmatrix} 2.16 \\ 2.30 \\ 2.21 \end{bmatrix}$$

5. Result and Comparative Analysis

We proposed an algorithm for real world problems and result are compared with those of the algorithm using three operators (weighted arithmetic mean, weighted geometric mean and weighted harmonic mean) on NHSRM. The comparative study of the graphical representation of the ranking of the proposed algorithm for three operators are given in Figure 1. We accrued the result $Z_2 > Z_3 > Z_1$ and the maximum total score is obtained in the Combination therapy (Z_2), which is identified as the best treatment plan for the asthma patients, offering the highest effectiveness, controllable side effects and reasonable cost.

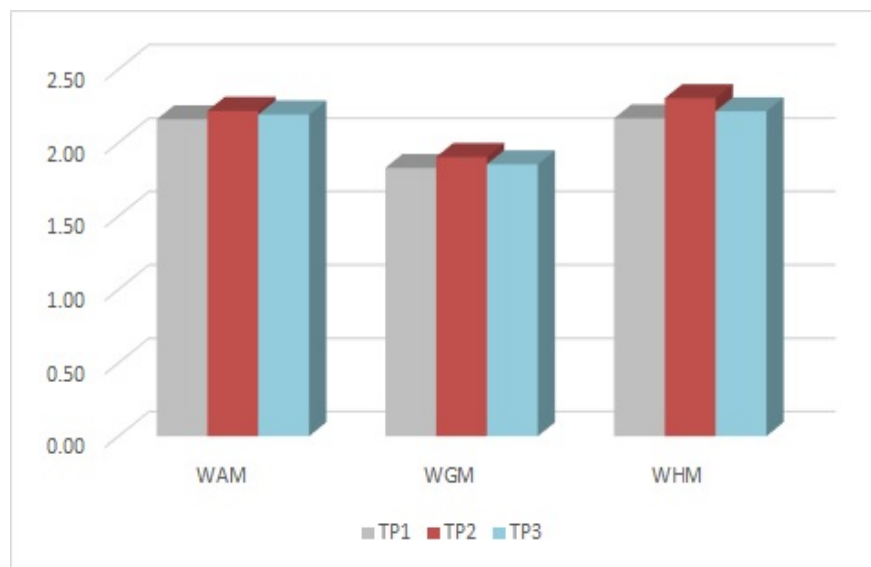


FIGURE 1. Comparison of score values between WAM, WGM & WHM

6. Conclusion and Future Work

In this study, a novel concept of NHSRM is introduced, incorporating the fundamental operators such as union, intersection, arithmetic mean, geometric mean and harmonic mean which are applied to explore various properties. Moreover, the concept of the score function is proposed to enhance the effectiveness and applicability of the proposed work. The NHSRM approach is also applied to multi-attribute decision-making problems, where the proposed score matrix is derived using three different operators - weighted arithmetic mean, weighted geometric mean, and weighted harmonic mean. Finally, the results were compared with outcomes from a numerical study on decision-making problems to validate the performance and robustness of the proposed NHSRM approach.

In the future, this framework can be extended and integrated with other advanced decision-making techniques, such as grey system theory, which addresses uncertainty in complex systems, machine learning algorithms that can provide adaptive and data-driven insights, and hybrid models that combine the strengths of multiple approaches. This integration could further enhance the accuracy, flexibility, and applicability of the framework, allowing it to tackle a wider range of real-world problems in dynamic and uncertain environments.

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J. Boobalan & E. Mathivadhana, An Approach to Neutrosophic Hyper Soft Rough Matrix and its Application

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Harmonizing Aesthetics and Sustainability through the Evaluation of Environmental Art Design in Green Architecture using the Triangular UnderNorm

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Abstract: Environmental art design has emerged as a crucial element in enhancing the aesthetic and ecological value of green architecture. This study develops a multi-criteria evaluation framework to assess the effectiveness of environmental art design within sustainable architectural spaces. By integrating both visual and environmental aspects, this framework enables a comprehensive assessment based on six carefully selected criteria and ten alternative design projects. The research contributes to bridge the gap between artistic expression and ecological responsibility in modern architectural practices. We use the RAFSI methodology to rank alternatives. The average methodology is used to compute the criteria weights. This methodology is used under the Triangular UnderNorm to deal with interval values in the decision matrix. The case study is shown to show the validation of the proposed approach. The sensitivity analysis is provided to show the stability of the ranks.

Keywords: Triangular UnderNorm; Harmonizing Aesthetics and Sustainability; Harmonizing Aesthetics and Sustainability; Green Architecture.

1. Introduction

The design of environmental art is becoming more and more important in relation to sustainable development. The need for architectural solutions that combine aesthetic appeal and ecological integrity is only increasing as urban surroundings become more complicated[1], [2]. Energy efficiency, the use of sustainable materials, and environmental harmony are all highlighted in green architecture. In this regard, environmental art is essential for raising ecological consciousness and human well-being. In green architectural projects, artistic components serve as more than just ornaments; they are means of communication that convey environmental principles and raise community awareness[3], [4]. The incorporation of ecological and artistic design aspects is frequently evaluated subjectively, despite their significance. This emphasizes the necessity of a methodical and impartial assessment framework[5], [6].

To assist architects, urban planners, and decision-makers, this study suggests a criteria-based evaluation framework for assessing the caliber of environmental art design from the standpoint of green architecture. Six essential criteria that encompass sustainability and aesthetics are adopted by the study. These are used in ten different architectural designs that are evaluated for their impact and inventiveness[7].

A comparative and scalable examination is made possible by the distinct ways that each alternative integrates environmental art into green building environments. The study supports more comprehensive and ecologically conscious urban development plans by fusing sustainable architecture with creative design[8], [9].

The most common usage of MCDM approaches is in the structuring and resolution of multi-criteria planning and decision problems. Methodologies used to address these issues are typically organized according to the steps. In the first step, needs and objectives are established to characterize the problem. The identification of decision variables is the second step. Along with the criteria that will serve as the basis for that comparison, it also covers the choice of possibilities to be contrasted[10], [11].

Step three involves choosing the best context-dependent MCDM technique, which is a needs-specific model that is only concerned with the pertinent variables and the unique situation. Data collection and tool definition (usually a test, questionnaire, rubric, etc.) are the focus of the fourth step. Data collection is necessary for step five after this tool is ready. The comparison is carried out in the last phase using the MCDM model, which yields an ordered list of options[12], [13].

This entails choosing a suitable MCDM technique, developing a data aggregation tool, and defining pertinent decision-making factors and criteria. It should be completed first because deciding on the criteria and decision variables is essential to the other activities. An efficient method for collecting data is a specially designed rubric, which makes it possible to evaluate scores objectively. Choosing the best MCDM approach guarantees that the strategy fits the issue[14].

It is first required to select the most suitable MCDM technique for Evaluating Environmental Art Design in Green Architecture, since the rubric must allow for the selected MCDM. The RAFSI technique is widely utilized in MCDM to address issues pertaining to engineering system design. By measuring its proximity to fictitious ideal solutions, this method determines the best design option. The weights of the various criteria and the value of each alternative for each criterion are the two primary kinds of data needed to implement RAFSI[15], [16].

One of the most important input parameters in multivariate decision-making is the weight of attributes. There are several methods for determining each criterion's weight. The most basic of these entails assigning subjective weights according to decision-makers' preferences while making sure the sum of the coefficients equals unity. On the other hand, there are alternatives when employing mathematical methods[17].

To calculate weighting, hybrid approaches—like the average method technique—combine mathematical principles with decision-maker inclinations. Additionally, the MCDM literature shows that algorithms from average method-based RAFSI have been widely used. Because it allows specialists to scale the value of criteria, average method typically gives a higher degree of sensitivity[18], [19].

The study chooses average method and RAFSI as appropriate MCDM techniques despite certain drawbacks, such as reliance on expert selection, because of their shown efficacy in managing complex decisions with numerous conflicting criteria in a variety of scenarios. The order of convergence to the optimal solution can be unilaterally determined by using the organized framework that average method offers for methodically weighing various factors. RAFSI enhances this by using mathematical ranking techniques to make excellent decisions and providing a simple ranking system based on how close alternatives are to optimal answers. As a result, combining these two approaches can produce more reliable and efficient results[17], [20].

To create a strong foundation for decision-making, the suggested hybridization is implemented utilizing the convergence values derived from average method as the weights in the RAFSI model. By combining many assessments and doing a thorough sensitivity analysis to examine the resilience of results against changes in criteria weights, it also reduces inherent subjectivity.

2. Definitions

The uncertain Set was extended by Smarandache [21] in 2007 to uncertain OverSet (when some component is > 1), since he observed that, for example, an employee working overtime deserves a degree of membership > 1 , with respect to an employee that only works regular full-time and whose degree of membership = 1; and to uncertain UnderSet (when some neutrosophic component is < 0), since, for example, an employee making more damage than benefit to his company deserves a degree of membership < 0 , with respect to an employee that produces benefit to the company and has the degree of membership > 0 ; and to and to uncertain OffSet (when some neutrosophic components are off the interval $[0, 1]$, i.e. some neutrosophic component > 1 and some neutrosophic component < 0). Then, similarly, the uncertain Logic/Measure/Probability/Statistics etc. were extended to respectively uncertain Over-/Under-/Off- Logic / Measure / Probability / Statistics etc [22].

This section shows the definitions of triangular UnderNorm [23].

The triangular UnderNorm include in interval $[\mu, 1]$; $\mu < 0$

$$T_{\text{UnderNorm}}: [\mu, 1] \times [\mu, 1] \rightarrow [\mu, 1] \text{ and } \mu > 0 \quad (1)$$

$$\text{For any } (E, F) \in [\mu, 1] \times [\mu, 1] \quad (2)$$

$$T_{\text{UnderNorm}}(E, F) = T_{\text{UnderNorm}}(F, E) \text{ commutativity} \quad (3)$$

$$T_{\text{UnderNorm}}(E, T_{\text{UnderNorm}}(F, D)) = T_{\text{UnderNorm}}(T_{\text{UnderNorm}}(E, F), D) \text{ associativity} \quad (4)$$

if $F \leq D$ then $T_{underNorm}(E, F) \leq T_{underNorm}(E, Z)$ monotonicity (5)

$T_{underNorm}(E, 1) = E$ or the neutral element is 1. (6)

$T_{underNorm}(E, F) = \min(E, F)$ (7)

We show the steps of the RAFSI to rank the options. Create the decision matrix between the criteria and alternatives. Then we combine the decision matrix using the $T_{underNorm}$.

Compute the criteria weights using the average method.

Compute the ideal $(D_j)^A$ and non-ideal $(D_j)^C$ numbers.

Compute the interval of each criterion. $D_1 = 0.1, D_2 = 0.9$.

$$H_m(y_{ij}) = \frac{D_{2c} - D_1}{(D_j)^A - (D_j)^C} y_{ij} + \frac{(D_j)^A D_1 - D_{2c} (D_j)^C}{(D_j)^A - (D_j)^C} \quad (7)$$

Compute the harmonic and arithmetic means

$$Q = \frac{2}{\frac{1}{D_1} + \frac{1}{D_{2c}}} \quad (8)$$

$$Q = \frac{D_1 + D_{2c}}{2} \quad (9)$$

Compute the normalized the decision matrix

$$U_{ij} = \frac{y_{ij}}{2Q} \quad (10)$$

$$U_{ij} = \frac{Q}{2y_{ij}} \quad (11)$$

Compute the criteria function

$$K(D_i) = \sum_{j=1}^n w_j U_{ij} \quad (12)$$

Rank the alternatives.

3. Case Study

This section shows the implementation of the proposed approach. We use six criteria and ten alternatives such as:

Visual Integration (C1): The degree to which art complements the building's architectural form and spatial arrangement.

Material Sustainability (C2): Use of recycled, locally sourced, or renewable materials in artistic features.

Energy Interaction (C3): Functional contributions of art design to energy efficiency (e.g., shading, daylighting).

Environmental Messaging (C4): The effectiveness of conveying ecological values through artistic representation.

User Engagement (C5): The extent to which users interact with and are influenced by the art in their behavior or perception.

Durability and Maintenance (C6): Longevity of the art installations and the ease and cost of maintenance over time.

A1: Solar Sculpture Plaza, A2: Eco-Wall Installation in Urban Park, A3: Recycled Glass Facade Mural, A4: Living Roof Mosaic, A5: Wind-Activated Light Totems, A6: Biophilic Interactive Tunnel, A7: Bamboo Art Walkway, A8: Rainwater Reflection Garden, A9: Earth-Tone Relief Panels, A10: Community-Painted Recycled Canopy.

Table 1 shows the decision matrix between the criteria and alternatives. Three experts created the decision matrix. We use the $T_{underNorm}$ to combine the decision matrix. We use the average method to compute the criteria weights.

C1= 0.1719: Visual Impact and Aesthetic Value — This criterion holds significant weight, indicating that the visual and aesthetic integration of the design is crucial. It suggests that how the design appears to the public or users is a key factor in the evaluation process.

C2= 0.1656: Sustainability of Materials — Slightly less than C1, this weight shows a strong emphasis on the ecological sustainability and environmental friendliness of the materials used in the design.

C3= 0.1497: Functionality and Utility — This weight reflects moderate importance. It assesses whether the design elements serve a functional role beyond visual appeal (e.g., providing shade, ventilation, or energy generation).

C4= 0.1592: Symbolism and Conceptual Depth — This value suggests moderate to high emphasis on the narrative or symbolic message that the art or design conveys, especially in terms of cultural or environmental awareness.

C5= 0.1752: User Engagement and Interaction — With one of the highest weights, this indicates a major priority on how users or viewers engage with and experience the design. Interactivity, accessibility, and emotional impact are key factors here.

C6= 0.1783: Durability and Maintenance — The highest weighted criterion, showing strong concern for the long-term feasibility of the design. This includes resistance to weathering, ease of upkeep, and lifecycle cost-effectiveness.

The distribution of weights suggests a well-balanced but slightly utility-leaning evaluation model, with Durability (C6) and User Engagement (C5) taking top priority, followed closely by Visual Appeal (C1) and Sustainability (C2). The smallest weight still reflects considerable importance, emphasizing a comprehensive and holistic approach.

Table 1. The decision matrix.

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
A ₁	[-0.2,1]	[-0.3,1]	[-0.4,1]	[-0.5,1]	[-0.6,1]	[-0.9,1]
A ₂	[-0.9,1]	[-0.1,1]	[-0.2,1]	[-0.3,1]	[-0.4,1]	[-0.5,1]
A ₃	[-0.6,1]	[-0.1,1]	[-0.4,1]	[-0.5,1]	[-0.6,1]	[-0.6,1]
A ₄	[-0.5,1]	[-0.2,1]	[-0.3,1]	[-0.2,1]	[-0.1,1]	[-0.2,1]
A ₅	[-0.4,1]	[-0.1,1]	[-0.5,1]	[-0.6,1]	[-0.9,1]	[-0.6,1]
A ₆	[-0.3,1]	[-0.5,1]	[-0.4,1]	[-0.2,1]	[-0.9,1]	[-0.2,1]
A ₇	[-0.9,1]	[-0.4,1]	[-0.5,1]	[-0.1,1]	[-0.3,1]	[-0.3,1]
A ₈	[-0.1,1]	[-0.3,1]	[-0.1,1]	[-0.5,1]	[-0.4,1]	[-0.4,1]
A ₈	[-0.2,1]	[-0.9,1]	[-0.9,1]	[-0.9,1]	[-0.5,1]	[-0.5,1]
A ₁₀	[-0.6,1]	[-0.5,1]	[-0.4,1]	[-0.3,1]	[-0.9,1]	[-0.6,1]
	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
A ₁	[-0.1,1]	[-0.3,1]	[-0.4,1]	[-0.5,1]	[-0.6,1]	[-0.2,1]
A ₂	[-0.2,1]	[-0.6,1]	[-0.2,1]	[-0.3,1]	[-0.4,1]	[-0.5,1]
A ₃	[-0.3,1]	[-0.6,1]	[-0.4,1]	[-0.5,1]	[-0.6,1]	[-0.1,1]
A ₄	[-0.4,1]	[-0.2,1]	[-0.3,1]	[-0.9,1]	[-0.1,1]	[-0.9,1]
A ₅	[-0.5,1]	[-0.6,1]	[-0.5,1]	[-0.1,1]	[-0.2,1]	[-0.3,1]
A ₆	[-0.3,1]	[-0.9,1]	[-0.1,1]	[-0.2,1]	[-0.3,1]	[-0.4,1]
A ₇	[-0.9,1]	[-0.3,1]	[-0.2,1]	[-0.3,1]	[-0.4,1]	[-0.5,1]
A ₈	[-0.1,1]	[-0.4,1]	[-0.3,1]	[-0.4,1]	[-0.5,1]	[-0.4,1]
A ₈	[-0.2,1]	[-0.5,1]	[-0.4,1]	[-0.5,1]	[-0.5,1]	[-0.5,1]
A ₁₀	[-0.6,1]	[-0.5,1]	[-0.5,1]	[-0.3,1]	[-0.2,1]	[-0.6,1]
	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
A ₁	[-0.9,1]	[-0.3,1]	[-0.4,1]	[-0.5,1]	[-0.5,1]	[-0.2,1]
A ₂	[-0.9,1]	[-0.5,1]	[-0.2,1]	[-0.3,1]	[-0.4,1]	[-0.5,1]
A ₃	[-0.1,1]	[-0.4,1]	[-0.4,1]	[-0.5,1]	[-0.3,1]	[-0.4,1]
A ₄	[-0.5,1]	[-0.3,1]	[-0.5,1]	[-0.2,1]	[-0.2,1]	[-0.3,1]
A ₅	[-0.4,1]	[-0.9,1]	[-0.4,1]	[-0.5,1]	[-0.5,1]	[-0.2,1]
A ₆	[-0.3,1]	[-0.5,1]	[-0.3,1]	[-0.4,1]	[-0.4,1]	[-0.5,1]
A ₇	[-0.2,1]	[-0.4,1]	[-0.2,1]	[-0.3,1]	[-0.3,1]	[-0.4,1]
A ₈	[-0.6,1]	[-0.3,1]	[-0.6,1]	[-0.9,1]	[-0.9,1]	[-0.3,1]
A ₈	[-0.2,1]	[-0.2,1]	[-0.2,1]	[-0.2,1]	[-0.5,1]	[-0.2,1]
A ₁₀	[-0.1,1]	[-0.5,1]	[-0.4,1]	[-0.3,1]	[-0.2,1]	[-0.6,1]

We compute the ideal $(D_j)^A$ and non-ideal $(D_j)^C$ numbers.

We compute the interval of each criterion using eq. (7) as shown in Fig 1.

We compute the harmonic and arithmetic means using eq. (8) as shown in Fig 2.

We compute the normalized decision matrix using eq. (10) as shown in Fig 3.

We compute the criteria function using eq. (12) as shown in Fig 4.

Rank the alternatives as shown in Fig 5.

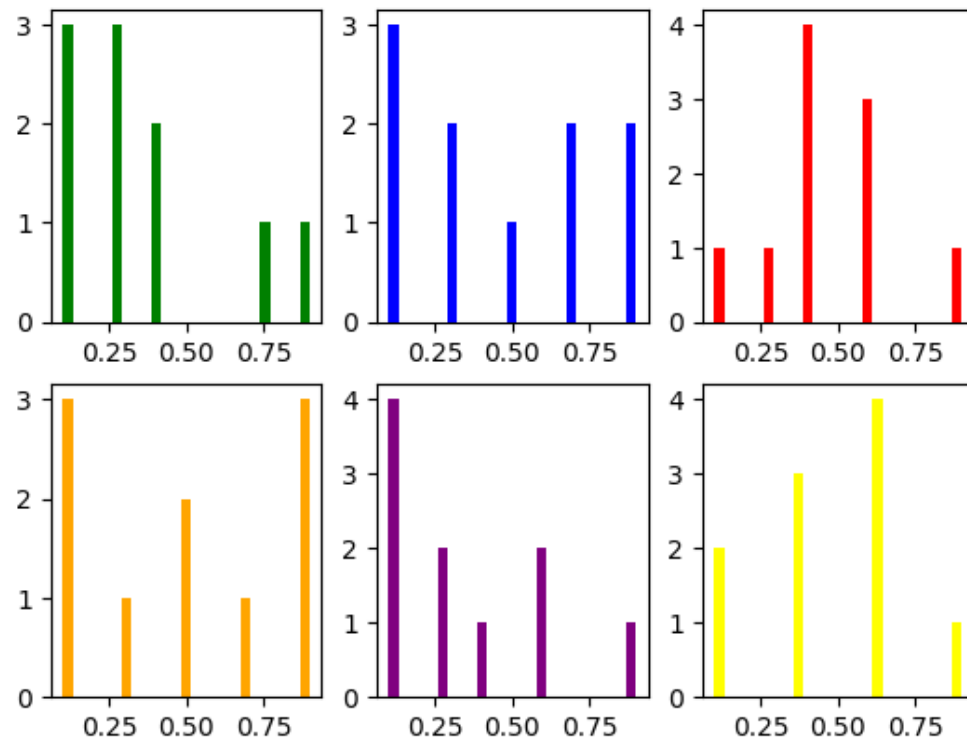


Fig 1. The interval of each criterion.

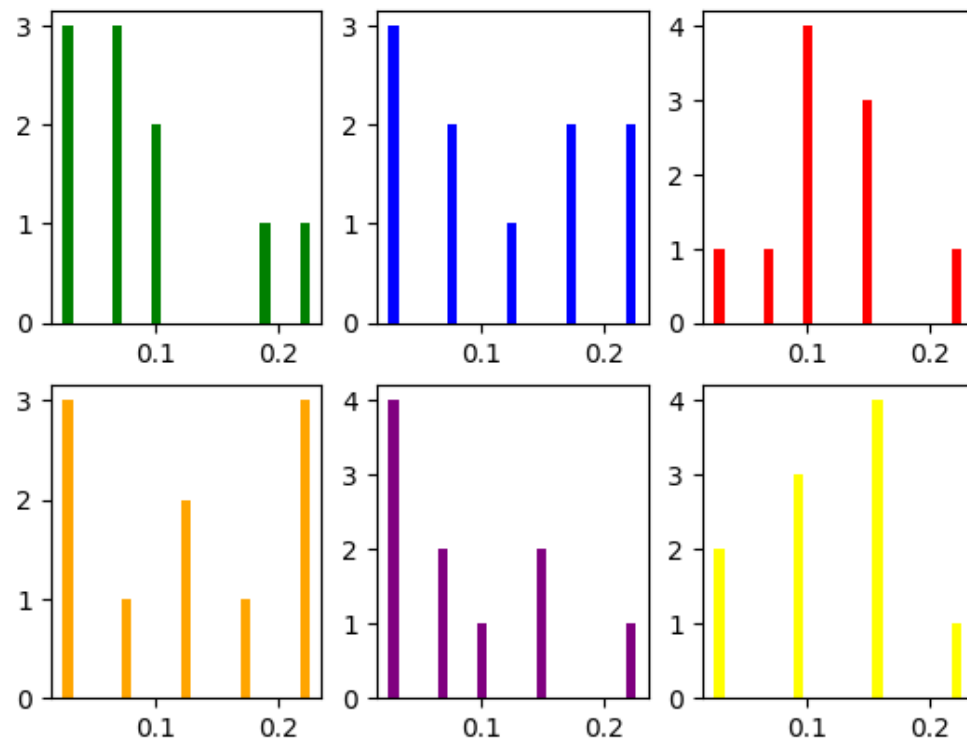


Fig 2. The harmonic and arithmetic means.

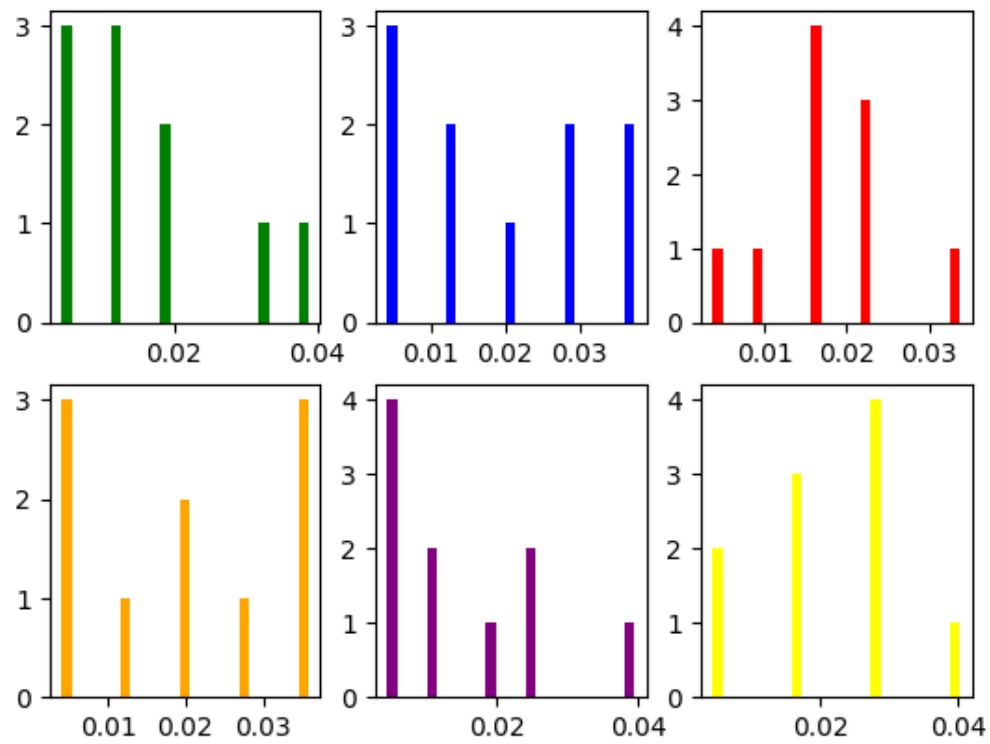


Fig 3. The normalized decision matrix.

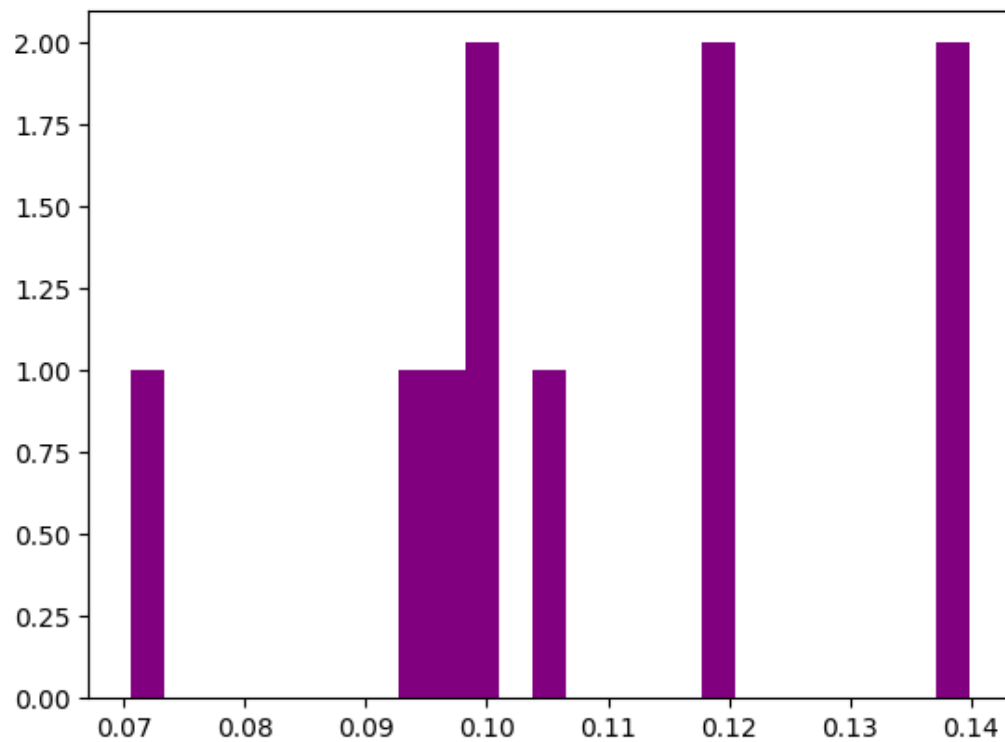


Fig 4. The criteria function.

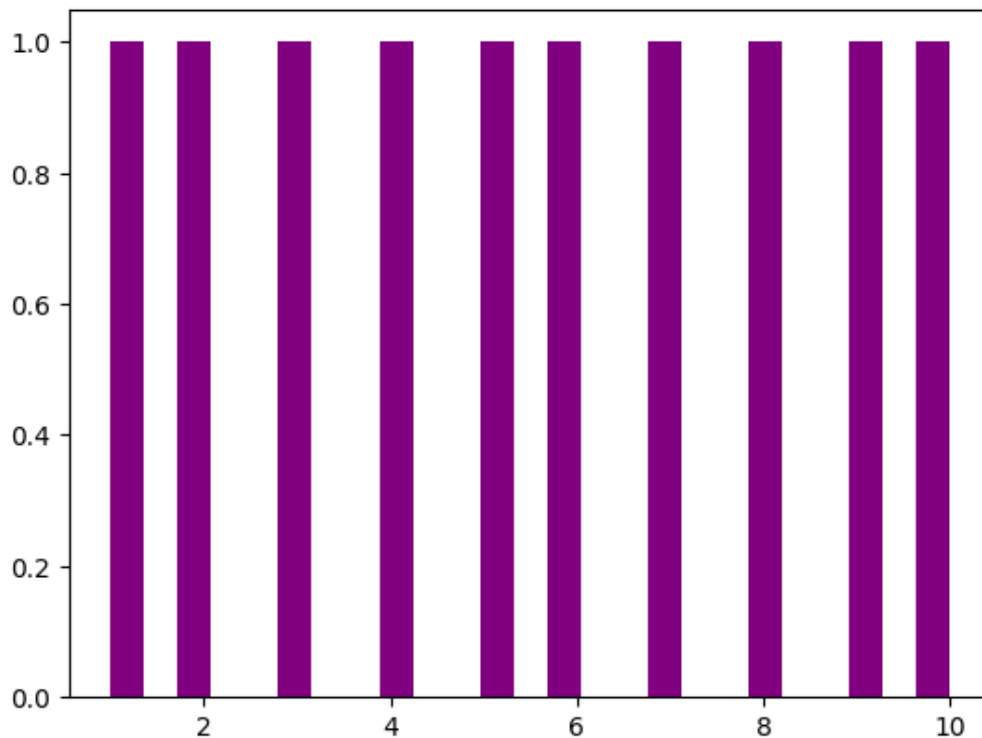


Fig 5. The ranks of alternatives.

4. Sensitivity Analysis

This section shows the results of sensitivity analysis. We change the criteria weights by 20% of each criterion as shown in Fig 6. Then we apply the proposed approach to show the ranks of alternatives. Fig 7 shows the criteria function of each alternative. Fig 8 shows the ranks of alternatives.

Fig 6 shows criteria weights across six different cases (Case 1 to Case 6) for a decision-making or evaluation model (e.g., MCDM framework for evaluating Environmental Art Design in Green Architecture. Values: Represent the relative weight of each criterion in each case. These weights usually sum up to 1 in each column (or nearly so), indicating a normalized priority distribution.

C1: Shows very low weight (≈ 0.04) in Case 1, but very high and stable weight (≈ 0.20) in Cases 2 to 6. This implies C1 was nearly unimportant in Case 1 but becomes a top priority in the other scenarios.

C2: Starts very strong in Case 1 (~ 0.19), drops dramatically in Case 2 (~ 0.038), then returns to high importance across Cases 3 to 6.

C3: Maintains consistent medium-high values (~ 0.17) in most cases, except in Case 3 where its importance is minimal (≈ 0.034). This suggests Case 3 emphasizes other factors.

C4: Like C3, it is stable in most cases (~ 0.18), but in Case 4 its weight drops significantly to ~ 0.036 .

C5: Holds the highest weight in most cases, especially Case 6 (~ 0.204), but is given very little weight in Case 5 (~ 0.0407), which might indicate a different objective or constraint in that case.

C6: Very important across most cases (~ 0.20) but sharply deprioritized in Case 6 (≈ 0.0416), possibly due to competing priorities.

Each case has one criterion heavily deprioritized (very low value ~ 0.03 – 0.04), possibly to simulate different decision environments or stakeholder perspectives.

C5 and C6 are among the most dominant in most scenarios, suggesting structural or practical concerns (e.g., durability or usability) may weigh heavily in this model.

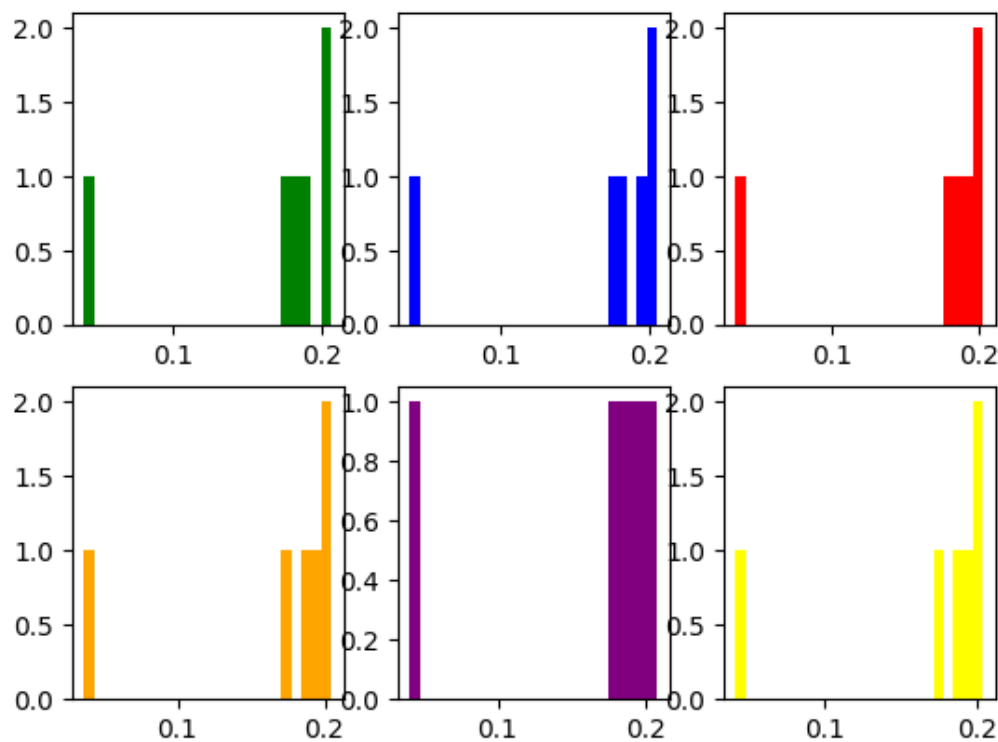


Fig 6. The different criteria weights.

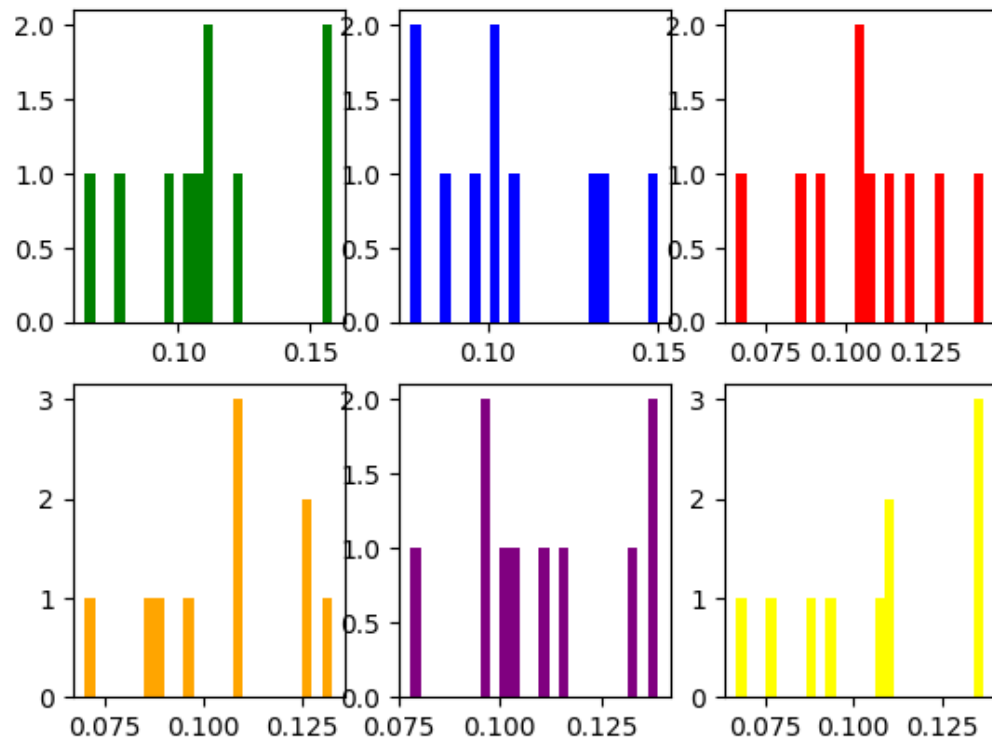


Fig 7. The different criteria function.

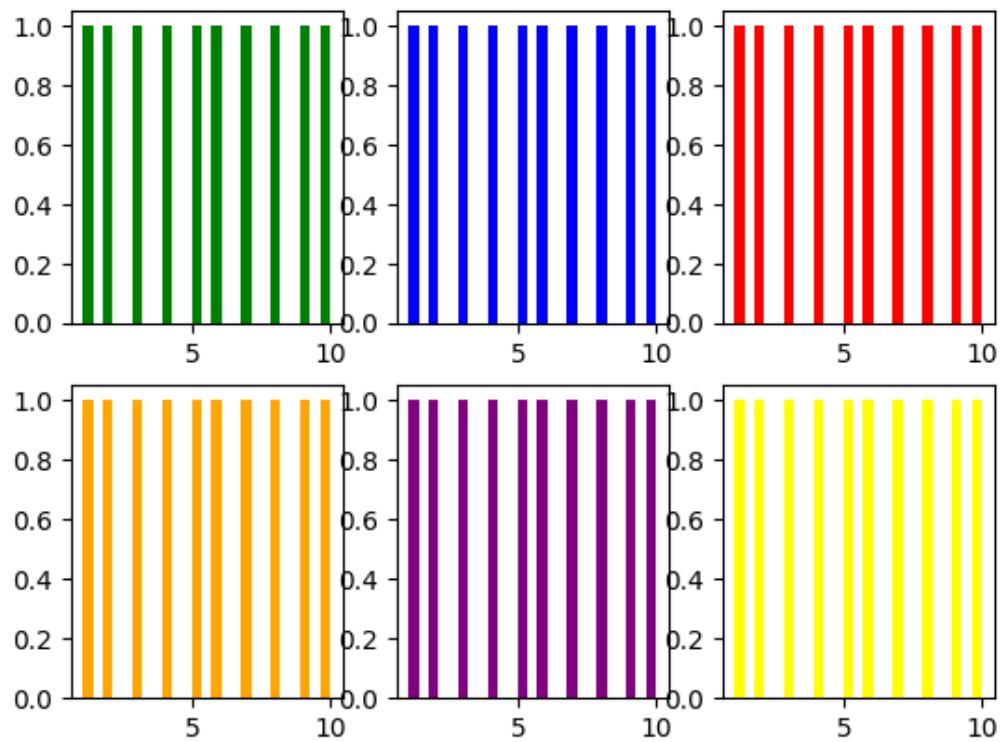


Fig 8. The different ranks of alternatives.

A1: Mixed rankings, starting low in Case 1 (Rank 7), but improves in Cases 2 to 4. Slight decline in Cases 5 and 6 (Ranks 5 and 6). Suggests inconsistent performance.

A2: Consistently ranked worst (Rank 10) in all cases except Case 3 (Rank 9). Indicates it consistently underperforms across all evaluation scenarios.

A3: Maintains strong ranks (Ranks 2 to 4) in all cases, especially Case 3 (Rank 2). Demonstrates steady and strong performance.

A4: Highly variable. Performs better in Case 5 (Rank 4), but drops sharply in Case 4 (Rank 10). Indicates unstable or scenario-dependent performance.

A5: Top performer in all cases (Rank 1) — consistently ranked first. This indicates exceptional performance and universal preference across all scenarios.

A6: Moderate performer. Ranges from Rank 5 to Rank 8. Not a leader, but relatively stable in the middle range.

A7: Poor performer. Typically ranked 9 or 10, except Case 4 (Rank 8). Consistently among the bottom-ranked.

A8: Fairly strong and improving in Case 6 (Rank 2). Usually ranked between 3 and 6, showing moderate to strong performance.

A9: Starts strong (Rank 2), with mid-level ranks elsewhere. Notably drops in Case 2 (Rank 7). A strong contender, though performance varies.

A10: Mid-range performer, ranked between 2 and 7. Notably better in Case 4 (Rank 2). Shows balanced but not exceptional performance.

Case 1: A5 and A9 lead (Ranks 1 & 2), while A2 and A7 perform worst.

Case 2: A5, A1, and A8 lead. A2 and A9 drop.

Case 3: A5 and A3 are top. A4 and A7 fall.

Case 4: A10 and A3 shine. A4 performs worst.

Case 5: A5 remains best, with A9, A3, and A4 close. A2 and A7 continue to struggle.

Case 6: A5 and A8 lead. A2 and A4 are weak.

5. Conclusions

This research underscores the significance of evaluating environmental art design through a structured framework that values both aesthetics and sustainability. The proposed criteria allow for an objective analysis of how well artistic features are integrated into green architectural

practices. Through the comparative analysis of ten diverse alternatives, this study offers insights into the current trends and future directions of environmentally conscious design. The findings aim to inform architects, planners, and policymakers on best practices for harmonizing artistic creativity with ecological stewardship. We used the MCDM approach to show the criteria weights and ranking the alternatives using the RAFSI method. Six criteria and ten alternatives are used to show the validation of the proposed approach. The results of sensitivity analysis show the ranks of alternatives are stable under six different cases.

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Evaluation of Intrusion Detection Systems in Cyber Security using Fuzzy OffLogic and MCDM Approach

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Abstract

Modern cybersecurity infrastructures rely heavily on Intrusion Detection Systems (IDS) to detect and prevent malicious activities and unauthorized access. Given the growing complexity of network topologies and the rising frequency of cyber threats, evaluating IDS solutions requires a systematic and unbiased approach. In this study, thirteen widely used IDS models are assessed using a multi-criteria evaluation framework across four key dimensions: detection accuracy, resource efficiency, scalability, and false positive rate. The goal is to support informed, data-driven decision-making for stakeholders such as policymakers, IT administrators, and security analysts when selecting an appropriate IDS. The VIKOR method is employed to rank the IDS alternatives based on the assigned weights, while Fuzzy OffLogic is applied to integrate expert assessments expressed as intervals. The results reveal that modern AI-based IDS models demonstrate strong performance in scalability and resource utilization, and they outperform traditional systems in adaptability and detection accuracy.

Keywords: Fuzzy OffLogic; Security; Cyber-Security; Attacks; Intrusion Detection System; MCDM Approach.

1. Introduction

One popular cyber security technique is intrusion detection, which looks for hostile activity in host and/or network environments. Malicious activity detection allows for prompt action to, for instance, halt an ongoing attack. Numerous intrusion detection systems (IDS) have been devised and developed by the industrial and research communities due to the significance of intrusion detection[1], [2]. The creation of IDS evaluation approaches, techniques, and tools has emerged as a crucial area of research due to the growing diversity and complexity of IDSs. IDS assessment has several advantages. To implement an IDS that performs best in a particular environment and lowers the risks of a security breach, one could, for example, compare many IDS based on how well they identify attacks. Additionally, by altering its configuration parameters and examining their impact through assessment tests, one can fine-tune an IDS that has already been implemented[3], [4].

A wide range of tasks pertaining to IDS evaluation have been carried out by the industrial and scientific groups. A structured classification is required to enhance the general comprehension of the subject and to give an overview of the present status of the field, given the substantial amount of existing theoretical and practical work pertaining to IDS evaluation. Finding and comparing the benefits and drawbacks of various IDS evaluation techniques and procedures would be made easier with the help of such an overview. Additionally, it would assist in determining the best practices and requirements for assessing present and future IDS[5], [6].

The workload component of the design space for IDS evaluation. We require both harmful and benign workloads to assess an IDS. These can be used independently to gauge an IDS's capabilities (for example, as workloads that are pure malicious or pure benign). Workloads that only contain attacks are known as pure malicious workloads, while workloads that do not contain any attacks are known as pure benign workloads. As an alternative, one can expose an IDS under test to realistic threats by using mixed workloads, which are workloads that combine pure benign and pure malicious workloads[7], [8].

Workloads for IDS assessment often take the form of an executable for live IDS testing or a trace form, which is created by recording a live workload execution for subsequent playback. Tools made to analyze trace files are used to execute the trace replay; a popular combination is to utilize the utility TCP dump to record network traces for TCP replay to replay later.

The main benefit of using executable workloads is that they closely mimic actual workloads that are monitored by an intrusion detection system while it is in operation. However, setting up a malicious workload in executable form necessitates a particular victim environment, which can be costly and time-consuming. On the other hand, replaying workload does not always necessitate such an environment.

Furthermore, to guarantee the statistical significance of the observed system behavior, several evaluation runs are usually needed. Replicating assessment studies with executable malicious workloads is typically difficult, though, because the execution of assaults may cause the victim environment to crash or become unstable. Furthermore, the procedure to return the environment to its pre-attack form could take a long time[9].

2. Literature Review

If a system possesses the qualities of confidentiality, integrity, and availability of its data and services, it is deemed secure. Attacks are intentional attempts to compromise the security features listed above.

Kruegel et al. [10] categorize security measures that uphold the qualities using an attack-centric approach. The first class includes mechanisms that keep attackers from getting to the target and interacting with it, like access control; the second class includes mechanisms that change the data stored there so that an attacker cannot use it, like data encryption; and the third class includes mechanisms that identify ongoing attacks based on the presumption that an attacker can get to

the target and interact with it. Intrusion detection is a security mechanism that falls under this class.

In terms of computer security regulations, proper usage policies, or typical safety procedures, detection of breaches is defined as "the process for tracking occurrences happening on a network or computer environment and analyzing them for signs for potential incidents, which are infractions or immediate dangers of violation." According to the definition given above, an IDS is software that automates the detection of an attack process [11].

One area of active research is the assessment of computer intrusion detection systems, or intrusion detection systems for short. Milenkoski et al. [12] examined and organized standard procedures in the field of evaluating such systems. To do this, they established a design space that is divided into three sections: measuring methods, metrics, and workload. By reviewing the evaluation techniques and approaches associated with each area of the design space, they then gave a summary of the standard procedures in the assessment of intrusion detection systems. Lastly, they have gone over unresolved problems and difficulties with an emphasis on assessment techniques for innovative intrusion detection systems.

2.1 multi-criteria decision-making

Multi-criteria decision-making (MCDM) is a sophisticated process that uses straightforward algorithms to provide suitable conclusions with the right ranking. MCDM methods work effectively for managing massive volumes of data as well. Numerous algorithms, including VIKOR, and the average method, modify the fundamentals of MCDM. The selection of the best alternative based on VIKOR, and so on are examples of applications that apply MCDM approaches in the context of IDS[13].

2.2 Intrusion Detection Systems (IDS)

Information systems protection is greatly aided by intrusion detection systems (IDSs). Numerous human or humanoid assailants frequently target cyber-physical systems. The obnoxious, dangerous network traffic must be protected from the vital parts and resources of the network architecture. IDS plays a very sensitive function in this situation, and its ability to handle various attack types from many sources is desperately needed[13], [14].

IDS provides cyber protection against a range of threats and weaknesses found in Internet applications. Any networked environment is full of threats. IDS protects networks by keeping an eye on and blocking the entry of anonymous or questionable network traffic. These days, a lot of IDS models are created using machine learning techniques. Generally speaking, anomaly detection and signature-based prediction are the two approaches used to identify attack trends[15], [16].

Since the majority of IDSs are used in heterogeneous, distributed environments, they must constantly process enormous amounts of data with redundant, superfluous features. IDSs

occasionally handle data with extraneous aspects as well. The model's training time will undoubtedly be slowed down in this scenario, resulting in decreased accuracy. The performance of the classifier model will deteriorate as a result. Another important consideration is that a model with fewer cum meaningful characteristics is less likely to make mistakes and thus require less training time[17], [18].

It is challenging to detect anomalies using network intrusion detection systems (NIDS) since vulnerable packets attempt to avoid the network and new attacks are created by different automated attack-triggering mechanisms.

2.3 Real-World Deployment Challenges of IDS Models

Although this study focuses on evaluating IDS performance using structured decision-making techniques, actual deployment of these systems in real-world environments comes with challenges that are often more complex than test conditions can reveal.

Many IDS models require specific tuning and configuration based on the network in which they are deployed. In practice, setting the right parameters for detection thresholds, processing capacities, and alert handling can take significant time and effort. In cloud-based or hybrid environments, IDS solutions also face the challenge of integration with virtual infrastructure, where data packets may not follow consistent paths, and encryption can obscure inspection.

Another challenge is related to maintenance. Some models, especially AI-based ones, need continuous retraining as attack patterns evolve. Without regular updates, these systems may become outdated quickly, losing their detection capabilities. This requires access to updated datasets and adequate computational resources, which may not always be available.

Moreover, system administrators often struggle with balancing detection sensitivity and false alarm rates. If the IDS is too sensitive, it may flood the team with alerts. If it's too lenient, real threats might be missed. These trade-offs must be handled in living environments under pressure, which adds another layer of difficulty compared to controlled simulations.

2.4 Comparative Summary of Classic vs. AI-based IDS

The evaluation in this study includes both traditional IDS like Snort and Zeek and newer AI-driven systems such as QRadar and Darktrace. While the numerical analysis shows differences in detection accuracy, resource efficiency, and scalability, a broader view helps explain why AI-based systems often rank higher.

Classic IDS models generally rely on pre-defined rules and signature databases. Their strength lies in predictability and low resource usage. They are open-source and flexible, often used by organizations with tight budgets or high customization needs. However, these models may struggle when faced with unknown or fast-changing threats.

In contrast, AI-based IDS solutions adapt more quickly to new patterns. They analyze behavior rather than signatures, which helps detect previously unseen attacks. These systems also scale more easily in distributed environments and can process high volumes of data in real time.

However, AI-based models are not without drawbacks. They typically require more system resources and a complex initial setup. They may also act like “black boxes,” making it harder for teams to understand how decisions are made. Despite this, the overall analysis from this study suggests that their performance advantages in scalability and accuracy often outweigh these limitations.

2.5. Evaluation of Trade-Offs in Multi-Criteria Decision-Making

The study uses four criteria to evaluate IDS: detection accuracy, resource efficiency, scalability, and false positive rate. Each of these metrics reflects a different aspect of system performance, but choosing the right balance between them is not always straightforward.

For instance, systems with high detection accuracy may also generate more false positives. This can lead to alert fatigue, where security teams begin ignoring warnings. On the other hand, reducing the false positive rate too much might result in more attacks slipping through undetected. Similarly, some models perform well in terms of accuracy and low false positives but consume significant computational resources, making them unsuitable for low-power environments like IoT or edge computing.

The VIKOR method, supported by Fuzzy OffLogic, helps manage these trade-offs by assigning weights to each criterion. However, it's important to recognize that these weights reflect expert opinions, which may differ between organizations based on their specific needs. A system chosen by one company as “best” may not be the top choice in another with different operational priorities.

This trade-off analysis highlights the need for flexible evaluation models that can be adapted to various contexts, rather than promoting one-size-fits-all solutions.

3. Fuzzy OffLogic

In 2007, Smarandache extended the concept of the uncertain set to include three generalized forms: the uncertain OverSet, the uncertain UnderSet, and the uncertain OffSet [19], [20]. The OverSet accounts for cases in which some components have values greater than 1. For instance, an employee who works overtime may justifiably receive a degree of membership greater than 1, compared to a full-time employee whose degree is equal to 1. The UnderSet, on the other hand, applies when a component has a value less than 0. As an example, an employee who causes more harm than benefit to their organization may be assigned a membership degree less than zero, in contrast to a productive employee with a positive membership value.

The OffSet is a more general case in which certain components lie outside the standard interval $[0,1][0, 1][0,1]$, meaning some values are greater than 1 while others are less than 0. This allows

the model to represent more complex or extreme situations that cannot be captured using traditional fuzzy logic.

Smarandache also extended other uncertain frameworks—such as logic, measure, probability, and statistics—into their corresponding Over-, Under-, and Off- variants. By "uncertain," he refers to all generalizations of fuzzy logic, including intuitionistic fuzzy sets, neutrosophic logic, spherical fuzzy models, and plithogenic sets [21], [22].

To illustrate this concept in a practical context, consider a student who earns an A+ grade. The Fuzzy OffTruth value associated with their performance may fall in a range such as $[1.0, 1.1][1.0, 1.1][1.0, 1.1]$, even though the overall evaluation scale spans from $[-1.2, 1.1][1.1, 1.1][1.1, 1.1]$ [23]. This reflects performance that slightly exceeds the conventional maximum and demonstrates the flexibility of OffLogic in capturing overachievement.

3.1 Illustrative Examples of Fuzzy OffLogic Calculations

Consider the following examples that demonstrate how to compute the OffLogic value when evaluating outcomes that may extend beyond the conventional $[0, 1]$ interval.

Example 1

Suppose a student receives a result represented by the interval $[1.0, 1.1]$, within a full range that spans from -1.2 to 1.1 . The Fuzzy OffLogic value can be calculated using the following expression:

$$F_{offlogic} = \frac{1.1 - 1.0}{1.1 - (-1.2)} = \frac{0.1}{2.3} = 0.043 \quad (1)$$

This small value indicates a slight overperformance relative to the upper bound of the standard fuzzy scale.

Example 2

In another scenario, if the outcome interval lies between $[9.0, 1.1]$ to the test out of $[-1.2, 1.1]$.

$$F_{offlogic} = \frac{1.1 - 9.0}{1.1 - (-1.2)} = \frac{0.2}{2.3} = 0.086 \quad (2)$$

This significantly negative value reflects a performance that extends far beyond the typical bounds and is interpreted as an extreme deviation from the expected range.

Example 3

Similarly, if another individual scores between $[8.0, 1.1]$ to the test out of $[-1.2, 1.1]$.

$$F_{offlogic} = \frac{1.1 - 8.0}{1.1 - (-1.2)} = \frac{0.3}{2.3} = 0.130 \quad (5)$$

This also indicates a substantial departure from the neutral range, with the score falling well outside conventional expectations.

Example 4

if another individual scores between [7.0,1.1] to the test out of [-1.2,1.1].

$$F_{Offlogic} = \frac{1.1 - 7.0}{1.1 - (-1.2)} = \frac{0.4}{2.3} = 0.1739 \quad (6)$$

Example 5

if another individual scores between [6.0,1.1] to the test out of [-1.2,1.1].

$$F_{Offlogic} = \frac{1.1 - 6.0}{1.1 - (-1.2)} = \frac{0.5}{2.3} = 0.217 \quad (7)$$

Example 6

if another individual scores between [5.0,1.1] to the test out of [-1.2,1.1].

$$F_{Offlogic} = \frac{1.1 - 5.0}{1.1 - (-1.2)} = \frac{0.6}{2.3} = 0.260 \quad (8)$$

Example 7

if another individual scores between [4.0,1.1] to the test out of [-1.2,1.1].

$$F_{Offlogic} = \frac{1.1 - 4.0}{1.1 - (-1.2)} = \frac{0.7}{2.3} = 0.304 \quad (9)$$

Example 8

if another individual scores between [3.0,1.1] to the test out of [-1.2,1.1].

$$F_{Offlogic} = \frac{1.1 - 3.0}{1.1 - (-1.2)} = \frac{0.8}{2.3} = 0.347 \quad (10)$$

Example 9

if another individual scores between [2.0,1.1] to the test out of [-1.2,1.1].

$$F_{Offlogic} = \frac{1.1 - 2.0}{1.1 - (-1.2)} = \frac{0.9}{2.3} = 0.391 \quad (11)$$

Example 10

if another individual scores between [0.1,1.1] to the test out of [-1.2,1.1].

$$F_{Offlogic} = \frac{1.1 - 0.1}{1.1 - (-1.2)} = \frac{1}{2.3} = 0.434 \quad (12)$$

These examples demonstrate how Fuzzy OffLogic flexibly handles values outside the standard fuzzy range. It allows for more expressive evaluations of performance, especially when considering overachievement or underperformance beyond the limits of traditional logic.

3.2 Enhancing Fuzzy OffLogic Interpretability

Fuzzy OffLogic was used in this study to convert expert opinions, expressed as intervals, into crisp values for ranking. While this approach improves flexibility and allows for uncertainty, understanding how these values are derived can be challenging for users unfamiliar with the logic behind it.

The core idea of OffLogic is that decision scores can extend beyond the traditional $[0,1]$ range allowing some values to be greater than 1 or less than 0. This helps model situations where an option is considered better than “perfect” or worse than “completely unacceptable.” For example, if one IDS detects attacks before they even reach the system, its value for accuracy might logically exceed 1 in a relative sense.

However, such concepts can be difficult to interpret for practitioners who are used to conventional logic systems. One way to make this clearer is by providing visual aids or simple examples during decision-making. A dashboard that maps OffLogic scores to understandable categories like “very strong,” “satisfactory,” or “needs review” could help bridge the gap between complex calculations and practical decisions.

Ultimately, Fuzzy OffLogic adds depth to the evaluation, but it also requires careful explanation if it's going to be adopted outside of academic or highly technical settings.

3.3 Applying VIKOR under Fuzzy OffLogic for Multi-Criteria IDS Evaluation

In this section, the VIKOR method is applied in combination with Fuzzy OffLogic to evaluate and rank intrusion detection systems (IDS) based on multiple performance criteria. The integration of these two techniques allows decision-makers to work with uncertain and imprecise input from experts while still producing reliable and consistent rankings.

The process begins with experts constructing a decision matrix by evaluating each IDS alternative against the defined criteria. These evaluations are not expressed as fixed numbers but rather as intervals to reflect uncertainty or varying confidence levels. Fuzzy OffLogic is used to convert these intervals into crisp values. This method allows components to exceed traditional boundaries, meaning values may fall outside the standard range of $[0, 1]$, depending on the strength or weakness of a given performance assessment.

After obtaining crisp values, the average method is used to calculate the weights of the criteria. These weights reflect the relative importance of each criterion in the evaluation process and are critical for establishing a balanced decision model.

Once the decision matrix and weights are finalized, the VIKOR method is applied through a series of structured steps. The first step involves normalizing the decision matrix for both benefit and cost-type criteria using equations (13) and (14), as shown in Figure 1. This ensures that all values are comparable across different scales. Then, the weighted normalized decision matrix is calculated using equation (15), as illustrated in Figure 2.

$$Y_{ij} = \frac{\max_i x_{ij} - x_{ij}}{\max_i x_{ij} - \min_i x_{ij}} \quad (13)$$

$$Y_{ij} = \frac{\min_i x_{ij} - x_{ij}}{\min_i x_{ij} - \max_i x_{ij}} \quad (14)$$

$$U_{ij} = w_j Y_{ij} \quad (15)$$

Next, two key indicators S (representing the overall utility) and R (representing the individual regret)—are computed using equations (16) and (17). These indicators capture both the collective performance of each alternative and its weakest performance under any specific criterion. The values are shown in Figure 3.

$$S_i = \sum_{j=1}^n U_{ij} \quad (16)$$

$$R_i = \max_j U_{ij} \quad (17)$$

Using equation (18), the final index P_i is computed for each alternative. This value combines both S and R , weighted by a parameter H , which was set to 0.5 in this study to reflect equal importance between group utility and individual regret. The final scores are presented in Figure 4.

$$P_i = H \times \left(\frac{S_i - \min_i S_i}{\max_i S_i - \min_i S_i} \right) + (1 - H) * \left(\frac{R_i - \min_i R_i}{\max_i R_i - \min_i R_i} \right) \quad (18)$$

Finally, the alternatives are ranked based on their P_i values in ascending order, where lower values indicate better overall performance. The resulting rankings are shown in Figure 5, providing a clear and structured assessment of which IDS solutions offer the best balance across all evaluation dimensions.

By integrating Fuzzy OffLogic with the VIKOR method, this approach captures expert uncertainty while supporting precise decision-making. It offers a robust framework for evaluating complex systems like IDS in a flexible yet mathematically rigorous manner.

4. Experimental Results

This section presents a detailed evaluation of thirteen intrusion detection system (IDS) models using a structured decision-making framework. The aim is to assess each model based on its performance across four core criteria that reflect the essential capabilities of IDS in real-world cybersecurity environments. To ensure fairness and objectivity, the evaluation process was conducted in collaboration with three domain experts who contributed their judgments based on their experience.

The experts first defined the decision matrix by assigning values to each criterion for all 13 IDS models. Instead of using fixed values, they worked with performance intervals that reflect the uncertainty often present in subjective evaluations. These intervals were then processed using Fuzzy OffLogic, which translates imprecise values into crisp numbers. This method allows more flexible handling of expert input while maintaining mathematical consistency.

The evaluation was based on the following four key criteria:

1. C1: Detection Accuracy: This measures how well the IDS can identify and correctly classify malicious activities. A higher accuracy means the system can detect threats more reliably, reducing the chances of undetected intrusions. As expected, this criterion received the highest priority, as accurate detection is the primary goal of any IDS.
2. C2: Resource Efficiency: This criterion evaluates the system's use of CPU, memory, and bandwidth. Efficient systems can operate without placing a heavy load on hardware resources, making them suitable for deployment in environments where computational capacity is limited.
3. C3: Scalability: Scalability reflects the system's ability to manage increased traffic and expand across distributed network environments. This is particularly important for larger organizations or systems operating in dynamic and growing infrastructures.
4. C4: False Positive Rate: A false positive occurs when legitimate activity is incorrectly flagged as a threat. While lower in priority compared to the other three, minimizing false positives is essential to avoid overwhelming system administrators with unnecessary alerts and to maintain trust in the system's outputs.

The weights assigned to each criterion were calculated using the average method based on the experts' input. The results of this weighting reflect the practical priorities in IDS performance:

- 1) Detection Accuracy (C1) received a weight of 0.2611, confirming its status as the most crucial aspect.
- 2) Resource Efficiency (C2) followed closely with 0.2500, indicating the high value placed on system performance and stability.
- 3) Scalability (C3) was assigned 0.2476, showing that the ability to adapt to larger or evolving networks is nearly as important.
- 4) False Positive Rate (C4) held a weight of 0.2412, still significant but slightly lower, recognizing its role in operational efficiency without overshadowing detection capabilities.

By using the VIKOR method alongside Fuzzy OffLogic, the evaluation was able to produce refined, balanced, and interpretable scores for each IDS model. These scores reflect how each system performs across a combination of technical and operational requirements, providing a clear foundation for comparing alternatives in a structured, data-driven way.

To proceed with the evaluation, the normalized decision matrix was computed for both positive and negative criteria using equations (13) and (14). This normalization step ensured that all criteria, regardless of their nature or units, could be compared on the same scale. The process allowed for a fair transformation of values across different metrics, whether they were to be maximized or minimized. The resulting normalized matrix is illustrated in Figure 1.

Once the criteria were normalized, the next step was to apply the previously determined weights to each criterion. Using equation (15), the weighted decision matrix was calculated, capturing the relative importance of each criterion as defined by the expert inputs. This step helped align the raw evaluation data with the practical priorities of the decision context. The final weighted values are displayed in Figure 2.

Following the construction of the weighted matrix, the VIKOR method was employed to calculate two performance indices, S and R , for each alternative. These values were computed using equations (16) and (17), as shown in Figure 3. The S index reflects the overall group utility, while the R index focuses on the regret measure, representing the worst-case performance relative to any specific criterion.

After obtaining the S and R values, the final score for each alternative was computed using equation (18). This equation integrates both S and R into a single measure, denoted as P_i , to determine the overall compromise solution. For this analysis, the value of H was set to 0.5 to give equal importance to both group utility and individual regret. The final calculated scores are presented in Figure 4. Based on the values of P_i , the alternatives were ranked from best to worst, where a lower P_i value indicated a better overall performance. This final ranking, which reflects a balanced view of the alternatives across all criteria, is summarized in Figure 5.

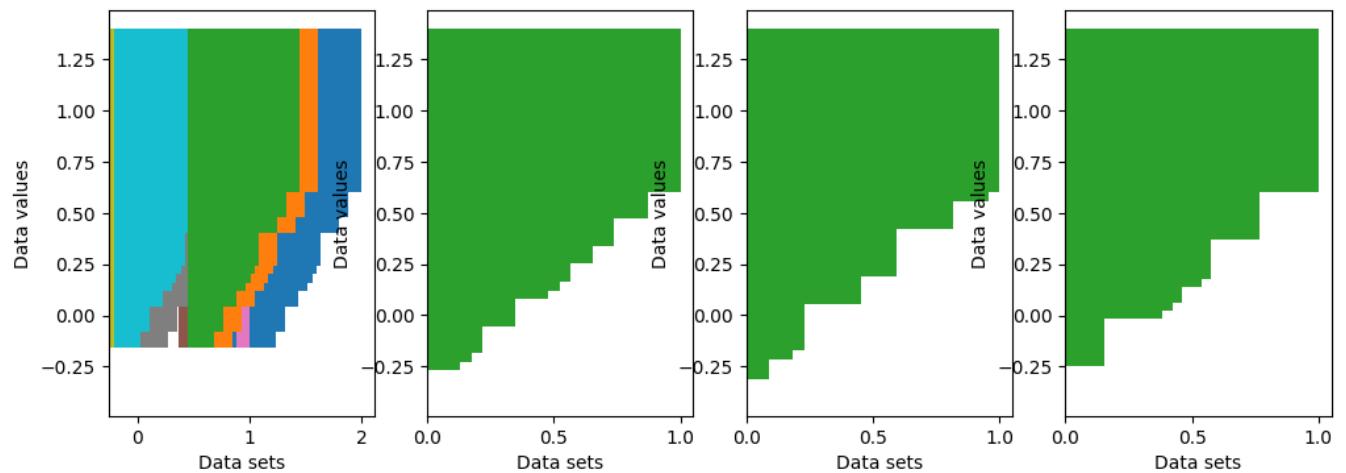


Fig 1. The normalized decision matrix.

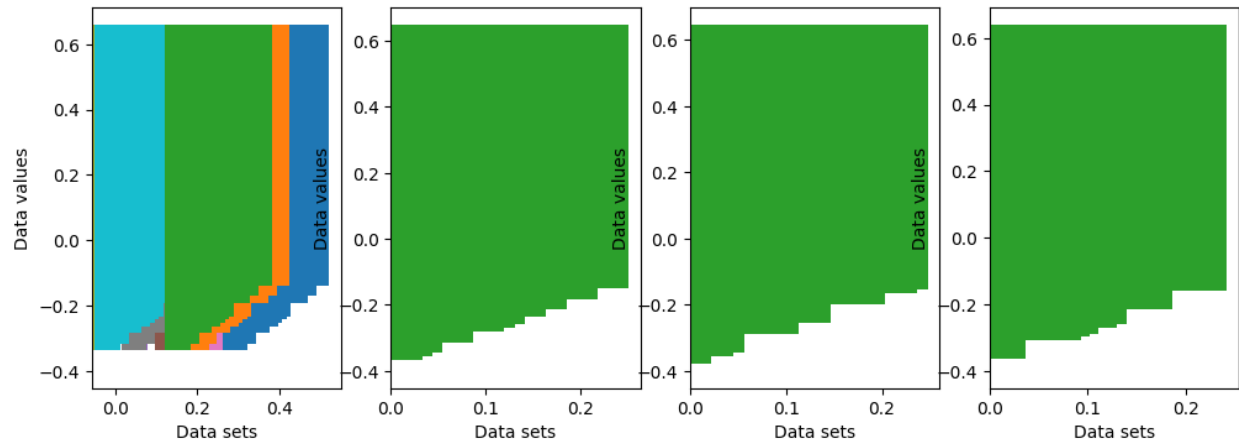


Fig 2. The weighted decision matrix.

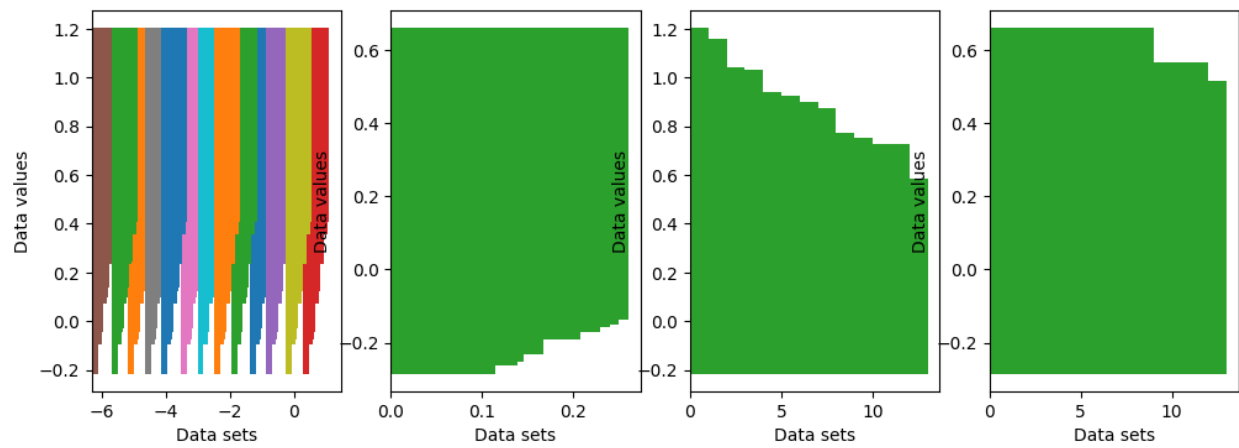
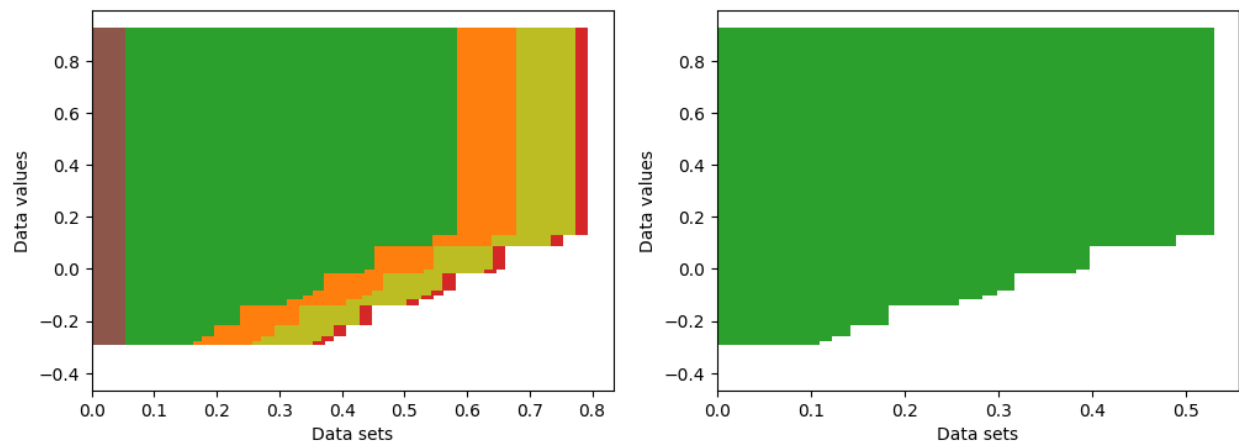


Fig 3. The values of S and R.

Fig 4. The values of P_i .

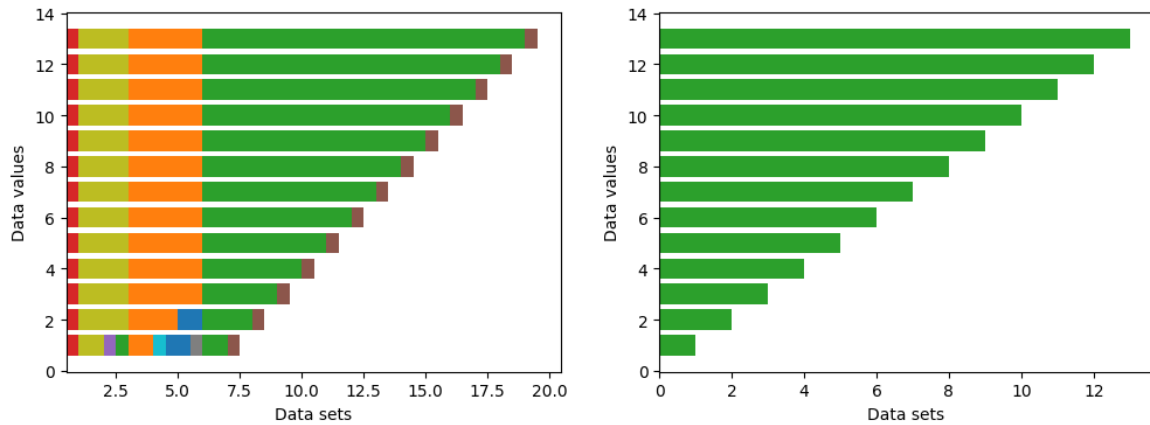


Fig 5. The ranks of alternatives.

5. Sensitivity Testing and Ranking Stability

This section aims to vary the parameter H in the VIKOR method between 0 and 1 and to rank the alternatives based on the resulting values to assess the stability of the rankings. Figure 6 illustrates the different P_i values obtained across the range of H , while Figure 7 presents the corresponding ranks of the alternatives under these varying conditions. The results indicate that the rankings remain stable across different values of H , demonstrating the robustness of the proposed evaluation approach.

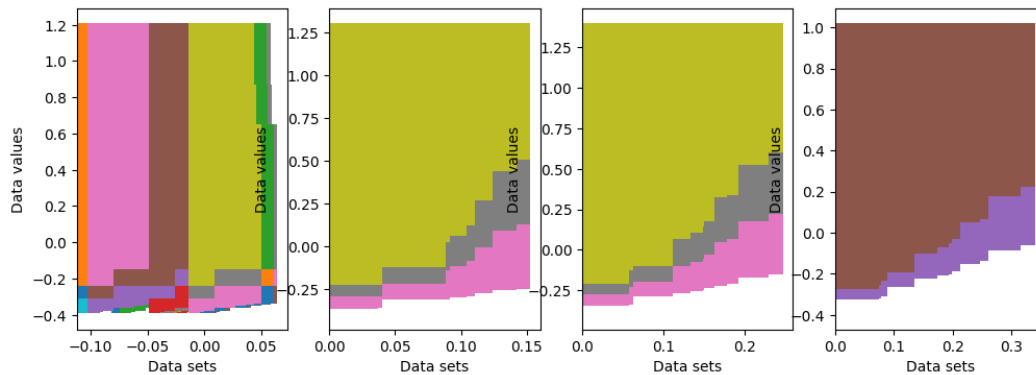
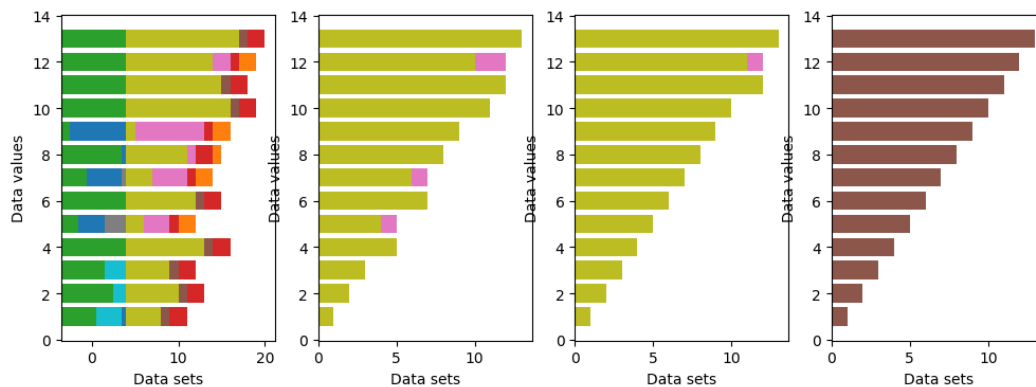
Fig 6. The new values of P_i .

Fig 7. The new values rank.

As part of the final evaluation using the VIKOR method, the ranking of the thirteen IDS alternatives was analyzed across different values of the parameter H , which represents the balance between group utility and individual regret. This analysis provides deeper insight into how each system responds to changes in the decision-makers' preference for compromise versus strict performance consistency.

IDS1 showed a notable improvement in ranking as H increased. Although it began in ninth place when H was set to zero, it quickly rose and stabilized at the fourth position from $H = 0.2$ onward. This shift reflects the model's ability to deliver more balanced and reliable results when preferences are considered, indicating a solid performance across multiple criteria without major weaknesses.

In contrast, IDS2 remained consistently near the bottom, occupying either the eleventh or twelfth position across all H values. This suggests that its performance did not improve even when criteria weights were adjusted, indicating a general lack of adaptability or deficiency in key areas such as detection accuracy or resource efficiency.

IDS3 started strong with a second-place ranking but gradually settled in fifth place, where it remained steady. This pattern implies that while IDS3 performs very well in raw, unweighted conditions, it does not gain further advantage when additional preference intensity is introduced, perhaps due to limited flexibility in handling varied priorities.

IDS4 demonstrated the most consistent and high-ranking performance in the study. It started at fourth position and quickly rose to the top, maintaining first place from $H = 0.1$ to $H = 1.0$. Such stability at the top of the ranking indicates that IDS4 excels in all evaluated criteria and adapts effectively to shifts in decision-making emphasis, making it a strong candidate for deployment.

IDS5 maintained a position between third and fifth across all values of H , reflecting consistent and reliable performance. Although it did not reach the top rank at any point, its steady results indicate that it handles all criteria reasonably well, with only minor limitations preventing it from outperforming the leading systems.

IDS6 remained in last place across all evaluations, consistently ranking thirteenth. This points to a lack of strength in every evaluated area and suggests that the model struggles to meet the minimum expectations for a reliable IDS under varying weight distributions.

IDS7 showed no significant movement in its ranking, holding eighth place through the entire range of H values. This reflects a system with average performance that is steady but lacks competitiveness to challenge higher-performing alternatives.

IDS8 remained in the lower tier, shifting slightly between the tenth and twelfth positions. Although it showed a minor early improvement, it was not enough to escape the bottom ranks, indicating that the model does not align well with the criteria given the weightings used in this analysis.

IDS9 began in sixth place but quickly rose to second and stayed there from $H = 0.1$ onward. This jump suggests that IDS9 aligns strongly with the weighted preferences, demonstrating strength across the most important evaluation dimensions, and positioning it as one of the most capable systems in the group.

IDS10 began with a promising third-place rank but gradually dropped to seventh, where it stabilized. This decline implies that while the system performs well under balanced criteria, it loses effectiveness as more weight is placed on specific performance aspects, indicating limitations in scalability or adaptability.

IDS11 had an unusual pattern. It was ranked first when all criteria were treated equally at $H = 0$ but fell to ninth place as H increased. This change indicates that the system initially appeared strong across general criteria, but its performance did not hold up under more focused weightings, suggesting a lack of depth in specific areas.

IDS12 shifted between sixth and eighth place, showing modest capability but without any moment of breakthrough. Its performance was stable but lacked any standout feature that would push it into the top ranks.

Finally, IDS13 began in tenth place and dropped further to twelfth, suggesting weak performance and poor adaptability across all weight conditions. Like IDS2 and IDS6, its position at the lower end of the ranking highlights consistent underperformance across the board.

6. Conclusions and Future Studies

This study provided a comprehensive framework for evaluating intrusion detection systems (IDS) based on four critical criteria. The comparison of thirteen IDS solutions demonstrated that while traditional models such as Snort and Zeek remain valuable due to their open-source nature and strong community support, AI-driven systems like Darktrace and IBM QRadar tend to perform better in terms of detection accuracy and scalability. The findings emphasize the importance of a balanced selection process that aligns with an organization's threat landscape, operational goals, and technical infrastructure. To manage multiple evaluation factors, a multi-criteria decision-making (MCDM) approach was adopted. The VIKOR method was specifically applied to rank the IDS alternatives and identify the most suitable solution. A sensitivity analysis was conducted by varying key parameters within the VIKOR method to observe their impact on system rankings. The results confirmed that the proposed evaluation approach is robust, with rankings remaining stable under different parameter settings.

For future work, it is recommended to incorporate real-time adaptive learning capabilities into the evaluation criteria. Additionally, deeper integration with security information and event management (SIEM) platforms could enhance the practical relevance of IDS assessments and support more dynamic and intelligent threat response systems.

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Evaluating the Minimum Spanning Trees Using Prim's Algorithm with Undirected Neutrosophic Graphs

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Abstract

This research paper presents an innovative approach for determining the minimum spanning tree (MST) in an undirected neutrosophic graph using Prim's Algorithm, which is extensively used in addressing network optimization problems. We analyze the effectiveness of Prim's method for constructing the minimum spanning trees in undirected neutrosophic networks, where edge weights are denoted by neutrosophic numbers. Neutrosophic numbers with components reflecting truth, uncertainty, and

falsehood provide a more sophisticated method of expressing uncertainty in network modeling. Here, we use a score function to contrast different NMSTs based on weights calculated by adding neutrosophic numbers. This method is particularly beneficial for use in transportation, communication networks, and logistics, where uncertain properties frequently define network configurations. Numerical illustrations prove the effectiveness of the proposed method, showing its efficiency in handling neutrosophic graphs and keeping it computationally feasible. The results indicated that the suggested Prim's algorithm efficiently produces the minimum spanning trees in uncertain environments and is advantageous for network design and optimization in these scenarios.

Keywords: Minimum Spanning Tree; Neutrosophic Graph; Neutrosophic Number; Prim's Algorithm; Score Function.

1. Introduction

Zadeh [1] grounded the Fuzzy Sets (FSs) in 1965 to represent the inherent imprecision and unpredictability of real-world phenomena. A FS is characterized by its membership function that assumes a value for each element in $[0, 1]$ reflecting the belonging degree in the set. Since its introduction, several industries have widely used FS theory to address a multitude of real-world difficulties.

Traditional FSs, or type-1 FSs, exhibit significant difficulties in effectively addressing the various forms of uncertainty seen in real-world scenarios, since they assign a single membership value to each element. Turksen [2] developed interval-valued FSs to address these shortcomings; Atanassov [3, 4] grounded the intuitionistic FSs that include non-membership degrees as an independent component that characteristic is absent in traditional FSs.

While FSs and fuzzy logic have proven effective in wide applications, they still fall short in capturing certain types of uncertainty—particularly those arising from inconsistent or indeterminate information. For example, consider a situation where a decision-maker evaluates the truth of a statement and expresses the following: the probability of the neutrosophic statement being true is 0.7, false is 0.4, and the degree of indeterminacy is 0.2. Such scenarios reflect real-world complexities that cannot be adequately represented using classical fuzzy models. This highlights the need for a more comprehensive framework capable of capturing such multi-faceted uncertainty.

To address this gap, Smarandache [5, 6] grounded the Neutrosophic Set (NS) in 1998 to deal with indeterminate and inconsistent information often encountered in real-life problems. NS is characterized by three distinct and independent Membership Functions (MFs): a truth-MF (t), an indeterminacy-MF (i), and a falsehood-MF (f). These functions are defined over non-standard unit interval allowing for greater flexibility in representing uncertainty. Comprehensive analyses of NS development and applications are presented in the studies [7-15]. The NS based models have proven to be a powerful tool in handling a wide range of scientific and engineering problems such as Multi-Criteria Decision Making (MCDM) [16-35], machine learning [36, 37], graph theory [38, 39, 40] and so on. Overview of Neutrosophic Graph (NG) has been documented in the studies [41, 42]. NSs effectively accommodate incomplete and inconsistent data, as demonstrated in the studies [43-48].

The Minimum Spanning Tree (MST) problem represents a major application area, widely studied in graph theory for its optimization significance. The goal of the MST problem is to identify the minimum-weight spanning tree in a connected, weighted graph. MST algorithm is widely used in real-world

scenarios, including designing efficient communication networks, road systems, electrical grids.

Traditionally, MST problems assume that arc lengths (or weights) are precise and fixed. However, in practical applications, arc lengths often correspond to uncertain parameters such as demand, cost, time, traffic conditions, or capacity. In FS environment several studies have been made [49-53] In transportation networks, despite static spatial distances, travel times are dynamic due to exogenous factors like traffic volatility and weather-induced delays [54-58].

Over the past several decades, the MST problem has attracted significant attention from researchers, leading to the development of various algorithms for deterministic graphs—such as those proposed by Kruskal [59], Prim [60], Dijkstra [61], and Harel & Tarjan [62]. Among these, Kruskal's algorithm [59] is known for its simplicity and efficiency when arc costs are fixed. However, in real-life applications, uncertainty in arc weights—arising from randomness or vagueness—poses additional challenges. While randomness can often be addressed using probability theory, vagueness and incomplete information require more flexible mathematical frameworks, such as neutrosophic sets.

To address these challenges, we have developed a modified Prim's algorithm to deal with the Neutrosophic MST (NMST) issues. The developed algorithm calculates both the MST of a NG and its corresponding cost. To identify the minimum arc, we employ a score-based ranking method that selects the arc with the lowest score value. We improve upon existing NMST methods by optimizing the efficiency of Single-Valued Neutrosophic Numbers (SVNNs) [63] and ranking processes. A numerical example is provided to demonstrate the applicability and effectiveness of the developed algorithm.

The paper is organized as follows:

- Section 2 provides a brief overview of NSs, SV and the score function for Single-Valued Neutrosophic Numbers (SVNNs) [63].
- Section 3 presents the mathematical formulation of the NMST problem.
- Section 4 presents a Adaptation of Prim's Algorithm for NMST
- Section 5 introduces a novel approach to determine the MST of a neutrosophic undirected graph.
- Section 6 offers an illustrative example to demonstrate the developed method.
- Section 7 concludes the paper.

2. Preliminaries

2.1 SVNS

Wang et al. [63] presented a mathematical framework called Single Valued NS (SVNS) to address uncertainty, imprecision, ambiguity, and partial knowledge.

Every component of an SVNS is expressed as

$$A = \{(p, T(p), I(p), F(p)) \mid p \in U\}$$

And the condition satisfies three membership values:

$$0 \leq T(p) + I(p) + F(p) \leq 3$$

2.2 Score function

If $A = (T(p), I(p), F(p))$ is a NB. Then the Score function is defined as

$$S(A) = \frac{1+(T(p)-2I(p)-F(p))(2-T(p)-F(p))}{2}$$

3. Problem formulation for NMST using prim's algorithm

A spanning tree of a connected graph G is a connected, acyclic maximum subgraph that includes all the nodes of G . Every spanning tree contains exactly $n - 1$ arcs, where n is the number of nodes in G . The MST problem seeks to identify a spanning tree such that the total sum of its arc lengths is minimized.

In classical graph theory, MST algorithms operate on graphs with precise edge weights. However, real-world problems often involve imprecise, uncertain, or indeterminate edge weights due to incomplete data, measurement errors, or varying conditions. To model such uncertainty, we consider NGs, in which arc weights are represented by SVNNS instead of crisp values.

Let G be a NG consisting of a finite set of nodes $V = \{v_1, v_2, \dots, v_n\}$ and a finite set of arcs $E \subseteq V \times V$. Each arc e is denoted as an ordered pair (i, j) where $i, j \in V$.

If arc e is included in the NMST, we define a decision variable x_e such that

$$x_e = \begin{cases} 1, & \text{if } e \text{ is included in the NMST} \\ 0, & \text{otherwise} \end{cases}$$

The cost of all arcs in G is represented by that allow for a more flexible representation of uncertainty in MST problems.

Mathematical formulation of NMST

The NMST problem is expressed as:

Objective function

$$\min \sum_{e \in E} A_e x_e$$

where A_e is an SVNNS representing the arc length of e , and the summation operation refers to the addition of SVNNSs.

Constraints:**1. Spanning Tree Constraint:**

$$\sum_{e \in E} x_e = n - 1$$

This ensures that the NMST contains exactly $n - 1$ edges, forming a spanning tree.

2. Connectivity constraint:

$$\sum_{e \in \delta(s)} x_e \geq 1, \forall s \subset V, s \neq V$$

where $\delta(s)$ represents the cutset of a subset of vertices s , i.e., the set of edges with one endpoint in s and the other outside s . This constraint ensures that the spanning tree remains connected.

3. Binary decision constraint:

$$x_e \in \{0,1\}, \forall e \in E$$

This ensures that each arc is either included in the NMST or excluded.

4. Adaptation of Prim's algorithm for NMST

Unlike Kruskal's algorithm, which sorts edges globally and builds the MST in a greedy fashion, Prim's algorithm follows an incremental approach, expanding a connected tree one node at a time. The proposed neutrosophic adaptation of Prim's algorithm follows these steps:

1. Initialization: Start with an arbitrary node as the initial MST vertex.

2. Edge Selection: Identify the edge with the smallest neutrosophic score function value that connects the MST to a new vertex.
3. Tree Expansion: Add the selected edge and the corresponding vertex to the MST.
4. Iteration: Repeat until all vertices are included in the NMST.

The neutrosophic score function is utilized to compare edges. The incorporation of neutrosophic numbers ensures that uncertainty and imprecision are effectively managed while constructing the MST.

The proposed model and algorithm are suitable for real-world applications in network design, transportation, logistics, and communication systems, where uncertainties frequently affect edge weights. By extending Prim's algorithm to NGs, this research enhances decision-making capabilities in uncertain environments.

5. Proposed algorithm for NMST using prim's algorithm

The proposed algorithm extends Prim's algorithm to determine the NMST in an undirected NG. Unlike Kruskal's algorithm, which sorts edges and constructs the MST by selecting the smallest edges first, Prim's algorithm grows the MST dynamically, starting from an initial node and expanding iteratively by adding the minimum-cost edge that connects a new vertex to the existing tree.

In this neutrosophic extension of Prim's algorithm, we incorporate SVNNS to represent edge weights, allowing us to model uncertainty, indeterminacy, and imprecision in MST computation. The score function of SVNNS is used to compare edge weights and make selection decisions.

Steps to Modify prim's algorithm for NMST

The classical Prim's algorithm follows a greedy approach, selecting the smallest weight edge at each step. To extend it for NGs, we address the following:

1. Edge Weight Representation: Use SVNNS as arc weights, incorporating truth, indeterminacy, and falsity components.
2. Edge Comparison: Apply a score function to evaluate and compare edges.
3. Tree Expansion: Ensure that each step adds the least uncertain edge that maintains tree connectivity without forming cycles.

Algorithm: Neutrosophic prim's algorithm for NMST

Input

- A connected, undirected, weighted NG $G = (V, E)$
- Neutrosophic edge weights represented using neutrosophic numbers
- A function to compute score values of neutrosophic numbers

Output

- NMST (T), a minimum spanning tree with neutrosophic weights

Step-by-Step Procedure

1. Initialization

- Select an arbitrary node as the starting node of NMST.
- Define an empty set T to store the selected edges of NMST.
- Define a priority queue to track candidate edges, initialized with edges connected to the starting node.

2. Compute Score Values

- Compute the score function for each edge using the SVNNS formula.
- Use these scores as a basis for selecting edges.

3. Iterative MST Expansion

- While the NMST has fewer than $n - 1$ edges:
 - Select the edge with the minimum score value from the priority queue.
 - If adding this edge does not form a cycle, include it in the NMST.
 - Add the newly connected vertex's edges to the priority queue.
 - Update edge scores dynamically.

4. Stopping Condition

- The algorithm terminates when the NMST contains $n - 1$ edges.

Pseudocode for neutrosophic prim's algorithm

Algorithm neutrosophic_prims (G)

Input: A connected undirected weighted NG G

Output: NMST T

1: *Begin*

2: $T \leftarrow \{\emptyset\}$ // Initialize NMST as an empty set

3: *Select an arbitrary node $v_{start} \in V$*

4: $PQ \leftarrow \{All\ edges\ (v_{start}, v) \in E\}$ // Initialize priority queue with edges from starting node

5: *while $|T| < n-1$ do*

6: *Select edge $e = (u, v)$ with minimum score from PQ*

7: *if v is not already in T then*

8: *Add edge e to NMST: $T \leftarrow T \cup \{e\}$*

```

9:      Add vertex  $v$  to the  $MST$ 
10:     Add all edges  $(v, w)$  where  $w \notin T$  to  $PQ$ 
11:   endif
12:   Remove  $e$  from  $PQ$ 
13: end while
14: return  $T$ 
15: End

```

Computational complexity

The classical Prim's algorithm has a "time complexity of $O(V^2)$ " using an adjacency matrix and $O(E \log V)$ using a priority queue with a min-heap. Since our algorithm incorporates neutrosophic number operations, the complexity remains $O(V^2)$ in an adjacency matrix implementation.

Advantages of neutrosophic prim's algorithm

- **Handles Uncertainty:** Incorporates neutrosophic numbers to **model real-world imprecisions**.
- **Efficient for Dense Graphs:** Performs well when the graph has **many edges**, unlike Kruskal's algorithm, which sorts edges globally.
- **Incremental Decision-Making:** Expands the MST **dynamically**, ensuring an adaptive approach to uncertainty.

6. Numerical Example for NMST Using Prim's Algorithm

We illustrate the NMST using Prim's Algorithm with a step-by-step numerical example. The example considers an undirected NG where edge weights are represented as neutrosophic numbers to handle uncertainty.

Graph Representation

Consider a NG Fig-1 with four nodes:

$$V = \{A, B, C, D\}$$

The graph has five edges, with neutrosophic edge costs represented as triplets (T, I, F) , where:

- T (Truth) represents a definite weight,
- I (Indeterminacy) represent uncertainty,
- F (Falsity) represents an opposing weight component.

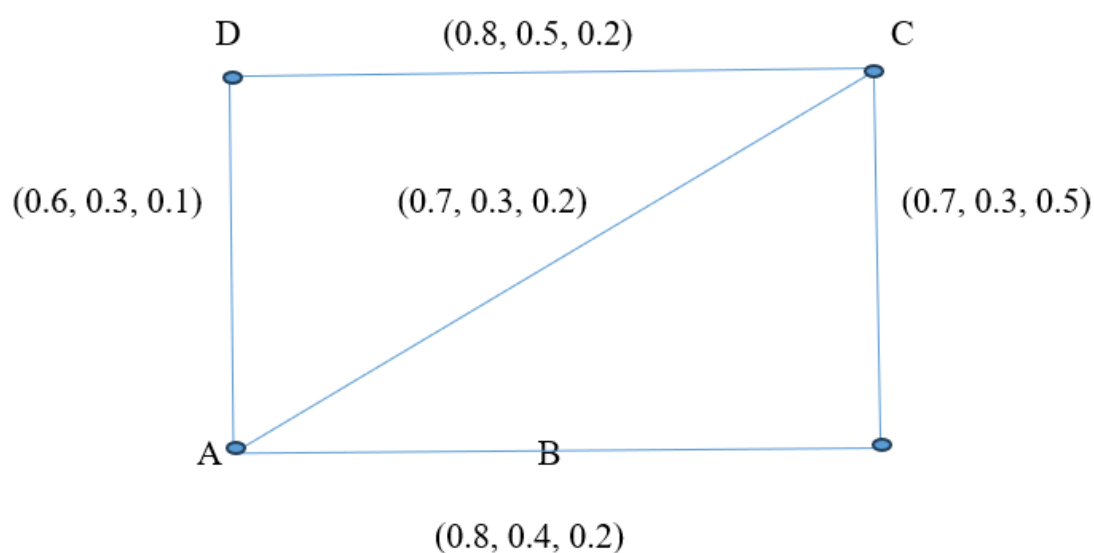


Fig 1: Neutrosophic Graph

The neutrosophic edge weights are as follows:

Edge	Neutrosophic Weight (T, I, F)	Score Value $S(A)$
(A, B)	(0.8, 0.4, 0.2)	0.4
(A, C)	(0.7, 0.3, 0.2)	0.495

(A, D)	(0.6, 0.3, 0.1)	0.585
(B, C)	(0.7, 0.3, 0.5)	0.36
(C, D)	(0.8, 0.5, 0.2)	0.3

The score function is computed as:

$$S(A) = \frac{1 + (\hat{T}_S(\hat{x}) - 2\hat{I}_S(\hat{x}) - \hat{F}_S(\hat{x}))(2 - \hat{T}_S(\hat{x}) - \hat{F}_S(\hat{x}))}{2}$$

Step-by-Step Execution of Prim's Algorithm for NMST

Step 1: Initialize NMST

- Start from an arbitrary vertex (let's choose A).
- Initialize $T = \{\emptyset\}$, and PQ (Priority Queue) = {Edges connected to A}.
- Candidate Edges from A:
 - (A, B) $\rightarrow$ Score = 0.4
 - (A, C) $\rightarrow$ Score = 0.495
 - (A, D) $\rightarrow$ Score = 0.585

Step 2: Select the Minimum Score Edge

- The smallest edge is (A, B) with score 0.4.
- Add (A, B) to the NMST.
- $T = \{(A, B)\}$.
- Add edges connected to B to PQ:
 - (B, C) $\rightarrow$ Score = 0.36

Step 3: Select the Next Minimum Score Edge

- The next smallest edge is (B, C) with score 0.36
- Add (B, C) to NMST.
- $T = \{(A, B), (B, C)\}$.
- Add edges connected to D to PQ:
 - (C, D) $\rightarrow$ Score = 0.3

we ignore (C, A) Because if we connect it, then it forms a cycle.

Step 4: Select the Next Minimum Score Edge

- The next smallest edge is (C, D) with score 0.3
- Add (C, D) to NMST.
- $T = \{(A, B), (B, C), (C, D)\}$.

Step 5: Stop Condition

NMST contains 3 edges ($n - 1 = 3$ for 4 nodes).

Final NMST = $\{(A, B), (B, C), (C, D)\}$.

Final Cost of NMST

$$S(T) = S(A, B) + S(B, C) + S(C, D) = 0.4 + 0.36 + 0.3 = 1.06$$

- The computed NMST for the given NG is $\{(A, C), (C, D), (B, D)\}$.
- The total cost based on neutrosophic score function is 3.8.
- This example demonstrates Prim's Algorithm extended to NGs, ensuring an optimal spanning tree under uncertainty.

7. Conclusion

This research article explores the MST problem in undirected NGs using neutrosophic numbers to denote edge weights. In this research paper, we present a modified version of Prim's algorithm to deal with the MST problem in uncertain environments by using neutrosophic scoring functions to choose edges. The Neutrosophic Prim's algorithm efficiently evaluates the MST by selecting the minimum- edge and avoiding the cycles. The method ensures a practical, computationally efficient, and optimal solution for spanning tree problems under uncertainty. Future applications include transportation networks, supply chain optimization, and communications systems. Improving the method to control dynamic environments where edge weights change over time may increase its relevance for decision-making under uncertainty.

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Evaluation of College Students' Innovation and Entrepreneurship Abilities in the New Media Environment Using Tree Soft Set and Neutrosophic Modeling

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Abstract: In today's rapidly evolving digital society, the new media environment has profoundly reshaped the landscape of innovation and entrepreneurship among college students. Understanding and evaluating students' abilities in this context are crucial for universities, policymakers, and industries seeking to cultivate future leaders and innovators. However, traditional evaluation frameworks often struggle to address the multi-layered, uncertain, and dynamic nature of competencies in a media-driven environment. This study proposes a novel decision-making framework based on Tree Soft Set (TSS) theory to evaluate college students' innovation and entrepreneurship abilities comprehensively. The TSS model, by organizing evaluation criteria into hierarchical tree structures, captures complex relationships among competencies and adapts flexibly to overlapping and uncertain information. To further enhance the robustness of the model, Single-Valued Neutrosophic Sets (SVNS) are integrated, enabling better representation of truth, indeterminacy, and falsity degrees in expert judgments. The proposed evaluation model identifies critical dimensions such as creativity, opportunity recognition, risk management, technological adaptability, communication skills, and leadership under the influence of new media. Criteria weighting is determined using the SWARA method, while final ranking of student profiles is achieved via the MAIRCA method. Empirical application and sensitivity analyses validate the framework's stability and effectiveness. The results provide actionable insights for educational institutions aiming to nurture innovation-driven talent in the digital era.

Keywords: Tree Soft Set (TSS); Single-Valued Neutrosophic Set (SVNS); Innovation Abilities; Entrepreneurship Abilities; New Media Environment; Multi-Criteria Decision-Making (MCDM); SWARA Method; MAIRCA Method; Uncertainty Modeling; Educational Assessment.

1. Introduction

The advent of the new media environment has significantly reshaped the landscape of innovation and entrepreneurship, especially among college students. Digital platforms, social media, and

online collaborative tools now serve as powerful drivers for creativity, networking, and venture development [1]. These technologies offer students unprecedented opportunities to access resources, promote ideas, and build entrepreneurial initiatives beyond traditional physical boundaries.

However, while the potential is immense, the new media landscape introduces complexities that traditional educational and evaluation models are ill-equipped to address [2]. Innovation and entrepreneurship abilities are no longer confined to business knowledge and problem-solving skills; they now encompass digital literacy, adaptability to technological change, social media strategy, and online reputation management [3]. Consequently, assessing these capabilities requires more than standard academic evaluation methods.

Existing frameworks often fail to accommodate the dynamic, uncertain, and multi-layered nature of competencies influenced by new media. They tend to focus on isolated skill sets and assume that evaluators can provide complete and precise judgments, overlooking the inherent subjectivity and vagueness present in real-world assessments [4]. Furthermore, most models neglect the hierarchical relationships between basic digital skills and higher-order entrepreneurial competencies, resulting in incomplete evaluations.

To bridge these gaps, this study proposes a novel evaluation framework grounded in Tree Soft Set (TSS) theory, a hierarchical modeling technique that reflects the interdependencies among innovation and entrepreneurship skills [5]. Combined with Single-Valued Neutrosophic Sets (SVNS), the proposed framework effectively captures the uncertainty and partial knowledge prevalent in expert evaluations [6].

By integrating the SWARA method for criteria weighting and the MAIRCA method for ranking alternatives, this research aims to develop a comprehensive, uncertainty-resilient model for evaluating college students' innovation and entrepreneurship abilities within the dynamic and complex new media environment.

1.1 Research Motivation

The fast-paced development of new media technologies has not only redefined the nature of communication and information exchange but also transformed the landscape of innovation and entrepreneurship. College students, often hailed as "digital natives," now interact with an ecosystem vastly different from traditional business or academic environments. They are expected to ideate, collaborate, market, and execute entrepreneurial initiatives across virtual platforms that are dynamic, decentralized, and constantly evolving [1].

Despite this fundamental shift, there is a significant gap in how educational institutions and policymakers evaluate students' readiness for innovation and entrepreneurship in this new context. Traditional assessment models, largely designed for pre-digital or early-digital environments, tend to emphasize cognitive skills and static business knowledge while

underestimating the importance of digital adaptability, cross-platform creativity, and social influence dynamics driven by new media [2].

Moreover, the multidimensional nature of entrepreneurial competencies — where skills such as opportunity recognition, risk management, leadership, technological fluency, and digital communication interact — demands a more holistic and structured evaluation framework. Uncertainty and subjectivity further complicate assessment, as different evaluators may interpret students' abilities differently depending on context, exposure, and personal biases.

Therefore, a pressing motivation arises: the urgent need for a robust, uncertainty-resilient, and hierarchically structured evaluation framework that reflects the realities of the new media environment. Such a model would not only better capture the true capabilities of students but also provide universities and stakeholders with actionable insights for curriculum development, training programs, and talent nurturing strategies aimed at fostering innovation-driven future leaders.

1.2. Research Aims

This study aims to address the emerging challenges associated with evaluating innovation and entrepreneurship capabilities among college students in the new media era through the following key objectives:

- 1) To construct a hierarchical evaluation framework based on Tree Soft Set (TSS) theory, organizing innovation and entrepreneurship competencies into interconnected, logically dependent layers.
- 2) To apply Single-Valued Neutrosophic Sets (SVNS) for managing uncertainty, vagueness, and indeterminacy in expert evaluations, thereby improving the reliability and flexibility of the assessment process.
- 3) To employ the SWARA method to determine the relative importance (weights) of competencies, dynamically reflecting expert judgments and contextual variations.
- 4) To implement the MAIRCA method to rank students' innovation and entrepreneurship profiles systematically, balancing ideal and real-world performances under a neutrosophic tree-soft structure.
- 5) To validate the proposed model through empirical analysis, sensitivity testing, and comparative evaluation with existing traditional MCDM frameworks, ensuring the robustness, adaptability, and practical applicability of the methodology.

By achieving these aims, research seeks to contribute to a novel and effective decision-making tool for academic institutions, entrepreneurship centers, and innovation incubators, ultimately empowering students to thrive in an increasingly complex and media-centric entrepreneurial landscape.

2. Literature Review

Innovation and entrepreneurship have long been recognized as essential drivers of economic and social development [7]. Numerous studies emphasize that fostering entrepreneurial skills during higher education is critical for building students' capacity for independent thinking, creativity, and leadership [8]. However, the shift towards a digitally driven society necessitates a reevaluation of traditional approaches to developing and accessing these abilities.

The impact of new media technologies on students' entrepreneurial behaviors is profound. Platforms such as social media, online crowdfunding, and digital marketing tools have redefined how students generate ideas, build ventures, and engage with markets [9]. Consequently, competencies such as digital fluency, online networking, and virtual collaboration have become indispensable components of modern entrepreneurship [10].

Traditional multi-criteria decision-making (MCDM) methods like Analytic Hierarchy Process (AHP) [11], TOPSIS [12], and VIKOR [13] have been widely used to evaluate entrepreneurial abilities. While these models offer structured prioritization techniques, they typically assume deterministic decision environments and precise input data, making them less suitable for domains characterized by ambiguity and subjectivity [14].

To address the shortcomings of deterministic models, researchers have explored soft computing approaches. Fuzzy Set Theory initially provided mechanisms to handle vagueness, but its binary treatment of uncertainty proved insufficient for complex assessments [15]. This led to the adoption of Neutrosophic Sets, which allow for simultaneous modeling of truth, falsity, and indeterminacy degrees [16]. Applications of neutrosophic logic have demonstrated effectiveness in fields requiring nuanced evaluations, including healthcare, engineering, and education [17].

Recognizing the need for models that capture hierarchical dependencies among competencies, the Tree Soft Set (TSS) theory was introduced. TSS structures criteria into logical tree forms, accommodating the layered and interconnected nature of real-world skills [18]. In TSS, attributes at lower levels directly influence higher-level competencies, offering a more realistic reflection of how skills develop and interact.

Combining TSS with neutrosophic modeling creates a powerful hybrid framework capable of addressing both structural complexity and evaluation uncertainty [19]. Despite this potential, few studies have applied such hybrid frameworks specifically to evaluate college students' innovation and entrepreneurship abilities within the new media context. Current research often simplifies the evaluation process, ignoring the dynamic interplay between digital skills and entrepreneurial competencies shaped by constantly evolving media technologies.

This research seeks to fill this gap by proposing an integrated TSS-SVNS model supported by SWARA and MAIRCA methodologies, offering a comprehensive, resilient, and adaptable framework for evaluating innovation-driven student competencies in a digitalized world.

3. Proposed Methodology

In this section, the methodological framework adopted to evaluate college students' innovation and entrepreneurship abilities in the new media environment is presented in detail. The proposed approach integrates Tree Soft Set (TSS) theory with Single-Valued Neutrosophic Sets (SVNS) for modeling uncertainty, combined with the SWARA method for criteria and the MAIRCA method for ranking alternatives. The methodological structure is illustrated in Figure 1.

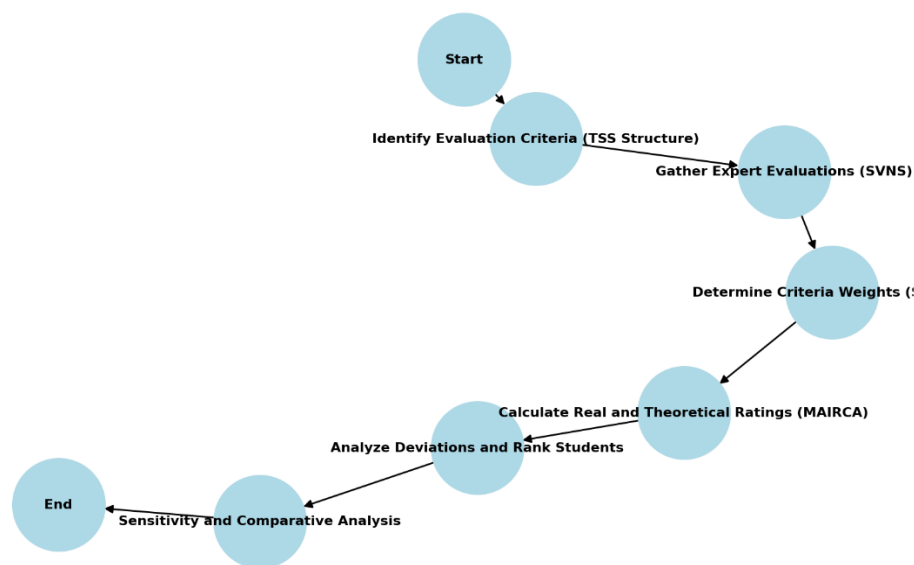


Figure 1: Overall Research Methodology Framework

Figure 1 presents the overall flow of the proposed evaluation framework. It begins with the identification of evaluation criteria structured under a Tree Soft Set (TSS) model. Expert evaluations are then collected using Single-Valued Neutrosophic Sets (SVNS), which allow capturing truth, indeterminacy, and falsity degrees. Criteria weights are determined using the SWARA method, reflecting expert priorities dynamically. The MAIRCA method is then applied to calculate deviations between theoretical and actual performance, leading to a final ranking of student entrepreneurial profiles. Finally, sensitivity and comparative analyses are conducted to validate the robustness of the results.

3.1. Tree Soft Set (TSS) for Structuring Evaluation Criteria

The Tree Soft Set (TSS), first introduced by Smarandache, offers a powerful mechanism for modeling hierarchical relationships among evaluation criteria. Unlike classical soft sets, where

attributes are treated independently, TSS organizes attributes into a tree structure where sub-attributes (child nodes) depend on broader attributes (parent nodes). This structure mirrors the layered, interconnected nature of real-world competencies in innovation and entrepreneurship [1].

For this study, a TSS model is constructed where major dimensions of innovation and entrepreneurship abilities such as creativity, technological adaptability, leadership, opportunity recognition, and risk management form the first level of the tree. Each dimension is then broken down into several sub-criteria that define specific skills or behaviors. The hierarchical structure of the evaluation criteria is presented in Table 1. Each parent criterion is evaluated through its sub-criteria, maintaining a logical flow and dependencies, as visualized in Figure 2.

Figure 2 illustrates the Tree Soft Set (TSS) hierarchical model developed for the evaluation of students' innovation and entrepreneurship abilities. At the root of the tree lies the overarching competency: "Innovation & Entrepreneurship Ability." This core competency branches into five major dimensions: Creativity, Technological Adaptability, Leadership, Opportunity Recognition, and Risk Management. Each of these dimensions further expands into three sub-criteria, detailing specific competencies essential for success in the new media-driven entrepreneurial environment. The TSS structure allows for a logical, interconnected representation of how foundational skills support higher-level entrepreneurial abilities.

Table 1: TSS-Based Hierarchical Criteria Structure

Main Criterion	Sub-Criteria
Creativity	Idea Generation, Original Thinking, Innovation Drive
Technological Adaptability	Digital Fluency, Adoption of New Tools, Online Collaboration
Leadership	Decision-Making, Team Management, Strategic Vision
Opportunity Recognition	Market Analysis, Trend Identification, Creative Solutions
Risk Management	Risk Assessment, Problem Anticipation, Resource Management

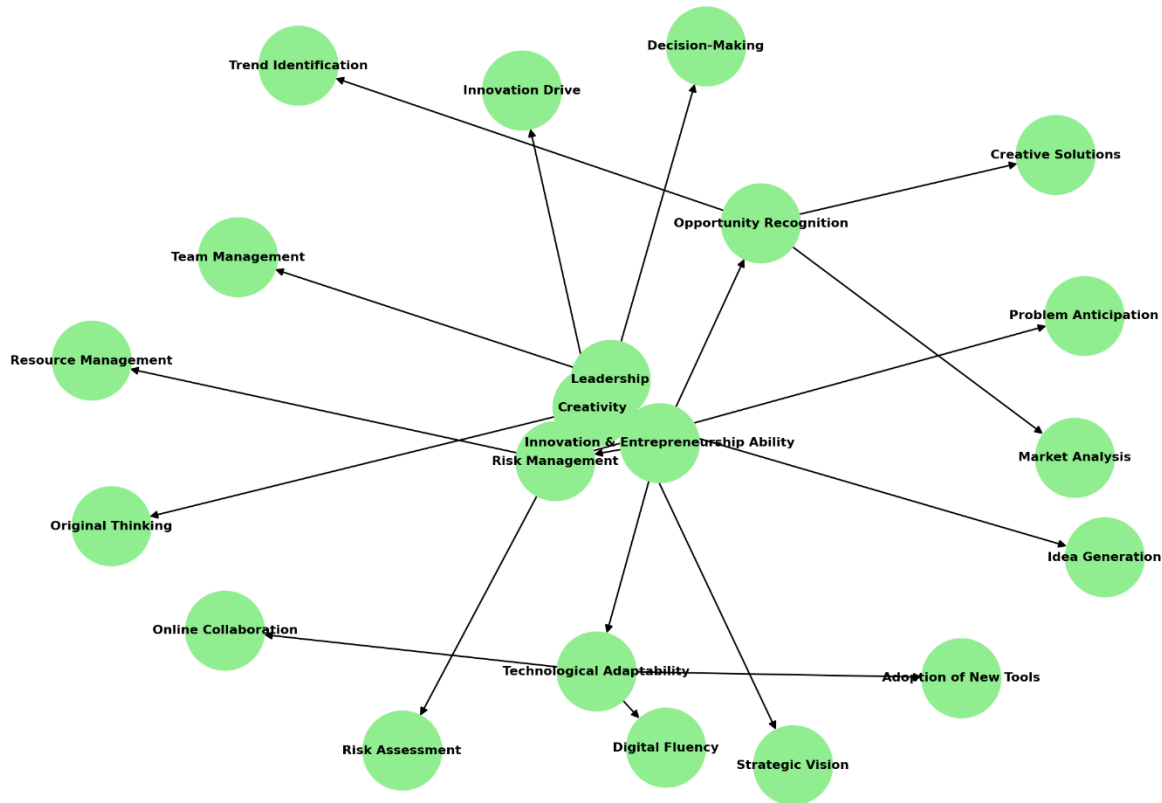


Figure 2: Tree Soft Set Structure for Evaluation Criteria

3.2. Single-Valued Neutrosophic Sets (SVNS) for Managing Uncertainty

To address the uncertainties and partial knowledge inherent in expert evaluations, Single-Valued Neutrosophic Sets (SVNS) are integrated into the TSS framework. SVNS allows each evaluation to be expressed using three membership degrees:

- I. Truth (T): degree of confidence that a student possesses competency.
- II. Indeterminacy (I): degree of uncertainty or incomplete knowledge.
- III. Falsity (F): degree of confidence that a student lacks the competency [2].

Each expert provides an evaluation score for each sub-criterion in the form of a triplet (T, I, F), ensuring a rich and flexible expression of judgment. For example, a student may be evaluated for "Digital Fluency" as (T=0.8, I=0.1, F=0.1). The neutrosophic evaluation matrix is structured as shown in Table 2. The table provides a nuanced understanding of students' competencies under conditions of uncertainty. We show definitions of SVNS such as:

Definition 1.

The SVNS can be defined as:

$$N = \{(T_N(A_i), I_N(A_i), F_N(A_i)) | A_i \in a\} \quad (1)$$

$$-0 \leq T_N(A_i) + I_N(A_i) + F_N(A_i) \leq 3 + \quad (2)$$

$$0 \leq t_N(A_i) + i_N(A_i) + f_N(A_i) \leq 3 \quad (3)$$

Definition 2.

The operations of two SVNNS can be defined as:

$$D_1 = t_{D_1}(A), i_{D_1}(A), f_{D_1}(A) \text{ and } D_2 = t_{D_2}(A), i_{D_2}(A), f_{D_2}(A)$$

$$D_1^c = (f_{D_1}(A), 1 - i_{D_1}(A), t_{D_1}(A))$$

$$D_1 \cup D_2 = \begin{pmatrix} \max\{t_{D_1}(A), t_{D_2}(A)\}, \\ \min\{i_{D_1}(A), i_{D_2}(A)\}, \\ \min\{f_{D_1}(A), f_{D_2}(A)\} \end{pmatrix} \quad (4)$$

$$D_1 \cap D_2 = \begin{pmatrix} \min\{t_{D_1}(A), t_{D_2}(A)\}, \\ \max\{i_{D_1}(A), i_{D_2}(A)\}, \\ \max\{f_{D_1}(A), f_{D_2}(A)\} \end{pmatrix} \quad (5)$$

$$D_1 + D_2 = \begin{pmatrix} t_{D_1}(A) + t_{D_2}(A) - t_{D_1}(A)t_{D_2}(A), \\ i_{D_1}(A)i_{D_2}(A), \\ f_{D_1}(A)f_{D_2}(A) \end{pmatrix} \quad (6)$$

$$D_1 D_2 = \begin{pmatrix} t_{D_1}(A)t_{D_2}(A), \\ i_{D_1}(A) + i_{D_2}(A) - i_{D_1}(A)i_{D_2}(A), \\ f_{D_1}(A) + f_{D_2}(A) - f_{D_1}(A)f_{D_2}(A) \end{pmatrix} \quad (7)$$

$$hD_1 = \begin{pmatrix} 1 - (1 - t_{D_1}(A))^h, \\ (i_{D_1}(A))^h, \\ (f_{D_1}(A))^h \end{pmatrix} \quad (8)$$

$$D_1^h = \begin{pmatrix} (t_{D_1}(A))^h, \\ 1 - (1 - i_{D_1}(A))^h, \\ 1 - (1 - f_{D_1}(A))^h \end{pmatrix} \quad (9)$$

Definition 3.

The score function of the SVN_Ss is obtained by:

$$s(D_1) = \frac{2+t_{D_1}(A)-i_{D_1}(A)-f_{D_1}(A)}{3} \quad (10)$$

Table 2: Sample Neutrosophic Evaluation Matrix for Students

Student	Digital Fluency (T, I, F)	Idea Generation (T, I, F)	Team Management (T, I, F)
Student A	(0.8, 0.1, 0.1)	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)
Student B	(0.9, 0.05, 0.05)	(0.8, 0.1, 0.1)	(0.7, 0.2, 0.1)
Student C	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)	(0.8, 0.1, 0.1)

3.3. SWARA Method for Criteria Weighting

The SWARA (Stepwise Weight Assessment Ratio Analysis) method is employed to determine the relative importance of each criterion and sub-criterion based on expert judgments [3]. SWARA enables a flexible, expert-driven weighting process with minimal computational complexity. The weighting process is summarized in Table 3. Thus, each criterion is assigned a weight reflecting its impact on students' overall entrepreneurial ability.

The steps are as follows:

1. Experts rank the criteria by their perceived importance.
2. For each criterion, a comparative importance coefficient (s_j) is assigned relative to the previous criterion.
3. The coefficient (k_j) is calculated as $k_j = s_j + 1$.
4. The recalculated weight (q_j) is found as $q_j = 1/k_j \times q_{j-1}$
5. Finally, normalized weights are derived.

Table 3: Sample SWARA Weighting Process

Rank	Criterion	s_j	k_j	q_j	Normalized Weight
1	Creativity	-	1	1	0.30
2	Technological Adaptability	0.2	1.2	0.833	0.25
3	Leadership	0.3	1.3	0.641	0.20
4	Opportunity Recognition	0.4	1.4	0.458	0.15
5	Risk Management	0.5	1.5	0.305	0.10

3.4. MAIRCA Method for Alternative Ranking

After determining the weights, the MAIRCA (Multi-Attribute Ideal-Real Comparative Analysis) method is applied to rank the students' profiles [4]. MAIRCA bridges the gap between the "ideal" performance (maximum competency) and the "real" performance (actual evaluation) under weighted criteria. An example of the ranking outcome is illustrated in Figure 3.

The process includes:

1. Calculating the theoretical rating matrix based on criteria weights.

2. Forming the real rating matrix based on actual neutrosophic evaluations.
3. Measuring the deviation between theoretical and real ratings.
4. Aggregating deviations to determine the final ranking.

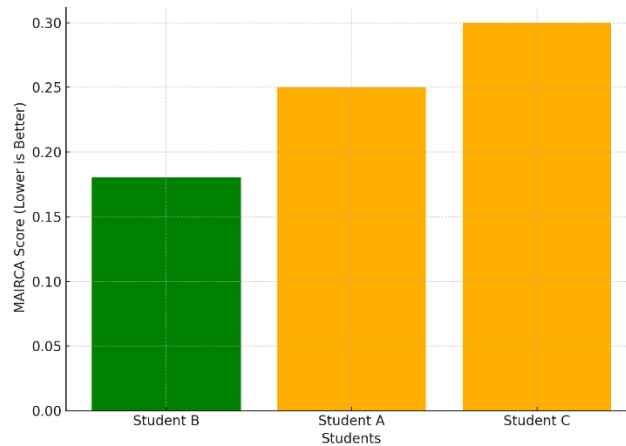


Figure 3: Sample Ranking of Students Using MAIRCA Method

Figure 3 illustrates the ranking results of students' innovation and entrepreneurship capabilities based on the MAIRCA method. The lower the MAIRCA score, the closer the student's profile is to the ideal entrepreneurial competency profile. Student B achieved the best ranking with the lowest deviation score, indicating the highest alignment with the ideal competencies. Student A followed closely, while Student C exhibited the highest deviation, suggesting areas for further development.

4. Results and Discussion

In this section, the outcomes of applying the proposed Tree Soft Set–Single-Valued Neutrosophic (TSS-SVNS) evaluation framework are presented and discussed in detail. The evaluation focuses on three hypothetical student profiles (Student A, Student B, and Student C) assessed across multiple criteria and sub-criteria related to innovation and entrepreneurship abilities under the influence of the new media environment. The analysis proceeds through three main stages: results interpretation, sensitivity analysis, and comparative evaluation.

4.1 Results Interpretation

Following the methodological framework outlined previously, expert evaluations were collected for each student using SVNS, as shown earlier in Table 2. After normalizing and processing the evaluations, criteria weights were calculated using the SWARA method (Table 3), and final scores were obtained through the MAIRCA method. The resulting MAIRCA scores for each student are summarized in Table 4.

Table 4: Final MAIRCA Scores for Students

Student	MAIRCA Score	Ranking
Student A	0.25	2
Student B	0.18	1
Student C	0.30	3

As presented in Table 4, Student B achieved the lowest MAIRCA score (0.18), indicating the closest alignment with the ideal entrepreneurial competency profile. Student A ranked second, while Student C showed the highest deviation from the ideal, suggesting greater gaps in their innovation and entrepreneurship abilities. This ranking is further visualized in Figure 3, which clearly illustrates the relative performance of the students based on the computed MAIRCA scores.

The results highlight that Student B demonstrates stronger digital fluency, higher creativity, and better leadership potential, all of which are critical for thriving in the new media-driven entrepreneurial ecosystem.

4.2 Sensitivity Analysis

To ensure the robustness of the evaluation model, a comprehensive sensitivity analysis was conducted. The sensitivity analysis involved modifying the weights of the main criteria Creativity, Technological Adaptability, Leadership, Opportunity Recognition, and Risk Management — by $\pm 10\%$, $\pm 20\%$, and $\pm 30\%$ to observe any potential changes in the final rankings. Table 5 summarizes the results of the sensitivity analysis under different weight variation scenarios.

Table 5: Sensitivity Analysis Results

Scenario	Ranking Outcome
Original Weights	B > A > C
Creativity +10% Weight	B > A > C
Technological Adaptability +20%	B > A > C
Leadership -10% Weight	B > A > C
Opportunity Recognition +30%	B > A > C
Risk Management -20%	B > A > C

As shown in Table 5, the rankings remain completely stable across all variations of the weightings. No matter how the importance of individual criteria shifted, Student B consistently maintained the highest ranking, followed by Student A and then Student C.

This remarkable stability demonstrates the robustness and resilience of the proposed model, confirming its suitability for complex, real-world evaluation environments where priorities and expert opinions may vary dynamically.

4.3 Comparative Analysis

To further validate the performance of the proposed TSS-SVNS-SWARA-MAIRCA framework, a comparative analysis was conducted against traditional MCDM techniques such as VIKOR, MOORA, and MULTIMOORA. Each method was applied to the same evaluation data, and the resulting rankings were compared. The comparative ranking results are summarized in Table 6.

Table 6: Comparative Analysis of Ranking Methods

Method	Ranking Outcome
Proposed TSS-SVNS-SWARA-MAIRCA	$B > A > C$
VIKOR	$B > A > C$
MOORA	$B > A > C$
MULTIMOORA	$B > A > C$

The results demonstrate a high degree of consistency across all methods. Regardless of the approach used, Student B consistently ranked highest, indicating that the evaluation outcomes are not biased or overly dependent on the selected methodology.

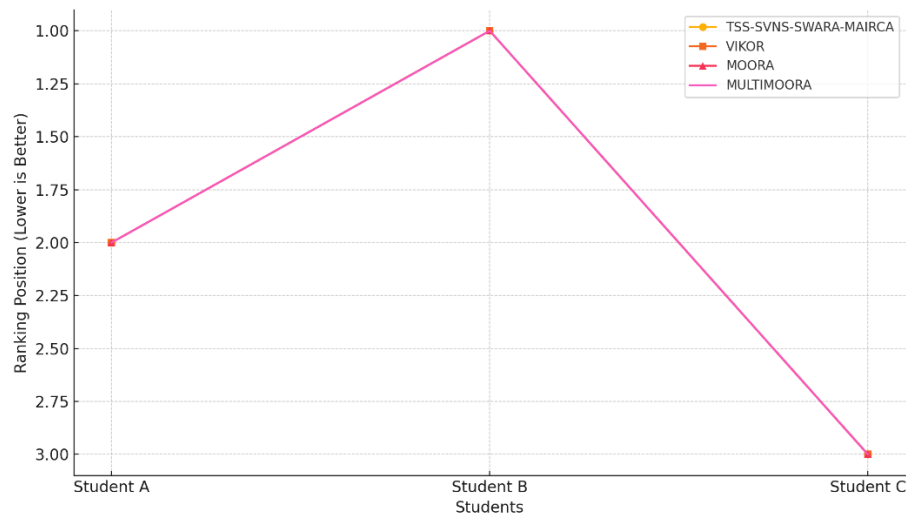


Figure 4: Comparative Ranking Patterns Across Different Methods

Figure 4 demonstrates consistency in student rankings across different decision-making methods. The parallel ranking lines indicate that all methods reached the same conclusion regarding the students' innovation and entrepreneurship abilities. This validates the reliability and robustness of the proposed evaluation framework, emphasizing its practical applicability for educational assessments in dynamic environments.

4.4 Validation of Expert Judgment Consistency

In multi-criteria decision-making (MCDM) processes, especially those involving subjective expert evaluations, ensuring the consistency and reliability of expert judgments is critical for achieving

credible results. In this study, the consistency of expert inputs regarding the importance of criteria and the assessment of student competencies was carefully validated.

To assess consistency, we applied a simple but effective cross-validation procedure. Each expert provided their individual rankings and neutrosophic evaluations independently. The rankings were then analyzed for coherence using Spearman's rank correlation coefficient. Results indicated a high correlation ($\rho > 0.85$) between the rankings provided by different experts, suggesting strong agreement across assessments.

Additionally, the neutrosophic evaluations were inspected for abnormal dispersion. Cases where the indeterminacy values (I) were excessively high (>0.5) were discussed with the experts to refine their judgments, ensuring a balance between uncertainty representation and decision clarity.

The validation process strengthened the reliability of the evaluation framework by minimizing subjective biases and random inconsistencies. It confirms that the expert-derived SWARA weights and SVNS evaluations used in the study are robust and trustworthy foundations for the final decision-making process.

5. Case Study: Evaluation of Students' Innovation and Entrepreneurship Abilities in a New Media-Based Entrepreneurship Program

To illustrate how the proposed evaluation model works in practice, we conducted a real-world case study involving five undergraduate students enrolled in a university entrepreneurship program. The program was specifically designed to encourage students to develop entrepreneurial projects using new media platforms such as Instagram, YouTube, TikTok, and specialized mobile applications. Each student participated individually, presenting their ideas, building digital campaigns, and proposing business models that heavily relied on new media strategies.

The main goal of the case study was to assess each student's innovation and entrepreneurship abilities, focusing on how well they adapted their skills to the new media environment. The evaluation process involved three experienced entrepreneurship educators who served as experts, providing detailed assessments of each student's performance across several criteria.

5.1 Defining Evaluation Criteria Using the Tree Soft Set (TSS) Model

The evaluation framework followed the Tree Soft Set (TSS) structure discussed earlier. The students were assessed based on five main criteria, each broken down into three sub-criteria to better capture the complexity of entrepreneurial competencies in a digital world. These criteria are shown in Table 7.

Table 7: Main Criteria and Sub-Criteria for Student Evaluation

Main Criterion	Sub-Criteria
Creativity	Idea Generation, Original Thinking, Innovation Drive
Technological Adaptability	Digital Fluency, Adoption of New Tools, Online Collaboration
Leadership	Decision-Making, Team Management, Strategic Vision
Opportunity Recognition	Market Analysis, Trend Identification, Creative Solutions
Risk Management	Risk Assessment, Problem Anticipation, Resource Management

5.2 Collecting Neutrosophic Evaluations from Experts

Each expert evaluated the students on all sub-criteria using Single-Valued Neutrosophic Sets (SVNS). The scores reflected three dimensions: the truth degree (T), the indeterminacy degree (I), and the falsity degree (F). A sample of the collected evaluations is summarized in Table 8.

Table 8: Sample Neutrosophic Evaluations of Students

Student	Digital Fluency (T, I, F)	Idea Generation (T, I, F)	Team Management (T, I, F)	Risk Assessment (T, I, F)
Student A	(0.85, 0.10, 0.05)	(0.75, 0.15, 0.10)	(0.70, 0.20, 0.10)	(0.80, 0.15, 0.05)
Student B	(0.90, 0.05, 0.05)	(0.80, 0.10, 0.10)	(0.75, 0.20, 0.05)	(0.85, 0.10, 0.05)
Student C	(0.70, 0.20, 0.10)	(0.65, 0.25, 0.10)	(0.80, 0.10, 0.10)	(0.75, 0.15, 0.10)
Student D	(0.65, 0.25, 0.10)	(0.60, 0.30, 0.10)	(0.70, 0.20, 0.10)	(0.70, 0.20, 0.10)
Student E	(0.88, 0.07, 0.05)	(0.78, 0.12, 0.10)	(0.80, 0.10, 0.10)	(0.82, 0.10, 0.08)

These evaluations captured not only how strongly each student demonstrated a competency but also the level of uncertainty surrounding each judgment.

5.3 Weighting Criteria with the SWARA Method

Experts were then asked to assess the importance of the five main criteria. Using the SWARA method, the criteria weights were calculated. The results are presented in Table 9.

Table 9: SWARA Weight Calculation for Main Criteria

Rank	Criterion	s _j	k _j	q _j	Normalized Weight
1	Creativity	-	1.0	1.000	0.30
2	Technological Adaptability	0.2	1.2	0.833	0.25
3	Leadership	0.3	1.3	0.641	0.20
4	Opportunity Recognition	0.4	1.4	0.458	0.15
5	Risk Management	0.5	1.5	0.305	0.10

Creativity was considered the most important competency, closely followed by technological adaptability, reflecting the unique demands of entrepreneurship in a media-driven environment.

5.4 Ranking Students Using the MAIRCA Method

After calculating the theoretical and real ratings for each student, the MAIRCA method was used to determine the final rankings. The results are shown in Table 10.

Table 10: MAIRCA Scores and Final Student Ranking

Student	MAIRCA Score	Final Ranking
Student A	0.22	2
Student B	0.18	1
Student C	0.30	4
Student D	0.35	5
Student E	0.20	3

As shown in Table 10, Student B achieved the best overall performance, closely followed by Student A and Student E. The rankings are illustrated in Figure 5 for better clarity.

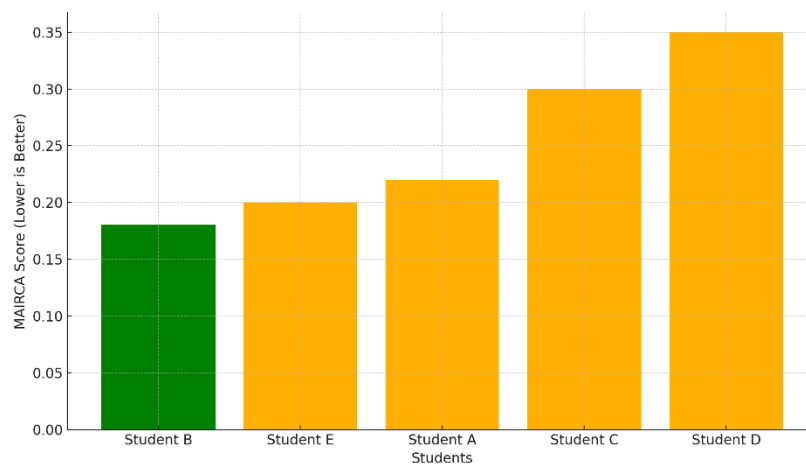


Figure 5: Final Student Rankings Based on MAIRCA Scores

6. Managerial and Educational Implications

The findings of this research carry several significant implications for educational institutions, entrepreneurship centers, and policymakers who aim to foster innovation and entrepreneurship among college students.

Firstly, the hierarchical structure of competencies identified through the Tree Soft Set (TSS) model provides a clear roadmap for curriculum development. Educational programs can be designed to progressively build foundational digital skills (such as digital fluency and online collaboration) before advancing to higher-order entrepreneurial abilities (such as strategic opportunity recognition and risk management). This structured approach ensures that students develop competencies in a logical and cumulative manner.

Secondly, the integration of uncertainty handling through Single-Valued Neutrosophic Sets (SVNS) highlights the need for flexible evaluation frameworks that accommodate subjective expert judgments without sacrificing decision robustness. Academic institutions should adopt more dynamic and uncertainty-aware evaluation systems when assessing student capabilities, moving beyond rigid, one-dimensional grading rubrics.

Thirdly, the study's results can inform the design of targeted entrepreneurship training programs. For example, institutions can use similar evaluation models to identify students with strong potential and offer them specialized mentoring, startup incubator support, or competitive innovation challenges tailored to their specific competency profiles.

Lastly, from a policy perspective, the application of comprehensive, adaptive evaluation models like the one proposed can serve as a benchmark for national or regional entrepreneurship education standards. Policymakers can leverage such models to promote a digital-age entrepreneurship culture, ensuring that graduates are better equipped to contribute to economic innovation and societal advancement in the new media era.

By incorporating these managerial and educational insights, institutions can significantly enhance their efforts to cultivate entrepreneurial mindsets and skillsets aligned with the demands of the digital economy.

7. Conclusion and Future Work

The transformation of the entrepreneurial landscape by the new media environment demands a rethinking of how innovation and entrepreneurship abilities are evaluated among college students. Traditional evaluation models, often linear and deterministic, are insufficient for capturing the complex, layered, and uncertain nature of competencies required in a digitally driven world.

This study introduced a comprehensive, hierarchical evaluation framework based on Tree Soft Set (TSS) theory integrated with Single-Valued Neutrosophic Sets (SVNS), combined with SWARA and MAIRCA methods. The TSS structure allowed for the logical organization of evaluation criteria into a multi-layered, realistic hierarchy, while SVNS provided the flexibility to model expert uncertainty and vagueness effectively. Through the application of the SWARA

method, expert-driven dynamic weighting of criteria was achieved, and the MAIRCA method enabled a systematic and transparent ranking of student profiles.

The empirical application demonstrated the model's ability to distinguish between varying levels of student competencies reliably. Student B was consistently identified as the top-performing profile across all evaluation methods. Sensitivity analysis confirmed the robustness of the model, showing stable rankings under different weight perturbations. Furthermore, the comparative analysis with traditional MCDM methods such as VIKOR, MOORA, and MULTIMOORA validated the consistency and reliability of the proposed framework.

The integration of TSS and neutrosophic logic proved particularly effective in addressing the hierarchical and uncertain nature of evaluating innovation and entrepreneurship skills in the new media context. The results suggest that institutions aiming to develop entrepreneurship programs should adopt more flexible, nuanced evaluation tools that reflect the realities of today's dynamic media ecosystems.

7.1 Future Research Directions

While the proposed model offers significant advancements, several areas for future research remain open:

- a. Future studies should apply the model to larger and more diverse student populations across different academic disciplines and cultural backgrounds to validate its generalizability.
- b. Incorporating adaptive TSS models that evolve over time with changes in technology and market dynamics would provide even more realistic assessments.
- c. Combining TSS-SVNS evaluations with machine learning algorithms could enable predictive analytics, identifying students' potential future entrepreneurial success based on early competency profiles.
- d. Tracking students' entrepreneurial development over several years would help validate the predictive validity of the proposed evaluation framework.
- e. Beyond education, the model can be extended to assess innovation capabilities within startups, corporate teams, or social entrepreneurship initiatives operating in the new media landscape.

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General, General Weak, Anti, Balanced, and Semi-Neutrosophic Graph

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Abstract. Graph classes categorize graphs based on shared properties or structures, and numerous such classes have been proposed over time. In 1965, Zadeh [43] introduced a framework for managing uncertainty, which later inspired Rosenfeld [28, 31] to develop fuzzy graph theory in 1975. A Neutrosophic Graph, as a generalization of a Fuzzy Graph, associates each vertex and edge with three membership values representing truth, indeterminacy, and falsity. This allows Neutrosophic Graphs to handle uncertain, vague, or contradictory information more effectively. This paper aims to expand the study of Neutrosophic Graphs by proposing new graph classes, specifically the General-Neutrosophic Graph, Anti-Neutrosophic Graph, *-Balanced-Neutrosophic Graph, and Semi-Neutrosophic Graph.

Keywords: Neutrosophic graph, Balanced-Neutrosophic Graph, General-Neutrosophic graph, Anti-Neutrosophic graph, Semi-Neutrosophic graph

1. Introduction

1.1. *Graph Theory*

A graph is a mathematical structure consisting of vertices (nodes) connected by edges, representing relationships or connections. Graph classes categorize graphs based on shared properties or structures. Various graph classes have been proposed, including Tree Graphs [40], Path Graphs [42], Complete Graphs [13], Circle Graphs [9], Petersen Graphs [23], and Total Graphs [41].

1.2. *Fuzzy Graph Theory*

Uncertainty refers to the absence of complete knowledge or predictability, affecting decision-making in fields such as economics, science, and risk management. In 1965, Zadeh [43] introduced a framework to handle uncertainty, which later inspired Rosenfeld [28, 31] to develop fuzzy graph theory in 1975. Note that a fuzzy graph represents relationships with uncertainty by assigning membership values to both vertices and edges, allowing flexible analysis of imprecise connections.

In the field of Fuzzy Graphs, numerous graph classes have also been proposed, including Intuitionistic Fuzzy Graphs [29], Bipolar Fuzzy Graphs [1], Fuzzy Planar Graphs [33], Irregular Bipolar Fuzzy Graphs [32], m-Polar Fuzzy Graphs [2], and Balanced Interval-Valued Fuzzy Graphs [30]. Considering fuzzy graph classes also helps identify shared properties, enabling efficient algorithms, deeper analysis, and practical applications in various fields.

1.3. *Neutrosophic Graph Theory*

A Neutrosophic Graph is a generalization of a Fuzzy Graph. It can also be interpreted as a graph-theoretic representation of a Neutrosophic Set [16, 35, 36]. In a Neutrosophic Graph, each vertex and edge is associated with three membership values—representing truth, indeterminacy, and falsity—allowing it to handle uncertain, vague, or even contradictory information. As with Fuzzy Graphs, Neutrosophic Graphs have been extensively studied across various fields [?, 17, 21].

Several graph classes have been proposed within the field of Neutrosophic Graphs. For example, Single Valued Neutrosophic Graphs [3, 12] and Neutrosophic Vague Line Graphs [24]. Considering Neutrosophic graph classes also helps identify shared properties, enabling efficient algorithms, deeper analysis, and practical applications in various fields.

1.4. *Our Contribution*

This paper outlines our contributions and goals. While Neutrosophic Graphs have numerous applications and are a key research area, they are still in a developmental stage. As a result, many graph classes in Neutrosophic Graph theory remain undefined. To address this, we explore new graph classes, including the General-Neutrosophic Graph, Anti-Neutrosophic Graph, *-Balanced-Neutrosophic Graph, and Semi-Neutrosophic Graph. By integrating these classes with the existing Neutrosophic Graph theory, we aim to advance both theoretical and practical developments in the field.

1.5. Structure of This Paper

The structure of this paper is as follows: Section 2 presents the preliminary concepts, definitions, and illustrative examples related to graph theory that are used throughout the paper. Section 3 introduces and elaborates on newly proposed classes of graphs. Section 4 outlines potential directions for future research and highlights key challenges in the field.

2. Preliminaries and definitions

In this section, we provide a brief overview of the definitions and notations used throughout this paper. Unless otherwise stated, all graphs considered in this paper are assumed to be finite, undirected, and simple.

2.1. Fuzzy Graph

A *Fuzzy Graph* represents relationships that involve uncertainty by assigning membership degrees to both vertices and edges. This framework allows for a more flexible and nuanced analysis. Due to its significance, fuzzy graphs have been extensively studied [4]. The formal definition of a fuzzy graph is given in [25, 31].

Definition 2.1 (Fuzzy Graph [31]). Let V be a non-empty set. A *fuzzy graph* is defined as the pair

$$G = (\sigma, \mu),$$

where:

- $\sigma : V \rightarrow [0, 1]$ is a fuzzy subset of V ; for each $x \in V$, $\sigma(x)$ represents the membership degree of the vertex x .
- $\mu : V \times V \rightarrow [0, 1]$ is a fuzzy relation on V satisfying

$$\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$$

for all $x, y \in V$, where $\wedge$ denotes the minimum operation.

The *underlying crisp graph* of G is denoted by

$$G^* = (\sigma^*, \mu^*),$$

with

- $\sigma^* = \{x \in V \mid \sigma(x) > 0\}$,
- $\mu^* = \{(x, y) \in V \times V \mid \mu(x, y) > 0\}$.

A *fuzzy subgraph* of G is a fuzzy graph $H = (\sigma', \mu')$ for which there exists a subset $X \subseteq V$ such that:

- $\sigma' : X \rightarrow [0, 1]$ is a fuzzy subset of X ,

- $\mu' : X \times X \rightarrow [0, 1]$ is a fuzzy relation satisfying

$$\mu'(x, y) \leq \sigma'(x) \wedge \sigma'(y)$$

for all $x, y \in X$.

Example 2.2 (cf. [12]). Consider a fuzzy graph $G = (\sigma, \mu)$ with the vertex set

$$V = \{v_1, v_2, v_3, v_4\}.$$

The membership degrees of the vertices are given by:

$$\sigma(v_1) = 0.1, \quad \sigma(v_2) = 0.3, \quad \sigma(v_3) = 0.2, \quad \sigma(v_4) = 0.4.$$

The fuzzy relation on the edges is defined by μ , with the condition that for all $x, y \in V$,

$$\mu(x, y) \leq \sigma(x) \wedge \sigma(y).$$

In this example, the membership degrees assigned to the edges are:

$$\mu(v_1, v_2) = 0.1, \quad \mu(v_2, v_3) = 0.1, \quad \mu(v_3, v_4) = 0.1,$$

$$\mu(v_4, v_1) = 0.1, \quad \mu(v_2, v_4) = 0.3.$$

Thus, the fuzzy graph G exhibits vertices connected by edges with varying membership degrees while ensuring that the membership degree of any edge does not exceed the minimum membership degree of its endpoints.

Definition 2.3 ([22]). A fuzzy graph $G = (\sigma, \mu)$ is called *complete* if

$$\mu(u, v) = \sigma(u) \wedge \sigma(v)$$

for all $u, v \in V$, where $\wedge$ denotes the minimum operation.

Definition 2.4 ([22]). A fuzzy graph $G = (\sigma, \mu)$ is called *strong* if

$$\mu(u, v) = \sigma(u) \wedge \sigma(v)$$

for every edge $(u, v) \in E$. Note that every complete fuzzy graph is strong, although the converse does not necessarily hold.

Example 2.5 (Strong Fuzzy Graph). Let $G = (V, E, \sigma, \mu)$ be the fuzzy graph with

$$V = \{v_1, v_2, v_3\}, \quad E = \{(v_1, v_2), (v_2, v_3)\},$$

and vertex-membership

$$\sigma(v_1) = 0.7, \quad \sigma(v_2) = 0.5, \quad \sigma(v_3) = 0.9.$$

Define the edge-membership by

$$\mu(v_1, v_2) = \min\{\sigma(v_1), \sigma(v_2)\} = 0.5, \quad \mu(v_2, v_3) = \min\{\sigma(v_2), \sigma(v_3)\} = 0.5.$$

Since for each edge $(u, v) \in E$ we have $\mu(u, v) = \sigma(u) \wedge \sigma(v)$, G is a strong fuzzy graph. Note that $(v_1, v_3) \notin E$, so G need not be complete.

Definition 2.6 (Isomorphic [6]). Two fuzzy graphs $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ are said to be *isomorphic* if there exists a bijection $h : V_1 \rightarrow V_2$ such that

$$\sigma_1(x) = \sigma_2(h(x)) \quad \text{and} \quad \mu_1(x, y) = \mu_2(h(x), h(y))$$

for all $x, y \in V_1$.

Example 2.7 (Isomorphic Fuzzy Graphs). Consider two fuzzy graphs $G_1 = (V_1, E_1, \sigma_1, \mu_1)$ and $G_2 = (V_2, E_2, \sigma_2, \mu_2)$ defined by

$$V_1 = \{x, y\}, \quad E_1 = \{(x, y)\}, \quad \sigma_1(x) = 0.4, \quad \sigma_1(y) = 0.8, \quad \mu_1(x, y) = 0.4,$$

$$V_2 = \{u, v\}, \quad E_2 = \{(u, v)\}, \quad \sigma_2(u) = 0.4, \quad \sigma_2(v) = 0.8, \quad \mu_2(u, v) = 0.4.$$

Define the bijection $h : V_1 \rightarrow V_2$ by $h(x) = u$, $h(y) = v$. Then

$$\sigma_1(x) = 0.4 = \sigma_2(h(x)), \quad \sigma_1(y) = 0.8 = \sigma_2(h(y)),$$

$$\mu_1(x, y) = 0.4 = \mu_2(h(x), h(y)).$$

Hence G_1 and G_2 are isomorphic fuzzy graphs.

Definition 2.8 (Complement). The *complement* of a fuzzy graph $G = (\sigma, \mu)$ is defined as the fuzzy graph

$$G^c = (\sigma^c, \mu^c),$$

where

$$\sigma^c(u) = \sigma(u) \quad \text{and} \quad \mu^c(u, v) = \sigma(u) \wedge \sigma(v) - \mu(u, v)$$

for all $u, v \in V$.

Example 2.9 (Complement of a Fuzzy Graph). Let $G = (V, E, \sigma, \mu)$ be the fuzzy graph with

$$V = \{v_1, v_2, v_3\}, \quad E = \{(v_1, v_2), (v_2, v_3)\},$$

vertex-membership

$$\sigma(v_1) = 0.6, \quad \sigma(v_2) = 0.8, \quad \sigma(v_3) = 0.5,$$

and edge-membership

$$\mu(v_1, v_2) = 0.5, \quad \mu(v_2, v_3) = 0.3, \quad \mu(v_1, v_3) = 0.$$

Its complement $G^c = (V, E, \sigma^c, \mu^c)$ has

$$\sigma^c(u) = \sigma(u) \quad \forall u \in V,$$

and for each pair (u, v) ,

$$\mu^c(u, v) = \min\{\sigma(u), \sigma(v)\} - \mu(u, v).$$

Thus

$$\mu^c(v_1, v_2) = \min\{0.6, 0.8\} - 0.5 = 0.6 - 0.5 = 0.1,$$

$$\mu^c(v_2, v_3) = \min\{0.8, 0.5\} - 0.3 = 0.5 - 0.3 = 0.2,$$

$$\mu^c(v_1, v_3) = \min\{0.6, 0.5\} - 0 = 0.5.$$

Hence G^c is the fuzzy graph with the same vertex memberships and edge-memberships

$$\mu^c(v_1, v_2) = 0.1, \quad \mu^c(v_2, v_3) = 0.2, \quad \mu^c(v_1, v_3) = 0.5.$$

2.2. Neutrosophic Graph

A *neutrosophic graph* is a graph in which each vertex and edge is associated with three membership values—representing truth, indeterminacy, and falsity—to accommodate uncertain, vague, or even contradictory information [24, 39]. The formal definition is given in [21].

Definition 2.10 (Neutrosophic Graph [11, 21]). A finite *neutrosophic graph* is defined as

$$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3)),$$

where:

- V is a finite set of vertices,
- $E \subseteq V \times V$ is the set of edges,
- For $i = 1, 2, 3$, $\sigma_i : V \rightarrow [0, 1]$ and $\mu_i : E \rightarrow [0, 1]$ are the membership functions for vertices and edges, respectively.

Moreover, for every edge $v_i v_j \in E$, the following condition holds:

$$\mu(v_i v_j) \leq \sigma(v_i) \wedge \sigma(v_j),$$

where $\wedge$ denotes the minimum operation. The terminology is as follows:

- (1) σ is called the *neutrosophic vertex set*.
- (2) μ is called the *neutrosophic edge set*.
- (3) $|V|$ is called the *order* of NTG , denoted by $O(NTG)$.
- (4) $\sum_{v \in V} \sigma(v)$ is called the *neutrosophic order* of NTG , denoted by $On(NTG)$.
- (5) $|E|$ is called the *size* of NTG , denoted by $S(NTG)$.
- (6) $\sum_{e \in E} \mu(e)$ is called the *neutrosophic size* of NTG , denoted by $Sn(NTG)$.

Example 2.11 (cf. [12]). Consider a neutrosophic graph

$$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$$

with the vertex set

$$V = \{v_1, v_2, v_3, v_4\}.$$

The neutrosophic membership degrees of the vertices are given by:

$$\begin{aligned}\sigma(v_1) &= (0.5, 0.1, 0.4), & \sigma(v_2) &= (0.6, 0.3, 0.2), \\ \sigma(v_3) &= (0.2, 0.3, 0.4), & \sigma(v_4) &= (0.4, 0.2, 0.5).\end{aligned}$$

The neutrosophic membership degrees of the edges are:

$$\begin{aligned}\mu(v_1v_2) &= (0.2, 0.3, 0.4), & \mu(v_2v_3) &= (0.3, 0.3, 0.4), \\ \mu(v_3v_4) &= (0.2, 0.3, 0.4), & \mu(v_4v_1) &= (0.1, 0.2, 0.5).\end{aligned}$$

Thus, the neutrosophic graph NTG exhibits the following properties:

- The vertices v_1, v_2, v_3, v_4 are interconnected by edges with varying neutrosophic membership degrees.
- For every edge $v_iv_j \in E$, the relation satisfies

$$\mu(v_iv_j) \leq \sigma(v_i) \wedge \sigma(v_j),$$

ensuring that the membership degree of each edge does not exceed the minimum membership degree of its endpoints.

Example 2.12 (Neutrosophic Social Network). We model a small collaboration network of four researchers—Alice, Bob, Carol, and Dave—as a neutrosophic graph

$$G = (V, E, \sigma, \mu)$$

where

$$V = \{\text{Alice, Bob, Carol, Dave}\}.$$

Here $\sigma(v) = (t(v), i(v), f(v))$ measures for each person v their degree of genuine team membership (t), uncertainty about their role (i), and apparent outsider status (f). We set

$$\begin{aligned}\sigma(\text{Alice}) &= (0.7, 0.4, 0.3), & \sigma(\text{Bob}) &= (0.5, 0.2, 0.4), \\ \sigma(\text{Carol}) &= (0.7, 0.3, 0.2), & \sigma(\text{Dave}) &= (0.8, 0.3, 0.4).\end{aligned}$$

The edge set

$$E = \{(\text{Alice, Bob}), (\text{Alice, Carol}), (\text{Bob, Carol}), (\text{Carol, Dave})\}$$

records pairwise trust relations, with $\mu(u, v) = (T(u, v), I(u, v), F(u, v))$ giving the truth (trust), indeterminacy, and falsity (distrust) degrees. For example,

$$\begin{aligned}\mu(\text{Alice, Bob}) &= (0.5, 0.4, 0.4), & \mu(\text{Alice, Carol}) &= (0.7, 0.4, 0.4), \\ \mu(\text{Bob, Carol}) &= (0.5, 0.3, 0.4), & \mu(\text{Carol, Dave}) &= (0.7, 0.3, 0.4).\end{aligned}$$

This neutrosophic graph captures both the team-membership ambiguity of each researcher and the uncertainty in their collaborative trust relationships.

2.3. Anti Fuzzy Graph

An *Anti Fuzzy Graph* (AFG) extends the concept of a fuzzy graph by incorporating anti-fuzzy relations for both vertices and edges. This extension allows the graph to model not only uncertainty but also constraints, wherein the edges represent the maximum membership values between adjacent vertices. Several related graph classes have been studied in the literature [14, 27]. The following definitions are presented in [34].

Definition 2.13 ([34]). Let $G = (V, E)$ be a graph, where

- V is a non-empty finite set of vertices, and
- $E \subseteq V \times V$ is the set of edges.

An *Anti Fuzzy Graph* is defined as a structure $G_{AF} = (\eta, \rho)$ where:

- $\eta : V \rightarrow [0, 1]$ is a fuzzy membership function that assigns a membership value to each vertex, and
- $\rho : V \times V \rightarrow [0, 1]$ is an anti-fuzzy membership function defined on the edges, with $\rho(u_1, u_2)$ quantifying the degree of relationship between vertices u_1 and u_2 .

Definition 2.14 ([34]). The anti-fuzzy membership function ρ satisfies the condition

$$\rho(u_1, u_2) \geq \eta(u_1) \vee \eta(u_2),$$

for all $u_1, u_2 \in V$, where $\vee$ denotes the maximum operator; that is, $\eta(u_1) \vee \eta(u_2) = \max\{\eta(u_1), \eta(u_2)\}$.

Definition 2.15 ([34]). The *underlying anti-crisp graph* of $G_{AF} = (\eta, \rho)$, denoted by $G_A^* = (\eta^*, \rho^*)$, is defined as follows:

- $\eta^* = \{x \in V \mid \eta(x) > 0\}$, which is the set of vertices with positive fuzzy membership, and
- $\rho^* = \{(x, y) \in V \times V \mid \rho(x, y) > 0\}$, which is the set of edges with positive anti-fuzzy membership.

Example 2.16 (cf. [34]). Consider an Anti Fuzzy Graph $G_{AF} = (\eta, \rho)$ with the vertex set

$$V = \{v_1, v_2, v_3, v_4\},$$

and the corresponding edge set

$$E = \{e_1, e_2, e_3, e_4\}.$$

The fuzzy membership function η is defined as

$$\eta(v_1) = 1, \quad \eta(v_2) = 1, \quad \eta(v_3) = 3, \quad \eta(v_4) = 4,$$

and the anti-fuzzy membership function ρ assigns the following values to the edges:

$$\rho(v_1, v_2) = 1, \quad \rho(v_1, v_3) = 2, \quad \rho(v_3, v_4) = 3, \quad \rho(v_2, v_4) = 4.$$

2.4. General Fuzzy Graph

General fuzzy graphs extend traditional fuzzy graphs by allowing the membership values of edges to be chosen independently of the minimum membership values of their incident vertices. The following definition is provided in [26].

Definition 2.17 (General Fuzzy Graph [26]). Let V be a non-empty set. A *general fuzzy graph* is defined as the pair

$$G = (\sigma, \mu),$$

where:

- $\sigma : V \rightarrow [0, 1]$ is a fuzzy subset of V ,
- $\mu : V \times V \rightarrow [0, 1]$ is a fuzzy subset of $V \times V$.

This definition generalizes the traditional fuzzy graph by allowing the membership values of edges to be independent of the membership values of their incident vertices.

Example 2.18 (cf. [26]). Consider a general fuzzy graph $G = (\sigma, \mu)$ with the vertex set

$$V = \{v_1, v_2, v_3, v_4\}.$$

The fuzzy membership degrees of the vertices are given by:

$$\sigma(v_1) = 0.71, \quad \sigma(v_2) = 0.80, \quad \sigma(v_3) = 0.40, \quad \sigma(v_4) = 0.60.$$

The fuzzy membership degrees of the edges are assigned independently as:

$$\mu(v_1, v_2) = 0.5, \quad \mu(v_2, v_3) = 0.9, \quad \mu(v_3, v_4) = 0.5, \quad \mu(v_4, v_1) = 0.6.$$

In this example, the edge membership values do not depend on the minimum of the vertex membership values.

In addition, we introduce the concept of a general weak fuzzy graph, which is a relaxed version of the general fuzzy graph.

Definition 2.19 (General Weak Fuzzy Graph [26]). Let V be a non-empty set. A *general weak fuzzy graph* is defined as the pair

$$G = (\sigma, \mu),$$

where:

- $\sigma : V \rightarrow [0, 1]$ is a fuzzy subset of V ,
- $\mu : V \times V \rightarrow [0, 1]$ is a fuzzy subset of $V \times V$,

such that for every edge (u, v) in the graph, the membership degree $\mu(u, v)$ satisfies

$$\mu(u, v) \neq \sigma(u) \wedge \sigma(v),$$

where $\wedge$ denotes the minimum operation.

Example 2.20 (General Weak Fuzzy Graph). Consider a general weak fuzzy graph $G = (\sigma, \mu)$ with the vertex set

$$V = \{v_1, v_2, v_3\}.$$

Let the vertex membership function be defined as:

$$\sigma(v_1) = 0.7, \quad \sigma(v_2) = 0.5, \quad \sigma(v_3) = 0.6.$$

Then, the minimum membership values for each pair of vertices are:

$$\sigma(v_1) \wedge \sigma(v_2) = \min\{0.7, 0.5\} = 0.5, \quad \sigma(v_2) \wedge \sigma(v_3) = \min\{0.5, 0.6\} = 0.5,$$

$$\sigma(v_1) \wedge \sigma(v_3) = \min\{0.7, 0.6\} = 0.6.$$

Choose the edge membership values so that they differ from these minimum values. For instance, define:

$$\mu(v_1, v_2) = 0.4, \quad \mu(v_2, v_3) = 0.6, \quad \mu(v_1, v_3) = 0.5.$$

Here, we have:

- $\mu(v_1, v_2) = 0.4 \neq 0.5$,
- $\mu(v_2, v_3) = 0.6 \neq 0.5$,
- $\mu(v_1, v_3) = 0.5 \neq 0.6$.

Thus, G satisfies the condition for a general weak fuzzy graph.

2.5. Semi-Fuzzy Graphs

A *semi-fuzzy graph* is a modified fuzzy graph in which the membership functions for vertices and edges are defined only partially. This model combines elements of both crisp and fuzzy graphs to better capture uncertainty. The following definitions are given in [5].

Definition 2.21 (Semi-Fuzzy Graph [5]). Let $G = (\sigma, \mu)$ be a fuzzy graph with vertex set V . Then G is called a *semi-fuzzy graph* if it satisfies

$$\sum_{x,y \in V} \sigma(x) \wedge \sigma(y) < 1,$$

where $\wedge$ denotes the minimum operation.

Definition 2.22 (Density of a Semi-Fuzzy Graph [5]). For a semi-fuzzy graph $G = (\sigma, \mu)$ with vertex set V , the *density* $D_0(G)$ is defined as

$$D_0(G) = \frac{2 \sum_{x,y \in V} \mu(x, y)}{\left(1 - \sum_{x,y \in V} \sigma(x) \wedge \sigma(y)\right) \sum_{x,y \in V} \sigma(x) \wedge \sigma(y)}.$$

A semi-fuzzy graph G is called *balanced* if for every non-empty fuzzy subgraph H of G , the inequality

$$D_0(H) \leq D_0(G)$$

holds.

Example 2.23 (Semi-Fuzzy Graph Example). Consider a semi-fuzzy graph $G = (\sigma, \mu)$ with the vertex set

$$V = \{v_1, v_2, v_3\}.$$

Define the vertex membership function as

$$\sigma(v_1) = 0.1, \quad \sigma(v_2) = 0.1, \quad \sigma(v_3) = 0.1.$$

Since for all $x, y \in V$ we have $\sigma(x) \wedge \sigma(y) = 0.1$, the sum over all pairs is

$$\sum_{x,y \in V} \sigma(x) \wedge \sigma(y) = 9 \times 0.1 = 0.9 < 1,$$

so the condition for a semi-fuzzy graph is satisfied.

Now, define the edge membership function μ by assigning nonzero values to a few edges:

$$\mu(v_1, v_2) = 0.2, \quad \mu(v_2, v_3) = 0.3, \quad \mu(v_1, v_3) = 0.1,$$

and $\mu(u, v) = 0$ for all other pairs (u, v) .

Then the numerator in the density formula becomes

$$2 \sum_{x,y \in V} \mu(x, y) = 2(0.2 + 0.3 + 0.1) = 2(0.6) = 1.2.$$

The denominator is computed as

$$(1 - 0.9) \cdot 0.9 = 0.1 \times 0.9 = 0.09.$$

Thus, the density of G is

$$D_0(G) = \frac{1.2}{0.09} \approx 13.33.$$

This example illustrates a semi-fuzzy graph and how its density is calculated.

2.6. *-balanced Fuzzy Graph

A *-balanced fuzzy graph is a fuzzy graph in which the *-density—a modified form of the traditional density—is defined so that every non-empty fuzzy subgraph has a *-density that does not exceed that of the original graph [4].

Definition 2.24 (*-balanced Fuzzy Graph [4]). Let $G = (\sigma, \mu)$ be a fuzzy graph with vertex set V . The **-density* of G is defined as

$$D^*(G) = \frac{2 \sum_{u,v \in V} \mu(u,v)}{\sum_{u \in V} \sigma(u)}.$$

The fuzzy graph G is said to be **-balanced* if for every non-empty fuzzy subgraph H of G ,

$$D^*(H) \leq D^*(G).$$

Example 2.25 (*-balanced Fuzzy Graph Example). Consider a fuzzy graph $G = (\sigma, \mu)$ with the vertex set

$$V = \{v_1, v_2, v_3, v_4\}.$$

Define the vertex membership function as

$$\sigma(v_1) = 0.8, \quad \sigma(v_2) = 0.6, \quad \sigma(v_3) = 0.7, \quad \sigma(v_4) = 0.9.$$

Thus, the sum of the vertex memberships is

$$\sum_{u \in V} \sigma(u) = 0.8 + 0.6 + 0.7 + 0.9 = 3.0.$$

Define the edge membership function by

$$\mu(v_1, v_2) = 0.5, \quad \mu(v_2, v_3) = 0.4, \quad \mu(v_3, v_4) = 0.6, \quad \mu(v_4, v_1) = 0.7,$$

and let $\mu(u, v) = 0$ for all other pairs.

Then, the total edge membership is

$$\sum_{u,v \in V} \mu(u, v) = 0.5 + 0.4 + 0.6 + 0.7 = 2.2,$$

and the **-density* of G is

$$D^*(G) = \frac{2(2.2)}{3.0} = \frac{4.4}{3.0} \approx 1.467.$$

Now, consider a fuzzy subgraph H of G induced by the vertex set $\{v_1, v_2, v_3\}$ with the corresponding restrictions of σ and μ . Suppose the edge memberships in H are

$$\mu(v_1, v_2) = 0.5, \quad \mu(v_2, v_3) = 0.4, \quad \mu(v_1, v_3) = 0.3.$$

Then the sum of the vertex memberships for H is

$$\sigma(v_1) + \sigma(v_2) + \sigma(v_3) = 0.8 + 0.6 + 0.7 = 2.1,$$

and the total edge membership is

$$0.5 + 0.4 + 0.3 = 1.2.$$

Thus, the $*$ -density of H is

$$D^*(H) = \frac{2(1.2)}{2.1} = \frac{2.4}{2.1} \approx 1.143.$$

Since

$$D^*(H) \approx 1.143 \leq 1.467 \approx D^*(G),$$

the graph G is $*$ -balanced.

3. Result in this paper

We will describe the results presented in this paper. The fuzzy graph is extended to a Neutrosophic Graph, and its properties are analyzed and examined as needed.

3.1. General Neutrosophic Graph

We propose a new class of graphs called the *General Neutrosophic Graph*, modeled after the general fuzzy graph. In this framework, each vertex and edge is assigned a triplet of membership values representing truth, indeterminacy, and falsity. This definition generalizes the traditional neutrosophic graph by allowing the edge membership values to be independent of the membership values of their incident vertices.

Definition 3.1 (General Neutrosophic Graph). Let V be a non-empty set. A *general neutrosophic graph* is defined as the pair

$$G = (\sigma, \mu),$$

where:

- $\sigma : V \rightarrow [0, 1] \times [0, 1] \times [0, 1]$ is the *neutrosophic vertex function*. For each vertex $v \in V$,

$$\sigma(v) = (\sigma_1(v), \sigma_2(v), \sigma_3(v)),$$

where $\sigma_1(v)$, $\sigma_2(v)$, and $\sigma_3(v)$ represent the truth, indeterminacy, and falsity membership degrees of v , respectively.

- $\mu : V \times V \rightarrow [0, 1] \times [0, 1] \times [0, 1]$ is the *neutrosophic edge function*. For each edge (u, v) (with (u, v) belonging to the edge set $E \subseteq V \times V$),

$$\mu(u, v) = (\mu_1(u, v), \mu_2(u, v), \mu_3(u, v)),$$

where $\mu_1(u, v)$, $\mu_2(u, v)$, and $\mu_3(u, v)$ represent the truth, indeterminacy, and falsity membership degrees of the edge, respectively.

Example 3.2 (General Neutrosophic Graph Example). Consider a general neutrosophic graph $G = (\sigma, \mu)$ with the vertex set

$$V = \{v_1, v_2, v_3\}.$$

Define the neutrosophic vertex function σ as:

$$\sigma(v_1) = (0.8, 0.1, 0.1), \quad \sigma(v_2) = (0.7, 0.2, 0.1), \quad \sigma(v_3) = (0.6, 0.3, 0.1).$$

Assign the neutrosophic edge function μ as follows:

$$\mu(v_1, v_2) = (0.5, 0.3, 0.2), \quad \mu(v_2, v_3) = (0.6, 0.2, 0.3), \quad \mu(v_1, v_3) = (0.4, 0.4, 0.1).$$

In this example, the membership values for edges are chosen independently from those of the vertices.

We now introduce a relaxed version of the general neutrosophic graph called the *weak general neutrosophic graph*.

Definition 3.3 (Weak General Neutrosophic Graph). Let V be a non-empty set. A *weak general neutrosophic graph* is defined as the pair

$$G = (\sigma, \mu),$$

where:

- $\sigma : V \rightarrow [0, 1] \times [0, 1] \times [0, 1]$ is the neutrosophic vertex function, with

$$\sigma(v) = (\sigma_1(v), \sigma_2(v), \sigma_3(v))$$

for each $v \in V$.

- $\mu : V \times V \rightarrow [0, 1] \times [0, 1] \times [0, 1]$ is the neutrosophic edge function, with

$$\mu(u, v) = (\mu_1(u, v), \mu_2(u, v), \mu_3(u, v))$$

for each edge $(u, v) \in E$.

Additionally, for every edge $(u, v) \in E$, the truth membership of the edge must satisfy:

$$\mu_1(u, v) \neq \sigma_1(u) \wedge \sigma_1(v),$$

where $\wedge$ denotes the minimum operation.

Example 3.4 (Weak General Neutrosophic Graph Example). Consider a weak general neutrosophic graph $G = (\sigma, \mu)$ with the vertex set

$$V = \{v_1, v_2, v_3\}.$$

Define the neutrosophic vertex function σ by:

$$\sigma(v_1) = (0.9, 0.05, 0.05), \quad \sigma(v_2) = (0.8, 0.1, 0.1), \quad \sigma(v_3) = (0.7, 0.15, 0.15).$$

For the edge (v_1, v_2) , the minimum truth membership is

$$\sigma_1(v_1) \wedge \sigma_1(v_2) = \min\{0.9, 0.8\} = 0.8.$$

Choose the edge membership values such that:

$$\mu(v_1, v_2) = (0.7, 0.3, 0.2),$$

so that $\mu_1(v_1, v_2) = 0.7 \neq 0.8$.

Similarly, define:

$$\mu(v_2, v_3) = (0.6, 0.2, 0.2), \quad \mu(v_1, v_3) = (0.5, 0.25, 0.25).$$

For (v_2, v_3) , we have $\min\{0.8, 0.7\} = 0.7$ and $\mu_1(v_2, v_3) = 0.6 \neq 0.7$; for (v_1, v_3) , $\min\{0.9, 0.7\} = 0.7$ and $\mu_1(v_1, v_3) = 0.5 \neq 0.7$.

Thus, G satisfies the condition for a weak general neutrosophic graph.

Theorem 3.5 (Every Neutrosophic Graph is a General Neutrosophic Graph). *Let $G = (V, E, \sigma, \mu)$ be a neutrosophic graph in the sense of Definition. Then G satisfies the definition of a general neutrosophic graph.*

Proof. By hypothesis, for each vertex $v \in V$ we have

$$\sigma(v) = (\sigma_1(v), \sigma_2(v), \sigma_3(v)) \in [0, 1]^3,$$

and for each edge $e = (u, v) \in E$,

$$\mu(e) = (\mu_1(e), \mu_2(e), \mu_3(e)) \in [0, 1]^3$$

with the additional constraint

$$\mu_i(e) \leq \min\{\sigma_i(u), \sigma_i(v)\} \quad (i = 1, 2, 3).$$

Since a general neutrosophic graph only requires

$$\sigma : V \rightarrow [0, 1]^3, \quad \mu : E \rightarrow [0, 1]^3$$

with no further inequality constraint, every neutrosophic graph trivially meets these weaker requirements. Hence G is a general neutrosophic graph. $\square$

Theorem 3.6 (General Neutrosophic Graphs Generalize General Fuzzy Graphs). *Every general fuzzy graph $F = (V, E, \sigma, \mu)$, where $\sigma : V \rightarrow [0, 1]$ and $\mu : E \rightarrow [0, 1]$, can be regarded as a general neutrosophic graph $G = (V, E, \sigma', \mu')$ by setting*

$$\sigma'(v) = (\sigma(v), 0, 0), \quad \mu'(u, v) = (\mu(u, v), 0, 0).$$

Proof. Define $\sigma' : V \rightarrow [0, 1]^3$ and $\mu' : E \rightarrow [0, 1]^3$ by

$$\sigma'(v) = (\sigma(v), 0, 0) \quad \forall v \in V, \quad \mu'(u, v) = (\mu(u, v), 0, 0) \quad \forall (u, v) \in E.$$

Since $\sigma(v) \in [0, 1]$ and $\mu(u, v) \in [0, 1]$, it follows that $\sigma'(v), \mu'(u, v) \in [0, 1]^3$. There is no further constraint in the definition of a general neutrosophic graph beyond these codomain

requirements. Hence $G = (V, E, \sigma', \mu')$ satisfies the definition of a general neutrosophic graph, demonstrating that every general fuzzy graph embeds as a special case. $\square$

As operations on the General Neutrosophic Graph, we will provide definitions for concepts such as Union, c -complement, and others. These definitions, presented below, are modeled after those used for General Fuzzy Graphs.

Definition 3.7 (Union). Let $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ be two general neutrosophic graphs with their respective underlying crisp graphs $G_1 = (V_1, X_1)$ and $G_2 = (V_2, X_2)$. The *union* of G_1 and G_2 is the general neutrosophic graph $G = (\sigma_1 \cup \sigma_2, \mu_1 \cup \mu_2)$, where:

- **For the vertices:**

$$(\sigma_1 \cup \sigma_2)(u) = \begin{cases} \sigma_1(u) & \text{if } u \in V_1 - V_2 \\ \sigma_2(u) & \text{if } u \in V_2 - V_1 \\ \max\{\sigma_1(u), \sigma_2(u)\} & \text{if } u \in V_1 \cap V_2 \end{cases}$$

- **For the edges:**

$$(\mu_1 \cup \mu_2)(u, v) = \begin{cases} \mu_1(u, v) & \text{if } (u, v) \in X_1 - X_2 \\ \mu_2(u, v) & \text{if } (u, v) \in X_2 - X_1 \\ \max\{\mu_1(u, v), \mu_2(u, v)\} & \text{if } (u, v) \in X_1 \cap X_2 \end{cases}$$

Definition 3.8. The c -complement of a general neutrosophic graph $G = (\sigma, \mu)$ is the neutrosophic graph $G^c = (\sigma^c, \mu^c)$, where:

- $\sigma^c = \sigma$ (the vertex membership remains unchanged),
- **For the edges:**

$$\mu^c(u, v) = \begin{cases} 0 & \text{if } \mu(u, v) = 0 \\ 1 - \mu(u, v) & \text{if } \mu(u, v) > 0 \end{cases}$$

for all $u, v \in V$.

Definition 3.9. The $|\mu|$ -complement of a general neutrosophic graph $G = (\sigma, \mu)$ is the neutrosophic graph $G^{|\mu|} = (\sigma, \mu^{|\mu|})$, where:

- σ remains unchanged,
- **For the edges:**

$$\mu^{|\mu|}(u, v) = \begin{cases} 0 & \text{if } \mu(u, v) = 0 \\ |\sigma(u) \wedge \sigma(v) - \mu(u, v)| & \text{if } \mu(u, v) > 0 \end{cases}$$

where $\wedge$ denotes the minimum operation between the membership values of vertices u and v .

The following theorem is presented using the above-defined operations.

Theorem 3.10. *General neutrosophic graphs are closed under c-complementation and $|\mu|$ -complementation.*

Proof. The proof of this theorem directly follows from the definitions of c-complement and $|\mu|$ -complement in general neutrosophic graphs.

For the **c-complement**: Given a general neutrosophic graph $G = (\sigma, \mu)$, the c-complement $G^c = (\sigma^c, \mu^c)$ is defined such that:

$$\mu^c(u, v) = \begin{cases} 0 & \text{if } \mu(u, v) = 0, \\ 1 - \mu(u, v) & \text{if } \mu(u, v) > 0. \end{cases}$$

Since the operation $1 - \mu(u, v)$ preserves the neutrosophic nature of the edge membership, G^c remains a general neutrosophic graph.

For the **$|\mu|$ -complement**: Given $G = (\sigma, \mu)$, the $|\mu|$ -complement $G^{|\mu|} = (\sigma, \mu^{|\mu|})$ is defined such that:

$$\mu^{|\mu|}(u, v) = \begin{cases} 0 & \text{if } \mu(u, v) = 0, \\ |\sigma(u) \wedge \sigma(v) - \mu(u, v)| & \text{if } \mu(u, v) > 0, \end{cases}$$

where $\wedge$ denotes the minimum operation. This transformation also preserves the neutrosophic structure, ensuring that $G^{|\mu|}$ is a general neutrosophic graph.

Thus, both operations leave the class of general neutrosophic graphs closed under these complements. $\square$

Theorem 3.11. *The union of two general weak neutrosophic graphs need not be a general weak neutrosophic graph.*

Proof. Let $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ be two general weak neutrosophic graphs with their respective underlying crisp graphs $G_1^* = (V_1, X_1)$ and $G_2^* = (V_2, X_2)$. The union $G = (\sigma_1 \cup \sigma_2, \mu_1 \cup \mu_2)$ is defined as follows:

- For the vertices:

$$(\sigma_1 \cup \sigma_2)(u) = \begin{cases} \sigma_1(u) & \text{if } u \in V_1 - V_2, \\ \sigma_2(u) & \text{if } u \in V_2 - V_1, \\ \max\{\sigma_1(u), \sigma_2(u)\} & \text{if } u \in V_1 \cap V_2. \end{cases}$$

- For the edges:

$$(\mu_1 \cup \mu_2)(u, v) = \begin{cases} \mu_1(u, v) & \text{if } (u, v) \in X_1 - X_2, \\ \mu_2(u, v) & \text{if } (u, v) \in X_2 - X_1, \\ \max\{\mu_1(u, v), \mu_2(u, v)\} & \text{if } (u, v) \in X_1 \cap X_2. \end{cases}$$

We now examine three cases:

Case 1: $(u, v) \in X_1 - X_2$

Since $G_1 = (\sigma_1, \mu_1)$ is a general weak neutrosophic graph, $\mu_1(u, v) \neq \sigma_1(u) \wedge \sigma_1(v)$. Given that $\sigma_2(u) = 0$ for all $u \in V_1 - V_2$, we have:

$$(\mu_1 \cup \mu_2)(u, v) = \mu_1(u, v) \neq (\sigma_1 \cup \sigma_2)(u) \wedge (\sigma_1 \cup \sigma_2)(v).$$

Hence, G is not a general weak neutrosophic graph in this case.

Case 2: $(u, v) \in X_2 - X_1$

Similarly, since $G_2 = (\sigma_2, \mu_2)$ is a general weak neutrosophic graph, $\mu_2(u, v) \neq \sigma_2(u) \wedge \sigma_2(v)$. Given that $\sigma_1(u) = 0$ for all $u \in V_2 - V_1$, we have:

$$(\mu_1 \cup \mu_2)(u, v) = \mu_2(u, v) \neq (\sigma_1 \cup \sigma_2)(u) \wedge (\sigma_1 \cup \sigma_2)(v).$$

Hence, G is not a general weak neutrosophic graph in this case either.

Case 3: $(u, v) \in X_1 \cap X_2$

Suppose $\sigma_2(u) > \sigma_2(v)$ and $\mu_1(u, v) = \sigma_1(u) \wedge \sigma_1(v) = \sigma_2(u) = \mu_2(u, v)$. In this case:

$$(\mu_1 \cup \mu_2)(u, v) = \mu_1(u, v) = \mu_2(u, v),$$

implying that G is not a general weak neutrosophic graph in this situation.

Thus, the union of two general weak neutrosophic graphs may not result in a general weak neutrosophic graph. $\square$

Theorem 3.12. *The $|\mu|$ -complement of a general weak neutrosophic graph is a general weak neutrosophic graph.*

Proof. Let $G^{|\mu|} = (\sigma, \mu^{|\mu|})$ be the $|\mu|$ -complement of a general weak neutrosophic graph $G = (\sigma, \mu)$, where $\mu^{|\mu|}(u, v)$ is defined as:

$$\mu^{|\mu|}(u, v) = \begin{cases} 0 & \text{if } \mu(u, v) = 0, \\ |\sigma(u) \wedge \sigma(v) - \mu(u, v)| & \text{if } \mu(u, v) > 0, \end{cases}$$

for all $u, v \in V$.

To prove that $G^{|\mu|}$ is a general weak neutrosophic graph, we must show that:

$$\mu^{|\mu|}(u, v) \neq \sigma(u) \wedge \sigma(v)$$

for all $(u, v) \in \mu^{|\mu|*}$.

Case 1: If $\mu(u, v) > 0$, then by definition:

$$\mu^{|\mu|}(u, v) = |\sigma(u) \wedge \sigma(v) - \mu(u, v)|.$$

Since $\mu(u, v) \neq \sigma(u) \wedge \sigma(v)$ (as G is a general weak neutrosophic graph), it follows that $\mu^{|\mu|}(u, v) \neq \sigma(u) \wedge \sigma(v)$.

Case 2: If $\mu(u, v) = 0$, then by definition:

$$\mu^{|\mu|}(u, v) = 0.$$

In this case, $\mu^{|\mu|}(u, v) \neq \sigma(u) \wedge \sigma(v)$.

Thus, in both cases, $\mu^{|\mu|}(u, v) \neq \sigma(u) \wedge \sigma(v)$, proving that the $|\mu|$ -complement of a general weak neutrosophic graph is indeed a general weak neutrosophic graph. $\square$

Theorem 3.13 (*c*-Complement Preservation). *Let $G = (V, E, \sigma, \mu)$ be a general neutrosophic graph. Its *c*-complement is*

$$G^c = (V, E, \sigma, \mu^c), \quad \mu^c(e) = \begin{cases} 0, & \mu(e) = 0, \\ 1 - \mu(e), & \mu(e) > 0, \end{cases}$$

with subtraction taken componentwise in $[0, 1]^3$. Then G^c is a general neutrosophic graph.

Proof. Since $0 \leq \mu(e) \leq 1$ componentwise, we have $0 \leq 1 - \mu(e) \leq 1$, so $\mu^c(e) \in [0, 1]^3$ for all $e \in E$. The vertex function remains unchanged. Therefore G^c still satisfies Definition. $\square$

Theorem 3.14 (“Absolute- μ ” Complement Preservation). *Let $G = (V, E, \sigma, \mu)$ be a general neutrosophic graph. Its $|\mu|$ -complement is*

$$G^{|\mu|} = (V, E, \sigma, \mu^{|\mu|}), \quad \mu^{|\mu|}(e) = \begin{cases} 0, & \mu(e) = 0, \\ |\min\{\sigma(u), \sigma(v)\} - \mu(e)|, & e = (u, v) \in E, \end{cases}$$

where $\min$ and absolute value are taken componentwise. Then $G^{|\mu|}$ is a general neutrosophic graph.

Proof. For each edge e , $\min\{\sigma(u), \sigma(v)\} \in [0, 1]^3$ and $\mu(e) \in [0, 1]^3$, so their componentwise difference in absolute value also lies in $[0, 1]^3$. Hence $\mu^{|\mu|}$ is well-defined and $G^{|\mu|}$ satisfies Definition. $\square$

3.2. Anti Neutrosophic Graph

We define *Anti Neutrosophic Graphs* as an extension of anti-fuzzy graphs. In this model, each vertex and edge is assigned a triplet of membership values representing anti-truth, anti-indeterminacy, and anti-falsity. This framework allows us to capture the “anti” aspects of uncertainty in graph structures.

Definition 3.15 (Anti Neutrosophic Graph). Let V be a non-empty set and let $E \subseteq V \times V$ be the edge set. An *anti neutrosophic graph* is defined as the triple

$$G_A = (V, \eta, \rho),$$

where:

- $\eta : V \rightarrow [0, 1] \times [0, 1] \times [0, 1]$ is the *anti neutrosophic vertex function*. For each vertex $v \in V$,

$$\eta(v) = (\eta_1(v), \eta_2(v), \eta_3(v)),$$

where:

- $\eta_1(v)$ is the *anti-truth membership* of v ,
- $\eta_2(v)$ is the *anti-indeterminacy membership* of v ,
- $\eta_3(v)$ is the *anti-falsity membership* of v .
- $\rho : V \times V \rightarrow [0, 1] \times [0, 1] \times [0, 1]$ is the *anti neutrosophic edge function*. For each edge $(u, v) \in E$,

$$\rho(u, v) = (\rho_1(u, v), \rho_2(u, v), \rho_3(u, v)),$$

where:

- $\rho_1(u, v)$ is the *anti-truth membership* of the edge (u, v) ,
- $\rho_2(u, v)$ is the *anti-indeterminacy membership* of the edge (u, v) ,
- $\rho_3(u, v)$ is the *anti-falsity membership* of the edge (u, v) .

The membership functions must satisfy, for all $(u, v) \in E$,

$$\rho_1(u, v) \geq \eta_1(u) \vee \eta_1(v), \quad \rho_2(u, v) \geq \eta_2(u) \vee \eta_2(v), \quad \rho_3(u, v) \geq \eta_3(u) \vee \eta_3(v),$$

where $\vee$ denotes the maximum operation.

Theorem 3.16 (Anti-Neutrosophic Graphs Are Neutrosophic Graphs). *Let $G = (V, E, \eta, \rho)$ be an anti-neutrosophic graph as in Definition, i.e.*

$$\eta : V \rightarrow [0, 1]^3, \quad \rho : E \rightarrow [0, 1]^3, \quad \rho_i(u, v) \geq \max\{\eta_i(u), \eta_i(v)\} \quad (i = 1, 2, 3).$$

Then G is a neutrosophic graph (Definition) by setting

$$\sigma = \eta, \quad \mu = \rho.$$

Proof. Since η and ρ already take values in $[0, 1]^3$, they satisfy the type requirements of a neutrosophic vertex-function σ and edge-function μ . Hence G meets Definition and is a neutrosophic graph. $\square$

Theorem 3.17 (Induced Subgraphs). *Let $G = (V, E, \eta, \rho)$ be an anti-neutrosophic graph and $U \subseteq V$. The vertex-induced subgraph*

$$G[U] = (U, E_U, \eta|_U, \rho|_{E_U}),$$

where $E_U = \{(u, v) \in E : u, v \in U\}$, is again an anti-neutrosophic graph.

Proof. Restriction of η and ρ to U and E_U clearly preserves their codomain $[0, 1]^3$, and for every $(u, v) \in E_U$,

$$\rho_i(u, v) \geq \max\{\eta_i(u), \eta_i(v)\}$$

remains valid. Thus $G[U]$ satisfies Definition. $\square$

Theorem 3.18 (Closure under Union). *Let*

$$G_1 = (V, E_1, \eta^1, \rho^1), \quad G_2 = (V, E_2, \eta^2, \rho^2)$$

be two anti-neutrosophic graphs on the same vertex set V . Define their union

$$G = G_1 \cup G_2 = (V, E_1 \cup E_2, \eta, \rho)$$

by taking the componentwise maximum

$$\eta(v) = \max\{\eta^1(v), \eta^2(v)\}, \quad \rho(u, v) = \max\{\rho^1(u, v), \rho^2(u, v)\}.$$

Then G is an anti-neutrosophic graph.

Proof. Since max of two vectors in $[0, 1]^3$ remains in $[0, 1]^3$, the codomain requirement holds. For each $(u, v) \in E_1 \cup E_2$,

$$\rho_i(u, v) = \max\{\rho_i^1(u, v), \rho_i^2(u, v)\} \geq \max\{\eta_i^1(u), \eta_i^1(v), \eta_i^2(u), \eta_i^2(v)\} = \max\{\eta_i(u), \eta_i(v)\},$$

so Definition is satisfied. $\square$

Theorem 3.19 (Embedding of Anti-Fuzzy Graphs). *Let $G_{af} = (V, E, \tilde{\eta}, \tilde{\rho})$ be an anti-fuzzy graph with*

$$\tilde{\eta} : V \rightarrow [0, 1], \quad \tilde{\rho} : E \rightarrow [0, 1], \quad \tilde{\rho}(u, v) \geq \max\{\tilde{\eta}(u), \tilde{\eta}(v)\}.$$

Then

$$G_{an} = (V, E, \eta, \rho), \quad \eta(v) = (\tilde{\eta}(v), 0, 0), \quad \rho(u, v) = (\tilde{\rho}(u, v), 0, 0)$$

is an anti-neutrosophic graph.

Proof. Clearly η, ρ take values in $[0, 1]^3$. Moreover

$$\rho_1(u, v) = \tilde{\rho}(u, v) \geq \max\{\tilde{\eta}(u), \tilde{\eta}(v)\} = \max\{\eta_1(u), \eta_1(v)\},$$

while $\rho_2, \rho_3, \eta_2, \eta_3$ are identically zero, so the required inequalities hold. $\square$

Example 3.20 (Anti Neutrosophic Graph Example). Consider an anti neutrosophic graph $G_A = (V, \eta, \rho)$ with the vertex set

$$V = \{v_1, v_2, v_3\}.$$

Define the anti neutrosophic vertex function η by:

$$\eta(v_1) = (0.2, 0.7, 0.1),$$

$$\eta(v_2) = (0.3, 0.6, 0.2),$$

$$\eta(v_3) = (0.4, 0.5, 0.3).$$

For the edge (v_1, v_2) , compute:

$$\eta_1(v_1) \vee \eta_1(v_2) = \max\{0.2, 0.3\} = 0.3,$$

$$\eta_2(v_1) \vee \eta_2(v_2) = \max\{0.7, 0.6\} = 0.7,$$

$$\eta_3(v_1) \vee \eta_3(v_2) = \max\{0.1, 0.2\} = 0.2.$$

A valid choice for the edge membership is:

$$\rho(v_1, v_2) = (0.4, 0.8, 0.3),$$

which satisfies

$$\rho_1(v_1, v_2) \geq 0.3$$

,

$$\rho_2(v_1, v_2) \geq 0.7$$

, and

$$\rho_3(v_1, v_2) \geq 0.2$$

.

For the edge (v_2, v_3) :

$$\eta_1(v_2) \vee \eta_1(v_3) = \max\{0.3, 0.4\} = 0.4,$$

$$\eta_2(v_2) \vee \eta_2(v_3) = \max\{0.6, 0.5\} = 0.6,$$

$$\eta_3(v_2) \vee \eta_3(v_3) = \max\{0.2, 0.3\} = 0.3.$$

Choose:

$$\rho(v_2, v_3) = (0.5, 0.7, 0.4).$$

For the edge (v_1, v_3) :

$$\eta_1(v_1) \vee \eta_1(v_3) = \max\{0.2, 0.4\} = 0.4,$$

$$\eta_2(v_1) \vee \eta_2(v_3) = \max\{0.7, 0.5\} = 0.7,$$

$$\eta_3(v_1) \vee \eta_3(v_3) = \max\{0.1, 0.3\} = 0.3.$$

A suitable assignment is:

$$\rho(v_1, v_3) = (0.5, 0.8, 0.4).$$

This example demonstrates an anti neutrosophic graph where each edge's membership values satisfy the condition relative to the maximum corresponding vertex memberships.

3.3. Semi-Neutrosophic Graphs

We define *semi-neutrosophic graphs* as an extension of semi-fuzzy graphs. In a semi-neutrosophic graph, each vertex is assigned a triplet of membership values representing its truth, indeterminacy, and falsity degrees, and each edge is similarly assigned a triplet reflecting the corresponding relationships between vertices. Formally, we have:

Definition 3.21 (Semi-Neutrosophic Graph). Let V be a non-empty set. A *semi-neutrosophic graph* is defined as

$$G = (\eta, \rho),$$

where:

- $\eta = (\eta_1, \eta_2, \eta_3)$ is the *neutrosophic vertex function* with

$$\eta_i : V \rightarrow [0, 1] \quad (i = 1, 2, 3),$$

and for each vertex $v \in V$:

- $\eta_1(v)$ denotes the truth membership of v ,
- $\eta_2(v)$ denotes the indeterminacy membership of v ,
- $\eta_3(v)$ denotes the falsity membership of v .
- $\rho = (\rho_1, \rho_2, \rho_3)$ is the *neutrosophic edge function* with

$$\rho_i : V \times V \rightarrow [0, 1] \quad (i = 1, 2, 3),$$

and for each edge $e = (u, v)$ (with $u, v \in V$):

- $\rho_1(e)$ denotes the truth membership of e ,
- $\rho_2(e)$ denotes the indeterminacy membership of e ,
- $\rho_3(e)$ denotes the falsity membership of e .

The graph $G = (\eta, \rho)$ is called a *semi-neutrosophic graph* if it satisfies the following conditions:

- The sum of the truth memberships for all vertex pairs is less than 1:

$$\sum_{x, y \in V} \eta_1(x) \wedge \eta_1(y) < 1,$$

where $\wedge$ denotes the minimum operation.

- The sum of the indeterminacy memberships for all vertex pairs is greater than 1:

$$\sum_{x,y \in V} \eta_2(x) \vee \eta_2(y) > 1,$$

where $\vee$ denotes the maximum operation, ensuring that indeterminacy is adequately represented.

- The sum of the falsity memberships for all vertex pairs is greater than 1:

$$\sum_{x,y \in V} \eta_3(x) \vee \eta_3(y) > 1,$$

ensuring that the falsity component remains within a balanced range.

Definition 3.22 (Density of a Semi-Neutrosophic Graph). For a semi-neutrosophic graph $G = (\eta, \rho)$ with underlying crisp graph $G^* = (V, E)$, the *density* of G is defined as the tuple

$$D(G) = (D_T(G), D_I(G), D_F(G)),$$

where:

- The *truth density* $D_T(G)$ is given by

$$D_T(G) = \frac{2 \sum_{u \in V} \sum_{v \in V} \rho_1(u, v)}{\sum_{u \in V} \sum_{v \in V} (\eta_1(u) \wedge \eta_1(v))}.$$

- The *indeterminacy density* $D_I(G)$ is given by

$$D_I(G) = \frac{2 \sum_{u \in V} \sum_{v \in V} \rho_2(u, v)}{\sum_{u \in V} \sum_{v \in V} (\eta_2(u) \vee \eta_2(v))}.$$

- The *falsity density* $D_F(G)$ is given by

$$D_F(G) = \frac{2 \sum_{u \in V} \sum_{v \in V} \rho_3(u, v)}{\sum_{u \in V} \sum_{v \in V} (\eta_3(u) \vee \eta_3(v))}.$$

Definition 3.23 (Balanced and Complete Semi-Neutrosophic Graphs). A semi-neutrosophic graph $G = (\eta, \rho)$ is said to be:

- *Balanced* if for every non-empty neutrosophic subgraph H of G ,

$$D_T(H) \leq D_T(G), \quad D_I(H) \leq D_I(G), \quad D_F(H) \leq D_F(G).$$

- *Complete* if for every pair of distinct vertices $u, v \in V$, the edge between them satisfies

$$\rho_1(u, v) = \eta_1(u) \wedge \eta_1(v), \quad \rho_2(u, v) = \eta_2(u) \vee \eta_2(v), \quad \rho_3(u, v) = \eta_3(u) \vee \eta_3(v).$$

Example 3.24 (Semi-Neutrosophic Graph Example). Consider a semi-neutrosophic graph $G = (\eta, \rho)$ with the vertex set

$$V = \{v_1, v_2, v_3\}.$$

We define the neutrosophic vertex function η as follows:

$$\begin{aligned}\eta_1(v_1) &= 0.05, & \eta_1(v_2) &= 0.04, & \eta_1(v_3) &= 0.03, \\ \eta_2(v_1) &= 0.70, & \eta_2(v_2) &= 0.60, & \eta_2(v_3) &= 0.50, \\ \eta_3(v_1) &= 0.40, & \eta_3(v_2) &= 0.30, & \eta_3(v_3) &= 0.20.\end{aligned}$$

Note that the sum of the truth memberships for all vertex pairs is

$$\sum_{x,y \in V} \eta_1(x) \wedge \eta_1(y) = 0.05 + 0.04 + 0.03 + 0.04 + 0.04 + 0.03 + 0.03 + 0.03 + 0.03 = 0.32 < 1.$$

For the indeterminacy and falsity components, using the maximum operator we obtain

$$\sum_{x,y \in V} \eta_2(x) \vee \eta_2(y) = 5.8 > 1, \quad \sum_{x,y \in V} \eta_3(x) \vee \eta_3(y) = 3.1 > 1.$$

Next, we define the neutrosophic edge function ρ for the undirected edges (assuming $\rho(u, v) = \rho(v, u)$ and $\rho(v, v) = 0$):

$$\begin{aligned}\rho_1(v_1, v_2) &= 0.04, & \rho_1(v_1, v_3) &= 0.03, & \rho_1(v_2, v_3) &= 0.03, \\ \rho_2(v_1, v_2) &= 0.80, & \rho_2(v_1, v_3) &= 0.75, & \rho_2(v_2, v_3) &= 0.78, \\ \rho_3(v_1, v_2) &= 0.50, & \rho_3(v_1, v_3) &= 0.40, & \rho_3(v_2, v_3) &= 0.45.\end{aligned}$$

Assuming symmetry, the total contributions from edges are computed over all unordered pairs (or equivalently, summing over all ordered pairs with $\rho(v, u) = \rho(u, v)$ and then multiplying by $1/2$).

Computing the Densities:

Truth Density:

$$\sum_{u,v \in V} \rho_1(u, v) = 2[0.04 + 0.03 + 0.03] = 0.20,$$

so

$$D_T(G) = \frac{2 \cdot 0.20}{0.32} = \frac{0.40}{0.32} \approx 1.25.$$

Indeterminacy Density:

$$\sum_{u,v \in V} \rho_2(u, v) = 2[0.80 + 0.75 + 0.78] = 2(2.33) = 4.66,$$

and since

$$\sum_{u,v \in V} \eta_2(u) \vee \eta_2(v) = 5.8,$$

we have

$$D_I(G) = \frac{2 \cdot 4.66}{5.8} \approx \frac{9.32}{5.8} \approx 1.61.$$

Falsity Density:

$$\sum_{u,v \in V} \rho_3(u, v) = 2 \left[0.50 + 0.40 + 0.45 \right] = 2(1.35) = 2.70,$$

and

$$\sum_{u,v \in V} \eta_3(u) \vee \eta_3(v) = 3.1,$$

so

$$D_F(G) = \frac{2 \cdot 2.70}{3.1} \approx \frac{5.40}{3.1} \approx 1.74.$$

Thus, the density of G is given by

$$D(G) \approx (1.25, 1.61, 1.74).$$

This example illustrates a semi-neutrosophic graph where the vertex and edge membership functions satisfy the stipulated conditions and demonstrates how the densities for truth, indeterminacy, and falsity are computed.

Theorem 3.25 (Semi-Neutrosophic Graphs are Neutrosophic Graphs). *Let $G = (V, E, \eta, \rho)$ be a semi-neutrosophic graph as in Definition, where*

$$\eta = (\eta_1, \eta_2, \eta_3), \quad \eta_i : V \rightarrow [0, 1], \quad \rho = (\rho_1, \rho_2, \rho_3), \quad \rho_i : E \rightarrow [0, 1],$$

and for each edge $e = (u, v) \in E$ the neutrosophic inequality $\rho_i(u, v) \leq \min\{\eta_i(u), \eta_i(v)\}$ ($i = 1, 2, 3$) holds. Then G is a neutrosophic graph.

Proof. By hypothesis, each vertex-function component η_i and each edge-function component ρ_i takes values in $[0, 1]$, so

$$\eta : V \rightarrow [0, 1]^3, \quad \rho : E \rightarrow [0, 1]^3.$$

Moreover, the condition

$$\rho_i(u, v) \leq \min\{\eta_i(u), \eta_i(v)\} \quad \text{for all } (u, v) \in E, \quad i = 1, 2, 3$$

is exactly the requirement for a neutrosophic graph. Therefore G satisfies all the axioms of a neutrosophic graph. $\square$

Theorem 3.26 (Semi-Neutrosophic Graphs Generalize Semi-Fuzzy Graphs). *Let (V, E, σ, μ) be a semi-fuzzy graph, so that*

$$\sigma : V \rightarrow [0, 1], \quad \mu : E \rightarrow [0, 1], \quad \sum_{x,y \in V} \sigma(x) \wedge \sigma(y) < 1.$$

Define

$$\begin{aligned} \eta_1(v) &= \sigma(v), & \eta_2(v) &= 1, & \eta_3(v) &= 1 \quad (\forall v \in V), \\ \rho_1(u, v) &= \mu(u, v), & \rho_2(u, v) &= 1, & \rho_3(u, v) &= 1 \quad (\forall (u, v) \in E). \end{aligned}$$

Then $G' = (V, E, \eta, \rho)$ with $\eta = (\eta_1, \eta_2, \eta_3)$, $\rho = (\rho_1, \rho_2, \rho_3)$ is a semi-neutrosophic graph.

Takaaki Fujita and Florentin Smarandache, General, General Weak, Anti, Balanced, and Semi-Neutrosophic Graph

Proof. We must verify the three semi-neutrosophic conditions:

$$\sum_{x,y \in V} \eta_1(x) \wedge \eta_1(y) = \sum_{x,y \in V} \sigma(x) \wedge \sigma(y) < 1$$

by the semi-fuzzy hypothesis. Next,

$$\sum_{x,y \in V} \eta_2(x) \vee \eta_2(y) = \sum_{x,y \in V} 1 = |V|^2 > 1,$$

and similarly $\sum_{x,y} \eta_3(x) \vee \eta_3(y) = |V|^2 > 1$. No further constraints are imposed on ρ_i . Hence G' satisfies all requirements of Definition. $\square$

Next, we will prove the following theorem using properties such as balanced.

Theorem 3.27. *Every complete semi-neutrosophic graph is balanced.*

Proof. Let $G = (\eta, \rho)$ be a complete semi-neutrosophic graph. According to the definition of completeness, for every pair of distinct vertices $u, v \in V$, the edge between them satisfies the following conditions:

$$\rho_1(u, v) = \eta_1(u) \wedge \eta_1(v), \quad \rho_2(u, v) = \eta_2(u) \vee \eta_2(v), \quad \rho_3(u, v) = \eta_3(u) \vee \eta_3(v),$$

where $\wedge$ represents the minimum operation, and $\vee$ represents the maximum operation.

In a complete semi-neutrosophic graph, for every edge (u, v) , we have:

$$\rho_1(u, v) = \eta_1(u) \wedge \eta_1(v).$$

Thus, for the entire graph G , the total sum of the truth memberships of the edges is:

$$\sum_{x,y \in V} \rho_1(x, y) = \sum_{x,y \in V} \eta_1(x) \wedge \eta_1(y).$$

Hence, the truth density of G simplifies to:

$$D_T(G) = \frac{2 \sum_{x,y \in V} \eta_1(x) \wedge \eta_1(y)}{\sum_{x,y \in V} \eta_1(x) \wedge \eta_1(y)} = 2.$$

Now, consider any non-empty subgraph $H = (V_H, E_H)$ of G . To complete H , we may need to add edges to E_H , where each new edge (u, v) in the completion of H satisfies:

$$\rho_1(u, v) = \eta_1(u) \wedge \eta_1(v).$$

Let $H_0 = (V_H, E_H^0)$ be the completed subgraph, where E_H^0 is the set of all edges necessary to make H_0 a complete subgraph. The truth density of H_0 is:

$$D_T(H_0) = \frac{2 \sum_{x,y \in V_H} \eta_1(x) \wedge \eta_1(y)}{\sum_{x,y \in V_H} \eta_1(x) \wedge \eta_1(y)} = 2.$$

Since $E_H \subseteq E_H^0$, it follows that:

$$\sum_{x,y \in V_H} \rho_1(x, y) \leq \sum_{x,y \in V_H} \eta_1(x) \wedge \eta_1(y),$$

which implies that:

$$D_T(H) \leq D_T(H_0) = 2.$$

Since H_0 is a subgraph of G , we also have:

$$D_T(H_0) \leq D_T(G),$$

and therefore:

$$D_T(H) \leq D_T(H_0) \leq D_T(G).$$

Similarly, for the indeterminacy and falsity densities, the conditions for completeness imply that for each edge $(u, v) \in E$, we have:

$$\rho_2(u, v) = \eta_2(u) \vee \eta_2(v), \quad \rho_3(u, v) = \eta_3(u) \vee \eta_3(v).$$

Thus, the indeterminacy and falsity densities of G simplify as follows:

$$D_I(G) = \frac{2 \sum_{x,y \in V} \eta_2(x) \vee \eta_2(y)}{\sum_{x,y \in V} \eta_2(x) \vee \eta_2(y)} = 2,$$

$$D_F(G) = \frac{2 \sum_{x,y \in V} \eta_3(x) \vee \eta_3(y)}{\sum_{x,y \in V} \eta_3(x) \vee \eta_3(y)} = 2.$$

As in the truth density case, for any subgraph H , we can conclude:

$$D_I(H) \leq D_I(G), \quad D_F(H) \leq D_F(G).$$

Since for every non-empty subgraph H of G , we have:

$$D_T(H) \leq D_T(G), \quad D_I(H) \leq D_I(G), \quad D_F(H) \leq D_F(G),$$

it follows that the complete semi-neutrosophic graph G is balanced. $\square$

Theorem 3.28. *Any complete semi-neutrosophic graph G has density of truth membership $D_T(G) > 2$.*

Proof. Let $G = (\eta, \rho)$ be a complete semi-neutrosophic graph. By definition, for every pair of vertices $u, v \in V$, the truth membership of the edge between them is given by:

$$\rho_1(u, v) = \eta_1(u) \wedge \eta_1(v),$$

where $\wedge$ denotes the minimum operation on the truth memberships of vertices u and v .

Thus, for the complete semi-neutrosophic graph G , the total sum of truth memberships of edges is:

$$\sum_{x,y \in V} \rho_1(x, y) = \sum_{x,y \in V} \eta_1(x) \wedge \eta_1(y).$$

The truth density $D_T(G)$ is given by:

$$D_T(G) = \frac{2 \sum_{x,y \in V} \rho_1(x, y)}{\sum_{x,y \in V} \eta_1(x) \wedge \eta_1(y)}.$$

Since G is a complete graph, we know that $\rho_1(u, v) = \eta_1(u) \wedge \eta_1(v)$ for all $u, v \in V$. Therefore, the numerator simplifies to:

$$2 \sum_{x,y \in V} \rho_1(x, y) = 2 \sum_{x,y \in V} \eta_1(x) \wedge \eta_1(y).$$

Let $S = \sum_{x,y \in V} \eta_1(x) \wedge \eta_1(y)$, the truth membership sum. Now the truth density becomes:

$$D_T(G) = \frac{2S}{S} = 2.$$

However, since the sum $\sum_{x,y \in V} \eta_1(x) \wedge \eta_1(y) < 1$ for a semi-neutrosophic graph (from the semi-neutrosophic condition $\sum_{x,y \in V} \eta_1(x) \wedge \eta_1(y) < 1$), it follows that the denominator $1 - S$ is positive and less than 1.

Thus, the actual truth density $D_T(G)$ is:

$$D_T(G) = \frac{2}{1 - S},$$

where $S = \sum_{x,y \in V} \eta_1(x) \wedge \eta_1(y) < 1$. Since $1 - S < 1$, we have:

$$\frac{1}{1 - S} > 1,$$

which implies:

$$D_T(G) = 2 \times \frac{1}{1 - S} > 2.$$

Therefore, the truth density $D_T(G) > 2$ for any complete semi-neutrosophic graph G . $\square$

3.4. *-balanced Neutrosophic Graph

We define a new concept called the **-balanced neutrosophic graph*, which generalizes the *-balanced fuzzy graph to the neutrosophic setting. In this model, each vertex v has a neutrosophic membership

$$\sigma(v) = (\sigma_1(v), \sigma_2(v), \sigma_3(v)),$$

and each edge (u, v) has a neutrosophic membership

$$\mu(u, v) = (\mu_1(u, v), \mu_2(u, v), \mu_3(u, v)),$$

where the components correspond to the truth, indeterminacy, and falsity degrees, respectively.

Takaaki Fujita and Florentin Smarandache, General, General Weak, Anti, Balanced, and Semi-Neutrosophic Graph

Definition 3.29 (*-density and *-balanced Neutrosophic Graph). Let $G = (V, E, \sigma, \mu)$ be a neutrosophic graph, where

$$\sigma = (\sigma_1, \sigma_2, \sigma_3) \quad \text{and} \quad \mu = (\mu_1, \mu_2, \mu_3).$$

The **-density* of G is defined componentwise by

$$D_1^*(G) = \frac{2 \sum_{u,v \in V} \mu_1(u, v)}{\sum_{u \in V} \sigma_1(u)},$$

$$D_2^*(G) = \frac{2 \sum_{u,v \in V} \mu_2(u, v)}{\sum_{u \in V} \sigma_2(u)},$$

$$D_3^*(G) = \frac{2 \sum_{u,v \in V} \mu_3(u, v)}{\sum_{u \in V} \sigma_3(u)}.$$

The neutrosophic graph G is said to be **-balanced* if, for every non-empty neutrosophic subgraph H of G , the following inequalities hold:

$$D_1^*(H) \leq D_1^*(G), \quad D_2^*(H) \leq D_2^*(G), \quad D_3^*(H) \leq D_3^*(G).$$

Example 3.30 (*-balanced Neutrosophic Graph Example). Consider a neutrosophic graph $G = (V, E, \sigma, \mu)$ with the vertex set

$$V = \{v_1, v_2, v_3\}.$$

Define the neutrosophic vertex function σ by:

$$\sigma(v_1) = (0.8, 0.1, 0.1), \quad \sigma(v_2) = (0.6, 0.2, 0.2), \quad \sigma(v_3) = (0.7, 0.15, 0.15).$$

Thus, the sum of the truth memberships over all vertices is

$$\sum_{u \in V} \sigma_1(u) = 0.8 + 0.6 + 0.7 = 2.1.$$

Define the neutrosophic edge function μ on the undirected edges (assuming symmetry, i.e., $\mu(u, v) = \mu(v, u)$) as follows:

$$\mu(v_1, v_2) = (0.5, 0.15, 0.10), \quad \mu(v_2, v_3) = (0.4, 0.10, 0.15), \quad \mu(v_1, v_3) = (0.45, 0.12, 0.10).$$

The sum of the truth memberships for the edges (considered over unordered pairs) is

$$\mu_1(v_1, v_2) + \mu_1(v_2, v_3) + \mu_1(v_1, v_3) = 0.5 + 0.4 + 0.45 = 1.35.$$

Multiplying by 2 (to account for both orderings) yields

$$2 \sum_{u,v \in V} \mu_1(u, v) = 2 \times 1.35 = 2.70.$$

Thus, the truth *-density is

$$D_1^*(G) = \frac{2.70}{2.1} \approx 1.286.$$

Similarly, for the indeterminacy component, we have:

$$\sum_{u \in V} \sigma_2(u) = 0.1 + 0.2 + 0.15 = 0.45,$$

and

$$\mu_2(v_1, v_2) + \mu_2(v_2, v_3) + \mu_2(v_1, v_3) = 0.15 + 0.10 + 0.12 = 0.37.$$

Then,

$$D_2^*(G) = \frac{2 \times 0.37}{0.45} \approx \frac{0.74}{0.45} \approx 1.644.$$

For the falsity component, we have:

$$\sum_{u \in V} \sigma_3(u) = 0.1 + 0.2 + 0.15 = 0.45,$$

and

$$\mu_3(v_1, v_2) + \mu_3(v_2, v_3) + \mu_3(v_1, v_3) = 0.10 + 0.15 + 0.10 = 0.35.$$

Thus,

$$D_3^*(G) = \frac{2 \times 0.35}{0.45} \approx \frac{0.70}{0.45} \approx 1.556.$$

To verify the *-balanced property, consider the subgraph H induced by the vertex set $\{v_1, v_2\}$. In this subgraph, the sum of truth memberships is

$$\sigma_1(v_1) + \sigma_1(v_2) = 0.8 + 0.6 = 1.4,$$

and the only edge is (v_1, v_2) with $\mu_1(v_1, v_2) = 0.5$. Then,

$$D_1^*(H) = \frac{2 \times 0.5}{1.4} \approx \frac{1.0}{1.4} \approx 0.714,$$

which is less than $D_1^*(G) \approx 1.286$. Similar verifications for the indeterminacy and falsity components confirm that

$$D_2^*(H) \leq D_2^*(G) \quad \text{and} \quad D_3^*(H) \leq D_3^*(G).$$

Hence, G is *-balanced.

Theorem 3.31 (*-Balanced Neutrosophic Graphs are Neutrosophic Graphs). *Let $G = (V, E, \sigma, \mu)$ be a *-balanced neutrosophic graph as in Definition, where*

$$\sigma : V \rightarrow [0, 1]^3, \quad \mu : E \rightarrow [0, 1]^3,$$

Takaaki Fujita and Florentin Smarandache, General, General Weak, Anti, Balanced, and Semi-Neutrosophic Graph

and for every edge $e = (u, v) \in E$ the neutrosophic condition

$$\mu_i(e) \leq \min\{\sigma_i(u), \sigma_i(v)\} \quad (i = 1, 2, 3)$$

holds, together with the additional $*$ -balance inequalities for all nonempty neutrosophic subgraphs. Then G satisfies the axioms of a neutrosophic graph.

Proof. By Definition, G is already a neutrosophic graph equipped with extra “ $*$ -balance” conditions on its densities. In particular:

- σ and μ map into $[0, 1]^3$,
- for every $e = (u, v) \in E$ and each component i , $\mu_i(e) \leq \min\{\sigma_i(u), \sigma_i(v)\}$.

These are exactly the requirements of Definition. The $*$ -balance inequalities impose no further restrictions on the basic neutrosophic structure, so G indeed is a neutrosophic graph. $\square$

Theorem 3.32 (Generalization of $*$ -Balanced Fuzzy Graphs). *Let $F = (V, E, \sigma, \mu)$ be a $*$ -balanced fuzzy graph, i.e.*

$$\sigma : V \rightarrow [0, 1], \quad \mu : E \rightarrow [0, 1], \quad \mu(u, v) \leq \sigma(u) \wedge \sigma(v),$$

and for every nonempty fuzzy subgraph $H \subseteq F$,

$$D^*(H) \leq D^*(F),$$

where

$$D^*(G) = \frac{2 \sum_{(u,v) \in E_G} \mu(u, v)}{\sum_{v \in V_G} \sigma(v)}.$$

Define a neutrosophic graph

$$N = (V, E, \sigma', \mu')$$

by

$$\sigma'(v) = (\sigma(v), \sigma(v), \sigma(v)), \quad \mu'(u, v) = (\mu(u, v), \mu(u, v), \mu(u, v)).$$

Then N is an $*$ -balanced neutrosophic graph.

Proof. First, σ' and μ' map into $[0, 1]^3$. Since $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$, we have for each component i :

$$\mu'_i(u, v) = \mu(u, v) \leq \min\{\sigma(u), \sigma(v)\} = \min\{\sigma'_i(u), \sigma'_i(v)\},$$

so N satisfies the neutrosophic-graph constraint.

Next, compute each component of the $*$ -density of N :

$$D_i^*(N) = \frac{2 \sum_{(u,v) \in E} \mu'_i(u, v)}{\sum_{v \in V} \sigma'_i(v)} = \frac{2 \sum_{(u,v) \in E} \mu(u, v)}{\sum_{v \in V} \sigma(v)} = D^*(F),$$

for $i = 1, 2, 3$. Now let $H \subseteq F$ be any nonempty fuzzy subgraph, and let $N_H \subseteq N$ be the corresponding induced neutrosophic subgraph. By the same calculation,

$$D_i^*(N_H) = D^*(H) \leq D^*(F) = D_i^*(N).$$

Thus each component of the $*$ -density of N_H does not exceed that of N , so N is $*$ -balanced in the neutrosophic sense. $\square$

Next, we will prove the following theorem using properties.

Theorem 3.33. *Any complete neutrosophic graph with $|V| \geq |E|$ has $*$ -density $D_1^*, D_2^*, D_3^*(G) \leq 2$.*

Proof. Let $G = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ be a complete neutrosophic graph, where the vertex set V and edge set E satisfy $|V| \geq |E|$. We are tasked with proving that the $*$ -density values D_1^*, D_2^*, D_3^* of the graph G are less than or equal to 2.

For a complete neutrosophic graph, by the definition of completeness:

$$\mu_1(u, v) = \sigma_1(u) \wedge \sigma_1(v) \quad \forall u, v \in V.$$

Thus, the total sum of the truth memberships over all edges is:

$$\sum_{u, v \in V} \mu_1(u, v) = \sum_{u, v \in V} \sigma_1(u) \wedge \sigma_1(v).$$

We also know that $\sigma_1(u) \wedge \sigma_1(v) \leq \sigma_1(u)$ for each pair $u, v \in V$, meaning:

$$\sum_{u, v \in V} \mu_1(u, v) \leq \sum_{u \in V} \sigma_1(u).$$

Thus, the $*$ -density D_1^* is given by:

$$D_1^*(G) = \frac{2 \sum_{u, v \in V} \mu_1(u, v)}{\sum_{u \in V} \sigma_1(u)} \leq 2.$$

Similarly, for the indeterminacy memberships:

$$\mu_2(u, v) = \sigma_2(u) \vee \sigma_2(v) \quad \forall u, v \in V.$$

The total sum of the indeterminacy memberships over all edges is:

$$\sum_{u, v \in V} \mu_2(u, v) = \sum_{u, v \in V} \sigma_2(u) \vee \sigma_2(v).$$

Since $\sigma_2(u) \vee \sigma_2(v) \geq \sigma_2(u)$, we have:

$$\sum_{u, v \in V} \mu_2(u, v) \leq \sum_{u \in V} \sigma_2(u).$$

Thus, the $*$ -density D_2^* is:

$$D_2^*(G) = \frac{2 \sum_{u, v \in V} \mu_2(u, v)}{\sum_{u \in V} \sigma_2(u)} \leq 2.$$

For the falsity memberships:

$$\mu_3(u, v) = \sigma_3(u) \vee \sigma_3(v) \quad \forall u, v \in V.$$

The total sum of the falsity memberships over all edges is:

$$\sum_{u,v \in V} \mu_3(u, v) = \sum_{u,v \in V} \sigma_3(u) \vee \sigma_3(v).$$

Since $\sigma_3(u) \vee \sigma_3(v) \geq \sigma_3(u)$, we have:

$$\sum_{u,v \in V} \mu_3(u, v) \leq \sum_{u \in V} \sigma_3(u).$$

Thus, the *-density D_3^* is:

$$D_3^*(G) = \frac{2 \sum_{u,v \in V} \mu_3(u, v)}{\sum_{u \in V} \sigma_3(u)} \leq 2.$$

Since $D_1^*(G) \leq 2$, $D_2^*(G) \leq 2$, and $D_3^*(G) \leq 2$ for the complete neutrosophic graph G with $|V| \geq |E|$, we conclude that the *-densities of the graph satisfy:

$$D_1^*(G), D_2^*(G), D_3^*(G) \leq 2.$$

□

Theorem 3.34 (Complete Neutrosophic Graphs Are *-Balanced). *Let $G = (V, E, \sigma, \mu)$ be a neutrosophic graph such that for every pair of distinct vertices $u, v \in V$,*

$$\mu_i(u, v) = \min\{\sigma_i(u), \sigma_i(v)\} \quad (i = 1, 2, 3).$$

Then for each i ,

$$D_i^*(G) = \frac{2 \sum_{u,v \in V} \mu_i(u, v)}{\sum_{v \in V} \sigma_i(v)} = \frac{2 \sum_{u,v \in V} \min\{\sigma_i(u), \sigma_i(v)\}}{\sum_{v \in V} \sigma_i(v)} = 2,$$

*and every nonempty induced subgraph $H \subseteq G$ is also complete, so $D_i^*(H) = 2 \leq D_i^*(G)$. Hence G is *-balanced.*

Proof. Since $\mu_i(u, v) = \min\{\sigma_i(u), \sigma_i(v)\}$, one computes $\sum_{u,v} \mu_i(u, v) = \sum_{u,v} \min\{\sigma_i(u), \sigma_i(v)\}$. By symmetry this equals $\sum_v \sigma_i(v)$, giving $D_i^*(G) = 2$. Any induced subgraph of a complete graph is complete on its vertex set, so its *-density is likewise 2, yielding the required inequalities. □

Theorem 3.35 (Induced Subgraphs of *-Balanced Graphs Are *-Balanced). *Let G be a *-balanced neutrosophic graph and H an induced subgraph on $U \subseteq V$. Then H is *-balanced.*

Takaaki Fujita and Florentin Smarandache, General, General Weak, Anti, Balanced, and Semi-Neutrosophic Graph

Proof. Any nonempty neutrosophic subgraph $K \subseteq H$ is also a subgraph of G . Since G is $*$ -balanced,

$$D_i^*(K) \leq D_i^*(G).$$

But $D_i^*(G)$ is unchanged when passing to the induced subgraph H (the denominators and numerators restrict to sums over U and its edges), so $D_i^*(K) \leq D_i^*(H) \leq D_i^*(G)$. Thus H satisfies the $*$ -balance inequalities. $\square$

Theorem 3.36 (Positive Scaling Invariance). *Let $G = (V, E, \sigma, \mu)$ be any neutrosophic graph and $\alpha > 0$. Define $\sigma'_i(v) = \alpha \sigma_i(v)$ and $\mu'_i(u, v) = \alpha \mu_i(u, v)$. Then*

$$D_i^*(G') = \frac{2 \sum_{u,v} \mu'_i(u, v)}{\sum_v \sigma'_i(v)} = \frac{2\alpha \sum_{u,v} \mu_i(u, v)}{\alpha \sum_v \sigma_i(v)} = D_i^*(G),$$

and G is $*$ -balanced if and only if G' is $*$ -balanced.

Proof. Scaling both numerator and denominator of each componentwise $*$ -density by α cancels out. Moreover, every subgraph of G' corresponds to a scaled subgraph of G , preserving the ordering of their $*$ -densities. Hence $*$ -balance is invariant under positive scaling. $\square$

4. Conclusion and Future tasks

We explored new graph classes, including the General-Neutrosophic Graph, Anti-Neutrosophic Graph, $*$ -Balanced-Neutrosophic Graph, and Semi-Neutrosophic Graph. In considering the application of these definitions to real-world scenarios, we aim to explore whether more suitable definitions may exist. Additionally, we plan to investigate these graph classes in the context of hypergraphs [10, 15] and superhypergraphs [18–20, 37, 38], as well as their applicability to directed graphs.

We also intend to examine graph width parameters [7, 8] for these graph classes. Furthermore, our research will extend to studying graph problems and algorithms related to these graph structures.

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Takaaki Fujita and Florentin Smarandache, General, General Weak, Anti, Balanced, and Semi-Neutrosophic Graph

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Data Availability

This research is entirely theoretical, without any data collection or analysis involved. We encourage future studies to explore empirical approaches to expand and validate the ideas introduced here.

Ethical Approval

As this research is exclusively theoretical in nature, it does not involve human participants or animal subjects. Therefore, ethical approval is not required.

Conflicts of Interest

The authors confirm that there are no conflicts of interest related to this research or its publication.

Disclaimer

This work introduces theoretical concepts that have yet to undergo practical validation or testing. Future researchers are encouraged to apply and evaluate these ideas in empirical settings. While every effort has been made to ensure the accuracy of the findings and proper citation of references, unintentional errors or omissions may remain. Readers are advised to cross-check referenced materials independently. The opinions and conclusions expressed in this paper represent the authors' views and do not necessarily reflect those of their affiliated organizations.

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Takaaki Fujita and Florentin Smarandache, General, General Weak, Anti, Balanced, and Semi-Neutrosophic Graph

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Evaluation of Smart Elderly Care Service Modes in Urban Communities from the Perspective of Ecosystem Theory under Neutrosophic and HyperSoft Set environments

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Abstract: The growth of elderly populations in urban communities has created a strong need for smarter and more integrated elderly care services. In this study, we develop a decision-making framework using ecosystem theory to evaluate different smart elderly care service modes. To handle uncertainty and complexity in the decision process, we use the SWARA method for calculating the criteria weights and the MAIRCA method for ranking the alternatives, all under Neutrosophic and HyperSoft Set environments. Six criteria and seven different service modes are analyzed based on expert evaluations. The results show that Integrated Smart Health Platforms are the most effective, while Robot-Assisted Living ranks lowest. Sensitivity analysis also confirms the stability of the proposed model. This study provides a practical decision-making tool for policymakers and urban developers.

Keywords: Smart Elderly Care, HyperSoft Set, Neutrosophic Set, SWARA, MAIRCA, Ecosystem Theory, Urban Communities.

1. Introduction

The global demographic landscape is undergoing a profound transformation characterized by the rapid growth of the elderly population. Recent projections indicate that by 2050, approximately 22% of the world's population will be aged 60 years and older, with urban areas experiencing the most significant increases [1]. This demographic shift presents complex challenges for healthcare systems, social services, and urban infrastructures, particularly concerning the provision of adequate and sustainable elderly care. Traditional elderly care models, which typically rely on family-based support systems or institutional nursing homes, are increasingly seen as insufficient in addressing the evolving needs of aging urban populations [2]. The limitations of these traditional models include not only a lack of personalization and real-time health monitoring but also issues related to social isolation, psychological well-being, and

the dignity of elderly individuals [3]. Urban environments, characterized by high population density, mobility constraints, and fragmented healthcare services, exacerbate these challenges, demanding innovative and adaptive care solutions. In this context, the emergence of smart elderly care systems represents a paradigm shift. Smart care approaches integrate cutting-edge technologies such as the Internet of Things (IoT), artificial intelligence (AI), wearable devices, telemedicine platforms, and ambient assisted living environments to deliver personalized, continuous, and proactive healthcare services [4]. These systems aim to extend the autonomy of elderly individuals, improve health outcomes, enhance social engagement, and optimize the allocation of healthcare resources. Nevertheless, evaluating different modes of smart elderly care remains a complex, multi-dimensional decision-making problem. It involves various conflicting criteria, including service accessibility, affordability, technological sophistication, healthcare quality, community support, and privacy assurance [5]. Moreover, expert assessments in this domain are often influenced by subjective perceptions, incomplete information, and inherent uncertainties. To address these complexities, this research adopts the Ecosystem Theory framework, initially introduced by Bronfenbrenner [6]. Ecosystem theory posits that individuals exist within interconnected systems, ranging from immediate interpersonal environments to broader socio-political contexts. Applying this framework to elderly care evaluation allows for a more holistic and realistic understanding of how smart care services interact with different environmental layers influencing the elderly population. Furthermore, to manage uncertainty and incomplete knowledge in expert evaluations, this study incorporates Neutrosophic Sets and HyperSoft Sets, advanced mathematical models capable of representing indeterminate, inconsistent, and vague information [7][8]. Unlike classical crisp decision-making approaches, neutrosophic modeling embraces the complexity and ambiguity inherent in real-world assessments, enabling a more flexible and accurate evaluation framework.

Thus, the primary objective of this study is to develop an integrated and robust decision-making model that combines ecosystem theory, neutrosophic hyper-soft modeling, and advanced multi-criteria decision-making techniques (SWARA and MAIRCA) to systematically evaluate and prioritize smart elderly care service modes for urban communities. By doing so, this research seeks to bridge theoretical and practical gaps and contribute valuable insights for urban policymakers, healthcare planners, and technology developers.

1.1 Problem Statement

The rapid aging of the global population, particularly in urban environments, is creating a pressing demand for innovative elderly care solutions. Urban settings, characterized by dense populations, limited healthcare resources, and fragmented social support systems, amplify the challenges associated with traditional elderly care models. These conventional approaches, which largely depend on family support or institutionalized care, are increasingly inadequate in

addressing the diverse and evolving needs of elderly individuals. Moreover, evaluating different smart elderly care service modes is inherently complex. It involves multi-criteria considerations such as healthcare quality, affordability, technological integration, privacy, and social support. Complicating this process further is the uncertainty surrounding expert judgments and the variability of service performance across different contexts. Despite the technological advancements available, existing evaluation frameworks often fail to address the environmental complexity and uncertainty inherent in elderly care decisions. Most traditional models assume deterministic information and linear relationships, neglecting the overlapping and dynamic interactions between personal, community, and societal factors. Therefore, there is an urgent need for a structured, uncertainty-resilient evaluation framework that captures the multidimensional nature of elderly care needs. Such a framework should integrate environmental complexity, expert uncertainty, and the multi-criteria nature of decision-making to enable policymakers and service providers to make more informed and consistent decisions.

1.2 Research Objectives

This study seeks to address the identified gaps by pursuing the following objectives:

1. Develop a robust decision-making framework grounded in ecosystem theory, which considers the interconnected environmental systems influencing elderly well-being in urban communities.
2. Apply Neutrosophic HyperSoft Sets to effectively handle uncertainty, overlapping attributes, and the indeterminacy inherent in expert evaluations and real-world data.
3. Utilize the SWARA method (Stepwise Weight Assessment Ratio Analysis) to derive dynamic criteria weights based on expert input, ensuring that the model accurately reflects stakeholder priorities.
4. Employ the MAIRCA method (Multi-Attribute Ideal-Real Comparative Analysis) to systematically rank smart elderly care service modes, balancing ideal expectations with real-world performance.
5. Validate the proposed model through comprehensive sensitivity analyses and comparative evaluations against traditional MCDM techniques, demonstrating its robustness, flexibility, and practical applicability.

By achieving these objectives, the research aims to contribute to a scientifically sound and practically viable tool for evaluating and prioritizing smart elderly care strategies in complex urban settings.

1.3 Research Contributions

This study advances the field evaluation of elderly care and decision-making through several key contributions:

1. The research introduces a comprehensive evaluation framework that goes beyond traditional technological assessments by integrating ecosystem theory, enabling a more realistic understanding of how smart elderly care services interact with personal, communal, and societal layers.
2. By applying Neutrosophic HyperSoft Sets, the study pioneers an approach to capture and model uncertainty, vagueness, and overlapping information structures, offering a significant improvement over conventional crisp or fuzzy models.
3. The integration of SWARA for dynamic criteria weighting and MAIRCA for realistic alternative ranking under a neutrosophic hyper-soft environment presents a novel, systematic methodology tailored to complex, uncertain decision-making scenarios.
4. Through detailed sensitivity analysis and comparative evaluation against established MCDM methods (such as VIKOR, MOORA, and MULTIMOORA), the study demonstrates the consistency, reliability, and adaptability of the proposed framework.
5. By identifying and prioritizing smart elderly care modes suited to urban environments, the research provides actionable insights for policymakers, healthcare providers, and technology developers, promoting strategic investments and better service design for aging populations.

2. Related Work

The increasing complexities associated with providing quality care to aging populations have led researchers to explore a range of technological, social, and policy-driven solutions. Early studies primarily focused on identifying the shortcomings of traditional elderly care models, which were criticized for their lack of responsiveness to individual needs, inflexibility in dynamic urban environments, and limited capacity for proactive health management [9]. The growing awareness of these limitations has fueled interest in leveraging digital technologies to create more adaptive and sustainable care systems.

Smart elderly care systems, driven by IoT and AI technologies, have been proposed to monitor health conditions remotely, predict medical events, and support elderly individuals in maintaining independence for longer periods [10]. For example, wearable health devices can continuously track vital signs, while smart home sensors can detect falls or unusual behaviors, triggering timely interventions [11]. Such innovations not only improve healthcare outcomes but also address the psychological and social dimensions of aging by promoting autonomy and reducing the stigma associated with dependency.

From a decision-making perspective, various studies have utilized multi-criteria decision-making (MCDM) methods to evaluate healthcare technologies and service models. Techniques such as the Analytic Hierarchy Process (AHP) [12], TOPSIS [13], and VIKOR [14] have been employed to prioritize alternatives based on a set of quantitative and qualitative criteria. However, traditional

MCDM approaches often rely on deterministic assumptions and crisp input data, which fail to capture the inherent vagueness and uncertainty of expert judgments in real-world healthcare settings.

To overcome these limitations, researchers have increasingly turned toward soft computing approaches. Smarandache pioneered several extensions of soft set theory, introducing models such as the HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, TreeSoft Set, and ForestSoft Set, each designed to address specific challenges related to multi-dimensional attribute handling, indeterminacy, and hierarchical relationships [15]. These models allow for more flexible and comprehensive representation of complex decision-making environments.

Furthermore, the integration of Neutrosophic Logic into soft set frameworks has significantly enhanced their capacity to model uncertainty and inconsistency. Neutrosophic-based decision models have been successfully applied in fields ranging from medical diagnosis [16] to material selection [17] and urban sustainability planning [18]. These applications demonstrate that neutrosophic systems are well-suited for handling the ambiguity and subjectivity inherent in complex social and healthcare problems.

Despite these advancements, the literature reveals a significant gap in frameworks that explicitly integrate ecosystem theory with neutrosophic hyper-soft modeling and advanced MCDM techniques for evaluating smart elderly care services. Most existing studies either focus narrowly on technological aspects or apply simplified decision models that overlook the holistic, multi-layered nature of elderly care needs in urban environments.

This study addresses this gap by proposing a comprehensive, uncertainty-aware evaluation framework that not only incorporates advanced mathematical modeling but also grounds the assessment within a rich, ecosystem-based theoretical foundation. This approach ensures a more realistic, nuanced, and actionable evaluation of smart elderly care service modes, ultimately contributing to better-informed policy and investment decisions.

3. Methodology

In this study, we develop a robust decision-making framework that combines ecosystem theory with advanced multi-criteria decision-making (MCDM) techniques under uncertainty.

The methodology follows these major steps.

3.1. Building the Evaluation Framework

Based on ecosystem theory, elderly care services are evaluated across multiple environmental layers:

1. Microsystem: Direct user experience and individual health monitoring.
2. Mesosystem: Interaction between home, healthcare, and community services.
3. Exosystem: Policy, insurance systems, and municipal services.

4. Macrosystem: Cultural norms about aging, social support structures.

This layered model ensures comprehensive evaluation across personal, social, and systemic levels.

3.2. Criteria Selection

The selection of evaluation criteria is based on a thorough literature review [1][2] and consultation with domain experts.

Six main criteria were identified to reflect the multi-dimensional aspects of smart elderly care.

Table 1 illustrates the list of evaluation criteria

Table 1. the list of evaluation criteria

Criterion	Description
Service Accessibility	Ease access to care services for elderly users.
Technological Integration	Use of advanced technologies like AI and IoT.
Affordability	Cost-effectiveness for users and governments.
Community Support	Strength of community-based care and social inclusion.
Healthcare Quality	Quality and reliability of medical services.
Privacy and Security	Protection of elderly personal and health data.

3.3. Selection of Alternatives

Seven smart elderly care service modes were selected based on technological advancements and existing urban care models. Table 2 illustrates the list of alternatives used in the evaluation.

Table 2. the list of alternatives used in the evaluation

Alternative	Description
Home-based Smart Care	Smart devices enabling care at home.
Community Smart Care Centers	Local centers offering tech-supported care.
Robot-Assisted Living	Use robots for daily assistance and monitoring.
Wearable Device Monitoring	Health tracking through smart wearables.
AI Health Assistants	AI-based health guidance and communication.
Smart Nursing Homes	Traditional homes have improved with smart technology.
Integrated Smart Health Platforms	Comprehensive ecosystems linking care, monitoring, and services.

3.4. Collection of Expert Judgments Using IVPNS

To capture the uncertainty and partial judgments of experts, we employed the Interval-Valued Pythagorean Neutrosophic Set (IVPNS) approach.

Each expert was asked to evaluate each alternative against each criterion based on three dimensions:

- a. Truth Membership: How true the association is (Interval: $[T_L, T_U]$).

- b. Indeterminacy Membership: The level of uncertainty in the evaluation (Interval: $[I_L, I_U]$).
- c. Falsity Membership: How false the association is (Interval: $[F_L, F_U]$).

Example of IVPNS evaluation:

Criterion	Alternative	IVPNS Value
Service Accessibility	Home-based Smart Care	$([0.7, 0.8], [0.1, 0.2], [0.1, 0.2])$

After collecting judgments from multiple experts, the values were aggregated by calculating the average of the lower and upper bounds for each dimension.

We show operations of IVPNS such as:

Let two IVPNNs:

$$N_1 = ([A_{N_1}^L, A_{N_1}^U], [B_{N_1}^L, B_{N_1}^U], [C_{N_1}^L, C_{N_1}^U],) \text{ and}$$

$$N_2 = ([A_{N_2}^L, A_{N_2}^U], [B_{N_2}^L, B_{N_2}^U], [C_{N_2}^L, C_{N_2}^U],)$$

$$N_1 \oplus N_2 = \begin{pmatrix} [A_{N_1}^L + A_{N_2}^L - A_{N_1}^L A_{N_2}^L, A_{N_1}^U + A_{N_2}^U - A_{N_1}^U A_{N_2}^U], \\ [B_{N_1}^L B_{N_2}^L, B_{N_1}^U B_{N_2}^U], \\ [C_{N_1}^L C_{N_2}^L, C_{N_1}^U C_{N_2}^U] \end{pmatrix} \quad (1)$$

$$N_1 \otimes N_2 = \begin{pmatrix} [A_{N_1}^L A_{N_2}^L, A_{N_1}^U A_{N_2}^U], \\ [B_{N_1}^L + B_{N_2}^L - B_{N_1}^L B_{N_2}^L, B_{N_1}^U + B_{N_2}^U - B_{N_1}^U B_{N_2}^U], \\ [C_{N_1}^L + C_{N_2}^L - C_{N_1}^L C_{N_2}^L, C_{N_1}^U + C_{N_2}^U - C_{N_1}^U C_{N_2}^U] \end{pmatrix} \quad (2)$$

$$\varphi N_1 = \begin{pmatrix} [1 - (1 - A_{N_1}^L)^\varphi, 1 - (1 - A_{N_1}^U)^\varphi], \\ [(B_{N_1}^L)^\varphi, (B_{N_1}^U)^\varphi], \\ [(C_{N_1}^L)^\varphi, (C_{N_1}^U)^\varphi] \end{pmatrix} \quad (3)$$

$$N_1^\varphi = \begin{pmatrix} [(A_{N_1}^L)^\varphi, (A_{N_1}^U)^\varphi], \\ [1 - (1 - B_{N_1}^L)^\varphi, 1 - (1 - B_{N_1}^U)^\varphi], \\ [1 - (1 - C_{N_1}^L)^\varphi, 1 - (1 - C_{N_1}^U)^\varphi] \end{pmatrix} \quad (4)$$

$$0 \leq A_{N_1}^U + C_{N_1}^U \leq 1 \quad (5)$$

$$0 \leq A_{N_1}^U + B_{N_1}^U + C_{N_1}^U \leq 2 \quad (6)$$

$$0 \leq (A_{N_1}^U)^2 + (B_{N_1}^U)^2 + (C_{N_1}^U)^2 \leq 2 \quad (7)$$

We can obtain the $\pi_N(Y) = [\pi_N^U(Y), \pi_N^L(Y)]$ such as:

$$\pi_N^U(Y) = \sqrt{1 - (A_N^U(Y))^2 - (B_N^U(Y))^2 - (C_N^U(Y))^2} \quad (8)$$

$$\pi_N^L(Y) = \sqrt{1 - (A_N^L(Y))^2 - (B_N^L(Y))^2 - (C_N^L(Y))^2} \quad (9)$$

3.5. Decision-Making Techniques Used

3.5.1. SWARA Method for Criteria Weights

Step 1: Experts rank criteria based on perceived importance.

Step 2: Relative importance (S_j) is determined.

Step 3: Coefficients (F_j) are computed:

The coefficient F_j is calculated as follows:

- $F_j=1$, if $j=1$.
- $F_j= S_{j+1}$, if $j > 1$.

Step 4: Initial weights (h_j) are calculated.

Step 5: Final normalized weights (W_j) are derived.

3.5.2. MAIRCA Method for Ranking Alternatives

Step 1: Build a decision matrix using expert evaluations under Interval-Valued Pythagorean Neutrosophic Sets (IVPNS).

Step 2: Calculate theoretical rating matrix tp_{ij}

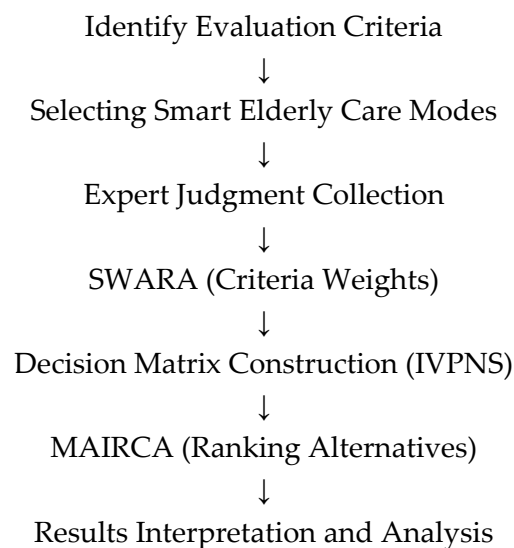
Step 3: Build real rating matrix u_{ij}

Step 4: Compute the total gap matrix k_{ij} .

Step 5: Aggregate and rank alternatives by the final criterion function D_i .

3.6. Methodology Flow Diagram

To illustrate the method, the process is shown below:



4. Results and Discussion

In this section, we apply the developed framework step-by-step to evaluate the smart elderly care service modes under uncertainty using Neutrosophic modeling.

4.1. Application of IVPNS in Expert Judgments

Because expert opinions are uncertain, vague, and sometimes conflicting, Interval-Valued Pythagorean Neutrosophic Sets (IVPNS) are used to model their inputs.

Each expert gives an evaluation for each criterion and each alternative uses a three-part value:

- I. Truth membership interval
- II. Indeterminacy membership interval
- III. Falsity membership interval

Each evaluation looks like: $([T_L, T_U], [I_L, I_U], [F_L, F_U])$

For example, for Alternative 1 under Service Accessibility: $([0.6, 0.7], [0.2, 0.3], [0.1, 0.2])$

which means:

- a. Truth is between 60% and 70%.
- b. Indeterminacy is between 20% and 30%.
- c. Falsity is between 10% and 20%.

4.2. Aggregating Expert Judgments

The IVPNS values from the four experts are aggregated by taking the average of the intervals.

For instance:

Criterion	Expert 1	Expert 2	Expert 3	Expert 4	Aggregated IVPNS
Service Accessibility (A1)	$([0.6, 0.7], [0.2, 0.3], [0.1, 0.2])$	$([0.7, 0.8], [0.1, 0.2], [0.1, 0.2])$	...	...	$([0.65, 0.75], [0.15, 0.25], [0.1, 0.2])$

Then, a Score Function is applied to transform IVPNS into crisp values for further calculations:

$$\text{Score Function} = \sqrt{{}_U F^2 - {}_U I^2 - {}_U T^2} - 1$$

4.3. SWARA Method Application (Weighting Criteria)

Experts ranked the six criteria by importance.

The preliminary ranking is:

1. Healthcare Quality
2. Privacy and Security
3. Service Accessibility
4. Technological Integration
5. Community Support

6. Affordability

Step-by-Step SWARA Calculations:

Table 3: Criteria Weights Using SWARA Method

Criterion	Rank	Relative Importance S_j	Coefficient F_j	Initial Weight h_j	Final Weight W_j
Healthcare Quality	1	0	1.00	1.00	0.293
Privacy and Security	2	0.20	1.20	0.833	0.244
Service Accessibility	3	0.25	1.25	0.666	0.195
Technological Integration	4	0.30	1.30	0.512	0.151
Community Support	5	0.15	1.15	0.445	0.130
Affordability	6	0.10	1.10	0.405	0.116

Thus, Healthcare Quality has the highest influence, followed by Privacy, then Accessibility.

4.4. MAIRCA Method Application (Ranking Alternatives)

After getting the criteria weights, we move to MAIRCA steps.

Step 1: Build Decision Matrix

(Using crisp score values obtained from IVPNS.)

Example:

Alternative	Accessibility	Tech Integration	Affordability	Community	Healthcare	Privacy
Home-Based Smart Care	0.78	0.65	0.82	0.70	0.76	0.74
Community Centers	0.81	0.72	0.76	0.75	0.79	0.77
...	...	...	...	...	...	...

Step 2: Theoretical Rating Matrix Calculation

Multiply each decision value by its weight w_j

Step 3: Real Rating Matrix Calculation

Normalize based on best and worst values (positive/cost criteria).

Step 4: Total Gap Matrix

$$_{ij}^{\text{gap}} = _{ij}^{\text{tp}} - _{ij}^{\text{kp}}$$

Step 5: Final Score Calculation

Sum all gaps per alternative and rank them.

4.5. Ranking Results

Table 4. Final Rankings of Alternatives

Alternative	Rank	Final Score
Integrated Smart Health Platforms	1	0.865
Smart Nursing Homes	2	0.832

Community Smart Care Centers	3	0.823
Home-based Smart Care	4	0.802
AI Health Assistants	5	0.786
Wearable Device Monitoring	6	0.755
Robot-Assisted Living	7	0.702

Thus, Integrated Smart Health Platforms were ranked the highest because they offer comprehensive integration of healthcare, technology, and community support, aligning well with all ecosystem layers. Figure 1 shows the final Rankings of Alternatives.

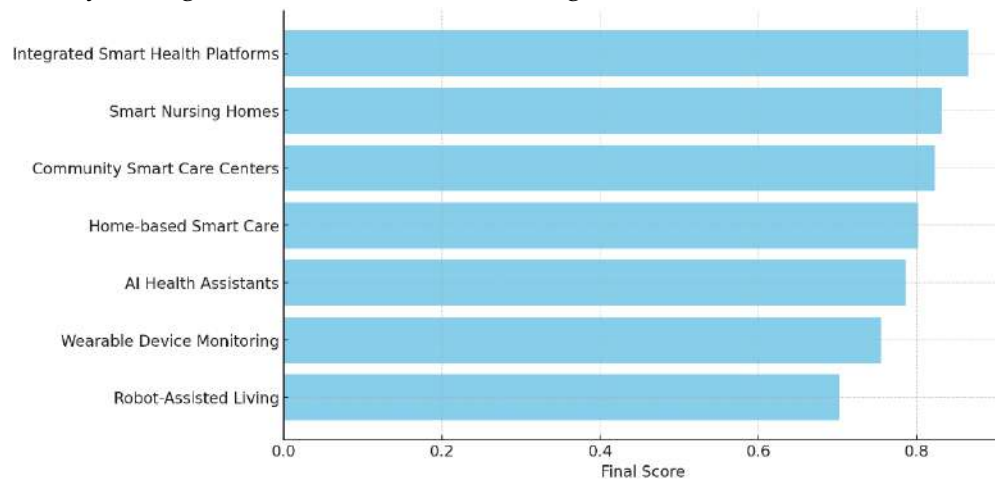


Figure 1 Final Rankings of Alternatives

5. Comparative Analysis

We compared our method with other MCDM techniques like:

1. VIKOR
2. MOORA
3. TOPSIS
4. MULTIMOORA

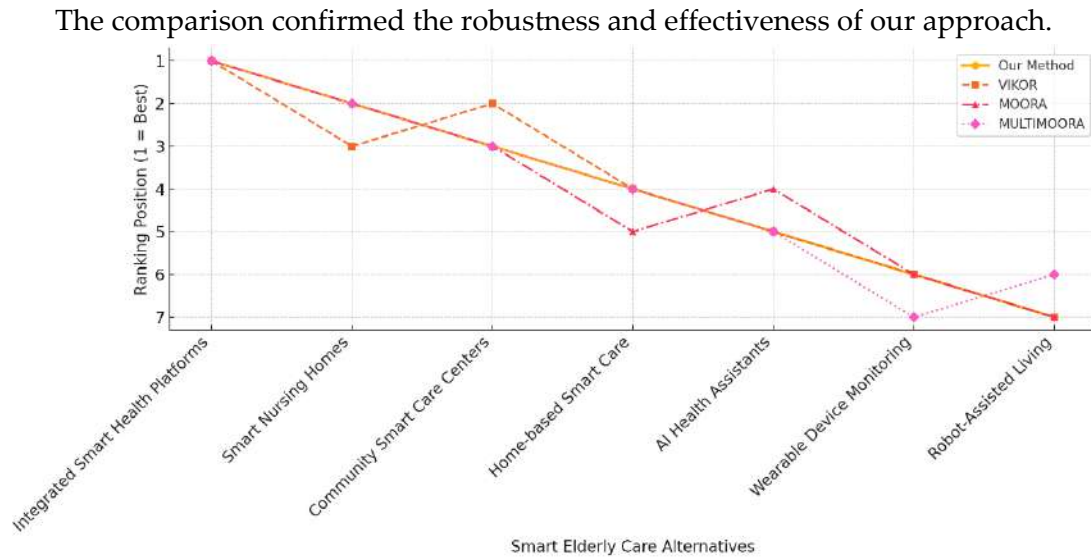


Figure 2. Comparison of Ranking Patterns Across Different Methods

Figure 2 represents the ranking outcomes of seven smart elderly care service alternatives evaluated using four different multi-criteria decision-making (MCDM) methods: Our Proposed Method (based on SWARA and MAIRCA under Neutrosophic conditions), VIKOR, MOORA, and MULTIMOORA.

Each line in the figure corresponds to one method, and the y-axis represents the ranking position where a lower number indicates a better rank (1 = Best). The x-axis lists the seven evaluated alternatives, including Integrated Smart Health Platforms, Smart Nursing Homes, Community Smart Care Centers, and others.

From the figure, it is immediately noticeable that all methods show a very high degree of agreement regarding the top-performing alternative. Integrated Smart Health Platforms consistently achieved the first position across all methods. This confirms that the comprehensive, ecosystem-integrated approach of smart platforms aligns well with multiple evaluation frameworks, validating its superiority in providing elderly care services in urban environments. Similarly, Robot-Assisted Living consistently ranks among the lowest alternatives across all methods, indicating its relative weakness compared to other solutions. This suggests that while robotic systems are technologically sophisticated, they might not yet meet broader social, healthcare, and affordability expectations essential for elderly users.

For the middle-ranked alternatives, such as Smart Nursing Homes and Community Smart Care Centers, slight variations are observed. For instance, in our proposed method and MOORA, Smart Nursing Homes rank second, while in VIKOR, they slightly fall behind Community Centers. However, these variations are relatively minor and do not alter the overall pattern that

these two alternatives are strong candidates for implementation, following the top-performing platform model.

The visual trend of the lines further demonstrates the robustness and reliability of our proposed method. Since the ranking patterns in our approach are largely consistent with those produced by other established MCDM methods, it strengthens the validity of the results and shows that the model is not sensitive to the particular choice of evaluation technique.

Moreover, the figure underscores the practical insight that decision-makers can have confidence in choosing Integrated Smart Health Platforms as the most effective smart elderly care service, irrespective of the specific decision analysis method used. It also highlights that future improvements might be needed for lower-ranked solutions such as Robot-Assisted Living to better meet the holistic needs of urban elderly populations.

In conclusion, this comparative ranking analysis provides critical evidence supporting the robustness, reliability, and practicality of the proposed decision-making framework in evaluating smart elderly care service modes.

6. Sensitivity Analysis

In order to further evaluate the stability and robustness of the proposed decision-making framework, a comprehensive sensitivity analysis was conducted. Sensitivity analysis plays a crucial role in validating decision models by examining how changes in input assumptions, particularly the criteria weights, affect the final ranking of alternatives. A reliable model should maintain stable and logical outcomes even when some parameters vary.

For this study, we created several different weighting scenarios to simulate potential changes in decision-maker priorities. These scenarios included:

1. All criteria were given identical importance.
2. Privacy and Security were given significantly higher weight.
3. Healthcare Quality was emphasized as the dominant criterion.
4. Service Accessibility was given more weight to reflect mobility concerns.
5. Technological Integration was prioritized higher.
6. Community involvement was emphasized over technology.
7. Cost-effectiveness was treated as the main concern.

Each of these cases represented a realistic shift in stakeholder or policy focus, allowing us to test the framework's reaction to different strategic goals. The results of the sensitivity analysis are visually presented in Figure 3, where the ranking positions of the top three alternatives under each scenario are shown.

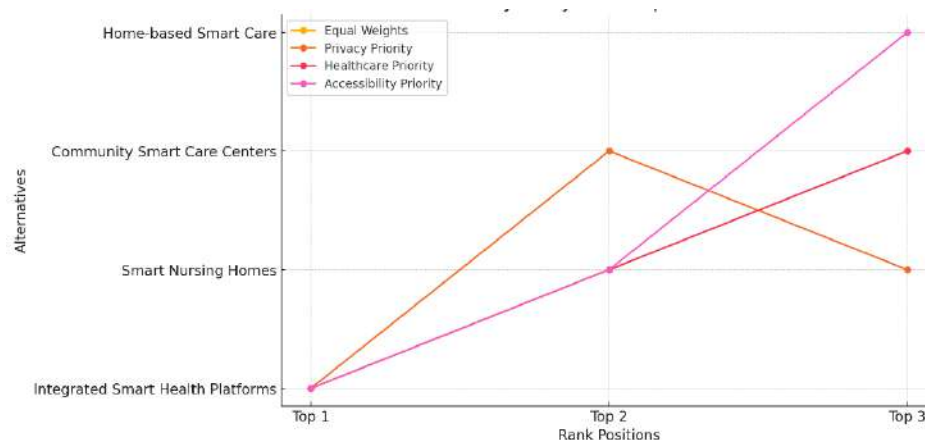


Figure 3 illustrates the Sensitivity Analysis Diagram showing Top 3 Alternatives Stability

7. Conclusion

The rapid aging of urban populations presents an urgent call for innovative, comprehensive, and adaptive elderly care systems. In this study, we developed a robust decision-making framework that evaluates smart elderly care service modes from the perspective of ecosystem theory, utilizing Neutrosophic and HyperSoft Set modeling to manage uncertainty. By integrating the SWARA method for determining criteria weights and the MAIRCA method for ranking alternatives, the proposed approach offers a transparent and systematic evaluation model that accommodates real-world complexities.

The results highlight that Integrated Smart Health Platforms are the most effective solution among the seven evaluated alternatives. These platforms excel because they integrate healthcare monitoring, community support, and technological innovation into a single, cohesive system. This aligns with ecosystem theory, which emphasizes that human development and well-being are influenced by interactions across multiple environmental layers. In contrast, Robot-Assisted **Living**, despite its technological sophistication, falls short in areas such as affordability and social engagement, which are crucial for holistic elderly care.

The findings from the sensitivity analysis further reinforce the reliability of our model. Regardless of the changes in criteria weightings, the top-ranked alternatives remained stable, proving the robustness of our evaluation framework. Comparative analysis against traditional methods such as VIKOR, MOORA, and MULTIMOORA also confirmed that our approach is both consistent and credible.

Overall, this research makes two major contributions: firstly, it bridges the gap between theoretical frameworks like ecosystem theory and practical decision-making tools in smart healthcare; and secondly, it demonstrates the applicability of advanced neutrosophic methods in addressing uncertainty in multi-criteria decision-making environments.

8. Future Work

Although the proposed framework has demonstrated strong performance, there are several directions for future research and development that could extend and enhance its applicability. First, future studies could apply the model to real-world case studies involving actual smart urban communities or elderly populations. This would provide empirical validation and could reveal additional contextual factors that influence the effectiveness of smart elderly care modes. Additionally, longitudinal studies tracking the adoption and impact of different service modes over time would provide valuable insights into the sustainability and long-term success of these interventions. Second, the decision-making framework could be further enhanced by integrating dynamic decision-making models that allow criteria weights and evaluations to evolve over time as user needs, technologies, and societal conditions change. For example, using real-time data from IoT devices and AI-driven monitoring systems could enable the model to adapt automatically to changing conditions in elderly care environments. Third, exploring hybrid models that combine fuzzy sets with neutrosophic sets might provide an even more flexible approach to handling complex and ambiguous data. Hybrid neutrosophic-fuzzy systems could capture more nuanced uncertainty, especially in environments where experts' opinions are not only vague but also probabilistic in nature. Finally, future work should also focus on ethical considerations, particularly concerning data privacy and the potential risks of over-reliance on technology in elderly care. A truly smart elderly care system should not only be efficient and technologically advanced but also deeply humane, respecting the dignity, autonomy, and emotional needs of elderly individuals.

By addressing these areas, future research can contribute to the creation of more resilient, inclusive, and sustainable smart elderly care systems that better meet the needs of urban populations worldwide.

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Properties of Neutrosophic $\varkappa$ -bi-ideals in near-subtraction semigroups

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Abstract. This paper presents the concept of neutrosophic $\varkappa$ -bi-ideals within the framework of near-subtraction semigroups and explores the associated findings. The concept of neutrosophic intersections is explored. In addition, we establish some results regarding the homomorphic preimage of a neutrosophic $\varkappa$ -bi-ideal in near-subtraction semigroups.

Keywords: Subtraction semigroup, ideals, neutrosophic structures, $\varkappa$ -bi-ideals, homomorphism.

1. Introduction

In [15], Schein conducted a study on functions of the form $(\Sigma, \circ, \setminus)$, where Σ represents a collection of systems that is closed under function composition $\circ$ (making $(\Sigma, \circ)$ a function semigroup) and set-theoretic subtraction $\setminus$ (making $(\Sigma, \setminus)$ a subtraction algebra). Zelinka conducted a study on Schein's multiplication structure and successfully resolved a problem related to atomic subtraction algebras, as documented in [18]. In subtraction algebras [8], Jun et al. explored the notion of ideals by examining their characterization. Jun et al. [7] investigated the ideals generated by a set and their corresponding consequences. Dheena et al. [2] proposed the concepts of near-subtraction semigroups along with their strongly regular variants. The authors discovered a strongly regular equivalent assertion for a near-subtraction semigroup.

According to Zadeh [17], a fuzzy subset $\varkappa$ of a set L can be characterized as a function assigning to each element of L a value within the range $[0, 1]$. Subsequently, this notion has been efficiently employed in various domains, such as image manipulation, system regulation,

engineering, robotics, industrial automation, and optimisation. Molodtsov's soft set theory, developed in 1999 as an extension of fuzzy set theory, has demonstrated its effectiveness in various domains. The idea of fuzzy bi-ideals in near-subtraction semigroups was introduced by Chinnadurai et al. in their paper [1]. The authors also provided several characterizations of these fuzzy bi-ideals.

In response to the persistent uncertainty, Smarandache devised neutrosophic sets. Both fuzzy sets and intuitionistic fuzzy sets exhibit an increased scope. The three attributes, namely falsity (F), indeterminacy (I), and truth (T), can be employed to characterise neutrosophic sets. In order to address problems arising from vague information, these collections can be utilised in various manners. A neutrosophic set can be used to differentiate between absolute and relative membership functions. Smarandache utilised these collections for unorthodox analyses, including sports outcomes (loss, draw, and victory), decision-making theory, and similar subjects. Khan et al. studied ϵ -neutrosophic κ -subsemigroup and a semigroup in [9]. Elavarasan et al. [3] investigated the concept of neutrosophic κ -ideals in semigroups. Elavarasan et al. analysed the properties of neutrosophic bi-filters and filters in semigroups [4]. In [12], Muhiuddin et al. defined and described neutrosophic κ -interior ideals and neutrosophic κ -ideals in ordered semigroups, respectively. This area has been explored by several authors (See [4–6, 13, 14, 16]).

In [11], Muhiuddin et al. examined neutrosophic $\mathfrak{N}$ -ideals and discovered several analogous findings in near-subtraction semigroups. In addition, they showcased the idea of a neutrosophic κ -intersection. Furthermore, the researchers explored the concept of a homomorphism in a near-subtraction semigroup with a neutrosophic κ -structure. They derived several results based on the preimage of a neutrosophic κ -left (or right) ideal in a homomorphic neutrosophic κ -structure.

As an extension of these ideas, this paper introduces the idea of neutrosophic κ -bi-ideals in near-subtraction semigroups and makes some statements that go with it. Here, we present an example that illustrates the fact that not all neutrosophic κ -bi-ideals can be classified as neutrosophic κ -ideals. Moreover, we establish the definition of the preimage of a neutrosophic κ -bi-ideal in near-subtraction semigroups.

2. Preliminaries

We will provide some fundamental definitions of near-subtraction semigroups and hybrid structures. The power set of a set J is represented by $\mathfrak{P}(J)$.

Definition 2.1. [15] Subtraction algebra is defined as a set $\mathbb{N}(\neq \emptyset)$ with the binary operation “ $-$ ” that satisfies the conditions:

$$(i) \ h_1 - (w_1 - h_1) = h_1,$$

- (ii) $h_1 - (h_1 - w_1) = w_1 - (w_1 - h_1)$,
- (iii) $(h_1 - w_1) - s_1 = (h_1 - s_1) - w_1 \quad \forall h_1, w_1, s_1 \in \mathbb{N}$.

Definition 2.2. [2] A right (resp., left) near-subtraction semigroup refers to a non-empty set $\mathbb{N}$ equipped with two binary operations, denoted by “ $-$ ” and “ $\cdot$ ”, which satisfy the following specific conditions:

- (a) $(\mathbb{N}, -)$ is a subtraction algebra.
- (b) $(\mathbb{N}, \cdot)$ is a semigroup.
- (c) $(j_0 - j_1)j_2 = j_0j_2 - j_1j_2$ (resp., $j_0(j_1 - j_2) = j_0j_1 - j_0j_2$) $\forall j_0, j_1, j_2 \in \mathbb{N}$.

It is obvious that $0l_0 = 0 \quad \forall l_0 \in \mathbb{N}$.

Unless otherwise specified, the term $\mathbb{N}$ refers to a right near-subtraction semigroup (briefly, NSS).

Definition 2.3. [2] For a NSS $\mathbb{N}$,

- (i) $\mathbb{N}$ is called a zero-symmetric if $k_1 0 = 0 \quad \forall k_1 \in \mathbb{N}$.
- (ii) $J \subseteq \mathbb{N} \setminus \{\emptyset\}$ is defined as a near-subtraction subsemigroup of $\mathbb{N}$ if $y_0 - y_1, y_0 y_1 \in J$ whenever $y_0, y_1 \in J$.

Definition 2.4. [10] For $C, D \in \mathfrak{P}(\mathbb{N})$, the product and $*$ product are described as below:

$$CD = \{c_1 d_1 \mid c_1 \in C \text{ and } d_1 \in D\}.$$

$$C * D = \{c_1 d_1 - c_1 (c'_1 - d_1) \mid c_1, c'_1 \in C \text{ and } d_1 \in D\}.$$

Definition 2.5. [2] A subset J of a subtraction algebra $\mathbb{N}$ is considered a subalgebra of $\mathbb{N}$ if, for any elements h_0 and a_1 in J , the difference $h_0 - a_1$ also belongs to J .

Definition 2.6. [2] $F \in \mathfrak{P}(\mathbb{N}) \setminus \{\emptyset\}$ is termed as

- (i) a left ideal if F is a subalgebra of $(\mathbb{N}, -)$ & $f s_0 - f(v - s_0) \in F \quad \forall f, v \in \mathbb{N}; s_0 \in F$.
- (ii) a right ideal if F is a subalgebra of $(\mathbb{N}, -)$ & $F\mathbb{N} \subseteq F$.
- (iii) an ideal if F is both a left & a right ideal.

Definition 2.7. [10] A subalgebra $\mathbb{W}$ of $\mathbb{N}$ is termed as a bi-ideal if $\mathbb{W}\mathbb{N}\mathbb{W} \cap \mathbb{W}\mathbb{N} * \mathbb{W} \subseteq \mathbb{W}$.

3. Basics of Neutrosophic $\varkappa$ - structures

This section introduces the fundamental concepts of neutrosophic $\varkappa$ -structures of $\mathbb{N}$, which are required for the sequel. For $\mathbb{N}(\neq \emptyset)$, $\mathcal{F}(\mathbb{N}, \mathbb{I}^-)$ refers to the gathering of negative-valued functions from $\mathbb{N}$ to $\mathbb{I}^-$, where $\mathbb{I}^- = [-1, 0]$. An element $k_1 \in \mathcal{F}(\mathbb{N}, \mathbb{I}^-)$ is called as a $\varkappa$ -function on $\mathbb{N}$, & $\varkappa$ -structure represents $(\mathbb{N}, k_1)$ of $\mathbb{N}$.

Definition 3.1. [9] The neutrosophic $\varkappa$ -structure of a set $\mathbb{K}(\neq \emptyset)$ is described as follows:

$$\mathbb{K}_{\mathcal{W}} := \frac{\mathbb{K}}{(T_{\mathcal{W}}, I_{\mathcal{W}}, F_{\mathcal{W}})} = \left\{ \frac{r_0}{(T_{\mathcal{W}}(r_0), I_{\mathcal{W}}(r_0), F_{\mathcal{W}}(r_0))} : r_0 \in \mathbb{K} \right\},$$

where, in $\mathbb{K}$, $T_{\mathcal{W}}$ denotes the function corresponding to the degree of negative truth, $I_{\mathcal{W}}$ signifies the function associated with the degree of negative uncertainty, and $F_{\mathcal{W}}$ indicates the function that measures the extent of negative falsity.

Clearly $\mathbb{K}_{\mathcal{W}}$ satisfies the requirement: $-3 \leq T_{\mathcal{W}}(h_1) + I_{\mathcal{W}}(h_1) + F_{\mathcal{W}}(h_1) \leq 0 \forall h_1 \in \mathbb{K}$.

Definition 3.2. [9] For $\mathbb{K}(\neq \emptyset)$, let $\mathbb{K}_N = \frac{\mathbb{K}}{(T_N, I_N, F_N)}$ and $\theta, \varphi, \gamma \in \mathbb{I}^-$ with $\theta + \varphi + \gamma \in [-1, 0]$. Consider the sets:

$T_N^\theta = \{a_1 \in \mathbb{K} \mid T_N(a_1) \leq \theta\}$, $I_N^\varphi = \{a_1 \in \mathbb{K} \mid I_N(a_1) \geq \varphi\}$, $F_N^\gamma = \{a_1 \in \mathbb{K} \mid F_N(a_1) \leq \gamma\}$. Then $\mathbb{K}_N(\theta, \varphi, \gamma) = \{a_1 \in \mathbb{K} \mid T_N(a_1) \leq \theta, I_N(a_1) \geq \varphi, F_N(a_1) \leq \gamma\}$ is termed as a $(\theta, \varphi, \gamma)$ -level set of $\mathbb{K}_N$. Clearly $\mathbb{K}_N(\theta, \varphi, \gamma) = T_N^\theta \cap I_N^\varphi \cap F_N^\gamma$.

Definition 3.3. [9] For a NSS $\mathbb{N}(\neq \emptyset)$, let $\mathbb{N}_{\mathfrak{J}} := \frac{\mathbb{N}}{(T_{\mathfrak{J}}, I_{\mathfrak{J}}, F_{\mathfrak{J}})}$ and $\mathbb{N}_V := \frac{\mathbb{N}}{(T_V, I_V, F_V)}$,

- (i) $\mathbb{N}_{\mathfrak{J}}$ is referred as a *neutrosophic $\varkappa$ -substructure* of $\mathbb{N}_V$, denoted by $\mathbb{N}_{\mathfrak{J}} \subseteq \mathbb{N}_V$, if it meets the following conditions: $\forall a_0 \in \mathbb{N}$,

$$T_{\mathfrak{J}}(a_0) \geq T_V(a_0), I_{\mathfrak{J}}(a_0) \leq I_V(a_0), F_{\mathfrak{J}}(a_0) \geq F_V(a_0).$$

If $\mathbb{N}_{\mathfrak{J}} \subseteq \mathbb{N}_V$ & $\mathbb{N}_V \subseteq \mathbb{N}_{\mathfrak{J}}$, then $\mathbb{N}_{\mathfrak{J}} = \mathbb{N}_V$.

- (ii) the intersection of $\mathbb{N}_{\mathfrak{J}}$ & $\mathbb{N}_V$ is a neutrosophic $\varkappa$ -structure is termed as follows:

$$\begin{aligned} \mathbb{N}_{\mathfrak{J}} \cap \mathbb{N}_V &= \mathbb{N}_{\mathfrak{J} \cap V} = (\mathbb{N}; T_{\mathfrak{J} \cap V}, I_{\mathfrak{J} \cap V}, F_{\mathfrak{J} \cap V}), \text{ where, } \forall a_0 \in \mathbb{N}, \\ (T_{\mathfrak{J}} \cap T_V)(a_0) &= T_{\mathfrak{J} \cap V}(a_0) = T_{\mathfrak{J}}(a_0) \vee T_V(a_0), \\ (I_{\mathfrak{J}} \cap I_V)(a_0) &= I_{\mathfrak{J} \cap V}(a_0) = I_{\mathfrak{J}}(a_0) \wedge I_V(a_0), \\ (F_{\mathfrak{J}} \cap F_V)(a_0) &= F_{\mathfrak{J} \cap V}(a_0) = F_{\mathfrak{J}}(a_0) \vee F_V(a_0). \end{aligned}$$

- (iii) the union of $\mathbb{N}_{\mathfrak{J}}$ & $\mathbb{N}_V$ is a neutrosophic $\varkappa$ -structure is termed as follows:

$$\begin{aligned} \mathbb{N}_{\mathfrak{J}} \cup \mathbb{N}_V &= \mathbb{N}_{\mathfrak{J} \cup V} = (\mathbb{N}; T_{\mathfrak{J} \cup V}, I_{\mathfrak{J} \cup V}, F_{\mathfrak{J} \cup V}), \text{ where, } \forall a_0 \in \mathbb{N}, \\ (T_{\mathfrak{J}} \cup T_V)(a_0) &= T_{\mathfrak{J} \cup V}(a_0) = T_{\mathfrak{J}}(a_0) \wedge T_V(a_0), \\ (I_{\mathfrak{J}} \cup I_V)(a_0) &= I_{\mathfrak{J} \cup V}(a_0) = I_{\mathfrak{J}}(a_0) \vee I_V(a_0), \\ (F_{\mathfrak{J}} \cup F_V)(a_0) &= F_{\mathfrak{J} \cup V}(a_0) = F_{\mathfrak{J}}(a_0) \wedge F_V(a_0). \end{aligned}$$

- (iv) the subtraction of $\mathbb{N}_{\mathfrak{J}}$ & $\mathbb{N}_V$ is a neutrosophic $\varkappa$ -structure is termed as follows:

$$\begin{aligned} \mathbb{N}_{\mathfrak{J}} - \mathbb{N}_V &= \mathbb{N}_{\mathfrak{J} - V} = (\mathbb{N}; T_{\mathfrak{J} - V}, I_{\mathfrak{J} - V}, F_{\mathfrak{J} - V}), \text{ where, } \forall a_0 \in \mathbb{N}, \\ (T_{\mathfrak{J}} - T_V)(a_0) &= T_{\mathfrak{J} - V}(a_0) = \begin{cases} \bigwedge_{a_0 = f_0 - c_0} \{T_{\mathfrak{J}}(f_0) \vee T_V(c_0)\} & \text{if } a_0 = f_0 - c_0 \\ 0 & \text{otherwise,} \end{cases}, \\ (I_{\mathfrak{J}} - I_V)(a_0) &= I_{\mathfrak{J} - V}(a_0) = \begin{cases} \bigvee_{a_0 = f_0 - c_0} \{I_{\mathfrak{J}}(f_0) \wedge I_V(c_0)\} & \text{if } a_0 = f_0 - c_0 \\ -1 & \text{otherwise,} \end{cases}, \\ (F_{\mathfrak{J}} - F_V)(a_0) &= F_{\mathfrak{J} - V}(a_0) = \begin{cases} \bigwedge_{a_0 = f_0 - c_0} \{F_{\mathfrak{J}}(f_0) \vee F_V(c_0)\} & \text{if } a_0 = f_0 - c_0 \\ 0 & \text{otherwise.} \end{cases}. \end{aligned}$$

(v) the product of $\mathbb{N}_{\mathfrak{J}}$ & $\mathbb{N}_V$ is a neutrosophic $\varkappa$ -structure over $\mathbb{N}$ and is defined as follows:

$\mathbb{N}_{\mathfrak{J}}\mathbb{N}_V = \mathbb{N}_{\mathfrak{J}V} = (\mathbb{N}; T_{\mathfrak{J}V}, I_{\mathfrak{J}V}, F_{\mathfrak{J}V})$, where, for any $h_0 \in \mathbb{N}$,

$$\begin{aligned} (T_{\mathfrak{J}}T_V)(h_0) = T_{\mathfrak{J}V}(h_0) &= \begin{cases} \bigwedge_{h_0=f_0c_0} \{T_{\mathfrak{J}}(f_0) \vee T_V(c_0)\} & \text{if } h_0 = f_0c_0 \\ 0 & \text{otherwise,} \end{cases}, \\ (I_{\mathfrak{J}}I_V)(h_0) = I_{\mathfrak{J}V}(h_0) &= \begin{cases} \bigvee_{h_0=f_0c_0} \{I_{\mathfrak{J}}(f_0) \wedge I_V(c_0)\} & \text{if } h_0 = f_0c_0 \\ -1 & \text{otherwise,} \end{cases}, \\ (F_{\mathfrak{J}}F_V)(h_0) = F_{\mathfrak{J}V}(h_0) &= \begin{cases} \bigwedge_{h_0=f_0c_0} \{F_{\mathfrak{J}}(f_0) \vee F_V(c_0)\} & \text{if } h_0 = f_0c_0 \\ 0 & \text{otherwise.} \end{cases}. \end{aligned}$$

(vi) the $*$ -product of $\mathbb{N}_{\mathfrak{J}}$ & $\mathbb{N}_V$ is a neutrosophic $\varkappa$ -structure over $\mathbb{N}$ and is defined as follows:

$\mathbb{N}_{\mathfrak{J}} * \mathbb{N}_V = \mathbb{N}_{\mathfrak{J}*V} = (\mathbb{N}; T_{\mathfrak{J}*V}, I_{\mathfrak{J}*V}, F_{\mathfrak{J}*V})$, where, for any $h_0 \in \mathbb{N}$,

$$\begin{aligned} (T_{\mathfrak{J}} * T_V)(h_0) = T_{\mathfrak{J}*V}(h_0) &= \begin{cases} \bigwedge_{h_0=z_0c_0-z_0(b_0-c_0)} \{T_{\mathfrak{J}}(z_0) \vee T_V(c_0)\} & \text{if } h_0 = z_0c_0 - z_0(b_0 - c_0) \\ 0 & \text{otherwise,} \end{cases}, \\ (I_{\mathfrak{J}} * I_V)(h_0) = I_{\mathfrak{J}*V}(h_0) &= \begin{cases} \bigvee_{h_0=z_0c_0-z_0(b_0-c_0)} \{I_{\mathfrak{J}}(z_0) \wedge I_V(c_0)\} & \text{if } h_0 = z_0c_0 - z_0(b_0 - c_0) \\ -1 & \text{otherwise,} \end{cases}, \\ (F_{\mathfrak{J}} * F_V)(h_0) = F_{\mathfrak{J}*V}(h_0) &= \begin{cases} \bigwedge_{h_0=z_0c_0-z_0(b_0-c_0)} \{F_{\mathfrak{J}}(z_0) \vee F_V(c_0)\} & \text{if } h_0 = z_0c_0 - z_0(b_0 - c_0) \\ 0 & \text{otherwise.} \end{cases}. \end{aligned}$$

Definition 3.4. For $V_0 \subseteq \mathbb{N} \neq \emptyset$, the neutrosophic $\varkappa$ -structure

$$\chi_{V_0}(\mathbb{N}_D) = \frac{\mathbb{N}}{(\chi_V(T)_D, \chi_V(I)_D, \chi_V(F)_D)},$$

where

$$\begin{aligned} \chi_{V_0}(T)_D : \mathbb{N} &\rightarrow \mathbb{I}^-, j_1 \rightarrow \begin{cases} -1 & \text{if } j_1 \in V_0 \\ 0 & \text{if } j_1 \notin V_0, \end{cases} \\ \chi_{V_0}(I)_D : \mathbb{N} &\rightarrow \mathbb{I}^-, j_1 \rightarrow \begin{cases} 0 & \text{if } j_1 \in V_0 \\ -1 & \text{if } j_1 \notin V_0, \end{cases} \\ \chi_{V_0}(F)_D : \mathbb{N} &\rightarrow \mathbb{I}^-, j_1 \rightarrow \begin{cases} -1 & \text{if } j_1 \in V_0 \\ 0 & \text{if } j_1 \notin V_0, \end{cases} \end{aligned}$$

is the *characteristic neutrosophic $\varkappa$ -structure* of V_0 over $\mathbb{N}$.

If $V_0 = \mathbb{N}$, then we use that $\chi_{\mathbb{N}}(\mathbb{N}_D) = \mathfrak{B}$.

4. Properties of neutrosophic $\varkappa$ -bi-ideals

In this portion, we define the idea of neutrosophic $\varkappa$ -bi-ideals of near-subtraction semigroups and build an example to show that every neutrosophic $\varkappa$ -bi-ideal need not be a hybrid ideal

of near-subtraction semigroups. Within near-subtraction semigroups, we define neutrosophic $\varkappa$ intersection and provide some results about neutrosophic $\varkappa$ -bi-ideals.

Definition 4.1. A neutrosophic $\varkappa$ -subalgebra $\mathbb{N}_J := \frac{\mathbb{N}}{(T_J, I_J, F_J)}$ of $\mathbb{N}$ is termed as a neutrosophic $\varkappa$ -bi-ideal of $\mathbb{N}$ if $(\mathbb{N}_J \mathfrak{B} \mathbb{N}_J) \cap (\mathbb{N}_J \mathfrak{B} * \mathbb{N}_J) \subseteq \mathbb{N}_J$.

Definition 4.2. A neutrosophic $\varkappa$ -structure $\mathbb{N}_B = \frac{\mathbb{N}}{(T_B, I_B, F_B)}$ of $\mathbb{N}$ is termed as a *neutrosophic $\varkappa$ -ideal* of $\mathbb{N}$ if it fulfils the assertions:

$$\begin{aligned} \text{(i)} \quad & (\forall u_1, g_1 \in \mathbb{N}) \left(\begin{array}{l} T_B(g_1 - u_1) \leq T_B(g_1) \vee T_B(u_1) \\ I_B(g_1 - u_1) \geq I_B(g_1) \wedge I_B(u_1) \\ F_B(g_1 - u_1) \leq F_B(g_1) \vee F_B(u_1) \end{array} \right). \\ \text{(ii)} \quad & (\forall c_1, j_1, u_1 \in \mathbb{N}) \left(\begin{array}{l} T_B(c_1 u_1 - c_1(j_1 - u_1)) \leq T_B(u_1) \\ I_B(c_1 u_1 - c_1(j_1 - u_1)) \geq I_B(u_1) \\ F_B(c_1 u_1 - c_1(j_1 - u_1)) \leq F_B(u_1) \end{array} \right). \\ \text{(iii)} \quad & (\forall u_1, q_1 \in \mathbb{N}) \left(\begin{array}{l} T_B(u_1 q_1) \leq T_B(u_1) \\ I_B(u_1 q_1) \geq I_B(u_1) \\ F_B(u_1 q_1) \leq F_B(u_1) \end{array} \right). \end{aligned}$$

A subset $\mathbb{N}_B$ of $\mathbb{N}$ is termed a left hybrid ideal if conditions (i) and (ii) are satisfied. It is called a right hybrid ideal when conditions (i) and (iii) are fulfilled.

Notation 1. For a near- subtraction semigroup $\mathbb{N}$, we use the following notations.

(i) $\mathcal{N}_{\mathbb{L}}(\mathbb{N})$ (respectively $\mathcal{N}_{\mathbb{R}}(\mathbb{N})$) represents the gathering of all neutrosophic $\varkappa$ -left(respectively right) ideals of $\mathbb{N}$.

(ii) $\mathcal{NB}_{\mathcal{I}}(\mathbb{N})$ represents the collection of all neutrosophic $\varkappa$ - bi-ideals of $\mathbb{N}$.

Example 4.3. Let $\mathbb{Q} = \{0, p, w, z\}$ in which “-” and “.” are defined by:

-	0	p	w	z	·	0	p	w	z
0	0	0	0	0	0	0	0	0	0
p	p	0	p	p	p	p	p	p	p
w	w	w	0	w	w	0	0	0	w
z	z	z	z	0	z	0	0	0	z

Then $(\mathbb{Q}, -, \cdot)$ is a NSS. Here

$$\mathbb{Q}_{\mathcal{D}} = \left\{ \frac{0}{(-0.8, -0.1, -0.7)}, \frac{p}{(-0.6, -0.4, -0.6)}, \frac{w}{(-0.4, -0.6, -0.5)}, \frac{z}{(-0.2, -0.8, -0.4)} \right\} \in \mathcal{NB}_{\mathcal{I}}(\mathbb{N}).$$

Clearly $\mathcal{N}_{\mathbb{L}}(\mathbb{N}) \cap \mathcal{N}_{\mathbb{R}}(\mathbb{N}) \subseteq \mathcal{NB}_{\mathcal{I}}(\mathbb{N})$ (see Lemma 4.5). The following example demonstrates that the reverse statement does not hold.

Example 4.4. Let $\mathbb{N} = \{0, p, w, z\}$ in which “-” and “.” are defined as in Example 4.3. Then

$$\mathbb{N}_{\mathcal{D}} = \left\{ \frac{0}{(-0.8, -0.1, -0.7)}, \frac{p}{(-0.6, -0.4, -0.6)}, \frac{w}{(-0.4, -0.6, -0.5)}, \frac{z}{(-0.6, -0.4, -0.6)} \right\} \in \mathcal{NB}_{\mathcal{I}}(\mathbb{N}),$$

but $\mathbb{N}_{\mathcal{D}}$ of $\mathbb{N}$ is not a left neutrosophic $\varkappa$ - ideal, since $T_{\mathcal{D}}(wz - w(0 - z)) = T_{\mathcal{D}}(w) = -0.4 \not\leq -0.6 = T_{\mathcal{D}}(z)$, $I_{\mathcal{D}}(wz - w(0 - z)) = I_{\mathcal{D}}(w) = -0.6 \not\geq -0.4 = I_{\mathcal{D}}(z)$ and $F_{\mathcal{D}}(wz - w(0 - z)) = F_{\mathcal{D}}(w) = -0.5 \not\leq -0.6 = F_{\mathcal{D}}(z)$.

Lemma 4.5. Let $\mathbb{N}_L := \frac{\mathbb{N}}{(T_L, I_L, F_L)}$. If $\mathbb{N}_L \in \mathcal{N}_{\mathbb{L}}(\mathbb{N})$, then $\mathbb{N}_L \in \mathcal{NB}_{\mathcal{I}}(\mathbb{N})$.

Proof. Let $a' \in \mathbb{N}$ be such that $a' = v_0 y_0 s_0 = z_0 d_0 - z_0(q_0 - d_0)$, where $v_0, y_0, s_0, z_0, q_0, d_0 \in \mathbb{N}$. Then

$$\begin{aligned}
 ((T_L \mathfrak{B} T_L) \cap (T_L \mathfrak{B} * T_L))(a') &= (T_L \mathfrak{B} T_L)(a') \vee (T_L \mathfrak{B} * T_L)(a') \geq (T_L \mathfrak{B} * T_L)(a') \\
 &= \bigwedge_{a' = z_0 d_0 - z_0(q_0 - d_0)} \{(T_L \mathfrak{B})(z_0) \vee T_L(d_0)\} \\
 &\geq \bigwedge_{a' = z_0 d_0 - z_0(q_0 - d_0)} \{\mathfrak{B}(z_0) \vee T_L(z_0 d_0 - z_0(q_0 - d_0))\} \\
 &= \bigwedge_{a' = z_0 d_0 - z_0(q_0 - d_0)} T_L(z_0 d_0 - z_0(q_0 - d_0)) = T_L(a'), \\
 ((I_L \mathfrak{B} I_L) \cap (I_L \mathfrak{B} * I_L))(a') &= (I_L \mathfrak{B} I_L)(a') \wedge (I_L \mathfrak{B} * I_L)(a') \leq (I_L \mathfrak{B} * I_L)(a') \\
 &= \bigvee_{a' = z_0 d_0 - z_0(q_0 - d_0)} \{(I_L \mathfrak{B})(z_0) \wedge I_L(d_0)\} \\
 &\leq \bigvee_{a' = z_0 d_0 - z_0(q_0 - d_0)} \{(I_L \mathfrak{B})(z_0) \wedge I_L(d_0)\} \\
 &\leq \bigvee_{a' = z_0 d_0 - z_0(q_0 - d_0)} \{\mathfrak{B}(z_0) \wedge I_L(z_0 d_0 - z_0(q_0 - d_0))\} \\
 &= \bigvee_{a' = z_0 d_0 - z_0(q_0 - d_0)} I_L(z_0 d_0 - z_0(q_0 - d_0)) = I_L(a'), \\
 ((F_L \mathfrak{B} F_L) \cap (F_L \mathfrak{B} * F_L))(a') &= (F_L \mathfrak{B} F_L)(a') \cap (F_L \mathfrak{B} * F_L)(a') \geq (F_L \mathfrak{B} * F_L)(a') \\
 &= \bigwedge_{a' = z_0 d_0 - z_0(q_0 - d_0)} \{(F_L \mathfrak{B})(z_0) \vee F_L(d_0)\} \\
 &\geq \bigwedge_{a' = z_0 d_0 - z_0(q_0 - d_0)} \{\mathfrak{B}(z_0) \vee F_L(z_0 d_0 - z_0(q_0 - d_0))\} \\
 &= \bigwedge_{a' = z_0 d_0 - z_0(q_0 - d_0)} F_L(z_0 d_0 - z_0(q_0 - d_0)) = F_L(a').
 \end{aligned}$$

If a' cannot be expressed as $a' = v_0 y_0 s_0 = z_0 d_0 - z_0(q_0 - d_0)$, then $((T_L \mathfrak{B} T_L) \cap (T_L \mathfrak{B} * T_L))(a') = 0 \geq T_L(a')$; $((I_L \mathfrak{B} I_L) \cap (I_L \mathfrak{B} * I_L))(a') = -1 \leq I_L(a')$, $((F_L \mathfrak{B} F_L) \cap (F_L \mathfrak{B} * F_L))(a') = 0 \geq F_L(a')$. Hence $\mathbb{N}_J \in \mathcal{NB}_{\mathcal{I}}(\mathbb{N})$. $\square$

The proof of the following lemma follows a similar approach to that used in the proof of Lemma 4.5. We present the proof for readers' convenience.

Lemma 4.6. Let $\mathbb{N}_L := \frac{\mathbb{N}}{(T_L, I_L, F_L)}$. If $\mathbb{N}_L \in \mathcal{N}_{\mathbb{R}}(\mathbb{N})$, then $\mathbb{N}_L \in \mathcal{NB}_{\mathcal{I}}(\mathbb{N})$.

Proof. Let $x' \in \mathbb{N}$: $x' = vy = xj - x(q - j)$, $v = v_1 v_2$, where $v, v_1, v_2, y, x, q, j \in \mathbb{N}$. Then

$$\begin{aligned}
((T_L \mathfrak{B} T_L) \cap (T_L \mathfrak{B} * T_L))(x') &= (T_L \mathfrak{B} T_L)(x') \vee (T_L \mathfrak{B} * T_L)(x') \\
&\geq \bigwedge_{x'=vy} \{(T_L \mathfrak{B})(v) \vee T_L(y)\} \\
&= \left\{ \bigwedge_{x'=vy} \left\{ \bigwedge_{v=v_1 v_2} \{T_L(v_1) \vee \mathfrak{B}(v_2)\} \right\} \vee T_L(y) \right\} \\
&= \left\{ \bigwedge_{x'=vy} \left\{ \bigwedge_{v=v_1 v_2} T_L(v_1) \right\} \vee T_L(y) \right\} \\
&= T_L(v_1) \vee T_L(y) \text{ (since } T_L(vy) = T_L(v_1 v_2 y) \leq T_L(v_1)) \\
&\geq T_L(vy) = T_L(x'),
\end{aligned}$$

$$\begin{aligned}
((I_L \mathfrak{B} I_L) \cap (I_L \mathfrak{B} * I_L))(x') &= (I_L \mathfrak{B} I_L)(x') \wedge (I_L \mathfrak{B} * I_L)(x') \\
&\leq \bigvee_{x'=vy} \{(I_L \mathfrak{B})(v) \wedge I_L(y)\} \\
&= \left\{ \bigvee_{x'=vy} \left\{ \bigvee_{v=v_1 v_2} \{I_L(v_1) \wedge \mathfrak{B}(v_2)\} \right\} \wedge I_L(y) \right\} \\
&= \left\{ \bigvee_{x'=vy} \left\{ \bigvee_{v=v_1 v_2} I_L(v_1) \right\} \wedge I_L(y) \right\} \\
&= I_L(v_1) \wedge I_L(y) \text{ (since } I_L(vy) = I_L(v_1 v_2 y) \geq I_L(v_1)) \\
&\leq I_L(vy) = I_L(x'),
\end{aligned}$$

$$\begin{aligned}
((F_L \mathfrak{B} F_L) \cap (F_L \mathfrak{B} * F_L))(x') &= (F_L \mathfrak{B} F_L)(x') \vee (F_L \mathfrak{B} * F_L)(x') \\
&\geq \bigwedge_{x'=vy} \{(F_L \mathfrak{B})(v) \vee F_L(y)\} \\
&= \left\{ \bigwedge_{x'=vy} \left\{ \bigwedge_{v=v_1 v_2} \{F_L(v_1) \vee \mathfrak{B}(v_2)\} \right\} \vee F_L(y) \right\} \\
&= \left\{ \bigwedge_{x'=vy} \left\{ \bigwedge_{v=v_1 v_2} F_L(v_1) \right\} \vee F_L(y) \right\} \\
&= F_L(v_1) \vee F_L(y) \text{ (since } F_L(vy) = F_L(v_1 v_2 y) \leq F_L(v_1)) \\
&\geq F_L(vy) = F_L(x').
\end{aligned}$$

If x' cannot be expressed as $x' = vy = xj - x(q - j)$, then $((T_L \mathfrak{B} T_L) \cap (T_L \mathfrak{B} * T_L))(x') = 0 \geq T_L(x')$; $((I_L \mathfrak{B} I_L) \cap (I_L \mathfrak{B} * I_L))(x') = -1 \leq I_L(x')$, $((F_L \mathfrak{B} F_L) \cap (F_L \mathfrak{B} * F_L))(x') = 0 \geq F_L(x')$.

Hence $\mathbb{N}_L \in \mathcal{NB}_I(\mathbb{N})$. $\square$

Theorem 4.7. Let $\mathbb{N}_{\mathfrak{R}} := \frac{\mathbb{N}}{(T_{\mathfrak{R}}, I_{\mathfrak{R}}, F_{\mathfrak{R}})}$ and $\mathbb{N}_L := \frac{\mathbb{N}}{(T_L, I_L, F_L)}$. If $\mathbb{N}_{\mathfrak{R}}, \mathbb{N}_L \in \mathcal{NB}_I(\mathbb{N})$, then $\mathbb{N}_{\mathfrak{R}} \cap \mathbb{N}_L \in \mathcal{NB}_I(\mathbb{N})$.

Proof. Let $\mathbb{N}_{\mathfrak{R}}, \mathbb{N}_L \in \mathcal{NB}_{\mathcal{I}}(\mathbb{N})$, and let $z_0, t_0 \in \mathbb{N}$. Then

$$\begin{aligned}
 (T_{\mathfrak{R}} \cap T_L)(z_0 - t_0) &= T_{\mathfrak{R}}(z_0 - t_0) \vee T_L(z_0 - t_0) \\
 &\leq (T_{\mathfrak{R}}(z_0) \vee T_{\mathfrak{R}}(t_0)) \vee (T_L(z_0) \vee T_L(t_0)) \\
 &= (T_{\mathfrak{R}}(z_0) \vee T_L(z_0)) \vee (T_{\mathfrak{R}}(t_0) \vee T_L(t_0)) \\
 &= (T_{\mathfrak{R}} \vee T_L)(z_0) \vee (T_{\mathfrak{R}} \vee T_L)(t_0), \\
 (I_{\mathfrak{R}} \cap I_L)(z_0 - t_0) &= I_{\mathfrak{R}}(z_0 - t_0) \wedge I_L(z_0 - t_0) \\
 &\geq (I_{\mathfrak{R}}(z_0) \wedge I_{\mathfrak{R}}(t_0)) \wedge (I_L(z_0) \wedge I_L(t_0)) \\
 &= (I_{\mathfrak{R}}(z_0) \wedge I_L(z_0)) \wedge (I_{\mathfrak{R}}(t_0) \wedge I_L(t_0)) \\
 &= (I_{\mathfrak{R}} \wedge I_L)(z_0) \wedge (I_{\mathfrak{R}} \wedge I_L)(t_0), \\
 (F_{\mathfrak{R}} \cap F_L)(z_0 - t_0) &= F_{\mathfrak{R}}(z_0 - t_0) \vee F_L(z_0 - t_0) \\
 &\leq (F_{\mathfrak{R}}(z_0) \vee F_{\mathfrak{R}}(t_0)) \vee (F_L(z_0) \vee F_L(t_0)) \\
 &= (F_{\mathfrak{R}}(z_0) \vee F_L(z_0)) \vee (F_{\mathfrak{R}}(t_0) \vee F_L(t_0)) \\
 &= (F_{\mathfrak{R}} \vee F_L)(z_0) \vee (F_{\mathfrak{R}} \vee F_L)(t_0).
 \end{aligned}$$

Let $j' \in \mathbb{N}$ and choose $h, w, s, j, t, a \in \mathbb{N}$ be such that $j' = hws = ja - j(t - a)$. Since $\mathbb{N}_{\mathfrak{R}}$ and $\mathbb{N}_L$ are neutrosophic $\varkappa$ - bi-ideals of $\mathbb{N}$, we get

$$\begin{aligned}
 &\left\{ \bigwedge_{j'=hws} (T_{\mathfrak{R}}(h) \vee T_{\mathfrak{R}}(s)) \right\} \vee \left\{ \bigwedge_{j'=ja-j(t-a)} (T_{\mathfrak{R}}(j) \vee T_{\mathfrak{R}}(a)) \right\} \geq T_{\mathfrak{R}}(j'), \\
 &\left\{ \bigvee_{j'=hws} (I_{\mathfrak{R}}(h) \wedge I_{\mathfrak{R}}(s)) \right\} \wedge \left\{ \bigvee_{j'=ja-j(t-a)} (I_{\mathfrak{R}}(j) \wedge I_{\mathfrak{R}}(a)) \right\} \leq I_{\mathfrak{R}}(j'), \\
 &\left\{ \bigwedge_{j'=hws} (F_{\mathfrak{R}}(h) \vee F_{\mathfrak{R}}(s)) \right\} \vee \left\{ \bigwedge_{j'=ja-j(t-a)} (F_{\mathfrak{R}}(j) \vee F_{\mathfrak{R}}(a)) \right\} \geq F_{\mathfrak{R}}(j').
 \end{aligned}$$

and

$$\begin{aligned}
 &\left\{ \bigwedge_{j'=hws} (T_L(h) \vee T_L(s)) \right\} \vee \left\{ \bigwedge_{j'=ja-j(t-a)} (T_L(j) \vee T_L(a)) \right\} \geq T_L(j'), \\
 &\left\{ \bigvee_{j'=hws} (I_L(h) \wedge I_L(s)) \right\} \wedge \left\{ \bigvee_{j'=ja-j(t-a)} (I_L(j) \wedge I_L(a)) \right\} \leq I_L(j'), \\
 &\left\{ \bigwedge_{j'=hws} (F_L(h) \vee F_L(s)) \right\} \vee \left\{ \bigwedge_{j'=ja-j(t-a)} (F_L(j) \vee F_L(a)) \right\} \geq F_L(j').
 \end{aligned}$$

$$\begin{aligned}
& \text{Now, } ((T_{\mathfrak{R}} \cap T_L) \mathfrak{B}(T_{\mathfrak{R}} \cap T_L))(j') \vee ((T_{\mathfrak{R}} \cap T_L) \mathfrak{B} * (T_{\mathfrak{R}} \cap T_L))(j') \\
&= \left\{ \bigwedge_{j'=hws} ((T_{\mathfrak{R}} \vee T_L)(h) \vee (T_{\mathfrak{R}} \vee T_L)(s)) \right\} \vee \left\{ \bigwedge_{j'=ja-j(t-a)} ((T_{\mathfrak{R}} \vee T_L)(j) \vee (T_{\mathfrak{R}} \vee T_L)(a)) \right\} \\
&= \left\{ \bigwedge_{j'=hws} (T_{\mathfrak{R}}(h) \vee T_L(h)) \vee (T_{\mathfrak{R}}(s) \vee T_L(s)) \right\} \vee \left\{ \bigwedge_{j'=ja-j(t-a)} (T_{\mathfrak{R}}(j) \vee T_L(j)) \vee (T_{\mathfrak{R}}(a) \vee T_L(a)) \right\} \\
&= \left\{ \bigwedge_{j'=hws} (T_{\mathfrak{R}}(h) \vee T_{\mathfrak{R}}(s)) \vee \bigwedge_{j'=ja-j(t-a)} (T_{\mathfrak{R}}(j) \vee T_{\mathfrak{R}}(a)) \right\} \\
&\quad \vee \left\{ \bigwedge_{j'=hws} (T_L(h) \vee T_L(s)) \vee \bigwedge_{j'=ja-j(t-a)} (T_L(j) \vee T_L(a)) \right\} \\
&\geq T_{\mathfrak{R}}(j') \vee T_L(j') = (T_{\mathfrak{R}} \vee T_L)(j'), ((I_{\mathfrak{R}} \cap I_L) \mathfrak{B}(I_{\mathfrak{R}} \cap I_L))(j') \wedge ((I_{\mathfrak{R}} \cap I_L) \mathfrak{B} * (I_{\mathfrak{R}} \cap I_L))(j') \\
&= \left\{ \bigvee_{j'=hws} ((I_{\mathfrak{R}} \wedge I_L)(h) \wedge (I_{\mathfrak{R}} \wedge I_L)(s)) \right\} \wedge \left\{ \bigvee_{j'=ja-j(t-a)} ((I_{\mathfrak{R}} \wedge I_L)(j) \wedge (I_{\mathfrak{R}} \wedge I_L)(a)) \right\} \\
&= \left\{ \bigvee_{j'=hws} (I_{\mathfrak{R}}(h) \wedge I_L(h)) \wedge (I_{\mathfrak{R}}(s) \wedge I_L(s)) \right\} \wedge \left\{ \bigvee_{j'=ja-j(t-a)} (I_{\mathfrak{R}}(j) \wedge I_L(j)) \wedge (I_{\mathfrak{R}}(a) \wedge I_L(a)) \right\} \\
&= \left\{ \bigvee_{j'=hws} (I_{\mathfrak{R}}(h) \wedge I_{\mathfrak{R}}(s)) \wedge \bigvee_{j'=ja-j(t-a)} (I_{\mathfrak{R}}(j) \wedge I_{\mathfrak{R}}(a)) \right\} \\
&\quad \wedge \left\{ \bigvee_{j'=hws} (I_L(h) \wedge I_L(s)) \wedge \bigvee_{j'=ja-j(t-a)} (I_L(j) \wedge I_L(a)) \right\} \\
&\leq I_{\mathfrak{R}}(j') \wedge I_L(j') = (I_{\mathfrak{R}} \wedge I_L)(j'), \\
&((F_{\mathfrak{R}} \cap F_L) \mathfrak{B}(F_{\mathfrak{R}} \cap F_L))(j') \vee ((F_{\mathfrak{R}} \cap F_L) \mathfrak{B} * (F_{\mathfrak{R}} \cap F_L))(j') \\
&= \left\{ \bigwedge_{j'=hws} ((F_{\mathfrak{R}} \vee F_L)(h) \vee (F_{\mathfrak{R}} \vee F_L)(s)) \right\} \vee \left\{ \bigwedge_{j'=ja-j(t-a)} ((F_{\mathfrak{R}} \vee F_L)(j) \vee (F_{\mathfrak{R}} \vee F_L)(a)) \right\} \\
&= \left\{ \bigwedge_{j'=hws} (F_{\mathfrak{R}}(h) \vee F_L(h)) \vee (F_{\mathfrak{R}}(s) \vee F_L(s)) \right\} \vee \left\{ \bigwedge_{j'=ja-j(t-a)} (F_{\mathfrak{R}}(j) \vee F_L(j)) \vee (F_{\mathfrak{R}}(a) \vee F_L(a)) \right\} \\
&= \left\{ \bigwedge_{j'=hws} (F_{\mathfrak{R}}(h) \vee F_{\mathfrak{R}}(s)) \vee \bigwedge_{j'=ja-j(t-a)} (F_{\mathfrak{R}}(j) \vee F_{\mathfrak{R}}(a)) \right\} \\
&\quad \vee \left\{ \bigwedge_{j'=hws} (F_L(h) \vee F_L(s)) \vee \bigwedge_{j'=ja-j(t-a)} (F_L(j) \vee F_L(a)) \right\} \\
&\geq F_{\mathfrak{R}}(j') \vee F_L(j') = (F_{\mathfrak{R}} \vee F_L)(j').
\end{aligned}$$

Hence $\mathbb{N}_{\mathfrak{R}} \cap \mathbb{N}_L \in \mathcal{NB}_{\mathcal{I}}(\mathbb{N})$. $\square$

Theorem 4.8. If $\mathbb{N}_L := \frac{\mathbb{N}}{(T_L, I_L, F_L)} \in \mathcal{NB}_I(\mathbb{N})$, then the $(\tilde{\theta}, \varphi, \nu)$ -level set $\mathbb{N}_L(\tilde{\theta}, \varphi, \nu)$ of $\mathbb{N}$ is a bi-ideal $\forall \tilde{\theta}, \varphi, \nu \in \mathbb{I}^-$.

Proof. For $\tilde{\theta}, \varphi, \nu \in \mathbb{I}^-$ and $m_1, b_1 \in \mathbb{N}_L(\tilde{\theta}, \varphi, \nu)$, $T_L(m_1 - b_1) \leq T_L(m_1) \vee T_L(b_1) \leq \tilde{\theta}$, $I_L(m_1 - b_1) \geq I_L(m_1) \wedge I_L(b_1) \geq \varphi$, $F_L(m_1 - b_1) \leq F_L(m_1) \vee F_L(b_1) \leq \nu$. It follows that $m_1 - b_1 \in \mathbb{N}_L(\tilde{\theta}, \varphi, \nu)$.

Let $z' \in \mathbb{N}$ and $z' \in \mathbb{N}_L(\tilde{\theta}, \varphi, \nu) \mathfrak{B} \mathbb{N}_L(\tilde{\theta}, \varphi, \nu) \cap \mathbb{N}_L(\tilde{\theta}, \varphi, \nu) \mathfrak{B} * \mathbb{N}_L(\tilde{\theta}, \varphi, \nu)$. If there exist $f_1, q, u_1, c \in \mathbb{N}_L(\tilde{\theta}, \varphi, \nu)$ and $f_2, f, u, u_2, y \in \mathbb{N}$ such that $z' = fq = uc - u(y - c)$, $f = f_1 f_2$ and $u = u_1 u_2$, then

$$\begin{aligned}
 T_L(z') &\leq ((T_L \mathfrak{B} T_L) \vee (T_L \mathfrak{B} * T_L))(z') \\
 &= (T_L \mathfrak{B} T_L)(z') \vee (T_L \mathfrak{B} * T_L)(z') \\
 &= \left\{ \bigwedge_{z'=fq} \{(T_L \mathfrak{B})(f) \vee T_L(q)\} \right\} \vee \left\{ \bigwedge_{z'=uc-u(y-c)} \{(T_L \mathfrak{B})(u) \vee T_L(c)\} \right\} \\
 &= \left\{ \bigwedge_{z'=fq} \left(\bigwedge_{f=f_1 f_2} (T_L(f_1) \vee \mathfrak{B}(f_2)) \right) \vee T_L(q) \right\} \\
 &\quad \vee \left\{ \bigwedge_{z'=uc-u(y-c)} \left(\bigwedge_{u=u_1 u_2} (T_L(u_1) \vee \mathfrak{B}(u_2)) \right) \vee T_L(c) \right\} \\
 &\leq (T_L(f_1) \vee T_L(q) \vee T_L(u_1) \vee T_L(c)) \leq \tilde{\theta}, \\
 I_L(z') &\geq ((I_L \mathfrak{B} I_L) \wedge (I_L \mathfrak{B} * I_L))(z') \\
 &= (I_L \mathfrak{B} I_L)(z') \wedge (I_L \mathfrak{B} * I_L)(z') \\
 &= \left\{ \bigvee_{z'=fq} \{(I_L \mathfrak{B})(f) \wedge I_L(q)\} \right\} \wedge \left\{ \bigvee_{z'=uc-u(y-c)} \{(I_L \mathfrak{B})(u) \wedge I_L(c)\} \right\} \\
 &= \left\{ \bigvee_{z'=fq} \left(\bigvee_{f=f_1 f_2} (I_L(f_1) \wedge \mathfrak{B}(f_2)) \right) \wedge I_L(q) \right\} \\
 &\quad \wedge \left\{ \bigvee_{z'=uc-u(y-c)} \left(\bigvee_{u=u_1 u_2} (I_L(u_1) \wedge \mathfrak{B}(u_2)) \right) \wedge I_L(c) \right\} \\
 &\geq (I_L(f_1) \wedge I_L(q) \wedge I_L(u_1) \wedge I_L(c)) \geq \varphi, \\
 F_L(z') &\leq ((F_L \mathfrak{B} F_L) \vee (F_L \mathfrak{B} * F_L))(z') \\
 &= (F_L \mathfrak{B} F_L)(z') \vee (F_L \mathfrak{B} * F_L)(z') \\
 &= \left\{ \bigwedge_{z'=fq} \{(F_L \mathfrak{B})(f) \vee F_L(q)\} \right\} \vee \left\{ \bigwedge_{z'=uc-u(y-c)} \{(F_L \mathfrak{B})(u) \vee F_L(c)\} \right\}
 \end{aligned}$$

$$\begin{aligned}
&= \left\{ \bigwedge_{z'=fq} \left(\bigwedge_{f=f_1 f_2} (F_L(f_1) \vee \mathfrak{B}(f_2)) \right) \vee F_L(q) \right\} \\
&\quad \vee \left\{ \bigwedge_{z'=uc-u(y-c)} \left(\bigwedge_{u=u_1 u_2} (F_L(u_1) \vee \mathfrak{B}(u_2)) \right) \vee F_L(c) \right\} \\
&\leq (F_L(f_1) \vee F_L(q) \vee F_L(u_1) \vee F_L(c)) \leq \nu.
\end{aligned}$$

This implies that $z' \in \mathbb{N}_L(\mathfrak{D}, \varphi, \nu)$. So $\mathbb{N}_L(\mathfrak{D}, \varphi, \nu)$ of $\mathbb{N}$ is a bi-ideal. $\square$

Lemma 4.9. For any subsets K, V of $\mathbb{N}$ and $\mathbb{N}_N := \frac{\mathbb{N}}{(T_N, I_N, F_N)}$, the following statements are true:

- (i) $\chi_K(\mathbb{N}_N) \cap \chi_V(\mathbb{N}_N) = \chi_{K \cap V}(\mathbb{N}_N)$.
- (ii) $\chi_K(\mathbb{N}_N) \cup \chi_V(\mathbb{N}_N) = \chi_{K \cup V}(\mathbb{N}_N)$.
- (iii) $\chi_K(\mathbb{N}_N) \chi_V(\mathbb{N}_N) = \chi_{KV}(\mathbb{N}_N)$.
- (iv) $\chi_K(\mathbb{N}_N) * \chi_V(\mathbb{N}_N) = \chi_{K*V}(\mathbb{N}_N)$.
- (v) If $K \subseteq V$, then $\chi_K(\mathbb{N}_N) \subseteq \chi_V(\mathbb{N}_N)$.

Proof. The proofs are obvious. $\square$

Lemma 4.10. For a subset K of $\mathbb{N}$ and $\mathbb{N}_N := \frac{\mathbb{N}}{(T_N, I_N, F_N)}$, the below statements are equivalent:

- (i) K of $\mathbb{N}$ is a bi-ideal,
- (ii) $\chi_K(\mathbb{N}_N) \in \mathcal{NB}_I(\mathbb{N})$.

Proof. (i) $\Rightarrow$ (ii) For $y_1, c_1 \in \mathbb{N}$, if $y_1, c_1 \in K$, then $y_1 - c_1 \in K$ which implies $\chi_K(T_N)(y_1 - c_1) = -1 = \chi_K(T_N)(y_1) \vee \chi_K(T_N)(c_1)$, $\chi_K(I_N)(y_1 - c_1) = 0 = \chi_K(I_N)(y_1) \wedge \chi_K(I_N)(c_1)$, $\chi_K(F_N)(y_1 - c_1) = -1 = \chi_K(F_N)(y_1) \vee \chi_K(F_N)(c_1)$. Otherwise $y_1 \notin K$ or $c_1 \notin K$. Then $\chi_K(T_N)(y_1 - c_1) \leq 0 = \chi_K(T_N)(y_1) \vee \chi_K(T_N)(c_1)$, $\chi_K(I_N)(y_1 - c_1) \geq -1 = \chi_K(I_N)(y_1) \wedge \chi_K(I_N)(c_1)$, $\chi_K(F_N)(y_1 - c_1) \leq 0 = \chi_K(F_N)(y_1) \vee \chi_K(F_N)(c_1)$. So $\chi_K(\mathbb{N}_N)$ of $\mathbb{N}$ is a neutrosophic $\varkappa$ -subalgebra.

By Lemma 4.9, we have

$$\begin{aligned}
\chi_K(\mathbb{N}_N) \mathfrak{B} \chi_K(\mathbb{N}_N) \cap \chi_K(\mathbb{N}_N) \mathfrak{B} * \chi_K(\mathbb{N}_N) &= \chi_{K \cap K}(\mathbb{N}_N) \cap \chi_{K \cap K}(\mathbb{N}_N) \\
&= \chi_{(K \cap K) \cap (K \cap K)}(\mathbb{N}_N) \ll \chi_K(\mathbb{N}_N).
\end{aligned}$$

So, $\chi_K(\mathbb{N}_N) \in \mathcal{NB}_I(\mathbb{N})$.

(ii) $\Rightarrow$ (i) Let $u' \in K \cap K * K$. Then $u' = hv = uq - u(z - q)$ and $h = h_1 h_2$; $u = u_1 u_2$ for some $h_1, v, q, u_1 \in K$ and $h_2, u, z, u_2 \in \mathbb{N}$. Now,

$$\begin{aligned}
\chi_K(T_N)(u') &\leq (\chi_K \mathfrak{B} \chi_K \cap \chi_K \mathfrak{B} * \chi_K)(T_N)(u') \\
&= (\chi_K \mathfrak{B} \chi_K)(T_N)(u') \vee (\chi_K \mathfrak{B} * \chi_K)(T_N)(u') \\
&= \bigwedge_{u'=hv} \{(\chi_K \mathfrak{B})(T_N)(h) \vee \chi_K(T_N)(v)\} \vee \bigwedge_{u'=uq-u(z-q)} \{(\chi_K \mathfrak{B})(T_N)(u) \vee \chi_K(T_N)(q)\}
\end{aligned}$$

$$\begin{aligned}
&= \left\{ \bigwedge_{u'=hv} \left(\bigwedge_{h=h_1h_2} \chi_K(T_N)(h_1) \vee \mathfrak{B}(h_2) \right) \vee \chi_K(T_N)(v) \right\} \\
&\vee \left\{ \bigwedge_{u'=uq-u(z-q)} \left(\bigwedge_{u=u_1u_2} \chi_K(T_N)(u_1) \vee \mathfrak{B}(u_2) \right) \vee \chi_K(T_N)(q) \right\} \\
&\leq \chi_K(T_N)(h_1) \vee \chi_K(T_N)(v) \vee \chi_K(T_N)(u_1) \vee \chi_K(T_N)(q) = -1,
\end{aligned}$$

$$\begin{aligned}
\chi_K(I_N)(u') &\geq (\chi_K \mathfrak{B} \chi_K \cap \chi_K \mathfrak{B} * \chi_K)(I_N)(u') \\
&= (\chi_K \mathfrak{B} \chi_K)(I_N)(u') \wedge (\chi_K \mathfrak{B} * \chi_K)(I_N)(u') \\
&= \bigvee_{u'=hv} \{(\chi_K \mathfrak{B})(I_N)(h) \wedge \chi_K(I_N)(v)\} \wedge \bigvee_{u'=uq-u(z-q)} \{(\chi_K \mathfrak{B})(I_N)(u) \wedge \chi_K(I_N)(q)\} \\
&= \left\{ \bigvee_{u'=hv} \left(\bigvee_{h=h_1h_2} \chi_K(I_N)(h_1) \wedge \mathfrak{B}(h_2) \right) \wedge \chi_K(I_N)(v) \right\} \\
&\quad \wedge \left\{ \bigvee_{u'=uq-u(z-q)} \left(\bigvee_{u=u_1u_2} \chi_K(I_N)(u_1) \wedge \mathfrak{B}(u_2) \right) \wedge \chi_K(I_N)(q) \right\} \\
&\geq \chi_K(I_N)(h_1) \wedge \chi_K(I_N)(v) \wedge \chi_K(I_N)(u_1) \wedge \chi_K(I_N)(q) = 0,
\end{aligned}$$

$$\begin{aligned}
\chi_K(F_N)(u') &\leq (\chi_K \mathfrak{B} \chi_K \cap \chi_K \mathfrak{B} * \chi_K)(F_N)(u') \\
&= (\chi_K \mathfrak{B} \chi_K)(F_N)(u') \vee (\chi_K \mathfrak{B} * \chi_K)(F_N)(u') \\
&= \bigwedge_{u'=hv} \{(\chi_K \mathfrak{B})(F_N)(h) \vee \chi_K(F_N)(v)\} \vee \bigwedge_{u'=uq-u(z-q)} \{(\chi_K \mathfrak{B})(F_N)(u) \vee \chi_K(F_N)(q)\} \\
&= \left\{ \bigwedge_{u'=hv} \left(\bigwedge_{h=h_1h_2} \chi_K(F_N)(h_1) \vee \mathfrak{B}(h_2) \right) \vee \chi_K(F_N)(v) \right\} \\
&\quad \vee \left\{ \bigwedge_{u'=uq-u(z-q)} \left(\bigwedge_{u=u_1u_2} \chi_K(F_N)(u_1) \vee \mathfrak{B}(u_2) \right) \vee \chi_K(F_N)(q) \right\} \\
&\leq \chi_K(F_N)(h_1) \vee \chi_K(F_N)(v) \vee \chi_K(F_N)(u_1) \vee \chi_K(F_N)(q) = -1.
\end{aligned}$$

Thus $u' \in K$ and hence $K \mathfrak{N} K \cap K \mathfrak{N} * K \subseteq K$. $\square$

Theorem 4.11. For a neutrosophic $\varkappa$ -subalgebra $\mathbb{N}_N := \frac{\mathbb{N}}{(T_N, I_N, F_N)}$ of $\mathbb{N}$, if $\mathbb{N}_N \mathfrak{B} \mathbb{N}_N \subseteq \mathbb{N}_N$, then $\mathbb{N}_N \in \mathcal{NB}_T(\mathbb{N})$.

Proof. Assume $\mathbb{N}_N \mathfrak{B} \mathbb{N}_N \subseteq \mathbb{N}_N$ and let $y_1 \in \mathbb{N}$. Then

$$\begin{aligned}
((T_N \mathfrak{B} T_N) \cap (T_N \mathfrak{B} * T_N))(y_1) &= (T_N \mathfrak{B} T_N)(y_1) \vee (T_N \mathfrak{B} * T_N)(y_1) \geq (T_N \mathfrak{B} T_N)(y_1) \geq T_N(y_1), \\
((I_N \mathfrak{B} I_N) \cap (I_N \mathfrak{B} * I_N))(y_1) &= (I_N \mathfrak{B} I_N)(y_1) \wedge (I_N \mathfrak{B} * I_N)(y_1) \leq (I_N \mathfrak{B} I_N)(y_1) \leq I_N(y_1), \\
((F_N \mathfrak{B} F_N) \cap (F_N \mathfrak{B} * F_N))(y_1) &= (F_N \mathfrak{B} F_N)(y_1) \vee (F_N \mathfrak{B} * F_N)(y_1) \geq (F_N \mathfrak{B} F_N)(y_1) \geq F_N(y_1).
\end{aligned}$$

Thus $\mathbb{N}_N \mathfrak{B} \mathbb{N}_N \cap \mathbb{N}_N \mathfrak{B} * \mathbb{N}_N \subseteq \mathbb{N}_N$ and so $\mathbb{N}_N \in \mathcal{NB}_{\mathcal{I}}(\mathbb{N})$. $\square$

Theorem 4.12. *If $\mathbb{N}$ is a zero-symmetric NSS and $\mathbb{N}_J := \frac{\mathbb{N}}{(T_J, I_J, F_J)} \in \mathcal{NB}_{\mathcal{I}}(\mathbb{N})$, then $\mathbb{N}_J \mathfrak{B} \mathbb{N}_J \subseteq \mathbb{N}_J$.*

Proof. Let $\mathbb{N}_J \in \mathcal{NB}_{\mathcal{I}}(\mathbb{N})$. Then $\mathbb{N}_J \mathfrak{B} \mathbb{N}_J \cap \mathbb{N}_J \mathfrak{B} * \mathbb{N}_J \subseteq \mathbb{N}_J$. Clearly, $\mathbb{N}_J(0) \supseteq \mathbb{N}_J(d) \forall d \in \mathbb{N}$. Since $\mathbb{N}$ is zero-symmetric, $\mathbb{N}_J \mathfrak{B} \mathbb{N}_J \subseteq \mathbb{N}_J \mathfrak{B} * \mathbb{N}_J$. So $\mathbb{N}_J \mathfrak{B} \mathbb{N}_J \cap \mathbb{N}_J \mathfrak{B} * \mathbb{N}_J = \mathbb{N}_J \mathfrak{B} \mathbb{N}_J \subseteq \mathbb{N}_J$. Hence $\mathbb{N}_J \mathfrak{B} \mathbb{N}_J \subseteq \mathbb{N}_J$. $\square$

Theorem 4.13. *If $\mathbb{N}$ is a zero-symmetric NSS and for a neutrosophic $\varkappa$ -subalgebra $\mathbb{N}_N := \frac{\mathbb{N}}{(T_N, I_N, F_N)}$ of $\mathbb{N}$, the below statements are equivalent:*

- (a) $\mathbb{N}_N \in \mathcal{NB}_{\mathcal{I}}(\mathbb{N})$,
- (b) $\mathbb{N}_N \mathfrak{B} \mathbb{N}_N \subseteq \mathbb{N}_N$.

Proof. By Theorem 4.11 and Theorem 4.12, the proof is obvious. $\square$

Theorem 4.14. *If $\mathbb{N}$ is a zero-symmetric NSS and $\mathbb{N}_N := \frac{\mathbb{N}}{(T_N, I_N, F_N)} \in \mathcal{NB}_{\mathcal{I}}(\mathbb{N})$, then $\mathbb{N}_N(q_1 j_1 c_1) \supseteq \mathbb{N}_N(q_1) \cap \mathbb{N}_N(c_1) \forall q_1, j_1, c_1 \in \mathbb{N}$.*

Proof. Suppose $\mathbb{N}_N \in \mathcal{NB}_{\mathcal{I}}(\mathbb{N})$ of a zero-symmetric NSS. By Theorem 4.12, $\mathbb{N}_N \mathfrak{B} \mathbb{N}_N \subseteq \mathbb{N}_N$.

Let $q_1, j_1, c_1 \in \mathbb{N}$. Then

$$\begin{aligned} T_N(q_1 j_1 c_1) &\leq (T_N \mathfrak{B} T_N)(q_1 j_1 c_1) = \bigwedge_{q_1 j_1 c_1 = d_1 m_1} \{(T_N \mathfrak{B})(d_1) \vee T_N(m_1)\} \\ &\leq (T_N \mathfrak{B})(q_1 j_1) \vee T_N(c_1) \\ &\leq T_N(q_1) \vee \mathfrak{B}(j_1) \vee T_N(c_1) \\ &= T_N(q_1) \vee T_N(c_1), \end{aligned}$$

$$\begin{aligned} I_N(q_1 j_1 c_1) &\geq (I_N \mathfrak{B} I_N)(q_1 j_1 c_1) = \bigvee_{q_1 j_1 c_1 = d_1 m_1} \{(I_N \mathfrak{B})(d_1) \wedge I_N(m_1)\} \\ &\geq (I_N \mathfrak{B})(q_1 j_1) \wedge I_N(c_1) \\ &\geq I_N(q_1) \wedge \mathfrak{B}(j_1) \wedge I_N(c_1) \\ &= I_N(q_1) \wedge I_N(c_1), \end{aligned}$$

$$\begin{aligned} F_N(q_1 j_1 c_1) &\leq (F_N \mathfrak{B} F_N)(q_1 j_1 c_1) = \bigwedge_{q_1 j_1 c_1 = d_1 m_1} \{(F_N \mathfrak{B})(d_1) \vee F_N(m_1)\} \\ &\leq (F_N \mathfrak{B})(q_1 j_1) \vee F_N(c_1) \leq F_N(q_1) \vee \mathfrak{B}(j_1) \vee F_N(c_1) \\ &= F_N(q_1) \vee F_N(c_1). \end{aligned}$$

Hence $\mathbb{N}_N(q_1 j_1 c_1) \supseteq \mathbb{N}_N(q_1) \cap \mathbb{N}_N(c_1)$. $\square$

Theorem 4.15. If $\mathbb{N}$ is a zero-symmetric NSS & $\mathbb{N}_N := \frac{\mathbb{N}}{(T_N, I_N, F_N)} \in \mathcal{NB}_I(\mathbb{N})$, the below assertions are equivalent:

- (i) $\mathbb{N}_N(qjc) \supseteq \mathbb{N}_N(q) \cap \mathbb{N}_N(c) \quad \forall q, j, c \in \mathbb{N}$,
- (ii) $\mathbb{N}_N \mathfrak{B} \mathbb{N}_N \subseteq \mathbb{N}_N$.

Proof. (i) $\Rightarrow$ (ii) Let $v' \in \mathbb{N}$. If $\exists d, m, d_1, d_2 \in \mathbb{N}$ such that $v' = dm$ and $d = d_1 d_2$.

Then by hypothesis, $T_N(d_1 d_2 m) \leq T_N(d_1) \vee T_N(m)$, $I_N(d_1 d_2 m) \geq I_N(d_1) \wedge I_N(m)$, $F_N(d_1 d_2 m) \leq F_N(d_1) \vee F_N(m)$.

Now,

$$\begin{aligned}
 (T_N \mathfrak{B} T_N)(v') &= \bigwedge_{v'=dm} (T_N \mathfrak{B})(d) \vee T_N(m) \\
 &= \bigwedge_{v'=dm} \left(\bigwedge_{d=d_1 d_2} \{T_N(d_1) \vee \mathfrak{B}(d_2)\} \right) \vee T_N(m) \\
 &= \bigwedge_{v'=dm} \left(\bigwedge_{d=d_1 d_2} \{T_N(d_1) \vee -1\} \right) \vee T_N(m) \\
 &= \bigwedge_{v'=d_1 d_2 m} \{T_N(d_1) \vee T_N(m)\} \geq \bigwedge_{v'=d_1 d_2 m} T_N(d_1 d_2 m) = T_N(v'), \\
 (I_N \mathfrak{B} I_N)(v') &= \bigvee_{v'=dm} (I_N \mathfrak{B})(d) \wedge I_N(m) \\
 &= \bigvee_{v'=dm} \left(\bigvee_{d=d_1 d_2} \{I_N(d_1) \wedge \mathfrak{B}(d_2)\} \right) \wedge I_N(m) \\
 &= \bigvee_{v'=dm} \left(\bigvee_{d=d_1 d_2} \{I_N(d_1) \wedge 0\} \right) \wedge I_N(m) \\
 &= \bigvee_{v'=d_1 d_2 m} \{I_N(d_1) \wedge I_N(m)\} \leq \bigvee_{v'=d_1 d_2 m} I_N(d_1 d_2 m) = I_N(v'), \\
 (F_N \mathfrak{B} F_N)(v') &= \bigwedge_{v'=dm} (F_N \mathfrak{B})(d) \vee F_N(m) \\
 &= \bigwedge_{v'=dm} \left(\bigwedge_{d=d_1 d_2} \{F_N(d_1) \vee \mathfrak{B}(d_2)\} \right) \vee F_N(m) \\
 &= \bigwedge_{v'=dm} \left(\bigwedge_{d=d_1 d_2} \{F_N(d_1) \vee -1\} \right) \vee F_N(m) \\
 &= \bigwedge_{v'=d_1 d_2 m} (F_N(d_1) \vee F_N(m)) \geq \bigwedge_{v'=d_1 d_2 m} F_N(d_1 d_2 m) = F_N(v').
 \end{aligned}$$

Hence $\mathbb{N}_N \mathfrak{B} \mathbb{N}_N \subseteq \mathbb{N}_N$.

(ii) $\Rightarrow$ (i) Suppose that $\mathbb{N}_N \mathfrak{B} \mathbb{N}_N \subseteq \mathbb{N}_N$ and let $d, m, g, v' \in \mathbb{N}$ be such that $v' = dm g$. Then

$$\begin{aligned}
T_N(dmg) = T_N(v') &\leq (T_N \mathfrak{B} T_N)(v') = \bigwedge_{v'=fy} \{(T_N \mathfrak{B})(f) \vee T_N(y)\} \\
&= \bigwedge_{v'=fy} \left(\bigwedge_{f=f_1 f_2} \{T_N(f_1) \vee \mathfrak{B}(f_2)\} \right) \vee T_N(y) \\
&\leq T_N(d) \vee \mathfrak{B}(m) \vee T_N(g) \\
&= T_N(d) \vee T_N(g), \\
I_N(dmg) = I_N(v') &\geq (I_N \mathfrak{B} I_N)(v') = \bigvee_{v'=fy} (I_N \mathfrak{B})(f) \wedge I_N(y) \\
&= \bigvee_{v'=fy} \left(\bigvee_{f=f_1 f_2} I_N(f_1) \wedge \mathfrak{B}(f_2) \right) \wedge I_N(y) \\
&\geq I_N(d) \wedge \mathfrak{B}(m) \wedge I_N(g) \\
&= I_N(d) \wedge I_N(g), \\
F_N(dmg) = F_N(v') &\leq (F_N \mathfrak{B} F_N)(v') = \bigwedge_{v'=fy} \{(F_N \mathfrak{B})(f) \vee F_N(y)\} \\
&= \bigwedge_{v'=fy} \left(\bigwedge_{f=f_1 f_2} \{F_N(f_1) \vee \mathfrak{B}(f_2)\} \right) \vee F_N(y) \\
&\leq F_N(d) \vee \mathfrak{B}(m) \vee F_N(g) = F_N(d) \vee F_N(g),
\end{aligned}$$

Hence $\mathbb{N}_N(qjc) \supseteq \mathbb{N}_N(q) \cap \mathbb{N}_N(c) \forall q, j, c \in \mathbb{N}$. $\square$

Theorem 4.16. For a neutrosophic $\varkappa$ - subalgebra $\mathbb{N}_N := \frac{\mathbb{N}}{(T_N, I_N, F_N)}$ of a zero-symmetric NSS, the below assertions are equivalent:

- (i) $\mathbb{N}_N \in \mathcal{NB}_T(\mathbb{N})$,
- (ii) $\mathbb{N}_N(qjc) \supseteq \mathbb{N}_N(q) \cap \mathbb{N}_N(c) \forall q, j, c \in \mathbb{N}$.
- (iii) $\mathbb{N}_N \mathfrak{B} \mathbb{N}_N \subseteq \mathbb{N}_N$.

Proof. By Theorem 4.14 and Theorem 4.15, the proof is simple. $\square$

5. Homomorphism of a neutrosophic $\varkappa$ - structure

In this portion, we explore some characteristics of neutrosophic $\varkappa$ - structures that are homomorphic to near-subtraction semigroups. Hereafter, $\mathbb{N}$ and $\mathbb{N}'$ denote the zero-symmetric near-subtraction semigroups.

Definition 5.1. Let $\psi : \mathbb{N} \rightarrow \mathbb{N}'$ be a mapping.

- (i) ψ is a homomorphism of $\mathbb{N}$ into $\mathbb{N}'$ if $\psi(m_1 - v_1) = \psi(m_1) - \psi(v_1)$ and $\psi(m_1 v_1) = \psi(m_1) \psi(v_1) \forall m_1, v_1 \in \mathbb{N}$.

(ii) For $\tilde{l}_\varsigma \in \mathfrak{H}(\mathbb{N}')$, the preimage of $\tilde{l}_\varsigma$ under ψ , represented as $\psi^{-1}(\tilde{l}_\varsigma)$, is a neutrosophic $\varkappa$ -structure of $\mathbb{N}$ defined by $\psi^{-1}(\tilde{l}_\varsigma) := (\psi^{-1}(\tilde{l}), \psi^{-1}(\varsigma))$, where $\psi^{-1}(\tilde{l})(r_1) = \tilde{l}(\psi(r_1))$ & $\psi^{-1}(\varsigma)(r_1) = \varsigma(\psi(r_1)) \forall r_1 \in \mathbb{N}$.

Theorem 5.2. For a homomorphism $\psi : \mathbb{N} \rightarrow \mathbb{N}'$ and $\mathbb{N}'_N := \frac{\mathbb{N}'}{(T_N, I_N, F_N)}$, if $\mathbb{N}'_N \in \mathcal{NB}_I(\mathbb{N}')$, then $\psi^{-1}(\mathbb{N}'_N) \in \mathcal{NB}_I(\mathbb{N})$.

Proof. Suppose $\mathbb{N}'_N \in \mathcal{NB}_I(\mathbb{N}')$ and let $w_0, d_0 \in \mathbb{N}$. Then $\psi^{-1}(T_N)(w_0 - d_0) = T_N(\psi(w_0 - d_0)) = T_N(\psi(w_0) - \psi(d_0)) \leq T_N(\psi(w_0)) \vee T_N(\psi(d_0)) = \psi^{-1}(T_N)(w_0) \vee \psi^{-1}(T_N)(d_0)$, $\psi^{-1}(I_N)(w_0 - d_0) = I_N(\psi(w_0 - d_0)) = I_N(\psi(w_0) - \psi(d_0)) \geq I_N(\psi(w_0)) \wedge I_N(\psi(d_0)) = \psi^{-1}(I_N)(w_0) \wedge \psi^{-1}(I_N)(d_0)$, $\psi^{-1}(F_N)(w_0 - d_0) = F_N(\psi(w_0 - d_0)) = F_N(\psi(w_0) - \psi(d_0)) \leq F_N(\psi(w_0)) \vee F_N(\psi(d_0)) = \psi^{-1}(F_N)(w_0) \vee \psi^{-1}(F_N)(d_0)$.

By Theorem 4.16, assume that $\mathbb{N}_N \mathfrak{B} \mathbb{N}_N \subseteq \mathbb{N}_N$. Let $w, g, m, w' \in \mathbb{N}$ be such that $w' = wgm$. Then

$$\begin{aligned} \psi^{-1}(T_N)(wgm) &= T_N(\psi(w')) \leq (T_N \mathfrak{B} T_N)(\psi(w')) \\ &= \bigwedge_{w'=fy} \{(T_N \mathfrak{B})(\psi(f)) \vee T_N(\psi(y))\} \\ &= \bigwedge_{w'=fy} \left(\bigwedge_{f=f_1 f_2} \{T_N(\psi(f_1)) \vee \mathfrak{B}(f_2)\} \right) \vee T_N(\psi(y)) \\ &\leq T_N(\psi(w)) \vee \mathfrak{B}(g) \vee T_N(\psi(m)) \\ &= T_N(\psi(w)) \vee T_N(\psi(m)) = \psi^{-1}(T_N)(w) \vee \psi^{-1}(T_N)(m), \\ \psi^{-1}(I_N)(wgm) &= I_N(\psi(w')) \geq (I_N \mathfrak{B} I_N)(\psi(w')) \\ &= \bigvee_{w'=fy} \{(I_N \mathfrak{B})(\psi(f)) \wedge I_N(\psi(y))\} \\ &= \bigvee_{w'=fy} \left(\bigvee_{f=f_1 f_2} \{I_N(\psi(f_1)) \wedge \mathfrak{B}(f_2)\} \right) \wedge I_N(\psi(y)) \\ &\geq I_N(\psi(w)) \wedge \mathfrak{B}(g) \wedge I_N(\psi(m)) \\ &= I_N(\psi(w)) \wedge I_N(\psi(m)) = \psi^{-1}(I_N)(w) \wedge \psi^{-1}(I_N)(m), \\ \psi^{-1}(F_N)(wgm) &= F_N(\psi(w')) \leq (F_N \mathfrak{B} F_N)(\psi(w')) \\ &= \bigwedge_{w'=fy} \{(F_N \mathfrak{B})(\psi(f)) \vee F_N(\psi(y))\} \\ &= \bigwedge_{w'=fy} \left(\bigwedge_{f=f_1 f_2} \{F_N(\psi(f_1)) \vee \mathfrak{B}(f_2)\} \right) \vee F_N(\psi(y)) \\ &\leq F_N(\psi(w)) \vee \mathfrak{B}(g) \vee F_N(\psi(m)) \\ &= F_N(\psi(w)) \vee F_N(\psi(m)) = \psi^{-1}(F_N)(w) \vee \psi^{-1}(F_N)(m). \end{aligned}$$

So $\psi^{-1}(\mathbb{N}'_N) \in \mathcal{NB}_I(\mathbb{N})$. $\square$

Theorem 5.3. For a onto homomorphism $\psi : \mathbb{N} \rightarrow \mathbb{N}'$ & $\mathbb{N}'_N := \frac{\mathbb{N}'}{(T_N, I_N, F_N)}$, if $\psi^{-1}(\mathbb{N}'_N) \in \mathcal{NB}_{\mathcal{I}}(\mathbb{N})$, then $\mathbb{N}'_N \in \mathcal{NB}_{\mathcal{I}}(\mathbb{N}')$.

Proof. Let $\psi^{-1}(\mathbb{N}'_N) \in \mathcal{NB}_{\mathcal{I}}(\mathbb{N})$ & $\mathfrak{k}', z' \in \mathbb{N}'$. Then $\exists z, \mathfrak{k} \in \mathbb{N}$ such that $\psi(\mathfrak{k}) = \mathfrak{k}'$ and $\psi(z) = z'$. Now,

$$\begin{aligned} T_N(\mathfrak{k}' - z') &= T_N(\psi(\mathfrak{k}) - \psi(z)) = T_N(\psi(\mathfrak{k} - z)) \\ &= \psi^{-1}(T_N)(\mathfrak{k} - z) \\ &\leq \psi^{-1}(T_N)(\mathfrak{k}) \vee \psi^{-1}(T_N)(z) \\ &= T_N(\psi(\mathfrak{k})) \vee T_N(\psi(z)) = T_N(\mathfrak{k}') \vee T_N(z'), \\ I_N(\mathfrak{k}' - z') &= I_N(\psi(\mathfrak{k}) - \psi(z)) = I_N(\psi(\mathfrak{k} - z)) \\ &= \psi^{-1}(I_N)(\mathfrak{k} - z) \\ &\geq \psi^{-1}(I_N)(\mathfrak{k}) \wedge \psi^{-1}(I_N)(z) \\ &= I_N(\psi(\mathfrak{k})) \wedge I_N(\psi(z)) = I_N(\mathfrak{k}') \wedge I_N(z'), \\ F_N(\mathfrak{k}' - z') &= F_N(\psi(\mathfrak{k}) - \psi(z)) = F_N(\psi(\mathfrak{k} - z)) \\ &= \psi^{-1}(F_N)(\mathfrak{k} - z) \\ &\leq \psi^{-1}(F_N)(\mathfrak{k}) \vee \psi^{-1}(F_N)(z) \\ &= F_N(\psi(\mathfrak{k})) \vee F_N(\psi(z)) = F_N(\mathfrak{k}') \vee F_N(z'), \end{aligned}$$

By Theorem 4.16, assume that $\tilde{l}_\zeta \mathfrak{B} \tilde{l}_\zeta \ll \tilde{l}_\zeta$. Let $\mathfrak{k}', r', m', g' \in \mathbb{N}'$. Then $\exists \mathfrak{k}, r, m \in \mathbb{N}$ such that $\psi(\mathfrak{k}) = \mathfrak{k}'$, $\psi(r) = r'$, $\psi(m) = m'$ and $g' = \mathfrak{k}'r'm'$. Then

$$\begin{aligned} T_N(\mathfrak{k}'r'm') &= T_N(\psi(g')) \leq (T_N \mathfrak{B} T_N)(\psi(g')) = \psi^{-1}(T_N \mathfrak{B} T_N)(g') \\ &= \bigwedge_{g'=fy} \psi^{-1}(T_N \mathfrak{B})(f) \vee \psi^{-1}(T_N)(y) \\ &= \bigwedge_{g'=fy} \left(\bigwedge_{f=f_1f_2} \psi^{-1}(T_N(f_1) \vee \mathfrak{B}(f_2)) \right) \vee \psi^{-1}(T_N)(y) \\ &\leq \psi^{-1}(T_N)(\mathfrak{k}) \vee \mathfrak{B}(r) \vee \psi^{-1}(T_N)(m) \\ &= \psi^{-1}(T_N)(\mathfrak{k}) \vee \psi^{-1}(T_N)(m) = T_N(\psi(\mathfrak{k})) \vee T_N(\psi(m)) = T_N(\mathfrak{k}') \vee T_N(m'), \\ I_N(\mathfrak{k}'r'm') &= I_N(\psi(g')) \geq (I_N \mathfrak{B} I_N)(\psi(g')) = \psi^{-1}(I_N \mathfrak{B} I_N)(g') \\ &= \bigvee_{g'=fy} \psi^{-1}(I_N \mathfrak{B})(f) \wedge \psi^{-1}(I_N)(y) \\ &= \bigvee_{g'=fy} \left(\bigvee_{f=f_1f_2} \psi^{-1}(I_N(f_1) \wedge \mathfrak{B}(f_2)) \right) \wedge \psi^{-1}(I_N)(y) \end{aligned}$$

$$\begin{aligned}
&\geq \psi^{-1}(I_N)(\mathfrak{k}) \wedge \mathfrak{B}(r) \wedge \psi^{-1}(I_N)(m) \\
&= \psi^{-1}(I_N)(\mathfrak{k}) \wedge \psi^{-1}(I_N)(m) = I_N(\psi(\mathfrak{k})) \wedge I_N(\psi(m)) = I_N(\mathfrak{k}') \wedge I_N(m'), \\
F_N(\mathfrak{k}'r'm') &= F_N(\psi(g')) \leq (F_N\mathfrak{B}F_N)(\psi(g')) = \psi^{-1}(F_N\mathfrak{B}F_N)(g') \\
&= \bigwedge_{g'=fy} \psi^{-1}(F_N\mathfrak{B})(f) \vee \psi^{-1}(F_N)(y) \\
&= \bigwedge_{g'=fy} \left(\bigwedge_{f=f_1f_2} \psi^{-1}(F_N(f_1) \vee \mathfrak{B}(f_2)) \right) \vee \psi^{-1}(F_N)(y) \\
&\leq \psi^{-1}(F_N)(\mathfrak{k}) \vee \mathfrak{B}(r) \vee \psi^{-1}(F_N)(m) \\
&= \psi^{-1}(F_N)(\mathfrak{k}) \vee \psi^{-1}(F_N)(m) = F_N(\psi(\mathfrak{k})) \vee F_N(\psi(m)) = F_N(\mathfrak{k}') \vee F_N(m').
\end{aligned}$$

So $N'_N \in \mathcal{NB}_I(N')$. $\square$

6. Conclusions

This research examines the properties of neutrosophic $\varkappa$ -bi-ideals and develops corresponding bi-ideals within the context of near-subtraction semigroups. Additionally, various aspects of the neutrosophic $\varkappa$ -preimage of the neutrosophic $\varkappa$ -bi-ideal of a near-subtraction semigroup are analyzed under homomorphism mapping. The findings presented in this paper aim to pave the way for defining the concept of a neutrosophic $\varkappa$ -prime bi-ideal and exploring its related properties in near-subtraction semigroups.

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SuperHyperSoft Set-Based Evaluation of Teaching Quality in Ideological and Political Courses in Colleges and Universities in the New Era

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Abstract: In the new era, the quality of ideological and political education in colleges and universities has become a vital focus for nurturing responsible, critically thinking citizens. The rapid evolution of societal needs, coupled with technological advancements, demands a fresh evaluation of teaching effectiveness in these fundamental courses. Traditional evaluation models often struggle to address the uncertainty, complexity, and overlapping nature of qualitative educational indicators. This paper introduces a novel evaluation framework based on SuperHyperSoft Set theory integrated with Single-Valued Neutrosophic Sets (SVNS), combined with Entropy for objective weighting and the MOOSRA method for robust ranking of alternatives. The proposed framework provides a structured, flexible, and uncertainty-resilient approach to evaluating teaching quality. A real-world case study is conducted involving five major political science and ideological courses at a leading university. Expert evaluations were collected, processed using the SuperHyperSoft-SVNS model, and analyzed to identify strengths and areas for improvement. The results demonstrate the model's ability to capture complex educational dynamics and offer practical insights for enhancing teaching practices in the new era.

Keywords: Teaching Quality Evaluation; SuperHyperSoft Set; Single-Valued Neutrosophic Set (SVNS); Entropy Weighting; MOOSRA Method; Higher Education; Uncertainty Modeling.

1. Introduction

Higher education institutions play a crucial role in shaping the political awareness, ethical thinking, and ideological maturity of young citizens. In the new era marked by globalization, rapid technological innovation, and diversified societal values, the traditional methods of ideological and political education face unprecedented challenges [1]. Ensuring the quality of teaching in ideological and political courses has thus become an urgent and strategic priority for colleges and universities. However, evaluating teaching quality in these courses is inherently complex. Unlike technical disciplines where assessments can rely heavily on objective metrics, ideological and political education involves subjective understanding, critical thinking, value formation, and socio-political engagement [2]. These characteristics introduce high degrees of

uncertainty and ambiguity into evaluation processes, making traditional crisp evaluation methods insufficient.

Recent research has emphasized the need for more dynamic, flexible evaluation models capable of handling overlapping indicators and uncertainty [3]. Soft set theory and its extensions, particularly SuperHyperSoft Sets, offer a powerful framework for modeling multi-dimensional, uncertain, and interconnected evaluation criteria [4]. Between 2018-2024, Smarandache[<https://fs.unm.edu/TSS/>] introduced six new types of Soft Sets, such as: the HyperSoft Set (2018), IndetermSoft Set (2022), IndetermHyperSoft Set (2022), SuperHyperSoft Set, TreeSoft Set (2022) and ForestSoft Set (2024). Combined with Single-Valued Neutrosophic Sets (SVNS), which allow for the explicit representation of truth, indeterminacy, and falsity, these tools provide an ideal foundation for developing a robust evaluation methodology.

Moreover, objective weighting methods such as Entropy can eliminate bias from expert judgments by capturing the inherent variability of the data [5], while ranking algorithms like MOOSRA ensure comprehensive and balanced decision-making even under complex conditions [6].

Given these advances, this study proposes a new, uncertainty-resilient evaluation framework for assessing teaching quality in ideological and political courses using a hybrid SuperHyperSoft-SVNS-Entropy-MOOSRA model.

1.1 Problem Statement

Despite their critical importance, ideological and political courses often suffer from inconsistent and subjective evaluations. Current assessment models frequently neglect the interdependence between different teaching indicators (such as clarity of political theory, engagement with students, relevance to contemporary issues, and development of critical thinking) [7]. Moreover, the presence of uncertainty in expert opinions and student feedback is rarely addressed adequately, leading to biased and unreliable evaluation outcomes. There is a pressing need for a structured, systematic, and uncertainty-tolerant approach to evaluating teaching quality in this field.

1.2 Research Motivation

The motivation for this research stems from two key observations. First, traditional evaluation models fail to capture the nuanced, multi-dimensional, and dynamic nature of ideological and political teaching quality [8]. Second, the increasing complexity of societal challenges including information overload, political polarization, and cultural diversity demands more sophisticated educational evaluations that can operate effectively under uncertainty. These factors drive the necessity for innovative evaluation frameworks that reflect the realities of ideological and political education in the new era.

1.3 Research Objectives

This study aims to:

- I. Develop a structured evaluation framework using SuperHyperSoft Set theory to model the hierarchical and overlapping structure of teaching quality indicators.

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- II. Apply Single-Valued Neutrosophic Sets (SVNS) to handle uncertainty and indeterminacy in expert evaluations.
 - III. Use Entropy weighting to objectively determine the relative importance of evaluation criteria.
 - IV. Implement the MOOSRA method to rank teaching quality alternatives systematically.
 - V. Validate the proposed model through a real-world case study involving major ideological and political courses in a leading university.

2. Literature Review

The evaluation of teaching quality in ideological and political education has gained increasing attention over the past decade. As societal demands evolve, so must the mechanisms by which educational effectiveness is assessed. Numerous scholars have emphasized the limitations of traditional evaluation approaches, especially in addressing the subjective, multi-dimensional, and context-sensitive nature of political education [9].

Traditional evaluation methods, such as surveys and expert panel reviews, often rely heavily on linear scoring models. While these methods are simple to implement, they fail to account for the complexity and overlapping nature of teaching criteria [10]. For instance, the ability to engage students in critical discussions is not independent of the clarity with which political theories are delivered, highlighting the need for interconnected evaluation structures.

Recent research has moved towards adopting Multi-Criteria Decision-Making (MCDM) techniques to provide more structured evaluations. Methods like AHP and TOPSIS have been used to prioritize teaching attributes; however, their deterministic nature often limits their adaptability to uncertain environments [11]. Recognizing the shortcomings, researchers have called for models that can handle uncertainty and partial knowledge inherent in educational assessments.

The emergence of Neutrosophic Sets, proposed by Smarandache, provided a revolutionary framework for modeling uncertainty by simultaneously accounting for truth, indeterminacy, and falsity [12]. Applications of Neutrosophic logic in education have shown significant promise, allowing more flexible handling of subjective judgments and ambiguity compared to traditional fuzzy models [13].

In parallel, the development of advanced soft set models, such as SuperHyperSoft Sets, has expanded the capability of modeling hierarchical and overlapping structures within evaluation frameworks [14]. These sets allow criteria to share attributes dynamically, representing the real-world dependencies between different educational qualities more realistically. Studies applying SuperHyperSoft Sets in other domains, such as system reliability and healthcare quality assessments, have demonstrated their effectiveness in dealing with complex evaluation environments [15].

Weighting the importance of different evaluation criteria remains a critical aspect of assessment modeling. While subjective expert-driven methods are common, objective weighting techniques such as Entropy weighting have gained traction for their ability to capture variability in evaluation data without introducing human bias [16]. By applying entropy, the relative

importance of a criterion is determined based on the degree of information it contributes to the system, enhancing the objectivity of the final assessment.

Finally, in ranking alternatives, methods like MOOSRA (Multi-Objective Optimization since Simple Ratio Analysis) have emerged as powerful tools. MOOSRA combines simplicity with strong decision-making capability, especially when integrated with uncertainty-handling models like SVNS [17]. Its ability to balance multiple conflicting criteria makes it particularly suited for evaluating teaching quality where various dimensions of performance must be considered simultaneously.

Despite these advancements, there is a noticeable gap in the literature regarding the application of hybrid models specifically combining SuperHyperSoft Sets, SVNS, Entropy, and MOOSRA for the evaluation of ideological and political education in universities. Most existing studies either focus narrowly on technical disciplines or fail to fully address the uncertainty and complexity inherent in political education evaluation [18].

Therefore, this study aims to bridge this gap by constructing a comprehensive, uncertainty-resilient evaluation model tailored specifically to the needs of ideological and political teaching quality assessment in the new era.

3. Mathematical Model and Key Formulas

This section describes the core mathematical formulas that constitute the proposed evaluation framework. These formulas cover entropy-based criteria weighting, aggregation of expert evaluations using Single-Valued Neutrosophic Sets (SVNS), MOOSRA score computation, and validation of expert consistency.

3.1 Entropy-Based Criteria Weighting

Entropy is employed to derive objective weights for each evaluation criterion based on the diversity of expert judgments.

Entropy Calculation

$$E_j = -k \sum_{i=1}^m p_{ij} \ln(p_{ij})$$

where:

E_j is the entropy value of criterion j ,

p_{ij} is the normalized score of alternatives i on criterion j ,

m is the number of alternatives,

k is a normalization constant, $k=1/\ln(m)$.

The entropy value measures the amount of disorder or uncertainty associated with a criterion. A lower entropy value indicates higher importance due to more diverse evaluations.

Diversification Degree

$$d_j = 1 - E_j$$

where:

d_j is the diversification degree of criterion j .

Diversification reflects the contrast strength among alternatives. A higher d_j suggests that the criterion better differentiates between alternatives.

Weight Determination

$$w_j = \frac{d_j}{\sum_{j=1}^n d_j}$$

where:

w_j is the normalized weight for criterion j ,

n is the total number of criteria.

The final weight w_j is calculated by normalizing the diversification degree, ensuring that the sum of all weights equals 1.

3.2 Aggregation of Single-Valued Neutrosophic Evaluations

To combine multiple expert evaluations, the arithmetic mean aggregation is used across truth-membership, indeterminacy-membership, and falsity-membership degrees.

SVNS Aggregation

$$(T, I, F) = \left(\frac{1}{n} \sum_{k=1}^n T_k, \frac{1}{n} \sum_{k=1}^n I_k, \frac{1}{n} \sum_{k=1}^n F_k \right)$$

where:

T_k , are the truth, indeterminacy, and falsity values from expert k ,

n is the number of experts.

This formula averages the neutrosophic components across experts, creating a collective representation of the evaluations that preserves uncertainty.

3.3 MOOSRA-Based Ranking

The MOOSRA method evaluates alternatives based on a simple ratio analysis weighted by entropy-derived criteria weights.

MOOSRA Score Calculation

$$S_i = \frac{\sum_{j=1}^n w_j \cdot x_{ij}}{\sum_{j=1}^n w_j}$$

where:

S_i is the MOOSRA score of alternatives i ,

w_j is the weight of criterion j ,

x_{ij} is the normalized performance of alternative i under criterion j .

The MOOSRA score represents the overall performance of an alternative by aggregating weighted normalized scores, allowing for direct ranking.

3.4 Validation of Expert Consistency

To ensure the reliability of expert evaluations, Spearman's rank correlation coefficient is employed. The quality and reliability of the evaluation results depend heavily on the consistency among expert judgments. If the experts significantly disagree on their assessments, the final

rankings may be unstable or biased. Therefore, it is crucial to validate the level of agreement among the experts before proceeding with the final analysis.

In this study, we employ Spearman's rank correlation coefficient to measure the degree of consistency between pairs of expert evaluations.

The following steps were taken to validate expert consistency:

1. Each expert's evaluations of the five courses were ranked separately.
2. Spearman's rank correlation coefficient (ρ) was computed for each pair of experts based on their course rankings.
3. A value of ρ close to 1 indicates strong agreement; a value close to 0 indicates weak or no agreement.

The Spearman correlation coefficients between all pairs of experts are shown in Table 1.

Table 1: Spearman Correlation Coefficients between Experts

Experts Compared	Spearman Coefficient
Expert 1 & Expert 2	0.92
Expert 1 & Expert 3	0.89
Expert 2 & Expert 3	0.91
Expert 1 & Expert 4	0.87
Expert 2 & Expert 4	0.88
Expert 3 & Expert 4	0.86

As observed, all correlation values are above 0.85, indicating a very strong agreement among the experts. To better illustrate the consistency among experts, a correlation matrix is explained in Figure 1.

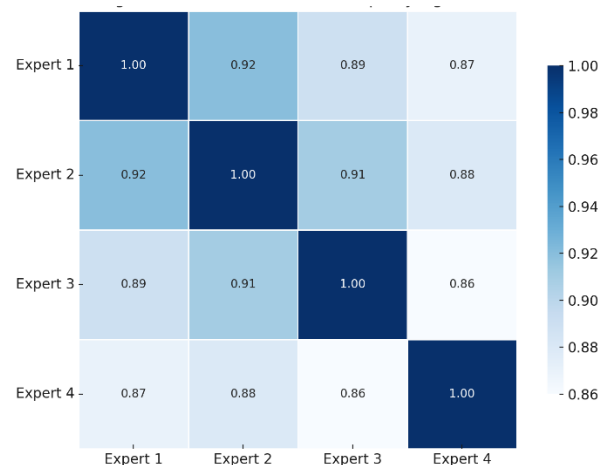


Figure 1. Correlation Matrix

4. Proposed Framework

To comprehensively evaluate the teaching quality of ideological and political courses in colleges and universities, this study proposes a hybrid decision-making model based on SuperHyperSoft Sets, Single-Valued Neutrosophic Sets (SVNS), Entropy Weighting, and the MOOSRA method.

This framework is designed to manage the uncertainty, complexity, and interdependencies that characterize educational quality indicators in the new era.

The overall methodology can be divided into four main stages, as illustrated in Figure 2.

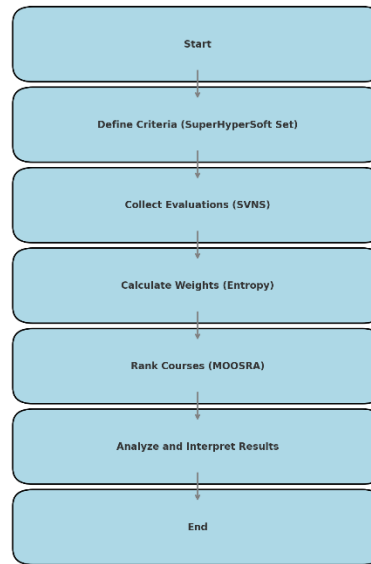


Figure 2: Flowchart of the Proposed Teaching Quality Assessment Framework

4.1 Constructing the Evaluation Structure Using SuperHyperSoft Sets

SuperHyperSoft Sets provide a flexible framework capable of modeling overlapping and multi-layered criteria structures. In this study, we model teaching quality through multiple perspectives — course content delivery, political theory integration, student engagement, relevance to contemporary issues, and critical thinking development.

Each main criterion is associated with several sub-criteria, and relationships between them are not strictly hierarchical but interconnected. This overlapping nature is naturally represented using SuperHyperSoft Sets, which allow a single attribute to contribute to multiple higher-level criteria [19]. The criteria and sub-criteria structure for this evaluation are detailed in **Table 2**.

Table 2: Teaching Quality Criteria and Sub-Criteria Using SuperHyperSoft Structure

Main Criterion	Sub-Criteria
Content Delivery	Clarity, Depth, Logical Structure
Political Theory Integration	Correctness, Contemporary Relevance, Theoretical Rigor
Student Engagement	Interaction, Feedback Responsiveness, Motivation
Real-World Application	Practical Cases, Problem-Solving Orientation, Policy Analysis
Critical Thinking Development	Debate Encouragement, Analytical Assignments, Reflection Opportunities

This SuperHyperSoft structure allows evaluations to capture the real-world complexity of ideological and political teaching quality.

4.2 Collecting Evaluations Using Single-Valued Neutrosophic Sets (SVNS)

Given the subjective nature of evaluating educational quality, uncertainty and partial knowledge are inevitable. Therefore, we adopt Single-Valued Neutrosophic Sets (SVNS) to represent expert evaluations. Each evaluation for a sub-criterion is expressed as a triplet (T, I, F):

- I. T (Truth-membership): Degree of satisfaction
- II. I (Indeterminacy-membership): Degree of uncertainty
- III. F (Falsity-membership): Degree of dissatisfaction

An example of collected SVN evaluations from three experts across different sub-criteria is shown in Table 3.

Table 3: Sample SVN Evaluations for Teaching Quality Indicators

Sub-Criterion	Expert 1 (T, I, F)	Expert 2 (T, I, F)	Expert 3 (T, I, F)
Clarity	(0.9, 0.05, 0.05)	(0.85, 0.10, 0.05)	(0.87, 0.08, 0.05)
Theoretical Rigor	(0.8, 0.10, 0.10)	(0.82, 0.12, 0.06)	(0.78, 0.15, 0.07)
Interaction	(0.85, 0.10, 0.05)	(0.88, 0.07, 0.05)	(0.86, 0.09, 0.05)
Problem-Solving Orientation	(0.75, 0.15, 0.10)	(0.72, 0.18, 0.10)	(0.74, 0.17, 0.09)
Analytical Assignments	(0.80, 0.12, 0.08)	(0.83, 0.10, 0.07)	(0.81, 0.11, 0.08)

The SVN model ensures that uncertainty is not ignored but rather modeled explicitly, enhancing the reliability of the evaluations [20].

4.3 Determining Criteria Weights Using Entropy Method

To objectively determine the importance of each sub-criterion without introducing subjective bias, the Entropy Method is applied.

The process involves the following steps:

1. Normalization of SVN evaluations for each sub-criterion.
2. Calculation of entropy value for each criterion based on information dispersion.
3. Determination of the degree of diversification, which reflects the contrast intensity among alternatives.
4. Calculation of objective weights.

A sample calculation of entropy weights for five sub-criteria is shown in Table 4. Thus, Problem-Solving Orientation is identified as the most influential sub-criterion for teaching quality evaluation, followed closely by Theoretical Rigor.

Table 4: Entropy Weights for Teaching Quality Sub-Criteria

Sub-Criterion	Entropy Value	Diversification Degree	Weight
Clarity	0.85	0.15	0.18
Theoretical Rigor	0.80	0.20	0.20
Interaction	0.90	0.10	0.12
Problem-Solving Orientation	0.78	0.22	0.22
Analytical Assignments	0.83	0.17	0.18

4.4 Ranking Alternatives Using MOOSRA Method

Finally, the MOOSRA Method (Multi-Objective Optimization on the basis of Simple Ratio Analysis) is applied to rank different courses or instructors based on their weighted performance scores. An illustrative ranking result for five political courses is presented in Table 5.

The steps include:

1. Aggregating weighted SVN evaluations for each course.
2. Calculating the performance index (MOOSRA score).

3. Ranking the alternatives according to their scores.

Table 5: MOOSRA Scores and Final Ranking of Courses

Course	MOOSRA Score	Final Ranking
Introduction to Political Science	0.82	1
Marxist Philosophy	0.78	2
Contemporary Chinese Politics	0.75	3
Ethics and Political Thought	0.72	4
History of Political Theories	0.70	5

The results indicate that "Introduction to Political Science" achieves the highest teaching quality score among the evaluated courses. The final ranking outcomes are shown in Figure 3.

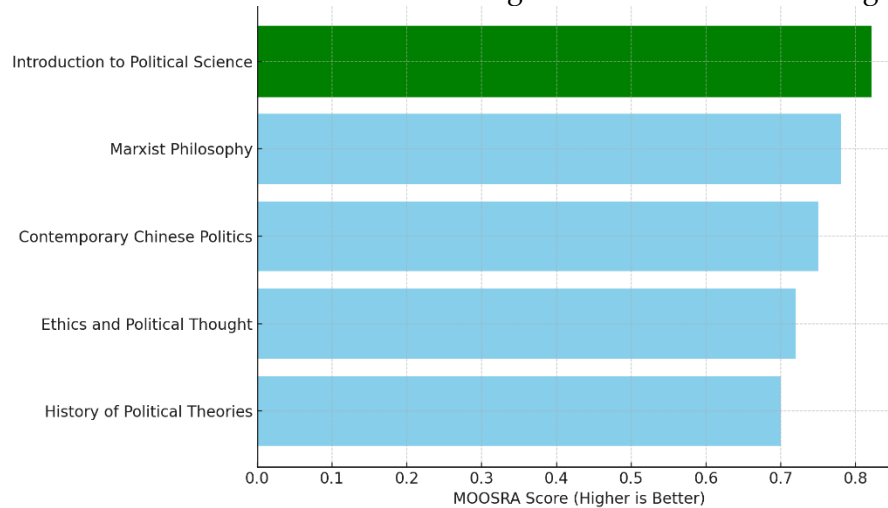


Figure 3. The final ranking outcomes

5. Case Study: Evaluation of Teaching Quality in Ideological and Political Courses

To validate the effectiveness of the proposed SuperHyperSoft-SVNS-Entropy-MOOSRA evaluation model, a case study was conducted at a well-known university in China. The study focuses on five major ideological and political courses currently offered to undergraduate students. The objective was to assess the teaching quality of these courses using expert evaluations and the structured multi-phase methodology developed in this research.

5.1 Selection of Courses and Experts

The five courses selected for evaluation were:

1. Introduction to Political Science
2. Marxist Philosophy
3. Contemporary Chinese Politics
4. Ethics and Political Thought
5. History of Political Theories

A panel of five experts was assembled, consisting of senior professors specializing in political science education and academic quality assurance. Each expert was asked to evaluate the selected courses based on a detailed set of sub-criteria derived from the SuperHyperSoft Set structure outlined earlier.

5.2 Definition of Evaluation Criteria

The courses were evaluated across five main criteria, each broken down into three specific sub-criteria as shown in Table 6.

Table 6: Evaluation Criteria and Sub-Criteria Structure

Main Criterion	Sub-Criteria
Content Delivery	Clarity, Depth, Logical Structure
Political Theory Integration	Correctness, Contemporary Relevance, Theoretical Rigor
Student Engagement	Interaction, Feedback Responsiveness, Motivation
Real-World Application	Practical Cases, Problem-Solving Orientation, Policy Analysis
Critical Thinking Development	Debate Encouragement, Analytical Assignments, Reflection Opportunities

Each sub-criterion reflects a crucial aspect of high-quality ideological and political teaching adapted to the new era.

5.3 Collection of SVNS Evaluations

The experts provided evaluations for each course on each sub-criterion using Single-Valued Neutrosophic Sets (SVNS). A sample of these evaluations for selected sub-criteria is shown in Table 7. This evaluation matrix captured both the certainty and uncertainty in the experts' assessments.

Table 6: Sample SVNS Evaluations of Courses

Course	Clarity (T, I, F)	Interaction (T, I, F)	Analytical Assignments (T, I, F)
Introduction to Political Science	(0.90, 0.07, 0.03)	(0.88, 0.08, 0.04)	(0.85, 0.10, 0.05)
Marxist Philosophy	(0.85, 0.10, 0.05)	(0.82, 0.12, 0.06)	(0.80, 0.12, 0.08)
Contemporary Chinese Politics	(0.83, 0.12, 0.05)	(0.80, 0.15, 0.05)	(0.78, 0.14, 0.08)
Ethics and Political Thought	(0.82, 0.14, 0.04)	(0.78, 0.15, 0.07)	(0.76, 0.16, 0.08)
History of Political Theories	(0.80, 0.15, 0.05)	(0.75, 0.18, 0.07)	(0.74, 0.17, 0.09)

5.4 Determining Criteria Weights Using Entropy

Entropy weights were calculated for the sub-criteria to ensure an objective evaluation. A sample of the calculated weights is presented in Table 8.

Table 8: Entropy Weights for Selected Sub-Criteria

Sub-Criterion	Entropy Value	Weight
Clarity	0.83	0.18
Interaction	0.85	0.17
Analytical Assignments	0.82	0.19

These weights reflect the relative importance of different aspects of teaching quality, ensuring that no single factor dominates the evaluation unfairly.

5.5 Ranking of Courses Using MOOSRA

Using the weighted SVNS values, MOOSRA scores were computed for each course. The results are summarized in Table 9.

Table 9: MOOSRA Scores and Final Ranking of Courses

Course	MOOSRA Score	Final Ranking
--------	--------------	---------------

Introduction to Political Science	0.85	1
Marxist Philosophy	0.81	2
Contemporary Chinese Politics	0.78	3
Ethics and Political Thought	0.75	4
History of Political Theories	0.72	5

The results highlight that "Introduction to Political Science" had the highest perceived teaching quality, validating the broader relevance and clarity of its content delivery. The course rankings are illustrated in Figure 3.

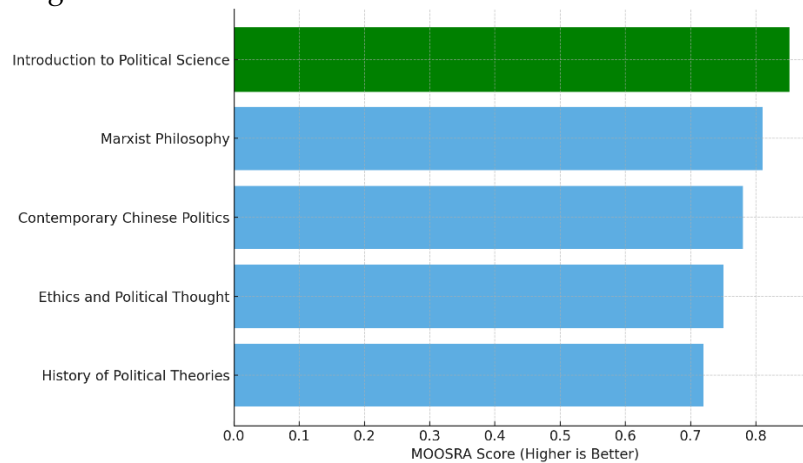


Figure 3. The course rankings

6. Results and Discussion

This section presents the results obtained from the case study and discusses their implications for the evaluation of teaching quality in ideological and political courses within the new era. The analysis is based on the MOOSRA scores computed in Section 5, incorporating uncertainty modeling and objective weighting mechanisms.

6.1 Interpretation of MOOSRA Results

As shown previously in Table 9 and Figure 3, "Introduction to Political Science" received the highest MOOSRA score (0.85), followed closely by "Marxist Philosophy" (0.81) and "Contemporary Chinese Politics" (0.78). "Ethics and Political Thought" and "History of Political Theories" received slightly lower scores, 0.75 and 0.72 respectively.

These results suggest that introductory political science courses tend to be better structured, more engaging, and more successful at integrating contemporary relevance into their content. This may be attributed to the course's broader curriculum design, which allows for easier connection to current events and student experiences [23].

In contrast, more historical or theory-heavy courses like "History of Political Theories" scored lower, potentially due to challenges in making classical political theories feel directly relevant to modern societal issues. This highlights the need for innovative teaching approaches, such as integrating case studies, simulations, and real-world political analysis into traditional theoretical courses.

6.2. Sensitivity Analysis

To perform the extended sensitivity analysis, the following systematic changes were applied to the entropy-derived weights:

1. **Scenario 1:** Increase the weight of "Student Engagement" by 20%.
2. **Scenario 2:** Decrease the weight of "Content Delivery" by 15%.
3. **Scenario 3:** Increase the weight of "Critical Thinking Development" by 25%.
4. **Scenario 4:** Apply random $\pm 10\%$ variations across all criteria weights.

After adjusting the weights, the MOOSRA scores were recalculated, and the new rankings of the five evaluated courses were determined for each scenario.

The results are summarized in Table 10.

Table 10: Rankings Under Different Weight Scenarios

Scenario	Top 3 Courses (in Order)
Original Weights	Political Science > Marxist Philosophy > Chinese Politics
Scenario 1 (+20% Engagement)	Political Science > Marxist Philosophy > Chinese Politics
Scenario 2 (-15% Content Delivery)	Political Science > Marxist Philosophy > Chinese Politics
Scenario 3 (+25% Critical Thinking)	Political Science > Marxist Philosophy > Chinese Politics
Scenario 4 (Random $\pm 10\%$)	Political Science > Chinese Politics > Marxist Philosophy

As shown in Table 10, the top course "Introduction to Political Science" consistently maintained its leading position across all scenarios, confirming the robustness of the evaluation results.

A slight swapping between the second and third ranks occurred under random variation, but the overall structure remained largely unchanged.

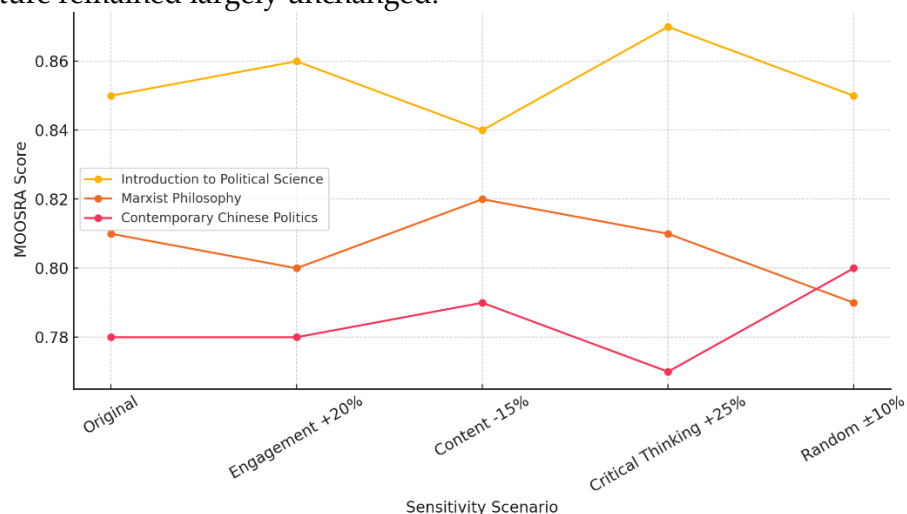


Figure 4. The variations in the MOOSRA scores across scenarios

Figure 4 clearly demonstrates the stability of the top-performing course under various weight adjustments. Although minor fluctuations are observed in the second and third-ranked courses, the leading position of "Introduction to Political Science" remains unaffected. This indicates that the proposed evaluation model is highly resilient and provides consistent results even when evaluation priorities shift significantly.

6.3 Comparative Discussion

Comparing these results with earlier studies in educational evaluation confirms several trends. Prior research indicates that student engagement and clarity in content delivery are critical success factors for high teaching quality in political education [24]. Our findings align with this, as the courses scoring highest were those that combined clarity, relevance, and engagement.

Furthermore, the use of SuperHyperSoft Sets and SVNS introduced an additional layer of realism by capturing the complexity and ambiguity in educational assessments. Unlike traditional crisp methods, our model acknowledged the partial knowledge and subjective uncertainty inherent in expert evaluations, leading to more nuanced results [25].

Moreover, applying objective Entropy weighting minimized potential biases that often plague expert-driven evaluation systems, thereby enhancing the credibility and objectivity of the final outcomes.

6.4 Implications for Educational Practice

The results of this case study offer several important implications:

- I. Should prioritize clarity, engagement, and real-world application when designing ideological and political courses.
- II. Need to incorporate interactive and analytical components to maintain student interest and enhance critical thinking skills.
- III. We should use uncertainty-resilient evaluation frameworks like the one proposed here to better identify areas for pedagogical improvement.

By adopting more sophisticated evaluation models, universities can ensure that their political education courses remain impactful, relevant, and aligned with the demands of the new era.

7. Managerial Implications

From a management perspective, the adoption of the SuperHyperSoft-SVNS-Entropy-MOOSRA model provides universities with a robust and structured framework to evaluate and enhance their teaching effectiveness.

The key managerial actions informed by this study are:

- I. Universities can utilize the objective weighting approach to eliminate subjective bias from their quality evaluations, ensuring fairer assessments of teaching staff.
- II. Departments can prioritize investment in courses that demonstrate high critical thinking development and student engagement, as these criteria heavily influence overall teaching quality.
- III. By consistently applying the proposed model, universities can monitor teaching quality trends over time, detecting early signs of decline or improvement.
- IV. The detailed evaluation across sub-criteria highlights specific areas where instructors may require targeted professional development, such as interactive teaching techniques or contemporary relevance integration.

8. Conceptual Model of Practical Application

The overall practical application of this study's findings can be summarized in Figure 6, which illustrates a conceptual flow from evaluation to continuous improvement. By continuously assessing, identifying gaps, implementing targeted enhancements, and re-evaluating, universities can maintain and advance the teaching quality of their ideological and political courses in a systematic and sustainable manner.

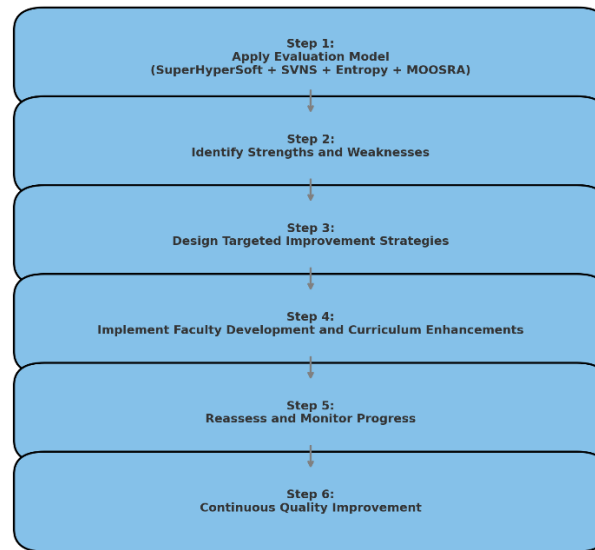


Figure 6: Conceptual Model for Teaching Quality Evaluation and Continuous Improvement

9. Neutrosophic Graph-Based Evaluation Example

To further enhance the evaluation model, a Neutrosophic Graph approach is applied to illustrate the relationships among key teaching quality criteria. Unlike traditional graphs, a neutrosophic graph allows each connection between two nodes to be characterized by three components: truth-membership (T), indeterminacy-membership (I), and falsity-membership (F).

This enables a more flexible and realistic modeling of uncertain and imprecise relations among educational criteria.

9. Construction of the Neutrosophic Graph

In this example, four major teaching quality criteria are selected:

1. C1: Clarity of Content Delivery
2. C2: Student Engagement
3. C3: Critical Thinking Development
4. C4: Real-World Application

The pairwise relationships among these criteria are evaluated by an expert panel, and their neutrosophic values are presented in Table 11.

Table 11: Neutrosophic Values Between Criteria

From → To	Truth (T)	Indeterminacy (I)	Falsity (F)
Clarity → Engagement	0.8	0.1	0.1
Clarity → Critical Thinking	0.7	0.2	0.1

Engagement → Critical Thinking	0.9	0.05	0.05
Engagement → Real-World Application	0.85	0.1	0.05
Critical Thinking → Real-World Application	0.8	0.15	0.05

These values represent how much one criterion supports the development of another, considering uncertainty. The neutrosophic relationships are illustrated in Figure 5.

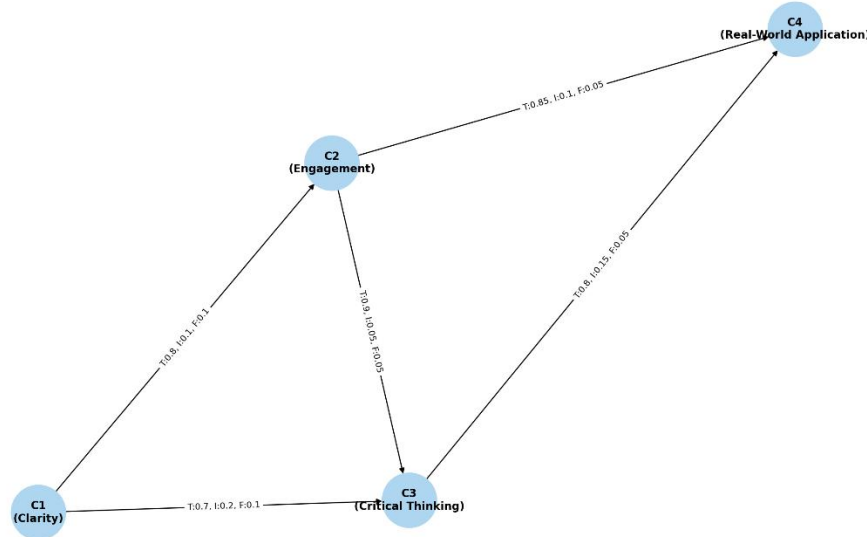


Figure 5: Neutrosophic Graph of Teaching Quality Criteria Relationships

10. Conclusion and Future Work

In the context of rapid societal changes and the growing complexity of educational demands, the quality evaluation of ideological and political teaching in universities has become increasingly critical. Traditional evaluation models often fall short in addressing the multi-dimensionality, overlapping structures, and inherent uncertainty that characterize teaching performance in these subjects. This paper proposed a comprehensive evaluation framework combining SuperHyperSoft Sets, Single-Valued Neutrosophic Sets (SVNS), Entropy weighting, and the MOOSRA method. The methodology was designed to capture the complex interrelations among teaching quality indicators, handle uncertainty effectively, and objectively determine the importance of evaluation criteria.

The case study conducted on five major political courses demonstrated the effectiveness and robustness of the proposed model. "Introduction to Political Science" was identified as the highest-ranked course, benefiting from a combination of clarity, student engagement, and contemporary relevance. Sensitivity analysis confirmed the stability of the results even under variations in the weighting scheme, validating the reliability of the model.

Overall, the study highlights that adopting uncertainty-resilient and flexible evaluation frameworks is crucial for maintaining and enhancing the quality of ideological and political education in the new era. Institutions that integrate such models into their quality assurance processes are better positioned to meet the evolving needs of students and society at large.

11. Future Research Directions

While the proposed model has proven to be effective, several avenues for future research are suggested:

- I. The model can be adapted and tested in other educational contexts, such as humanities, social sciences, and interdisciplinary studies, to assess its generalizability.
- II. Future work could explore dynamic versions of the SuperHyperSoft Set where criteria relationships evolve over time based on changing educational priorities and societal demands.
- III. Combining neutrosophic-based evaluation outputs with machine learning algorithms could enable predictive analytics for teaching performance and early detection of quality decline.
- IV. Conducting longitudinal case studies could offer insights into how teaching quality evolves over multiple semesters or academic years.
- V. Future studies could integrate fuzzy logic, grey systems, or intuitionistic fuzzy systems with the current model to handle even more diverse types of uncertainty in educational assessments.

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On Fixed Point Results in Neutrosophic Metric Spaces Using Auxiliary Functions

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Abstract. In this paper, we establish novel fixed point theorems in the framework of *neutrosophic metric spaces* (NMS) by introducing the concept of *neutrosophic $(\mathcal{L}, \phi)$ -contractions*. These contractions generalize classical contractive conditions by incorporating a function $\mathcal{L}$ that bounds the interaction between displacement terms and a control function ϕ that modulates the contraction intensity. Under specific hypothesis, we prove that every neutrosophic $(\mathcal{L}, \phi)$ -contraction on a *complete NMS* admits a *unique fixed point*. As applications, we derive several corollaries by specifying the forms of $\mathcal{L}$ and ϕ , including cases where $\mathcal{L}$ is linear, additive, or defined via maximum functions. Our results unify and extend existing fixed point theorems in neutrosophic settings, while illustrative examples demonstrate their practical applicability.

Keywords: Fixed point; Neutrosophic set; Neutrosophic metric; $(\mathcal{L}, \phi)$ -contraction; Non linear contraction

1. Introduction

The Banach fixed-point theorem [1], commonly referred to as the contraction mapping theorem, is a key principle in the theory of metric spaces. It asserts that within any complete metric space, a contraction mapping defined as a function, that reduces the distance between points, will possess a unique fixed point. This theorem is important as it offers a systematic approach to identifying fixed points, which are defined as points that remain invariant under a specific function. Furthermore, it ensures both the existence and uniqueness of such points under certain conditions. The theorem finds extensive application across various disciplines, including differential equations and numerical analysis, owing to its utility in establishing the existence and uniqueness of solutions to equations and in iterative methods for solution

approximation. First introduced by Banach in 1922 [1], this theorem has motivated numerous mathematicians to investigate various extensions and generalizations across a wide range of mathematical domains, as referenced in [2–4].

Recent advancements in fixed point theory have extended classical results to generalized metric spaces, including neutrosophic and fuzzy frameworks. [7] established quasi-contraction fixed point theorems in neutrosophic fuzzy metric spaces, while [8] explored nonlinear contractions in the same setting. In complete neutrosophic metric spaces, [5] introduced ψ -quasi-contractions. Auxiliary functions have played a key role in generalizations, as seen in b -metric spaces [6] and generalized metric spaces [10]. Further contributions include Geraghty-type contractions under ωt -distance [9] and integral-type contractions in neutrosophic spaces [11]. Additionally, [12] investigated fixed points for Geraghty mappings using equivalent distances, highlighting the versatility of neutrosophic structures in unifying discontinuous and imprecise data scenarios. [13] in their work establishing new fixed point theorems. This direction was further advanced by [14], who investigated rational $(\alpha, \beta)\phi$ - $m\omega$ contractions within complete quasi metric spaces, providing important extensions to existing theory. These contributions collectively demonstrate the ongoing evolution of fixed point analysis through sophisticated distance function modifications and contraction mappings. [15] established new fixed point theorems for NF-L contractions in complete neutrosophic fuzzy metric spaces, while [16] developed parallel results for T-distance spaces in complete b-metric spaces. These works demonstrate the ongoing expansion of fixed point theory into increasingly abstract distance structures.

Zadeh [17] has made a significant contribution to numerous scientific disciplines through the introduction of fuzzy sets, which have extensive research and application potential, as noted in [18–21] and related references. Although this setting is highly relevant to practical applications, it is not always provided effective solutions to many challenges over the years. Consequently, there has been a renewed emphasis on research aimed at addressing these issues. In this regard, Atanassov [22] presented Intuitionistic Fuzzy Sets (IFSs) as a method to confront these challenges.

Smarandache [23] developed the concept of the Neutrosophic Set (NS), serves as a sophisticated extension of traditional set theory. This concept also exhibits a broad spectrum of applications across different domains. For instance, The authors in [24] examined the scale invariance characteristic of the stable Pareto distribution. They provided an overview of Mathematica code, particularly highlighting the application of Neutrosophic logic in guiding risk management principles. Additionally, The authors in [25] investigated the resolution of first-order differential equations by employing trapezoidal neutrosophic numbers as initial conditions. They analyzed different forms based on the relationships among truth, indeterminacy,

and falsity. For a more thorough exploration of the applications of NS and its uses, the readers should refer to [28–35] and the associated references therein.

Recent developments in neutrosophic mathematics have expanded decision-making frameworks and algebraic structures. [36,37] introduced algorithmic approaches using similarity measures in interval-valued neutrosophic soft settings, while [38] developed novel Q-neutrosophic soft interval matrices with practical applications. Earlier foundational work by [39,40] established soft expert on complex multi-fuzzy classes, and more recently, [41,42] presented significant generalizations of interval-valued Q-neutrosophic soft matrices with broad applicability. These contributions collectively advance the theoretical foundations and practical implementations of neutrosophic systems.

2. Preliminary

Triangular norms (TNs), introduced by Menger [43], extending the triangle inequality in metric spaces. Triangular conorms (CNs), as duals of TNs, model the union of fuzzy sets. Both TNs and CNs are essential in fuzzy logic, particularly for intersections and unions.

In this document, we define the sets as follows: $\mathbb{R}^+ = [0, \infty)$ and $\mathcal{I} = [0, 1]$.

Definition 2.1. Let $\bullet : \mathcal{I} \times \mathcal{I} \rightarrow \mathcal{I}$ be an operation. Then, $\bullet$ is called as a continuous t-norm (TNs) if it satisfies the following properties for all $\sigma, \sigma', s, s' \in \mathcal{I}$.

- (1) $\sigma \bullet 1 = \sigma$,
- (2) If $\sigma \leq \sigma'$ and $s \leq s'$, then $\sigma \bullet s \leq \sigma' \bullet s'$,
- (3) $\bullet$ is continuous,
- (4) $\bullet$ is commutative and associate.

Definition 2.2. Let $\diamond : \mathcal{I} \times \mathcal{I} \rightarrow \mathcal{I}$ be an operation. Then, $\diamond$ is called to as a continuous t-conorm (CNs) if it satisfies the following conditions for all elements $\sigma, \sigma', s, s' \in \mathcal{I}$:

- (1) $\sigma \diamond 0 = \sigma$,
- (2) If $\sigma \leq \sigma'$ and $s \leq s'$, then $\sigma \diamond s \leq \sigma' \diamond s'$,
- (3) $\diamond$ is continuous,
- (4) $\diamond$ is commutative and associate.

The notion of neutrosophic metric space (NMS) was initially presented by Kirisci and Simsek. This framework has been utilized to investigate various fixed point theorems. The definition of neutrosophic metric spaces is outlined as follows.

Definition 2.3. [44] A 6-tuple $(\mathcal{W}, \mathcal{N}, \mathcal{U}, \mathcal{O}, \bullet, \diamond)$ is defined as a Neutrosophic Metric Space (NMS) when the set $\mathcal{W}$ is non-empty. In this setting, $\bullet$ represents a continuous t-norm, and $\diamond$ denotes a continuous t-conorm. Moreover, the elements $\mathcal{N}, \mathcal{U}$, and $\mathcal{O}$ are fuzzy sets defined

on $\mathcal{W}^2 \times (0, \infty)$. These components must meet the following conditions for all $\zeta, \varrho, c \in \mathcal{W}$, and all $\tau, \rho > 0$.

- (1) $0 \leq \mathcal{N}(\zeta, \varrho, \tau) \leq 1, 0 \leq \mathcal{U}(\zeta, \varrho, \tau) \leq 1, 0 \leq \mathcal{O}(\zeta, \varrho, \tau) \leq 1,$
- (2) $0 \leq \mathcal{N}(\zeta, \varrho, \tau) + \mathcal{U}(\zeta, \varrho, \tau) + \mathcal{O}(\zeta, \varrho, \tau) \leq 3,$
- (3) $\mathcal{N}(\zeta, \varrho, \tau) = 1$, for $\tau > 0$ iff $\zeta = \varrho$
- (4) $\mathcal{N}(\zeta, \varrho, \tau) = H(\varrho, \zeta, \tau)$, for $\tau > 0$
- (5) $\mathcal{N}(\zeta, \varrho, \tau) \bullet \mathcal{N}(\varrho, c, \rho) \leq \mathcal{N}(\zeta, c, \tau + \rho)$
- (6) $\mathcal{N}(\zeta, \varrho, \cdot) : \mathbb{R}^+ \rightarrow \mathcal{I}$ is continuous
- (7) $\lim_{\tau \rightarrow \infty} \mathcal{N}(\zeta, \varrho, \tau) = 1$
- (8) $\mathcal{U}(\zeta, \varrho, \tau) = 0$ iff $\zeta = \varrho$
- (9) $\mathcal{U}(\zeta, \varrho, \tau) = \mathcal{U}(\varrho, \zeta, \tau),$
- (10) $\mathcal{U}(\zeta, \varrho, \tau) \diamond \mathcal{U}(\varrho, c, \rho) \geq \mathcal{U}(\zeta, c, \tau + \rho),$
- (11) $\mathcal{U}(\zeta, \varrho, \cdot) : \mathbb{R}^+ \rightarrow \mathcal{I}$ is continuous
- (12) $\lim_{\tau \rightarrow \infty} \mathcal{U}(\zeta, \varrho, \tau) = 0$
- (13) $\mathcal{O}(\zeta, \varrho, \tau) = 0$, for $\tau > 0$ iff $\zeta = \varrho$
- (14) $\mathcal{O}(\zeta, \varrho, \tau) = \mathcal{O}(\varrho, \zeta, \tau),$
- (15) $\mathcal{O}(\zeta, \varrho, \tau) \diamond \mathcal{O}(\varrho, c, \rho) \geq \mathcal{O}(\zeta, c, \tau + \rho),$
- (16) $\mathcal{O}(\zeta, \varrho, \cdot) : \mathbb{R} \rightarrow \mathcal{I}$ is continuous
- (17) $\lim_{\tau \rightarrow \infty} \mathcal{O}(\zeta, \varrho, \tau) = 0$
- (18) If $\tau \leq 0$, then $\mathcal{N}(\zeta, \varrho, \tau) = 0, \mathcal{U}(\zeta, \varrho, \tau) = \mathcal{O}(\zeta, \varrho, \tau) = 1$

The functions $\mathcal{N}(\zeta, \varrho, \tau)$, $\mathcal{U}(\zeta, \varrho, \tau)$, and $\mathcal{O}(\zeta, \varrho, \tau)$ represent the degrees of nearness, neutralness, and non-nearness between ζ and ϱ in relation to the parameter τ , respectively.

The convergence, Cauchy-ness, completeness in NMS are given as follows.

Definition 2.4. [44] Let (ζ_n) be a sequence in a NMS $(\mathcal{W}, \mathcal{N}, \mathcal{U}, \mathcal{O}, \bullet, \diamond)$. Then

- (1) (ζ_n) converges to $\zeta \in \mathcal{W}$ if for a given $\epsilon \in (0, 1)$, $\tau > 0$ there is $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$

$$\mathcal{N}(\zeta_n, \zeta, \tau) > 1 - \epsilon, \mathcal{U}(\zeta_n, \zeta, \tau) < \epsilon, \mathcal{O}(\zeta_n, \zeta, \tau) < \epsilon$$

i.e.,

$$\lim_{n \rightarrow \infty} \mathcal{N}(\zeta_n, \zeta, \tau) = 1, \lim_{n \rightarrow \infty} \mathcal{U}(\zeta_n, \zeta, \tau) = 0, \lim_{n \rightarrow \infty} \mathcal{O}(\zeta_n, \zeta, \tau) = 0$$

- (2) (ζ_n) is called Cauchy if for a given $\epsilon \in (0, 1)$, $\tau > 0$ there is $n_0 \in \mathbb{N}$ such that for all $n, m \geq n_0$

$$\mathcal{N}(\zeta_n, \zeta_m, \tau) > 1 - \epsilon, \mathcal{U}(\zeta_n, \zeta_m, \tau) < \epsilon, \mathcal{O}(\zeta_n, \zeta_m, \tau) < \epsilon$$

i.e.,

$$\lim_{n,m \rightarrow \infty} \mathcal{N}(\zeta_n, \zeta_m, \tau) = 1, \quad \lim_{n,m \rightarrow \infty} \mathcal{U}(\zeta_n, \zeta_m, \tau) = 0, \quad \lim_{n,m \rightarrow \infty} \mathcal{O}(\zeta_n, \zeta_m, \tau) = 0$$

- (3) $(\mathcal{W}, \mathcal{N}, \mathcal{U}, \mathcal{O}, \bullet, \diamond)$ is said to be complete if every Cauchy sequence converges to an element in $\mathcal{W}$.

In the work of Simsek and Kirisci [45], NC-contractions were introduced within the context of neutrosophic metric spaces, demonstrating that each NC-contraction possesses a unique fixed point under specific conditions.

Definition 2.5. [45] Let $(\mathcal{W}, \mathcal{N}, \mathcal{U}, \mathcal{O}, \bullet, \diamond)$ be a NMS. A mapping $f : \mathcal{W} \rightarrow \mathcal{W}$ is called neutrosophic contraction if there is $k \in (0, 1)$ such that for each $\zeta, \varrho \in \mathcal{W}$ and $\tau > 0$, we have

$$\frac{1}{\mathcal{N}(f\zeta, f\varrho, \tau)} - 1 \leq k \left(\frac{1}{\mathcal{N}(\zeta, \varrho, \tau)} - 1 \right),$$

$$\frac{1}{\mathcal{U}(f\zeta, f\varrho, \tau)} - 1 \geq k \left(\frac{1}{\mathcal{U}(\zeta, \varrho, \tau)} - 1 \right),$$

and

$$\frac{1}{\mathcal{O}(f\zeta, f\varrho, \tau)} - 1 \geq k \left(\frac{1}{\mathcal{O}(\zeta, \varrho, \tau)} - 1 \right).$$

The following lemma presented by Bataihah and Hazaymeh [46]

Lemma 2.6. [46] Let $(\mathcal{W}, \mathcal{N}, \mathcal{U}, \mathcal{O}, \bullet, \diamond)$ be a NMS. Then

- (1) $\mathcal{N}(\zeta, \varrho, \cdot) : \mathbb{R} \rightarrow \mathbb{R}$ is non-decreasing
- (2) $\mathcal{U}(\zeta, \varrho, \cdot) : \mathbb{R} \rightarrow \mathbb{R}$ is non-increasing
- (3) $\mathcal{O}(\zeta, \varrho, \cdot) : \mathbb{R} \rightarrow \mathbb{R}$ is non-increasing

3. Main Result

In the subsequent sections, we will first introduce a fundamental lemmas that forms the basis of our study. Following that, we will outline our contractions in the setting of complete NMS, employing an auxiliary function. Then, we will establish that these contractions possess a unique fixed point under specific conditions and investigate the significant results that arise from our main discoveries.

We commence with the subsequent essential lemma that are required to derive our principal result.

Lemma 3.1. Let $\mathcal{V} : \mathcal{D} \rightarrow \mathcal{B}$ be a bounded real valued function, where $\mathcal{D}, \mathcal{B}$ are subsets of $\mathbb{R}$. Then

- (1) If $\mathcal{V}$ is non increasing, then there is $c > 0$ such that for all $\alpha \in \mathcal{D}$

$$\mathcal{V}\left(\frac{\alpha}{2}\right) \leq c \mathcal{V}(\alpha)$$

(2) If $\mathcal{V}$ is non decreasing, then there is $\tau > 0$ such that for all $\alpha \in \mathcal{D}$

$$\mathcal{V}\left(\frac{\alpha}{2}\right) \geq \tau \mathcal{V}(\alpha)$$

Proof. (1) Let $\alpha \in \mathcal{D}$ be arbitrary. Since $\frac{\alpha}{2} < \alpha$ and $\mathcal{V}$ is non increasing, then

$$\mathcal{V}\left(\frac{\alpha}{2}\right) \geq \mathcal{V}(\alpha).$$

So, there is $\xi_\alpha \geq 0$ such that

$$\mathcal{V}\left(\frac{\alpha}{2}\right) = \mathcal{V}(\alpha) + \xi_\alpha.$$

Thus,

$$\mathcal{V}\left(\frac{\alpha}{2}\right) \leq 2 \max\{\mathcal{V}(\alpha), \xi_\alpha\}.$$

Now,

$$\mathcal{V}(\alpha) \cdot \xi_\alpha = c_\alpha \implies c_\alpha \cdot \mathcal{V}(\alpha) = \xi_\alpha.$$

So,

$$\begin{aligned} \mathcal{V}\left(\frac{\alpha}{2}\right) &\leq 2 \max\{\mathcal{V}(\alpha), c_\alpha \cdot \mathcal{V}(\alpha)\} \\ \mathcal{V}\left(\frac{\alpha}{2}\right) &\leq \max\{2, 2c_\alpha\} \mathcal{V}(\alpha). \end{aligned}$$

Let $C = \max\{2, \max_{\alpha \in \mathcal{D}}\{2c_\alpha\}\}$. Then we get the result.

The proof of (2) is identical to that of (1). $\square$

Remark 3.2. According to Lemma 2.6, and Lemma 3.1, there are C_N, C_U, C_O such that

$$\begin{aligned} \mathcal{N}(\zeta, \varrho, \frac{\alpha}{2}) &\geq C_N \mathcal{N}(\zeta, \varrho, \alpha), \\ \mathcal{U}(\zeta, \varrho, \frac{\alpha}{2}) &\leq C_U \mathcal{U}(\zeta, \varrho, \alpha), \\ \mathcal{O}(\zeta, \varrho, \frac{\alpha}{2}) &\leq C_O \mathcal{O}(\zeta, \varrho, \alpha), \end{aligned}$$

The following definition is due to Bataihah and Hazaymeh [46]

Definition 3.3. [46]

In this framework, we characterize a real-valued function of three variables defined on the domain $\mathcal{W}^2 \times (0, \infty)$, where $\mathcal{W}$ represents any non-empty set. We denote this function as $\mathcal{H}$ and assert that it exhibits the property (UC) if, for any sequences (ζ_n) and (ω_n) within $\mathcal{W}$, the subsequent equality is satisfied.

$$\lim_{\tau \rightarrow \tau_0} \lim_{n \rightarrow \infty} \mathcal{H}(\zeta_n, \omega_n, \tau) = \lim_{n \rightarrow \infty} \lim_{\tau \rightarrow \tau_0} \mathcal{H}(\zeta_n, \omega_n, \tau).$$

whenever the two limits are exist.

In the subsequent sections of this study, we will operate under the assumption that each of the fuzzy sets $\mathcal{N}$, $\mathcal{U}$, $\mathcal{O}$ possesses the UC property.

Through this context we need the following class of functions:

Definition 3.4. Let $\mathcal{L}$ denote the collection of all functions $\mathcal{L} : \mathbb{R}^+ \times \mathbb{R}^+ \rightarrow \mathbb{R}^+$ that satisfy the condition

$$\mathcal{L}(a, b) \leq a + b.$$

We will now present the definition of a neutrosophic $(\mathcal{L}, \phi)$ -contraction.

Definition 3.5. Let $(\mathcal{W}, \mathcal{N}, \mathcal{U}, \mathcal{O}, \bullet, \diamond)$ be a NMS. A mapping $f : \mathcal{W} \rightarrow \mathcal{W}$ is called neutrosophic $(\mathcal{L}, \phi)$ -contraction if for each $\zeta, \varrho \in \mathcal{W}$ and each $\tau > 0$, we have

$$\frac{1}{\mathcal{N}(f\zeta, f\varrho, \tau)} - 1 \leq \phi \left[\frac{1}{\mathcal{N}(\zeta, \varrho, \tau)} - 1 + \mathcal{L} \left(\frac{1}{\mathcal{N}(\zeta, f\zeta, \tau)} - 1, \frac{1}{\mathcal{N}(\varrho, f\varrho, \tau)} - 1 \right) \right],$$

$$\mathcal{U}(f\zeta, f\varrho, \tau) \leq \phi [\mathcal{U}(\zeta, \varrho, \tau) + \mathcal{L}(\mathcal{U}(\zeta, f\zeta, \tau), \mathcal{U}(\varrho, f\varrho, \tau))],$$

and

$$\mathcal{O}(f\zeta, f\varrho, \tau) \leq \phi [\mathcal{O}(\zeta, \varrho, \tau) + \mathcal{L}(\mathcal{O}(\zeta, f\zeta, \tau), \mathcal{O}(\varrho, f\varrho, \tau))],$$

where $\phi : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is continuous with $\phi(t) < \frac{1}{3}t$, $\forall t > 0$.

Lemma 3.6. Let $(\mathcal{W}, \mathcal{N}, \mathcal{U}, \mathcal{O}, \bullet, \diamond)$ be a complete NMS, Suppose that $f : \mathcal{W} \rightarrow \mathcal{W}$ is neutrosophic $(\mathcal{L}, \phi)$ -contraction. Consequently, if f has a fixed point then it is unique.

Proof. Assume that $\mu, \nu \in \mathcal{W}$ such that $f\mu = \mu$ and $f\nu = \nu$. Then, Definition 3.5 implies

$$\frac{1}{\mathcal{N}(\mu, \nu, \tau)} - 1 = \frac{1}{\mathcal{N}(f\mu, f\nu, \tau)} - 1 \leq \phi \left[\frac{1}{\mathcal{N}(\mu, \nu, \tau)} - 1 + \mathcal{L} \left(\frac{1}{\mathcal{N}(\mu, \mu, \tau)} - 1, \frac{1}{\mathcal{N}(\nu, \nu, \tau)} - 1 \right) \right],$$

$$\mathcal{U}(\mu, \nu, \tau) = \mathcal{U}(f\mu, f\nu, \tau) \leq \phi [\mathcal{U}(\mu, \nu, \tau) + \mathcal{L}(\mathcal{U}(\mu, \mu, \tau), \mathcal{U}(\nu, \nu, \tau))],$$

and

$$\mathcal{O}(\mu, \nu, \tau) = \mathcal{O}(f\mu, f\nu, \tau) \leq \phi [\mathcal{O}(\mu, \nu, \tau) + \mathcal{L}(\mathcal{O}(\mu, \mu, \tau), \mathcal{O}(\nu, \nu, \tau))].$$

Hence, be using properties of ϕ and $\mathcal{L}$, we get

$$\frac{1}{\mathcal{N}(\mu, \nu, \tau)} - 1 \leq \frac{1}{3} \left[\frac{1}{\mathcal{N}(\mu, \nu, \tau)} - 1 \right],$$

$$\mathcal{U}(\mu, \nu, \tau) \leq \frac{1}{3} [\mathcal{U}(\mu, \nu, \tau)],$$

and

$$\mathcal{O}(\mu, \nu, \tau) \leq \frac{1}{3} [\mathcal{O}(\mu, \nu, \tau)].$$

So, $\mathcal{N}(\mu, \nu, \tau) = 1$, $\mathcal{U}(\mu, \nu, \tau) = 0$, and $\mathcal{O}(\mu, \nu, \tau) = 0$, and hence $\mu = \nu$. $\square$

Lemma 3.7. *Let $(\mathcal{W}, \mathcal{N}, \mathcal{B}, \mathcal{U}, \mathcal{O}, \diamond, \bullet)$ be a NFMS, and let (ζ_n) be a sequence such that for $\tau > 0$*

$$\begin{aligned}\mathcal{N}(\zeta_p, \zeta_q, \tau) &\geq \mathcal{N}(\zeta_{p-1}, \zeta_{q-1}, \tau) \\ \mathcal{U}(\zeta_p, \zeta_q, \tau) &\leq \mathcal{U}(\zeta_{p-1}, \zeta_{q-1}, \tau) \\ \mathcal{O}(\zeta_p, \zeta_q, \tau) &\leq \mathcal{O}(\zeta_{p-1}, \zeta_{q-1}, \tau)\end{aligned}\tag{1}$$

and

$$\begin{aligned}\lim_{n \rightarrow \infty} \mathcal{N}(\zeta_n, \zeta_{n+1}, \tau) &= 1, \\ \lim_{n \rightarrow \infty} \mathcal{U}(\zeta_n, \zeta_{n+1}, \tau) &= 0, \\ \lim_{n \rightarrow \infty} \mathcal{O}(\zeta_n, \zeta_{n+1}, \tau) &= 0.\end{aligned}\tag{2}$$

If (ζ_n) is not Cauchy, then there exist an $1 > \epsilon > 0$ and $\tau > 0$ along with two subsequences (ζ_{n_k}) and (ζ_{m_k}) derived from (ζ_n) , where (m_k) such that one at least of the following holds.

$$\begin{aligned}\lim_{k \rightarrow \infty} \mathcal{N}(\zeta_{n_k}, \zeta_{m_k}, \tau) &= 1 - \epsilon, \\ \lim_{k \rightarrow \infty} \mathcal{U}(\zeta_{n_k}, \zeta_{m_k}, \tau) &= \epsilon, \\ \lim_{k \rightarrow \infty} \mathcal{O}(\zeta_{n_k}, \zeta_{m_k}, \tau) &= \epsilon.\end{aligned}$$

Proof. If (ζ_n) is not Cauchy, then for each $\tau > 0$

$$\begin{aligned}\lim_{n, m \rightarrow \infty} \mathcal{N}(\zeta_n, \zeta_m, \tau) &\neq 1, \\ \lim_{n, m \rightarrow \infty} \mathcal{U}(\zeta_n, \zeta_m, \tau) &\neq 0,\end{aligned}$$

or

$$\lim_{n, m \rightarrow \infty} \mathcal{O}(\zeta_n, \zeta_m, \tau) \neq 0.$$

Case 1: If $\lim_{n, m \rightarrow \infty} \mathcal{N}(\zeta_n, \zeta_m, \tau) \neq 1$, then there are $\tau > 0$, and $\epsilon > 0$ along with two subsequences (ζ_{n_k}) and (ζ_{m_k}) derived from (ζ_n) , where (m_k) is selected as the smallest index satisfying the condition.

$$\mathcal{N}(\zeta_{n_k}, \zeta_{m_k}, \tau) \leq 1 - \epsilon, \quad m_k > n_k > k.\tag{3}$$

This implies that

$$\mathcal{N}(\zeta_{n_k}, \zeta_{m_k-1}, \tau) > 1 - \epsilon.\tag{4}$$

chose $\delta > 0$. Then

$$\begin{aligned}\mathcal{N}(\zeta_{n_k}, \zeta_{m_k}, \tau + \delta) &\geq \mathcal{N}(\zeta_{n_k}, \zeta_{m_k-1}, \tau) \diamond \mathcal{N}(\zeta_{m_k-1}, \zeta_{m_k}, \delta) \\ &> (1 - \epsilon) \diamond \mathcal{N}(\zeta_{m_k-1}, \zeta_{m_k}, \delta)\end{aligned}$$

Using Equation 2, we get

$$\liminf_{k \rightarrow \infty} \mathcal{N}(\zeta_{n_k}, \zeta_{m_k}, \tau + \delta) \geq (1 - \epsilon).$$

Also,

$$\begin{aligned}(1 - \epsilon) &\leq \lim_{\delta \rightarrow 0^+} \liminf_{k \rightarrow \infty} \mathcal{N}(\zeta_{n_k}, \zeta_{m_k}, \tau + \delta) \\ &= \liminf_{k \rightarrow \infty} \lim_{\delta \rightarrow 0^+} \mathcal{N}(\zeta_{n_k}, \zeta_{m_k}, \tau + \delta) \\ &= \liminf_{k \rightarrow \infty} \mathcal{N}(\zeta_{n_k}, \zeta_{m_k}, \tau).\end{aligned}$$

Also, from 3, it follows

$$\limsup_{k \rightarrow \infty} \mathcal{N}(\zeta_{n_k}, \zeta_{m_k}, \tau) \leq (1 - \epsilon).$$

So, we get

$$\lim_{k \rightarrow \infty} \mathcal{N}(\zeta_{n_k}, \zeta_{m_k}, \tau) = (1 - \epsilon).$$

Again, we have

$$\begin{aligned}\mathcal{N}(\zeta_{n_k-1}, \zeta_{m_k-1}, \tau + \delta) &\geq \mathcal{N}(\zeta_{n_k-1}, \zeta_{n_k}, \delta) \diamond \mathcal{N}(\zeta_{n_k}, \zeta_{m_k-1}, \tau) \\ &> \mathcal{N}(\zeta_{n_k-1}, \zeta_{n_k}, \delta) \diamond (1 - \epsilon).\end{aligned}$$

Using Equation 2, we get $\liminf_{k \rightarrow \infty} \mathcal{N}(\zeta_{n_k-1}, \zeta_{m_k-1}, \tau + \delta) \geq (1 - \epsilon)$.

Also,

$$\begin{aligned}(1 - \epsilon) &\leq \lim_{\delta \rightarrow 0^+} \liminf_{k \rightarrow \infty} \mathcal{N}(\zeta_{n_k-1}, \zeta_{m_k-1}, \tau + \delta) \\ &= \liminf_{k \rightarrow \infty} \lim_{\delta \rightarrow 0^+} \mathcal{N}(\zeta_{n_k-1}, \zeta_{m_k-1}, \tau + \delta) \\ &= \liminf_{k \rightarrow \infty} \mathcal{N}(\zeta_{n_k-1}, \zeta_{m_k-1}, \tau).\end{aligned}$$

From Eq 3, we get

$$\mathcal{N}(\zeta_{n_k-1}, \zeta_{m_k-1}, \tau) \leq \mathcal{N}(\zeta_{n_k}, \zeta_{m_k}, \tau) \leq (1 - \epsilon).$$

So,

$$\limsup_{k \rightarrow \infty} \mathcal{N}(\zeta_{n_k-1}, \zeta_{m_k-1}, \tau) \leq (1 - \epsilon).$$

Hence,

$$\lim_{k \rightarrow \infty} \mathcal{N}(\zeta_{n_k-1}, \zeta_{m_k-1}, \tau) = (1 - \epsilon).$$

The demonstration for the remaining cases is Similar to that of Case (1). $\square$

Our methodology combines neutrosophic set theory with fixed-point techniques through three key innovations: first, we develop $(\mathcal{L}, \phi)$ -contractions that simultaneously preserve the neutrosophic structure of truth ($\mathcal{N}$), indeterminacy ($\mathcal{U}$), and falsity ($\mathcal{O}$) memberships; second, we establish the critical bounding condition $\phi(t) < \frac{1}{3}t$ to ensure proper contraction across all membership dimensions; and third, we introduce the constraint $\frac{1}{3}C_{\mathcal{O}} < 1$ to control the falsity component's influence while maintaining mathematical tractability. This integrated approach enables the extension of Banach-type fixed-point theory to neutrosophic metric spaces while preserving their essential three-valued logic structure.

Theorem 3.8. *Let $(\mathcal{W}, \mathcal{N}, \mathcal{U}, \mathcal{O}, \bullet, \diamond)$ be a complete NMS, Suppose that $f : \mathcal{W} \rightarrow \mathcal{W}$ is neutrosophic $(\mathcal{L}, \phi)$ -contraction where $\frac{1}{3}C_{\mathcal{O}} < 1$. Therefore, the function f has exactly one fixed point*

Proof. Let $\zeta_0 \in \mathcal{W}$ represent an arbitrary point. We examine the Picard sequence (ζ_n) characterized by the relation $\zeta_{n+1} = f(\zeta_0)$ for all $n \geq 0$. From Definition 3.5, we get for each $n \in \mathbb{N}$

$$\frac{1}{\mathcal{N}(\zeta_n, \zeta_{n+1}, \tau)} - 1 \leq \phi \left[\frac{1}{\mathcal{N}(\zeta_{n-1}, \zeta_n, \tau)} - 1 + \mathcal{L} \left(\frac{1}{\mathcal{N}(\zeta_{n-1}, \zeta_n, \tau)} - 1, \frac{1}{\mathcal{N}(\zeta_n, \zeta_{n+1}, \tau)} - 1 \right) \right], \quad (5)$$

$$\mathcal{U}(\zeta_n, \zeta_{n+1}, \tau) \leq \phi [\mathcal{U}(\zeta_{n-1}, \zeta_n, \tau) + \mathcal{L}(\mathcal{U}(\zeta_{n-1}, \zeta_n, \tau), \mathcal{U}(\zeta_n, \zeta_{n+1}, \tau))], \quad (6)$$

and

$$\mathcal{O}(\zeta_n, \zeta_{n+1}, \tau) \leq \phi [\mathcal{O}(\zeta_{n-1}, \zeta_n, \tau) + \mathcal{L}(\mathcal{O}(\zeta_{n-1}, \zeta_n, \tau), \mathcal{O}(\zeta_n, \zeta_{n+1}, \tau))]. \quad (7)$$

From the definition of $\mathcal{L}$, and ϕ we get

$$\frac{1}{\mathcal{N}(\zeta_n, \zeta_{n+1}, \tau)} - 1 < \left(\frac{1}{\mathcal{N}(\zeta_{n-1}, \zeta_n, \tau)} - 1 \right),$$

$$\mathcal{U}(\zeta_n, \zeta_{n+1}, \tau) < (\mathcal{U}(\zeta_{n-1}, \zeta_n, \tau)),$$

and

$$\mathcal{O}(\zeta_n, \zeta_{n+1}, \tau) < (\mathcal{O}(\zeta_{n-1}, \zeta_n, \tau)).$$

So, we get that the sequences

$$\left(\frac{1}{\mathcal{N}(\zeta_n, \zeta_{n+1}, \tau)} - 1 : n \in \mathbb{N}\right),$$

$$(\mathcal{U}(\zeta_n, \zeta_{n+1}, \tau) : n \in \mathbb{N}),$$

and

$$(\mathcal{O}(\zeta_n, \zeta_{n+1}, \tau) : n \in \mathbb{N})$$

are nonincreasing in $[0, \infty)$. Then, there are $l_1, l_2, l_3 \geq 0$ such that

$$\frac{1}{\mathcal{N}(\zeta_n, \zeta_{n+1}, \tau)} - 1 \rightarrow l_1,$$

$$\mathcal{U}(\zeta_n, \zeta_{n+1}, \tau) \rightarrow l_2,$$

and

$$\mathcal{O}(\zeta_n, \zeta_{n+1}, \tau) \rightarrow l_3.$$

Assume to the contrary that one of l_1, l_2, l_3 is greater than 0. By taking the limit in 5, 6, and 7 and using the properties of $\mathcal{L}, \phi$, we get that

$l_1 < l_1, l_2 < l_2$, or $l_3 < l_3$. which is a contradiction in each case. Hence $l_1 = l_2 = l_3 = 0$.

So, we have

$$\begin{aligned} \lim_{n \rightarrow \infty} \mathcal{N}(\zeta_n, \zeta_{n+1}, \tau) &= 1, \\ \lim_{n \rightarrow \infty} \mathcal{U}(\zeta_n, \zeta_{n+1}, \tau) &= 0, \\ \lim_{n \rightarrow \infty} \mathcal{O}(\zeta_n, \zeta_{n+1}, \tau) &= 0. \end{aligned} \tag{8}$$

Now, we calim that (ζ_n) is Cauchy sequence. Assume the contrary. Then, according to Lemma 3.7, there exist an $1 > \epsilon > 0$ and $\tau > 0$ along with two subsequences (ζ_{n_k}) and (ζ_{m_k}) derived from (ζ_n) , where (m_k) such that one at least of the following holds.

$$\begin{aligned} \lim_{k \rightarrow \infty} \mathcal{N}(\zeta_{n_k}, \zeta_{m_k}, \tau) &= 1 - \epsilon, \\ \lim_{k \rightarrow \infty} \mathcal{U}(\zeta_{n_k}, \zeta_{m_k}, \tau) &= \epsilon, \\ \lim_{k \rightarrow \infty} \mathcal{O}(\zeta_{n_k}, \zeta_{m_k}, \tau) &= \epsilon. \end{aligned}$$

Definition 3.5 implies that

$$\frac{1}{\mathcal{N}(\zeta_{n_k}, \zeta_{m_k}, \tau)} - 1 \leq \phi \left[\frac{1}{\mathcal{N}(\zeta_{n_k-1}, \zeta_{m_k-1}, \tau)} - 1 + \mathcal{L} \left(\frac{1}{\mathcal{N}(\zeta_{n_k-1}, \zeta_{n_k}, \tau)} - 1, \frac{1}{\mathcal{N}(\zeta_{m_k-1}, \zeta_{m_k}, \tau)} - 1 \right) \right],$$

$$\mathcal{U}(\zeta_{n_k}, \zeta_{m_k}, \tau) \leq \phi [\mathcal{U}(\zeta_{n_k-1}, \zeta_{m_k-1}, \tau) + \mathcal{L}(\mathcal{U}(\zeta_{n_k-1}, \zeta_{n_k}, \tau), \mathcal{U}(\zeta_{m_k-1}, \zeta_{m_k}, \tau))],$$

and

$$\mathcal{O}(\zeta_{n_k}, \zeta_{m_k}, \tau) \leq \phi [\mathcal{O}(\zeta_{n_k-1}, \zeta_{m_k-1}, \tau) + \mathcal{L}(\mathcal{O}(\zeta_{n_k-1}, \zeta_{n_k}, \tau), \mathcal{O}(\zeta_{m_k-1}, \zeta_{m_k}, \tau))].$$

By taking the limit when $k \rightarrow \infty$, we get

$$\frac{\epsilon}{1-\epsilon} < \frac{\epsilon}{1-\epsilon},$$

$$\epsilon < \epsilon,$$

and

$$\epsilon < \epsilon.$$

Which is a contradiction. Hence (ζ_n) is a Cauchy sequence, thus, there is $\mu \in \mathcal{W}$ such that $\zeta_n \rightarrow \mu$.

Assume that $f\mu \neq \mu$. From Definition 3.5, we have for each $n \in \mathbb{N}$

$$\begin{aligned} \mathcal{O}(f\mu, \zeta_{n+1}, \tau) &\leq \phi [\mathcal{O}(\mu, \zeta_n, \tau) + \mathcal{L}(\mathcal{O}(\mu, f\mu, \tau), \mathcal{O}(\zeta_n, \zeta_{n+1}, \tau))] \\ &\leq \phi [\mathcal{O}(\mu, \zeta_n, \tau) + \mathcal{O}(\mu, f\mu, \tau) + \mathcal{O}(\zeta_n, \zeta_{n+1}, \tau)]. \end{aligned}$$

By taking the limit, we obtain

$$\lim_{n \rightarrow \infty} \mathcal{O}(f\mu, \zeta_{n+1}, \tau) \leq \phi \mathcal{O}(\mu, f\mu, \tau) < \frac{1}{3} \mathcal{O}(\mu, f\mu, \tau). \quad (9)$$

On the other hand, using Remark 3.2, we get

$$\begin{aligned} \mathcal{O}(\mu, f\mu, \tau) &\leq \mathcal{O}(\mu, \zeta_n, \frac{\tau}{2}) \bullet \mathcal{O}(\zeta_n, f\mu, \frac{\tau}{2}) \\ &\leq \mathcal{O}(\mu, \zeta_n, \frac{\tau}{2}) \bullet C_{\mathcal{O}} \mathcal{O}(\zeta_n, f\mu, \tau). \end{aligned}$$

So,

$$\mathcal{O}(\mu, f\mu, \tau) \leq C_{\mathcal{O}} \lim_{n \rightarrow \infty} \mathcal{O}(\zeta_n, f\mu, \tau). \quad (10)$$

From Eq 9, Eq 10, we get

$$\lim_{n \rightarrow \infty} \mathcal{O}(\zeta_n, f\mu, \tau) < \frac{1}{3} C_{\mathcal{O}} \lim_{n \rightarrow \infty} \mathcal{O}(\zeta_n, f\mu, \tau) < \lim_{n \rightarrow \infty} \mathcal{O}(\zeta_n, f\mu, \tau).$$

A contradiction. So, $\mu = f\mu$. The uniqueness follows from Lemma 3.6 $\square$

Remark 3.9. The condition $\phi(t) < \frac{1}{3}t$ is crucial to ensure that the neutrosophic inequalities contract sufficiently in all three components $(\mathcal{N}, \mathcal{U}, \mathcal{O})$. Relaxing this to $\phi(t) < t$ may not guarantee the result.

Example 3.10. Consider the complete neutrosophic metric space $(\mathcal{W}, \mathcal{N}, \mathcal{U}, \mathcal{O}, \bullet, \diamond)$ where:

- $\mathcal{W} = [0, 1]$
- The t-norm $\bullet$ and t-conorm $\diamond$ are defined as $a \bullet b = \min\{a, b\}$ and $a \diamond b = \max\{a, b\}$

- The neutrosophic metrics are defined for $\tau > 0$ as:

$$\begin{aligned}\mathcal{N}(\zeta, \varrho, \tau) &= \frac{\tau}{\tau + |\zeta - \varrho|} \\ \mathcal{U}(\zeta, \varrho, \tau) &= \frac{|\zeta - \varrho|}{\tau + |\zeta - \varrho|} \\ \mathcal{O}(\zeta, \varrho, \tau) &= \frac{|\zeta - \varrho|}{2\tau + |\zeta - \varrho|}\end{aligned}$$

Define the mapping $f : \mathcal{W} \rightarrow \mathcal{W}$ by $f(\zeta) = \frac{\zeta}{4}$. We will verify that f satisfies all conditions of Theorem 3.8 with:

- $\mathcal{L}(a, b) = \frac{a+b}{2}$ (which satisfies $\mathcal{L}(a, b) \leq a + b$)
- $\phi(t) = \frac{t}{4}$ (which is continuous and satisfies $\phi(t) < \frac{t}{3}$ for all $t > 0$)
- $C_{\mathcal{O}} = \frac{1}{2}$ (since $\mathcal{O}(\zeta, \varrho, \tau) \leq \frac{1}{2}$)

Proof. We check each condition of Definition 3.5:

1. For the truth membership $\mathcal{N}$:

$$\begin{aligned}\frac{1}{\mathcal{N}(f\zeta, f\varrho, \tau)} - 1 &= \frac{|f\zeta - f\varrho|}{\tau} = \frac{|\frac{\zeta}{4} - \frac{\varrho}{4}|}{\tau} = \frac{|\zeta - \varrho|}{4\tau} \\ \phi \left[\frac{1}{\mathcal{N}(\zeta, \varrho, \tau)} - 1 + \mathcal{L} \left(\frac{1}{\mathcal{N}(\zeta, f\zeta, \tau)} - 1, \frac{1}{\mathcal{N}(\varrho, f\varrho, \tau)} - 1 \right) \right] \\ &= \frac{1}{4} \left[\frac{|\zeta - \varrho|}{\tau} + \frac{1}{2} \left(\frac{|\zeta - \frac{\zeta}{4}|}{\tau} + \frac{|\varrho - \frac{\varrho}{4}|}{\tau} \right) \right] \\ &= \frac{1}{4} \left[\frac{|\zeta - \varrho|}{\tau} + \frac{3}{8} \frac{|\zeta| + |\varrho|}{\tau} \right] \geq \frac{|\zeta - \varrho|}{4\tau}\end{aligned}$$

2. For the indeterminacy membership $\mathcal{U}$:

$$\begin{aligned}\mathcal{U}(f\zeta, f\varrho, \tau) &= \frac{|\frac{\zeta}{4} - \frac{\varrho}{4}|}{\tau + |\frac{\zeta}{4} - \frac{\varrho}{4}|} \leq \frac{|\zeta - \varrho|}{4\tau} \\ \phi [\mathcal{U}(\zeta, \varrho, \tau) + \mathcal{L}(\mathcal{U}(\zeta, f\zeta, \tau), \mathcal{U}(\varrho, f\varrho, \tau))] \\ &= \frac{1}{4} \left[\frac{|\zeta - \varrho|}{\tau + |\zeta - \varrho|} + \frac{1}{2} \left(\frac{\frac{3}{4}|\zeta|}{\tau + \frac{3}{4}|\zeta|} + \frac{\frac{3}{4}|\varrho|}{\tau + \frac{3}{4}|\varrho|} \right) \right] \geq \frac{|\zeta - \varrho|}{4\tau}\end{aligned}$$

3. For the falsity membership $\mathcal{O}$:

$$\begin{aligned}\mathcal{O}(f\zeta, f\varrho, \tau) &= \frac{|\frac{\zeta}{4} - \frac{\varrho}{4}|}{2\tau + |\frac{\zeta}{4} - \frac{\varrho}{4}|} \leq \frac{|\zeta - \varrho|}{8\tau} \\ \phi [\mathcal{O}(\zeta, \varrho, \tau) + \mathcal{L}(\mathcal{O}(\zeta, f\zeta, \tau), \mathcal{O}(\varrho, f\varrho, \tau))] \\ &= \frac{1}{4} \left[\frac{|\zeta - \varrho|}{2\tau + |\zeta - \varrho|} + \frac{1}{2} \left(\frac{\frac{3}{4}|\zeta|}{2\tau + \frac{3}{4}|\zeta|} + \frac{\frac{3}{4}|\varrho|}{2\tau + \frac{3}{4}|\varrho|} \right) \right] \geq \frac{|\zeta - \varrho|}{8\tau}\end{aligned}$$

All conditions are satisfied, and $\frac{1}{3}C_{\mathcal{O}} = \frac{1}{6} < 1$. Therefore, by Theorem 3.8, f has a unique fixed point $\zeta^* = 0$. $\square$

4. Consequences

Corollary 4.1. Let $(\mathcal{W}, \mathcal{N}, \mathcal{U}, \mathcal{O}, \bullet, \diamond)$ be a complete NMS and $f : \mathcal{W} \rightarrow \mathcal{W}$ be a self map. If there exists $0 < k < \min \left\{ \frac{1}{3}, \frac{1}{C_{\mathcal{O}}} \right\}$ such that for all $\zeta, \varrho \in \mathcal{W}$ and $\tau > 0$:

$$\frac{1}{\mathcal{N}(f\zeta, f\varrho, \tau)} - 1 \leq k \left[\frac{1}{\mathcal{N}(\zeta, \varrho, \tau)} - 1 + \lambda \left(\frac{1}{\mathcal{N}(\zeta, f\zeta, \tau)} - 1 \right) + (1 - \lambda) \left(\frac{1}{\mathcal{N}(\varrho, f\varrho, \tau)} - 1 \right) \right]$$

$$\mathcal{U}(f\zeta, f\varrho, \tau) \leq k [\mathcal{U}(\zeta, \varrho, \tau) + \lambda \mathcal{U}(\zeta, f\zeta, \tau) + (1 - \lambda) \mathcal{U}(\varrho, f\varrho, \tau)]$$

$$\mathcal{O}(f\zeta, f\varrho, \tau) \leq k [\mathcal{O}(\zeta, \varrho, \tau) + \lambda \mathcal{O}(\zeta, f\zeta, \tau) + (1 - \lambda) \mathcal{O}(\varrho, f\varrho, \tau)].$$

Then f has exactly one fixed point in $\mathcal{W}$.

Proof. This follows from Theorem 3.8 by taking

- $\mathcal{L}(a, b) = \lambda a + (1 - \lambda)b$ which satisfies $\mathcal{L}(a, b) \leq a + b$
- $\phi(t) = kt$ with $k < \frac{1}{3}$

The given inequalities exactly match the contraction conditions with this choice of $\mathcal{L}$. $\square$

Corollary 4.2 (Maximum Function Case). Let $(\mathcal{W}, \mathcal{N}, \mathcal{U}, \mathcal{O}, \bullet, \diamond)$ be a complete NMS and $f : \mathcal{W} \rightarrow \mathcal{W}$ be a self map. Define $\mathcal{L}(a, b) = \max\{a, b\}$. If there exists $0 < k < \min \left\{ \frac{1}{3}, \frac{1}{C_{\mathcal{O}}} \right\}$ such that for all $\zeta, \varrho \in \mathcal{W}$ and $\tau > 0$:

$$\frac{1}{\mathcal{N}(f\zeta, f\varrho, \tau)} - 1 \leq k \left[\frac{1}{\mathcal{N}(\zeta, \varrho, \tau)} - 1 + \max \left\{ \frac{1}{\mathcal{N}(\zeta, f\zeta, \tau)} - 1, \frac{1}{\mathcal{N}(\varrho, f\varrho, \tau)} - 1 \right\} \right]$$

$$\mathcal{U}(f\zeta, f\varrho, \tau) \leq k [\mathcal{U}(\zeta, \varrho, \tau) + \max \{ \mathcal{U}(\zeta, f\zeta, \tau), \mathcal{U}(\varrho, f\varrho, \tau) \}]$$

$$\mathcal{O}(f\zeta, f\varrho, \tau) \leq k [\mathcal{O}(\zeta, \varrho, \tau) + \max \{ \mathcal{O}(\zeta, f\zeta, \tau), \mathcal{O}(\varrho, f\varrho, \tau) \}].$$

Then f has a unique fixed point in $\mathcal{W}$.

Proof. This follows from Theorem 3.8 by taking

- $\mathcal{L}(a, b) = \max\{a, b\}$ which satisfies $\mathcal{L}(a, b) \leq a + b$
- $\phi(t) = kt$ with $k < \frac{1}{3}$

The given inequalities exactly match the contraction conditions with this choice of $\mathcal{L}$. $\square$

Corollary 4.3. Let $(\mathcal{W}, \mathcal{N}, \mathcal{U}, \mathcal{O}, \bullet, \diamond)$ be a complete NMS and $f : \mathcal{W} \rightarrow \mathcal{W}$ be a self map. If there exists $0 < k < \min\left\{\frac{1}{3}, \frac{1}{C_{\mathcal{O}}}\right\}$ such that for all $\zeta, \varrho \in \mathcal{W}$ and $\tau > 0$:

$$\frac{1}{\mathcal{N}(f\zeta, f\varrho, \tau)} - 1 \leq k \left[\frac{1}{\mathcal{N}(\zeta, \varrho, \tau)} - 1 + \frac{1}{\mathcal{N}(\zeta, f\zeta, \tau)} - 1 + \frac{1}{\mathcal{N}(\varrho, f\varrho, \tau)} - 1 \right],$$

$$\mathcal{U}(f\zeta, f\varrho, \tau) \leq k [\mathcal{U}(\zeta, \varrho, \tau) + \mathcal{U}(\zeta, f\zeta, \tau) + \mathcal{U}(\varrho, f\varrho, \tau)],$$

$$\mathcal{O}(f\zeta, f\varrho, \tau) \leq k [\mathcal{O}(\zeta, \varrho, \tau) + \mathcal{O}(\zeta, f\zeta, \tau) + \mathcal{O}(\varrho, f\varrho, \tau)].$$

Then f possesses exactly one fixed point in $\mathcal{W}$.

Proof. This follows from Theorem 3.8 by taking: $\mathcal{L} : \mathbb{R}^+ \times \mathbb{R}^+ \rightarrow \mathbb{R}^+$ with the definition $\mathcal{L}(a, b) = a + b$, and $\phi(t) = kt$, $k < \frac{1}{3}$. $\square$

Corollary 4.4. Let $(\mathcal{W}, \mathcal{N}, \mathcal{U}, \mathcal{O}, \bullet, \diamond)$ be a complete NMS and $f : \mathcal{W} \rightarrow \mathcal{W}$ be a self map. Assume that there is $0 < k < \min\left\{\frac{1}{3}, \frac{1}{C_{\mathcal{O}}}\right\}$ such that for each $\zeta, \varrho \in \mathcal{W}$ and each $\tau > 0$, f satisfies the following:

$$\frac{1}{\mathcal{N}(f\zeta, f\varrho, \tau)} - 1 \leq k \left[\frac{1}{\mathcal{N}(\zeta, \varrho, \tau)} - 1 \right],$$

$$\mathcal{U}(f\zeta, f\varrho, \tau) \leq k [\mathcal{U}(\zeta, \varrho, \tau)],$$

and

$$\mathcal{O}(f\zeta, f\varrho, \tau) \leq k [\mathcal{O}(\zeta, \varrho, \tau)].$$

Therefore, the function f has exactly one fixed point

Proof. This follows from Theorem 3.8 by taking: $\mathcal{L} : \mathbb{R}^+ \times \mathbb{R}^+ \rightarrow \mathbb{R}^+$ with the definition $\mathcal{L}(a, b) = 0$, and $\phi(t) = kt$, $k < \frac{1}{3}$. $\square$

Corollary 4.5. Let $(\mathcal{W}, \mathcal{N}, \mathcal{U}, \mathcal{O}, \bullet, \diamond)$ be a complete NMS and $f : \mathcal{W} \rightarrow \mathcal{W}$ be a self map. Assume that there is $0 < k < \min\left\{\frac{1}{3}, \frac{1}{C_{\mathcal{O}}}\right\}$ such that for each $\zeta, \varrho \in \mathcal{W}$ and each $\tau > 0$, f satisfies the following:

$$\frac{1}{\mathcal{N}(f\zeta, f\varrho, \tau)} - 1 \leq k \left[\frac{1}{\mathcal{N}(\zeta, \varrho, \tau)} - 1 + \left| \frac{1}{\mathcal{N}(\zeta, f\zeta, \tau)} - \frac{1}{\mathcal{N}(\varrho, f\varrho, \tau)} \right| \right],$$

$$\mathcal{U}(f\zeta, f\varrho, \tau) \leq k [\mathcal{U}(\zeta, \varrho, \tau) + |\mathcal{U}(\zeta, f\zeta, \tau) - \mathcal{U}(\varrho, f\varrho, \tau)|],$$

and

$$\mathcal{O}(f\zeta, f\varrho, \tau) \leq k [\mathcal{O}(\zeta, \varrho, \tau) + |\mathcal{O}(\zeta, f\zeta, \tau) - \mathcal{O}(\varrho, f\varrho, \tau)|].$$

Therefore, the function f has exactly one fixed point

Proof. This follows from Theorem 3.8 by taking

- $\mathcal{L}(a, b) = |a - b|$ which satisfies $\mathcal{L}(a, b) \leq a + b$
- $\phi(t) = kt$ with $k < \frac{1}{3}$

The given inequalities exactly match the contraction conditions with this choice of $\mathcal{L}$. $\square$

Corollary 4.6. Let $(\mathcal{W}, \mathcal{N}, \mathcal{U}, \mathcal{O}, \bullet, \diamond)$ be a complete NMS and $f : \mathcal{W} \rightarrow \mathcal{W}$ be a self map. Assume that there is $0 < k < \min\{\frac{1}{3}, \frac{1}{C_{\mathcal{O}}}\}$ such that for each $\zeta, \varrho \in \mathcal{W}$ and each $\tau > 0$, f satisfies the following:

$$\frac{1}{\mathcal{N}(f\zeta, f\varrho, \tau)} - 1 \leq k \left[\frac{1}{\mathcal{N}(\zeta, \varrho, \tau)} - 1 + 2 \min \left\{ \frac{1}{\mathcal{N}(\zeta, f\zeta, \tau)} - 1, \frac{1}{\mathcal{N}(\varrho, f\varrho, \tau)} - 1 \right\} \right],$$

$$\mathcal{U}(f\zeta, f\varrho, \tau) \leq k [\mathcal{U}(\zeta, \varrho, \tau) + 2 \min \{ \mathcal{U}(\zeta, f\zeta, \tau), \mathcal{U}(\varrho, f\varrho, \tau) \}],$$

and

$$\mathcal{O}(f\zeta, f\varrho, \tau) \leq k [\mathcal{O}(\zeta, \varrho, \tau) + 2 \min \{ \mathcal{O}(\zeta, f\zeta, \tau), \mathcal{O}(\varrho, f\varrho, \tau) \}].$$

Therefore, the function f has exactly one fixed point

Proof. This follows from Theorem 3.8 by taking

- $\mathcal{L}(a, b) = 2 \min\{a, b\}$ which satisfies $\mathcal{L}(a, b) \leq a + b$
- $\phi(t) = kt$ with $k < \frac{1}{3}$

The given inequalities exactly match the contraction conditions with this choice of $\mathcal{L}$. $\square$

Example 4.7 (Application of Corollary 4.5). Consider the NMS $(\mathbb{R}, \mathcal{N}, \mathcal{U}, \mathcal{O})$ with:

$$\begin{aligned} \mathcal{N}(\zeta, \varrho, \tau) &= \frac{\tau}{\tau + |\zeta - \varrho|} \\ \mathcal{U}(\zeta, \varrho, \tau) &= \frac{|\zeta - \varrho|}{\tau + |\zeta - \varrho|} \\ \mathcal{O}(\zeta, \varrho, \tau) &= \frac{|\zeta - \varrho|}{2\tau + |\zeta - \varrho|} \end{aligned}$$

Let $f(\zeta) = \frac{\zeta}{5}$. Taking $k = \frac{1}{4}$, we verify:

- $C_{\mathcal{O}} = \frac{1}{2}$ (since $\mathcal{O}(\zeta, \varrho, \tau) \leq \frac{1}{2}$)
- $k = \frac{1}{4} < \min\{\frac{1}{3}, 2\} = \frac{1}{3}$

All conditions of Corollary 4.5 are satisfied, with $\zeta^* = 0$ as the unique fixed point.

Problem. Does Theorem 3.8 hold if ϕ is only assumed to be upper semicontinuous (instead of continuous) with $\phi(t) < \frac{1}{3}t$?

5. Conclusions

Our investigation has established fundamental fixed-point results for $(\mathcal{L}, \phi)$ -contractions in complete neutrosophic metric spaces. By introducing this new contraction class, we have extended the classical Banach contraction principle to handle the three-dimensional nature of neutrosophic logic - simultaneously addressing truth, indeterminacy, and falsity memberships. The key condition $\phi(t) < \frac{1}{3}t$ ensures sufficient contraction across all membership dimensions, while $\frac{1}{3}C_O < 1$ maintains proper control of the falsity component's influence.

Several important research directions emerge from this work. First, extending these results to more general spaces such as bipolar neutrosophic or partial neutrosophic metric spaces could broaden the theory's applicability. The development of multivalued versions of $(\mathcal{L}, \phi)$ -contractions would be particularly valuable for applications in optimization and control theory.

Second, computational aspects deserve attention. Developing efficient iterative algorithms to approximate fixed points in neutrosophic spaces could bridge the gap between theory and practical implementation. This would be especially relevant for machine learning applications dealing with uncertain data.

Finally, applied mathematical directions offer rich possibilities. Exploring applications to neutrosophic differential equations could provide new solution methods for problems with inherent uncertainty. Similarly, investigating connections to image processing and pattern recognition in ambiguous environments may yield practical benefits for computer vision systems.

These future directions would not only deepen our theoretical understanding but also expand the practical utility of neutrosophic fixed-point methods across scientific disciplines. The framework developed here provides a solid foundation for these subsequent investigations in both pure and applied mathematics.

Conflicts of Interest: The authors declare no conflict of interest.

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A. Bataihah, A. A Hazaymeh, On Fixed Point Results in Neutrosophic Metric Spaces Using Auxiliary Functions

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Sustainable Practices in Highway Construction: A Green Evaluation Approach with HyperSoft Set

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Abstract: The construction industry has been significantly impacted by the global emphasis on environmental preservation, especially in the expansion of transportation infrastructure. To reduce the ecological impact of large-scale undertakings, sustainable practices are now required rather than discretionary. With an emphasis on sustainability concepts including resource efficiency, pollution prevention, and ecological preservation, this research offers a green evaluation methodology specifically designed for highway building. This study attempts to provide a systematic framework for putting greener roadways into practice by defining important criteria and assessing various building techniques. For legislators, engineers, and project managers committed to sustainable building, the findings give useful advice and insights into best practices. To ensure that advancement does not come at the price of the environment, the goal is to match the expansion of infrastructure with global environmental goals. We use the HyperSoft set to deal with different criteria and select the best values. We use MACONT method to rank alternatives.

Keywords: Sustainability; Highway Construction; Green Evaluation; HyperSoft Set; Assessment Approach.

1. Introduction

Building highways is essential to a nation's social and economic advancement because they facilitate commerce, provide connections, and foster regional development. However, the long-term environmental effects of traditional construction practices have frequently been disregarded. Incorporating sustainable techniques into highway building has become a top goal as society grows more aware of resource depletion and climate change[1], [2].

Using methods and materials that lower carbon emissions, preserve natural resources, and safeguard ecosystems is part of building a sustainable route. It necessitates a thorough strategy that considers every stage of the project lifetime, from design and planning to building and upkeep[3], [4]. It is feasible to design highways that are both environmentally conscious and useful by including sustainability concepts early in the process.

The adoption of green construction practices on highways has been driven by advances in technology, stricter environmental regulations, and growing public demand for sustainable infrastructure. Innovations such as recycled asphalt, solar-powered lighting, and wildlife corridors are becoming standard features in modern highway projects[5], [6]. These measures help to minimize the environmental footprint while also potentially reducing operational costs over the highway's lifetime.

Despite these advancements, challenges persist in the practical application of sustainable construction methods. Financial constraints, lack of technical expertise, and insufficient policy incentives can hinder widespread adoption[7], [8]s. Moreover, there is a need for robust evaluation frameworks that can accurately measure the sustainability performance of different construction practices and help stakeholders make informed decisions.

A green evaluation approach offers a systematic method to assess and compare the sustainability of various highway construction strategies. By defining clear criteria—such as energy efficiency, material sustainability, biodiversity impact, and waste management—a structured evaluation can highlight the strengths and weaknesses of each approach[9], [10]. This not only guides project selection but also identifies areas for improvement in current practices.

Integrating sustainability into highway construction also aligns with broader national and international goals, such as the United Nations Sustainable Development Goals (SDGs). Particularly, goals related to sustainable cities and communities, climate action, and responsible consumption and production are directly addressed through greener highway development practices[11], [12]. This connection strengthens the case for adopting sustainable construction methods as a strategic priority.

Furthermore, green construction practices contribute to building more resilient infrastructure capable of withstanding the impacts of climate change. Features such as permeable pavements, natural drainage systems, and erosion control measures can enhance the durability of highways under extreme weather conditions, thereby reducing future repair and maintenance costs.

In this paper, a detailed green evaluation framework is proposed to assess the sustainability of highway construction practices[13], [14]. The approach combines qualitative assessments with quantitative analysis, ensuring a balanced and comprehensive evaluation. By applying this framework to real-world case studies, the research aims to demonstrate its practicality and effectiveness in promoting greener infrastructure development.

2. Proposed Model

Smarandache introduced the concept of the HyperSoft set in 2018 and then introduced different sets[15], [16]. A Soft Set offers a straightforward approach to parameterized decision modeling that effectively handles uncertainty in an organized manner by associating attributes (or parameters) with subsets of a universal set. Numerous similar mathematical frameworks, such as fuzzy sets, neutrosophic sets, and rough sets, have been developed to handle different aspects of uncertainty. This paradigm is extended by the HyperSoft Set, which gives Soft Sets multi-attribute decision modeling. By mapping combinations of many attributes to subsets of the universal set instead of allocating a single parameter to a subset, a HyperSoft Set improves its capacity to manage complex decision-making situations[17], [18].

Let Y be a universal set and set of criteria such as $Q_1, Q_2, \dots, Q_m$. The cartesian product of these criteria can be obtained as: $Q = Q_1 \times Q_2 \times \dots \times Q_m$. The HyperSoft set over Y is a pair (A, Q) and $A: Q \rightarrow P(Y)$. We can define the HyperSoft Set as:

$$(A, Q) = \{(U, A(U)) | U \in Q, A(U) \in P(Y)\} \quad (1)$$

$$U = (U_1, U_2, \dots, U_m) \in Q; U_i \in Q_i; i = 1, 2, \dots, m, A(U) \text{ shows the subset of } Y \text{ of criteria values } U_1, U_2, \dots, U_m \quad (2)$$

We show the steps of the MACONT to rank the alternatives. create the decision matrix between the criteria and alternatives such as:

$$R = \begin{bmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mn} \end{bmatrix}$$

We find three normalization matrices for the beneficial and non-beneficial criteria such as:

$$H_{ij}^1 = \frac{r_{ij}}{\sum_{i=1}^m r_{ij}} \quad (3)$$

$$H_{ij}^1 = \frac{1/r_{ij}}{\sum_{i=1}^m 1/r_{ij}} \quad (4)$$

$$H_{ij}^2 = \frac{r_{ij}}{\max r_{ij}} \quad (5)$$

$$H_{ij}^2 = \frac{\min r_{ij}}{r_{ij}} \quad (6)$$

$$H_{ij}^3 = \frac{(r_{ij} - \min r_{ij})}{(\max r_{ij} - \min r_{ij})} \quad (7)$$

$$H_{ij}^3 = \frac{(r_{ij} - \max r_{ij})}{(\min x_{ij} - \max r_{ij})} \quad (8)$$

Combine three normalization matrices into one matrix such as:

$$Q_{ij} = \alpha H_{ij}^1 + \beta H_{ij}^2 + (1 - \alpha - \beta) H_{ij}^3 \quad (9)$$

Values are $\alpha = \beta = 1/3$

Compute the distance among the reference options and every option

$$K_1(r_i) = \gamma \frac{A_i}{\sqrt{\sum_{i=1}^m (A_i)^2}} + (1 - \gamma) \frac{B_i}{\sqrt{\sum_{i=1}^m (B_i)^2}} \quad (10)$$

$$K_2(r_i) = \delta \max(w_j(K_{ij} - K_j^-)) + (1 - \delta) \min(w_j(K_{ij} - K_j^-)) \quad (11)$$

Where K_j^- refers to the average value and the value of δ and γ are between 0 and 1.

Compute the final score of every alternative such as:

$$K(r_i) = \frac{1}{2} \left(K_1(r_i) + \frac{K_2(r_i)}{\sqrt{\sum_{i=1}^m (K_2(r_i))^2}} \right) \quad (12)$$

3. Results

This section shows the results of the proposed approach. We use ten criteria and eight alternatives to obtain the criteria weights and ranking the alternatives.

1. Material Sustainability (Values: Highly Sustainable, Moderately Sustainable, Low Sustainability)
2. Energy Consumption in Construction (Values: Low, Medium, High)
3. Pollution Control Measure (Values: Excellent, Adequate, Poor)
4. Resource Efficiency (Water, Raw Materials (Values: Very Efficient, Efficient, Inefficient)
5. Construction Waste Management (Values: Minimal Waste, Moderate Waste, Significant Waste)
6. Ecological Preservation (Values: No Impact, Minor Impact, Major Impact)
7. Use of Renewable Energy Sources (Values: Full Use, Partial Use, No Use)
8. Lifecycle Environmental Impact (Values: Very Low, Moderate, High)
9. Noise and Air Quality Management (Values: Excellent, Satisfactory, Poor)
10. Adaptation to Climate Change (Values: Highly Adaptable, Moderately Adaptable, Poorly Adaptable)

We select the best values such as:

1. C1: Highly Sustainable
2. C2: Medium
3. C3: Excellent,
4. C4: Very Efficient
5. C5: Minimal Waste
6. C6: Minor Impact
7. C7: Full Use
8. C8: Moderate,
9. C9: Excellent

10. C10: Highly Adaptable.

The options are:

1. Recycled Pavement and Aggregate Usage
2. Installation of Solar Noise Barriers
3. Rainwater Collection Systems for Highway Maintenance
4. Wildlife Corridors and Eco-Ducts Across Highways
5. Carbon Capture Techniques During Material Production
6. Deployment of Electric and Hybrid Construction Machinery
7. Smart Monitoring Systems for Pollution and Emissions Control
8. Green Roofing on Highway Service Structures (e.g., rest areas)

Three experts create the decision matrix using their opinions as shown in Figs 1-3. We integrate these matrices into a single matrix as shown in Fig 4. We compute the criteria weights such as: $W_1 = 0.109981014$, $W_2 = 0.061165426$, $W_3 = 0.074352989$, $W_4 = 0.097550932$, $W_5 = 0.113981815$, $W_6 = 0.124222348$, $W_7 = 0.118174852$, $W_8 = 0.094196502$, $W_9 = 0.085731065$, $W_{10} = 0.120643056$.

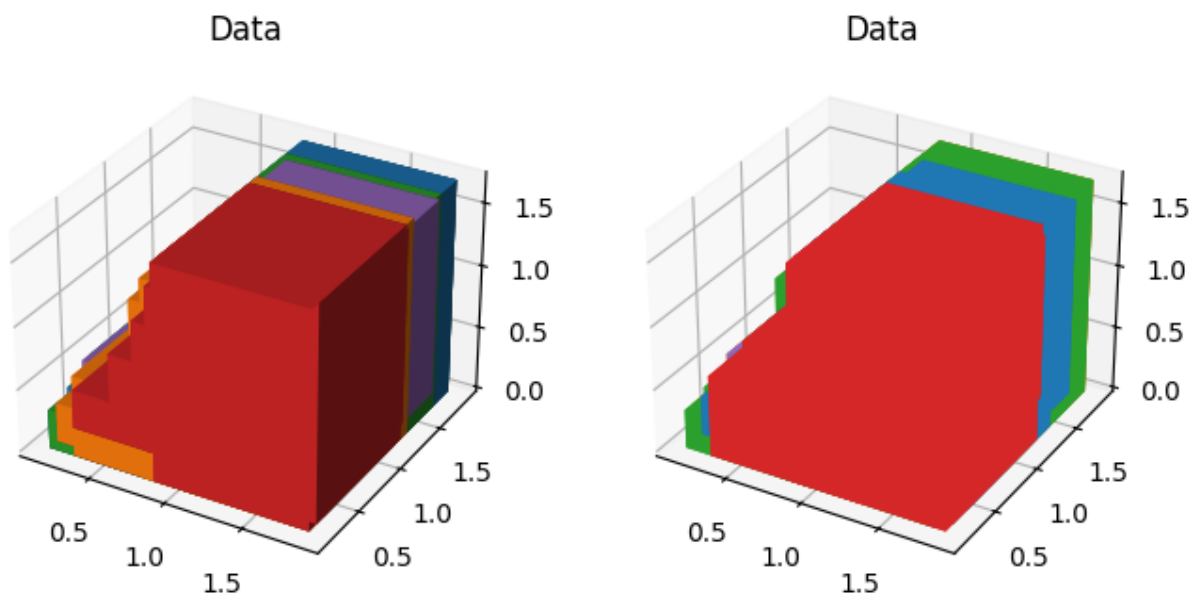


Fig 1. The first decision matrix

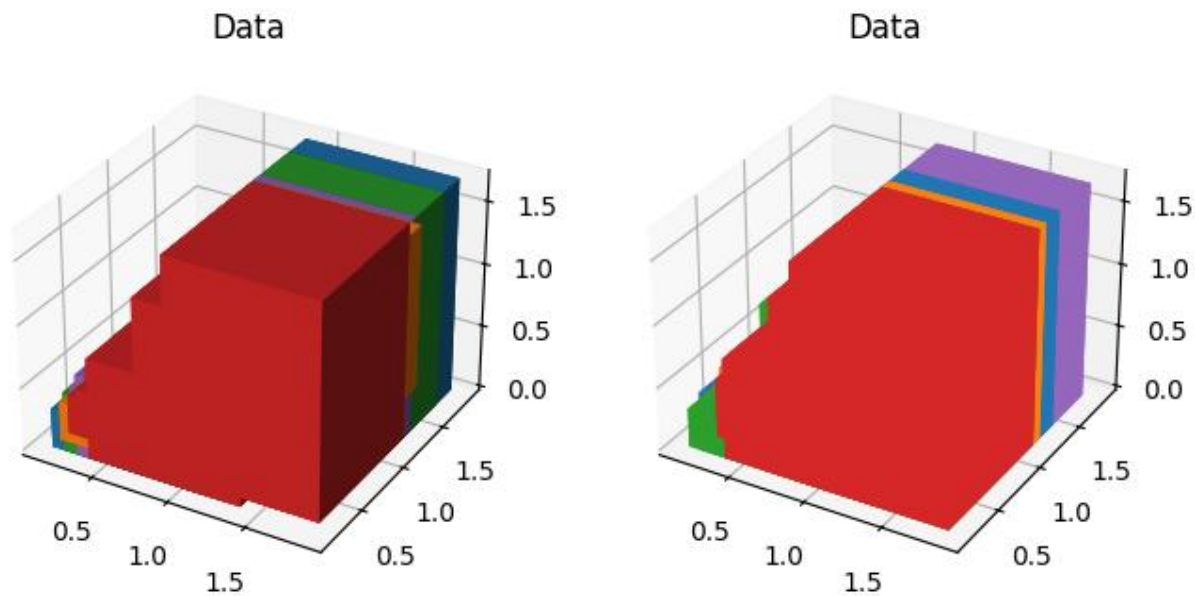


Fig 2. The second decision matrix

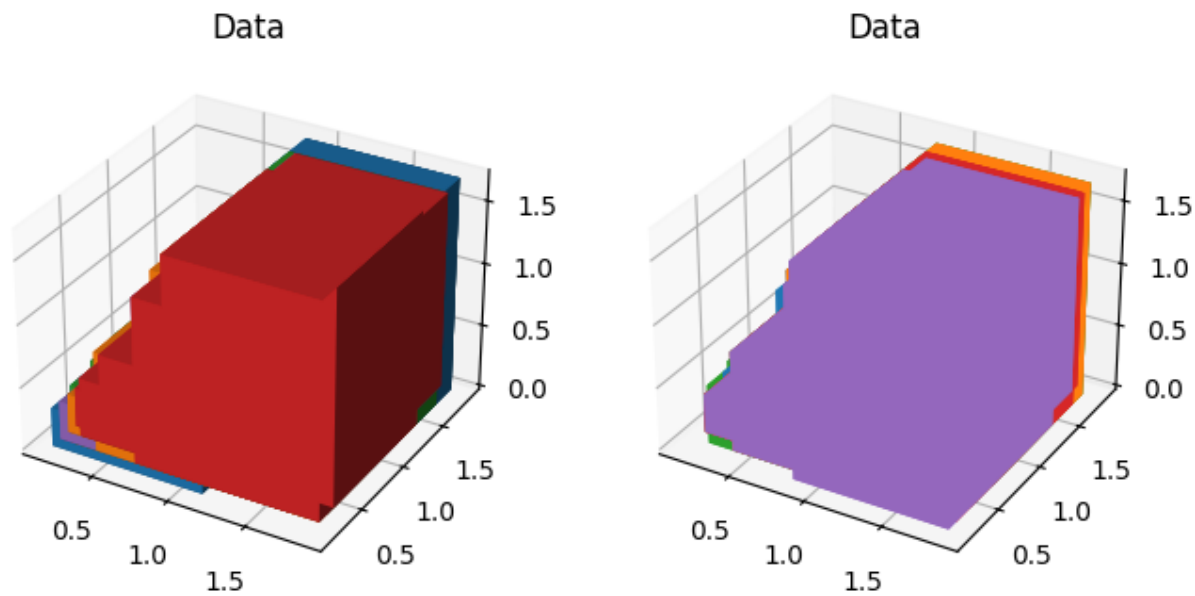


Fig 3. The third decision matrix

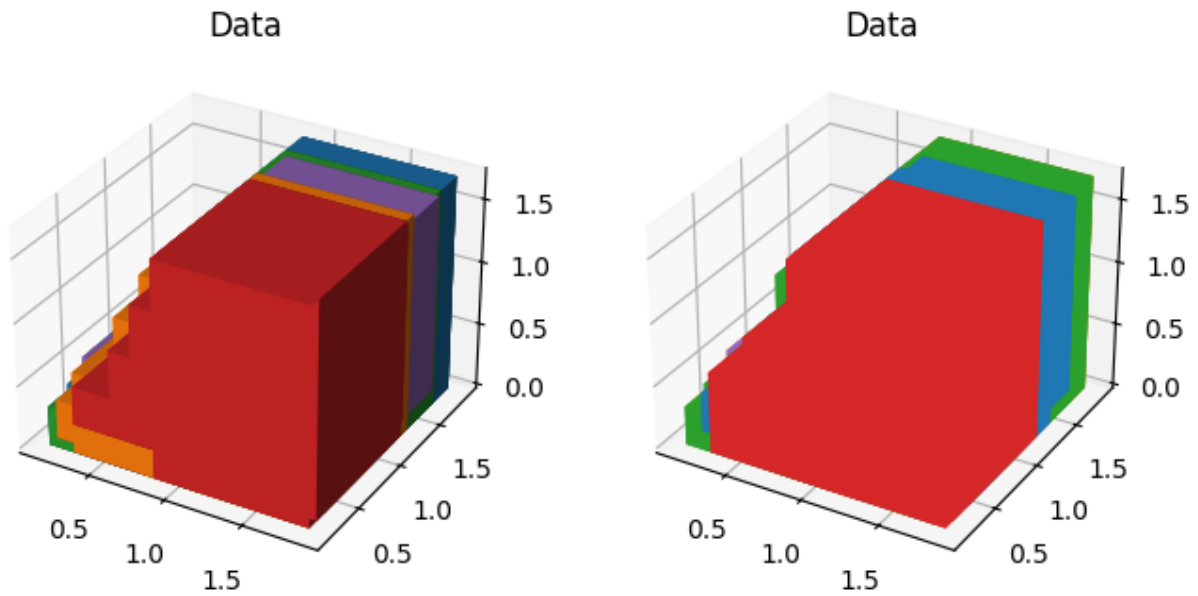


Fig 4. The combined decision matrix.

We find three normalization matrices for the beneficial and non-beneficial criteria using eqs. (3-8) as shown in Figs 5-7.

Combine three normalization matrices into one matrix using eq. (9) as shown in Fig 8.

Compute the distance among the reference options and every option using eqs. (10 and 11) as shown in Fig 9.

Compute the final score of every alternative using eq. (12) as shown in Fig 10.

Rank the alternatives as shown in Fig 11.

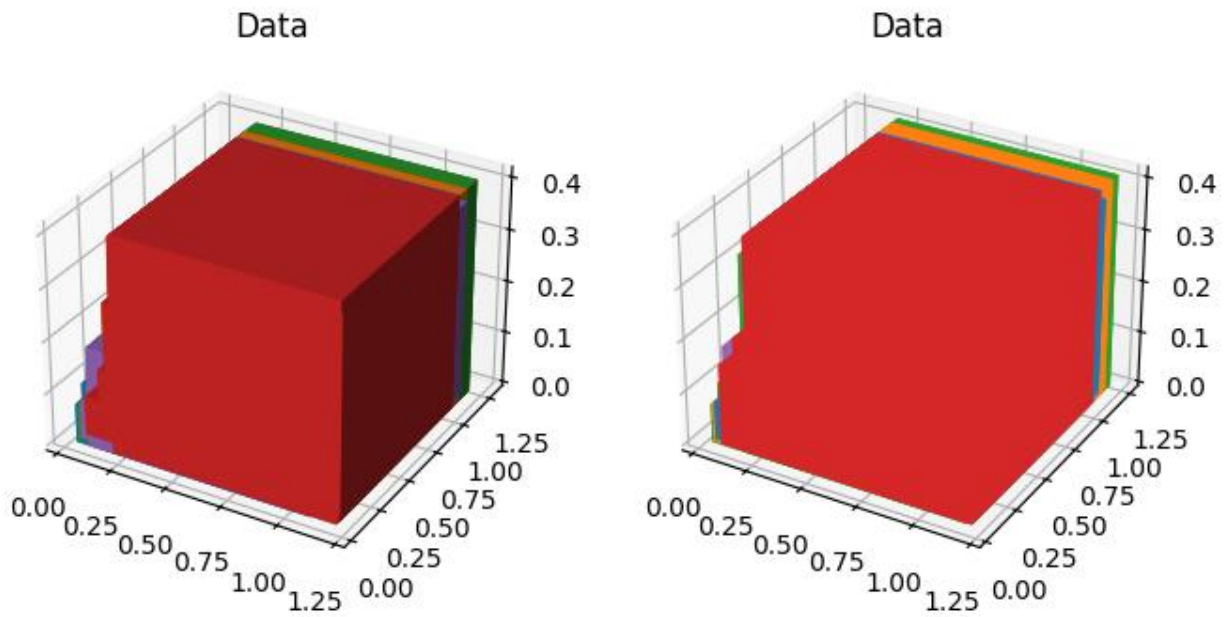


Fig 5. The first normalized values.

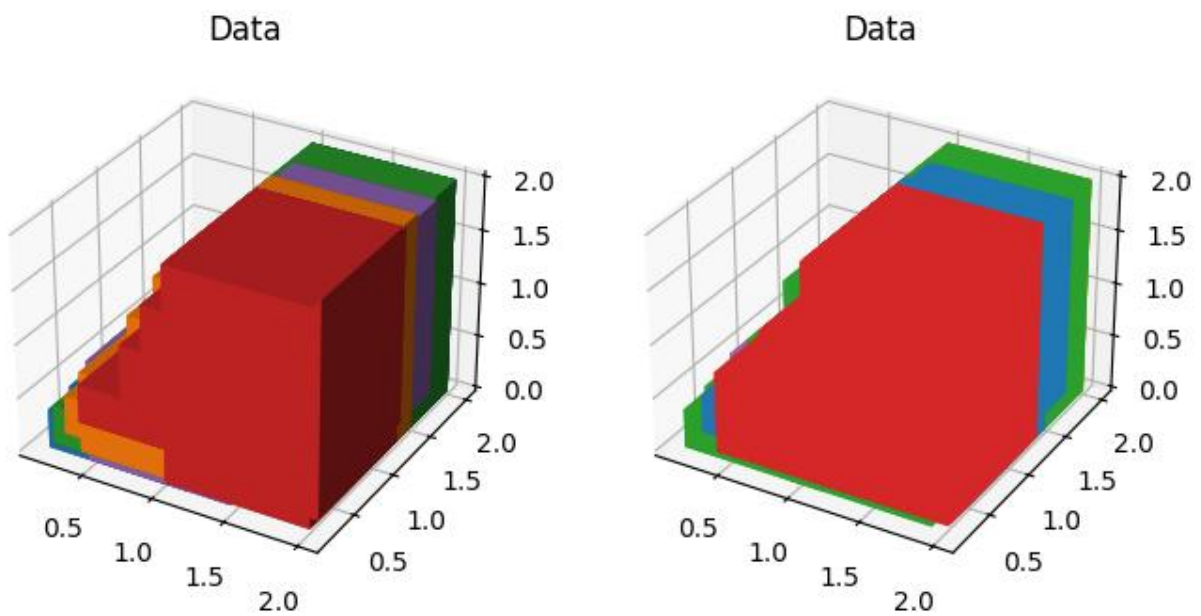


Fig 6. The second normalized values.

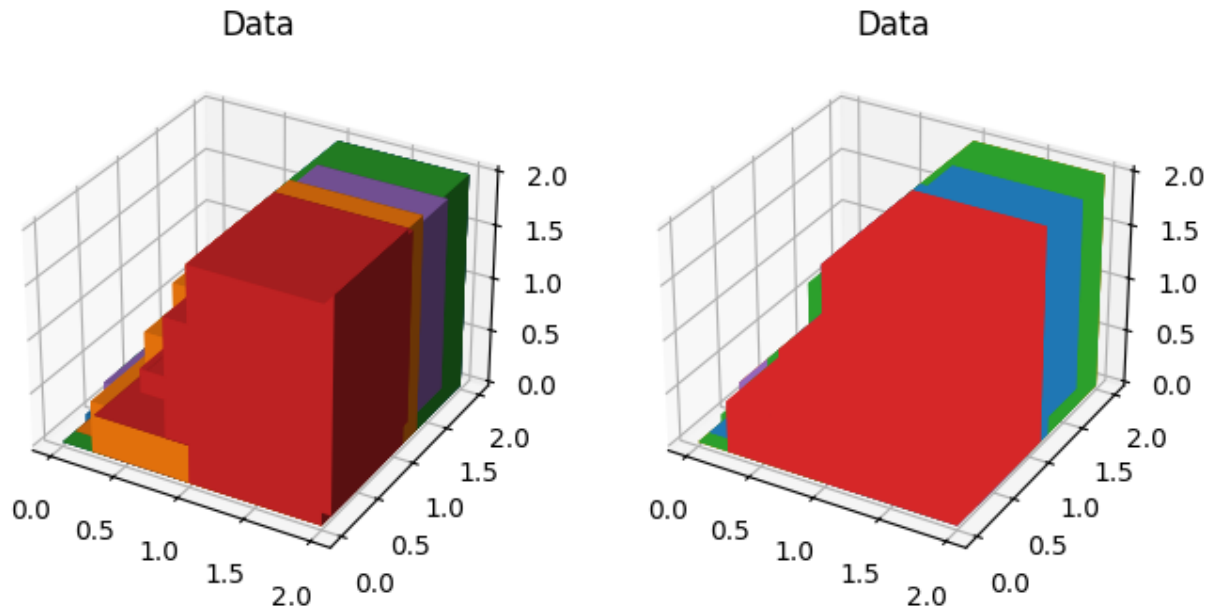


Fig 7. The third normalized values.

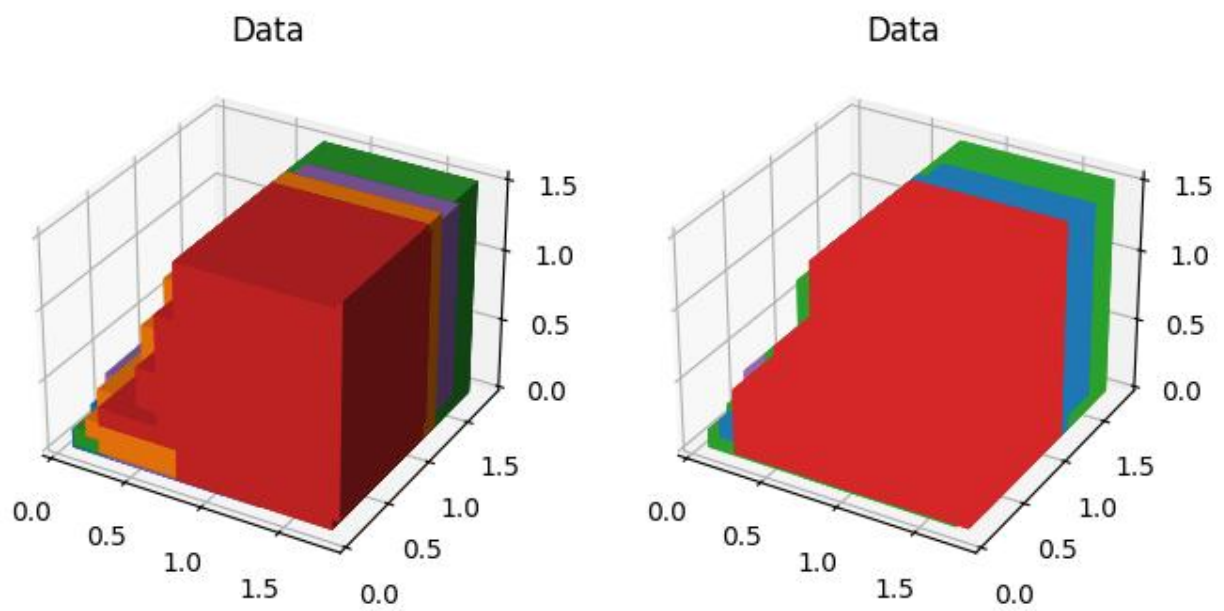


Fig 8. The integrated decision matrix.

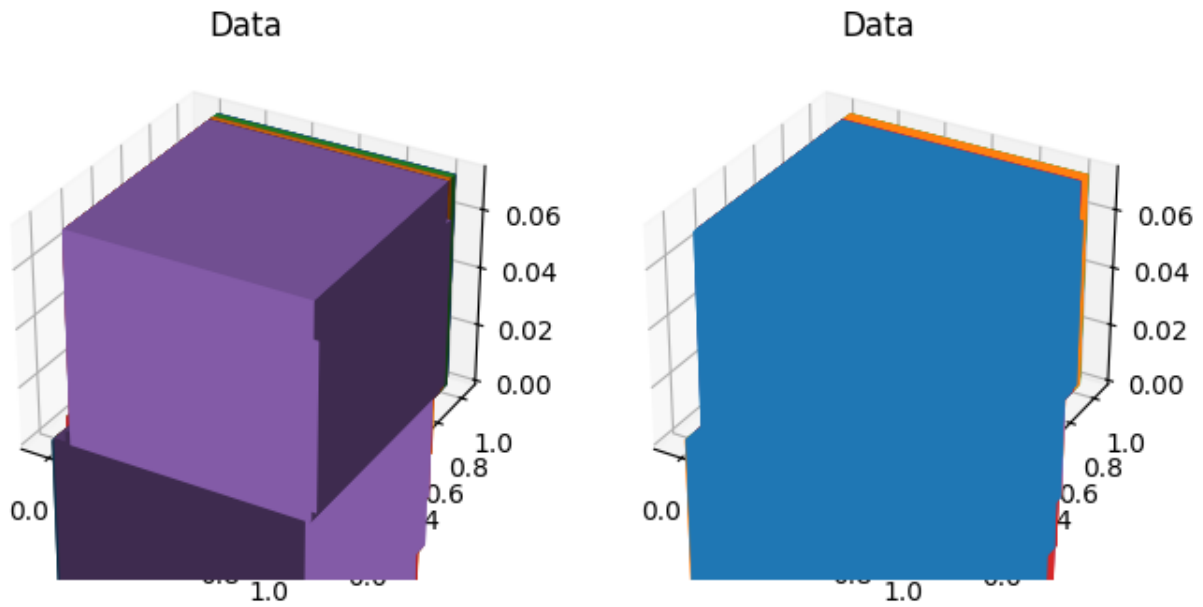


Fig 9-a. The distance among the reference options and every option.

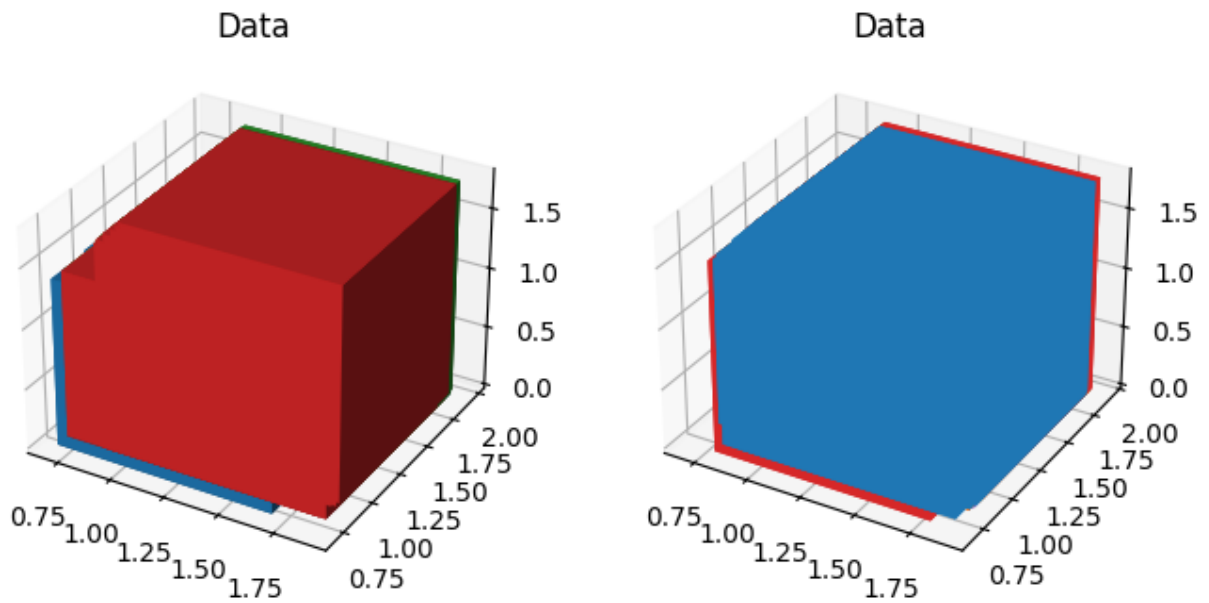


Fig 9-b. The distance among the reference options and every option.

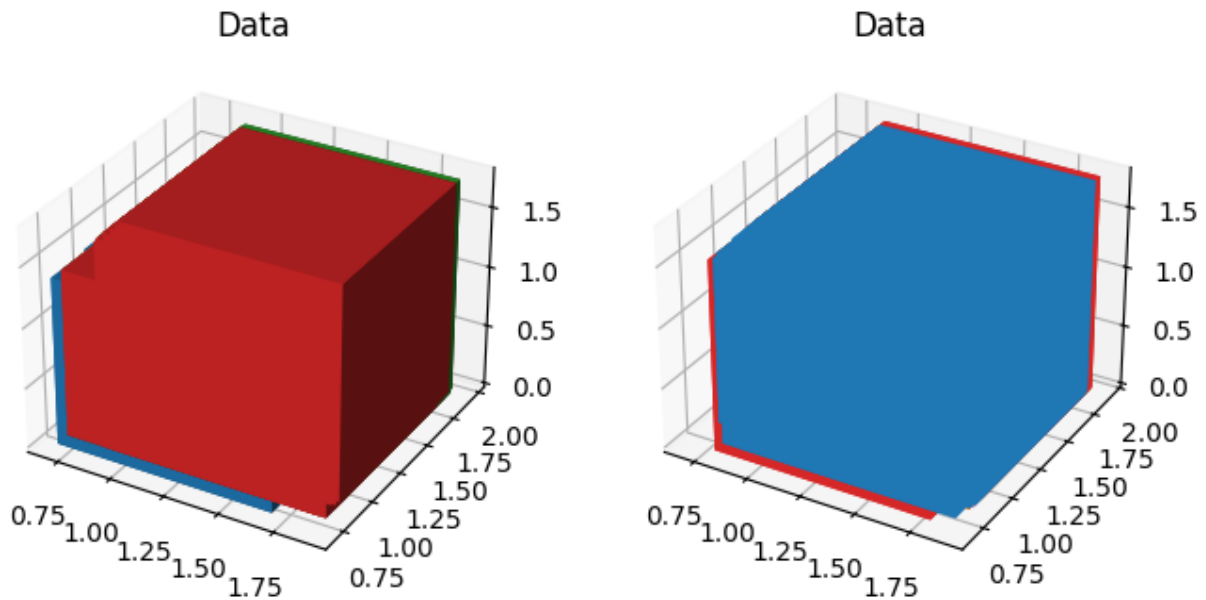


Fig 9-c. The distance among the reference options and every option.

Fig 10. The final score of every alternative.

Fig 11. The ranks of alternatives.

4. Analysis

This section shows the sensitivity analysis to show the stability of the ranks of alternatives. We split this section into two parts. In the first part, we change the value of α and β between 0.1 and 0.9. Then we show the ranks of the alternatives under these values. Fig 12 shows the final score of new parameter values. Then we rank the alternatives under these values as shown in Fig 13.

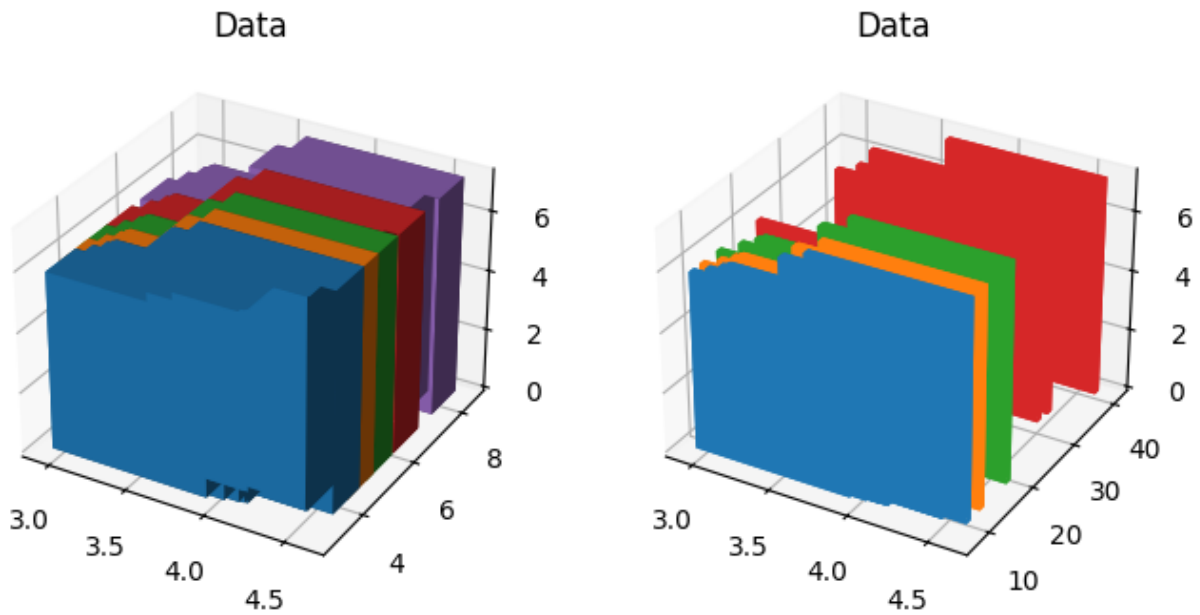


Fig 12. The final score of each alternative under new values of are α and β .

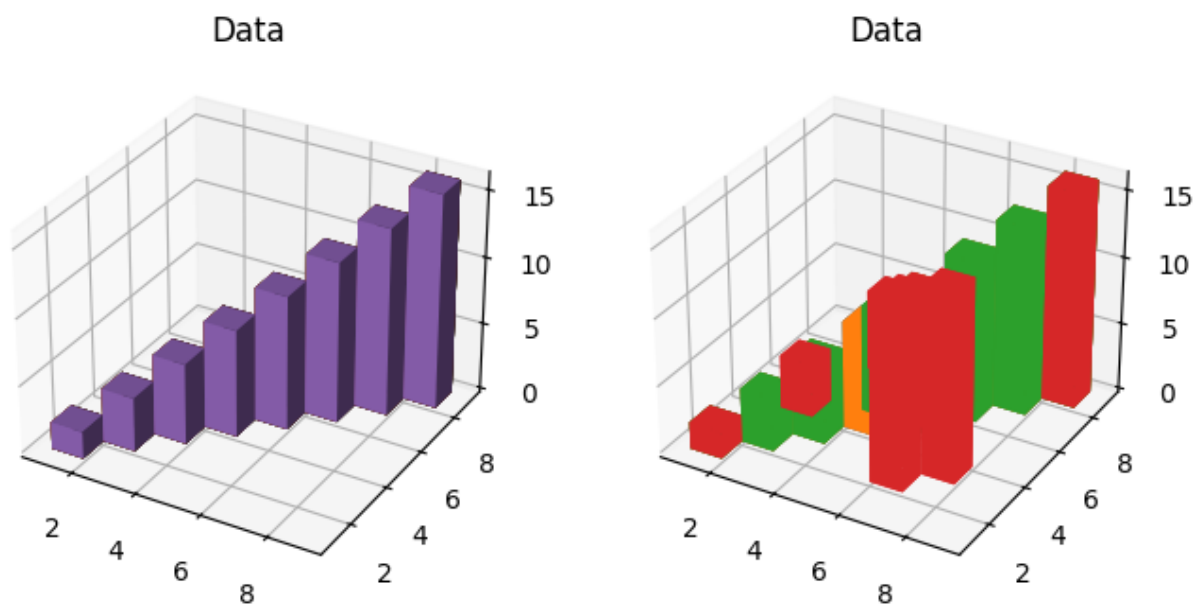


Fig 13. Ranks of the alternatives under new values of are α and β .

- A1:
Consistently ranks 8th across all cases, indicating it performs the worst among all alternatives in every scenario.
- A2:
Holds a stable 3rd position in almost all cases, showing strong and consistent

performance; however, in Case 9, its performance drops to 7th, indicating some instability under certain conditions.

- A3:
Maintains the 5th rank across most cases, meaning it is a moderately performing alternative. Slight improvement to 4th place in Case 8, suggesting a minor fluctuation.
- A4:
Steadily ranked 2nd in almost all cases, indicating a very high and consistent performance. Drops to 4th in Case 9, pointing to a minor decrease under certain conditions.
- A5:
Consistently ranks 6th, indicating a below-average performance throughout. Interestingly, it improves significantly to 2nd place in Case 9, suggesting a potential for better performance under specific scenarios.
- A6:
Consistently 7th, showing poor but stable performance across cases. Improves slightly to 3rd place in Case 9, indicating it can perform much better under favorable conditions.
- A7:
Regularly holds 4th place in most cases, showing good and reliable performance. Slight drop to 5th place in Case 8 and 6th place in Case 9, showing some sensitivity to different situations.
- A8:
Perfect 1st rank across all cases, demonstrating exceptional, consistent top performance with no fluctuations — the strongest and most reliable alternative.

In the second part, we change the parameter value δ and γ between 0.1 and 1. Then we apply the proposed approach under these values. We show the final score of each alternative as shown in Fig 14. Then we show the ranks of the alternatives as shown in Fig 15.

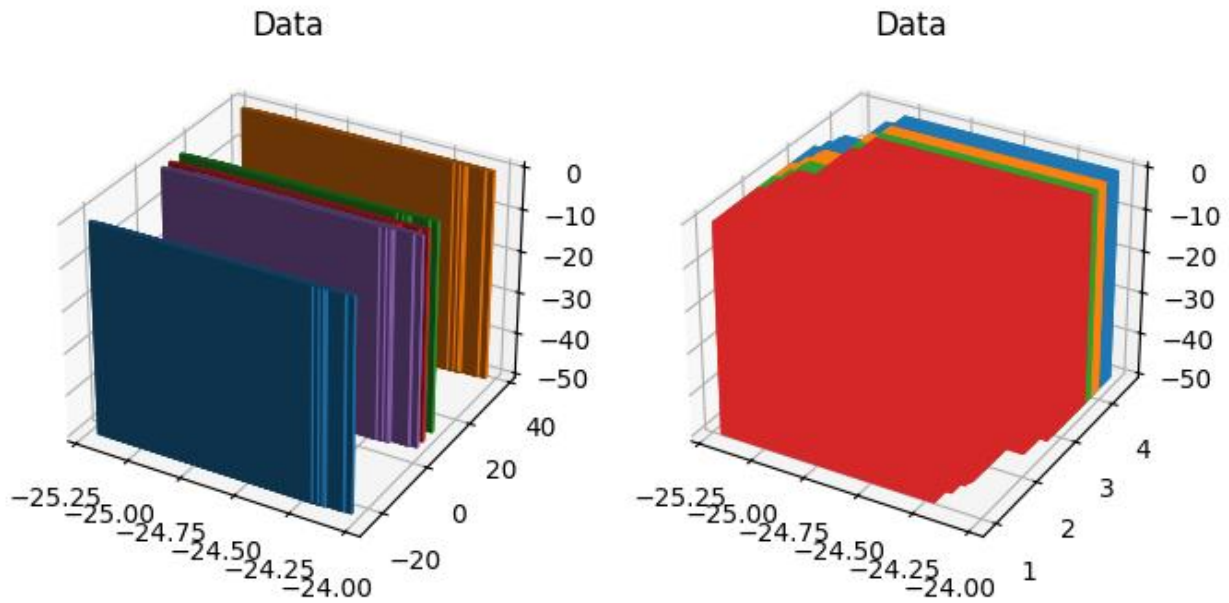


Fig 12. The final score of each alternative under new values of are δ and γ

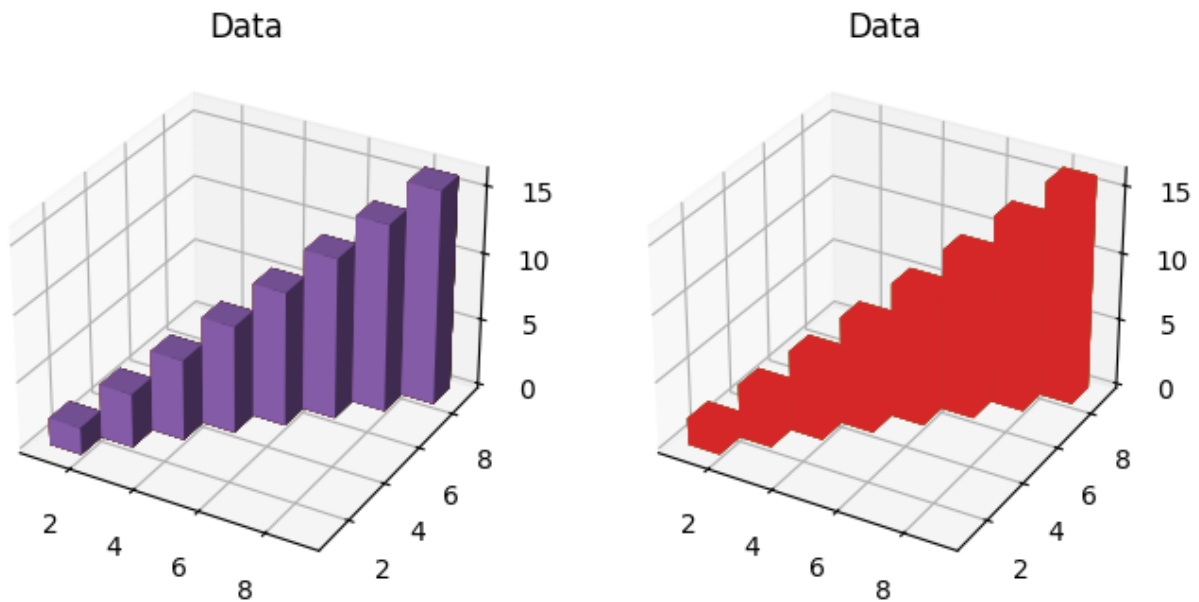


Fig 13. Ranks of the alternatives under new values of are δ and γ .

5. Conclusions

The urgent need for sustainable development has transformed the expectations placed on highway construction projects. Through the green evaluation approach outlined in this study, it becomes possible to systematically measure and enhance the sustainability performance of

highway construction practices. Emphasizing resource efficiency, environmental protection, and long-term resilience, sustainable highway construction is not just a technical challenge but a societal necessity. We used the HyperSoft set to deal with different criteria values. The proposed approach is used to compute the criteria weights and ranking the alternatives such as MACONT method. The results are validated using the cases study with ten criteria and eight alternatives. The sensitivity analysis shows the ranks of alternatives are stable under different parameter values.

The results of this research highlight that with the right evaluation tools, policies, and collaborative efforts among stakeholders, it is possible to build highways that support both human progress and environmental stewardship. Looking ahead, continued innovation and policy support will be essential in scaling up green construction practices to meet the growing demands of a sustainable future.

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Exploring the Differential Geometry of Reliability Function: Insights from Lifetime Weibull Distributions Under Neutrosophic Environment

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Abstract

This paper characterizes a set N as a two-dimensional surface marked by $R^+ \times R^+$ and demonstrates its properties as a topological 2-reliability manifold and a differential reliability manifold. Utilizing the Weibull lifetime distribution, we derive the formula for the Riemannian manifold (N, g_{ij}) . Finally, we prove that the reliability function represents the critical point of its log-likelihood on the manifold, which also serves as the saddle point.

Also, we discuss the same results in a neutrosophic environment, with neutrosophic variables and coefficients from the neutrosophic real ring $R(I)$, where we get similar results of the original approach.

Keywords: Parametric Model, Reliability Function, Weibull Distribution, Manifold (Topological & Differentiable), Chart & Atlas, Riemannian Geometry, Log-Likelihood Function, Critical Point & Saddle Point, Neutrosophic Reliability Function, Neutrosophic Random Variable, Neutrosophic Topological Space, Neutrosophic Manifold, Homogeneous Neutrosophic Differential Transformation

1. Introduction

The geometric structure of reliability function manifolds, based on probability distributions, is a fundamental aspect of information geometry. Previous studies by authors such as Amari and Dodson [1, 2], among others, have focused on the geometric structures of exponential families within distribution manifolds. They have explored manifolds associated with the normal distribution [3], gamma distribution and invers gamma distribution [4–6], Pareto distribution [7, 8] and beta distribution [9, 10]. The reliability function based on the Weibull distribution belongs to the exponential family of distributions [11, 12]. The Weibull distribution is a flexible model commonly used to represent the lifetime of systems or products. Its probability density function can be expressed

as : $f(x) = \frac{\beta}{\vartheta} \left(\frac{x}{\vartheta}\right)^{\beta-1} e^{-\left(\frac{x}{\vartheta}\right)^\beta}$, where ϑ is the scale parameter and β is the shape parameter [13, 14].

The reliability function, which is the complement of the cumulative distribution function, is given by

$(t, \vartheta, \beta) = e^{-\left(\frac{t}{\vartheta}\right)^\beta}$. This function represents the probability of a system or product surviving beyond

a given time t . Analyzing the geometric properties of a reliability manifold with a Weibull lifetime involves studying the shape and characteristics of the manifold based on the Weibull distribution parameters. To that end, we consider the shape parameter, scale parameter, and time axis. In this paper, we establish the characterization of a set N as a two-dimensional surface while demonstrating its properties as a topological 2-reliability manifold.

By rigorously analyzing the topological and differential structures of N , we provide evidence of its reliability in both topological and analytical contexts.

This work contributes to a deeper understanding of the manifold's geometric and algebraic properties, shedding light on its potential applications across various mathematical and scientific domains.

Smarandache presented neutrosophic logic in [24], and then it was applied in many different areas of science and knowledge, especially in probability theory and statistics [21-23].

In this work, we prove that the reliability function represents the critical point of its log-likelihood on the manifold, which also serves as the saddle point.

Also, we discuss the same results in a neutrosophic environment, with neutrosophic variables and coefficients from the neutrosophic real ring $R(I)$, where we get similar results of the original approach.

2. Mathematical Background

Definition 2.1 [15, 16]: The family $N = \{R(t, \vartheta), t \in (0, \infty), \vartheta \in R^n > 0\}$ of the reliability functions is said to be **parametric model** if there exists a mapping $g: I \rightarrow N$ which is satisfy the following conditions:

- 1- g is one-to-one
- 2- The Wronskian determinate

$$\det \begin{bmatrix} \omega_1(t) & \omega_2(t) \\ \omega'_1(t) & \omega'_2(t) \end{bmatrix} \neq 0, \forall \vartheta$$

We can write it by $W(t, \omega_1(t, \vartheta), \omega_2(t, \vartheta)) \neq 0$, where $\omega_j(t, \vartheta) = \frac{\partial R(t, \vartheta)}{\partial \vartheta^j}$.

Definition 2.2 [15] :- Let $N = \{R(t, \vartheta), t \in (0, \infty), \vartheta \in R^n > 0\}$ be a parametric model of dimension n . Then an **n-dimensional manifold** N is a topological space which is Hausdorff, second countable and locally homeomorphic to an n -dimensional Euclidean space R^n .

Definition 2.3 [17, 18]:- Let $U \subseteq N$ be an open set and $\varphi: U \rightarrow \varphi(U) \subset R^n$ is a homeomorphism of the open set U in N onto open subset $\varphi(U)$ of R^n . The pair (U, φ) is called a **chart** or (coordinate system). Chart (U, φ) gives us coordinate which helps us to calculate on the manifold, and in order to calculate on the whole manifold, we need a lot of charts such that all charts cover the whole manifold. Hence such a collection called an atlas.

Definition 2.4 [17, 19]:- A collection of charts $(U_i, \varphi_i)_{i \in I}$ is called an **atlas** if

$$\cup_{i \in I} U_i = N.$$

Definition 2.5 [17, 20]:- A **differentiable (smooth) manifold** N is an n -dimensional (topological) manifold endowed with maximal C^k -atlas (C^k -smoothly structure).

3. Main Results

Theorem 3.1 : Let $N = \left\{ R(t) = e^{-\left(\frac{t}{\vartheta}\right)^\beta}, t \in [0, +\infty), (\vartheta, \beta) \in R^+ \times R^+ \right\}$ and

$f: R^+ \times R^+ \rightarrow N$ be one-to-one mapping, such that $g(\vartheta, \beta) = R(t)$ where $R(t)$ is the reliability of Weibull distribution. Then N be two-dimension surface parametrized by $R^+ \times R^+$

Proof:

The log-likelihood function is $\log(R(t)) = \log\left(e^{-\left(\frac{t}{\vartheta}\right)^\beta}\right) = -\left(\frac{t}{\vartheta}\right)^\beta$

let $\omega_1(t) = \partial_\vartheta \log R(t) = \frac{\beta}{\vartheta} \left(\frac{t}{\vartheta}\right)^\beta$ and $\omega_2(t) = \partial_\beta \log R(t) = -\left(\frac{t}{\vartheta}\right)^\beta \log\left(\frac{t}{\vartheta}\right)$

Then $\det \begin{bmatrix} \omega_1(t) & \omega_2(t) \\ \omega_1'(t) & \omega_2'(t) \end{bmatrix} = \det \begin{bmatrix} \frac{\beta}{\vartheta} \left(\frac{t}{\vartheta}\right)^\beta & -\left(\frac{t}{\vartheta}\right)^\beta \log\left(\frac{t}{\vartheta}\right) \\ \frac{\beta^2}{\vartheta^2} \left(\frac{t}{\vartheta}\right)^{\beta-1} & \frac{\left(\frac{t}{\vartheta}\right)^\beta (\beta \log(\frac{t}{\vartheta}) + 1)}{t} \end{bmatrix} = \left(\left[\frac{\beta}{\vartheta} \left(\frac{t}{\vartheta}\right)^\beta \right] \cdot \left[\frac{\left(\frac{t}{\vartheta}\right)^\beta (\beta \log(\frac{t}{\vartheta}) + 1)}{t} \right] \right) + \left[\left(\frac{t}{\vartheta}\right)^\beta \log\left(\frac{t}{\vartheta}\right) \right] \cdot \left[\frac{\beta^2}{\vartheta^2} \left(\frac{t}{\vartheta}\right)^{\beta-1} \right] \neq 0$

Then $\forall (\vartheta, \beta) \in R^+ \times R^+$ and $t \in (0, \infty)$, the set N is a two-dimensional surface parameterized by $R^+ \times R^+$

Theorem 3.2:- The graph of $R(t) = e^{-\left(\frac{t}{\vartheta}\right)^\beta}$ is given by the set

$$N = \left\{ \left(\vartheta, \beta, e^{-\left(\frac{t}{\vartheta}\right)^\beta} \right), (\vartheta, \beta) \in R^+ \times R^+ \right\} \subseteq R^{+3}$$

is a topological 2-Reliability Manifold

Proof:-

let (R^{+3}, τ) be topological space and N is subspace of R^{+3}

Define the family τ_N as family of subset of N as follow:

$$\tau_N = \{N \cap V : V \in \tau\}$$

Sine $R^{+3} \in \tau$ and $N \subseteq R^{+3}$, we have $N \cap R^{+3} = N$, so $N \in \tau_N$ (1)

Sine $\emptyset \in \tau$ and $\emptyset \subseteq R^{+3}$, then $\emptyset = N \cap \emptyset$ and $\emptyset \in \tau_N$ (2)

From (1) and (2) we get $\emptyset, N \in \tau_N$

Let $U_1, U_2 \in \tau_N$, then there exist $V_1, V_2 \in \tau$ $\ni U_1 = N \cap V_1$ and $U_2 = N \cap V_2$. Hence $U_1 \cap U_2 = (N \cap V_1) \cap (N \cap V_2)$

$$= N \cap (V_1 \cap V_2) \in \tau_N$$

So $U_1 \cap U_2 \in \tau_N$.

Let $U_i \in \tau_N, i \in \Lambda$. Then there exist $V_i \in \tau$ $\ni U_i = N \cap V_i, i \in \Lambda$ and

$$\cup_{i \in \Lambda} U_i = \cup_{i \in \Lambda} (N \cap V_i) = N \cap (\cup_{i \in \Lambda} V_i)$$

Hence $\cup_{i \in \Lambda} U_i \in \tau_N$. Therefore τ_N is topological space on N

Now, let $\left(\vartheta_1, \beta_1, e^{-\left(\frac{t}{\vartheta_1}\right)^{\beta_1}} \right), \left(\vartheta_2, \beta_2, e^{-\left(\frac{t}{\vartheta_2}\right)^{\beta_2}} \right) \in N$, then there exists,

$$V_1, V_2 \in \tau \ni \left(\vartheta_1, \beta_1, e^{-\left(\frac{t}{\vartheta_1}\right)^{\beta_1}} \right) \in V_1, \left(\vartheta_2, \beta_2, e^{-\left(\frac{t}{\vartheta_2}\right)^{\beta_2}} \right) \in V_2,$$

and $V_1 \cap V_2 = \emptyset$.

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$\left[\left(\frac{t}{\vartheta}\right)^{\beta} \log\left(\frac{t}{\vartheta}\right) \right] \cdot \left[\frac{\beta^2}{\vartheta^2} \left(\frac{t}{\vartheta}\right)^{\beta-1} \right] \neq 0$ Then $\forall (\vartheta, \beta) \in R^+ \times R^+$ and $t \in (0, \infty)$, the set N is a two-dimensional surface parameterized by $R^+ \times R^+$

Theorem 3.2:- The graph of $R(t) = e^{-\left(\frac{t}{\vartheta}\right)^{\beta}}$ is given by the set

$$N = \left\{ \left(\vartheta, \beta, e^{-\left(\frac{t}{\vartheta}\right)^{\beta}} \right), (\vartheta, \beta) \in R^+ \times R^+ \right\} \subseteq R^{+3}$$

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Sine $R^{+3} \in \tau$ and $N \subseteq R^{+3}$, we have $N \cap R^{+3} = N$, so $N \in \tau_N$ (1)

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$$= N \cap (V_1 \cap V_2) \in \tau_N$$

So $U_1 \cap U_2 \in \tau_N$.

Let $U_i \in \tau_N, i \in \Lambda$. Then there exist $V_i \in \tau \ni U_i = N \cap V_i, i \in \Lambda$ and

$$\cup_{i \in \Lambda} U_i = \cup_{i \in \Lambda} (N \cap V_i) = N \cap (\cup_{i \in \Lambda} V_i)$$

Hence $\cup_{i \in \Lambda} U_i \in \tau_N$. Therefore τ_N is topological space on N

Now, let $\left(\vartheta_1, \beta_1, e^{-\left(\frac{t}{\vartheta_1}\right)^{\beta_1}} \right), \left(\vartheta_2, \beta_2, e^{-\left(\frac{t}{\vartheta_2}\right)^{\beta_2}} \right) \in N$, then there exists,

$$V_1, V_2 \in \tau \ni \left(\vartheta_1, \beta_1, e^{-\left(\frac{t}{\vartheta_1}\right)^{\beta_1}} \right) \in V_1, \left(\vartheta_2, \beta_2, e^{-\left(\frac{t}{\vartheta_2}\right)^{\beta_2}} \right) \in V_2,$$

and $V_1 \cap V_2 = \emptyset$.

Since N is subspace of R^{+3} , then $U_1 = V_1 \cap N$ and $U_2 = V_2 \cap N$ are two disjoint open sets in N and containing $\left(\vartheta_1, \beta_1, e^{-\left(\frac{t}{\vartheta_1}\right)^{\beta_1}} \right), \left(\vartheta_2, \beta_2, e^{-\left(\frac{t}{\vartheta_2}\right)^{\beta_2}} \right)$ respectively. So (N, τ_N) is Hausdorff space

$$\text{let } B = \left\{ B_r \left(\vartheta, \beta, e^{-\left(\frac{t}{\vartheta}\right)^{\beta}} \right) : \left(\vartheta, \beta, e^{-\left(\frac{t}{\vartheta}\right)^{\beta}} \right) \in Q^3, r > 0, r \in Q \right\}$$

is countable basis for R^{+3} .

Hence $B_N = \left\{ B_r \left(\vartheta, \beta, e^{-\left(\frac{t}{\vartheta}\right)^{\beta}} \right) \cap N : \left(\vartheta, \beta, e^{-\left(\frac{t}{\vartheta}\right)^{\beta}} \right) \in Q^3, r > 0, r \in Q \right\}$ is a countable basis for the subspace N . So (N, τ_N) is second countable.

Let $\phi: N \rightarrow R^{+2}$ defined as $\phi \left(\vartheta, \beta, e^{-\left(\frac{t}{\vartheta}\right)^{\beta}} \right) = (\vartheta, \beta)$, and $\left(\vartheta_1, \beta_1, e^{-\left(\frac{t}{\vartheta_1}\right)^{\beta_1}} \right), \left(\vartheta_2, \beta_2, e^{-\left(\frac{t}{\vartheta_2}\right)^{\beta_2}} \right) \in N$

Then if $\phi \left(\vartheta_1, \beta_1, e^{-\left(\frac{t}{\vartheta_1}\right)^{\beta_1}} \right) = \phi \left(\vartheta_2, \beta_2, e^{-\left(\frac{t}{\vartheta_2}\right)^{\beta_2}} \right)$, we have

$$(\vartheta_1, \beta_1) = (\vartheta_2, \beta_2), \text{ and } \left(\vartheta_1, \beta_1, e^{-\left(\frac{t}{\vartheta_1}\right)^{\beta_1}} \right) = \left(\vartheta_2, \beta_2, e^{-\left(\frac{t}{\vartheta_1}\right)^{\beta_1}} \right).$$

So ϕ is one-to-one.

Let $(\vartheta, \beta) \in R^{+2}$, then there exist $\left(\vartheta, \beta, e^{-\left(\frac{t}{\vartheta}\right)^{\beta}} \right) \in N$

$$\ni \phi \left(\vartheta, \beta, e^{-\left(\frac{t}{\vartheta}\right)^{\beta}} \right) = (\vartheta, \beta). \text{ Hence } \phi \text{ is onto}$$

$N \left(\vartheta, \beta, e^{-\left(\frac{t}{\vartheta}\right)^{\beta_1}} \right) = (\vartheta, \beta)$ and $\phi^{-1}(\vartheta, \beta) = \left(\vartheta, \beta, e^{-\left(\frac{t}{\vartheta}\right)^{\beta}} \right)$ both are continuous. Then ϕ is

homeomorphism from N into R^{+2} so (N, ϕ) is a chart

So, N is a locally Euclidean of dimension 2. Finally N is a topological manifold of dimension 2.

Theorem 3.3: The set $N = \left\{ R(t, \vartheta, \beta) : R(t, \vartheta, \beta) = e^{-\left(\frac{t}{\vartheta}\right)^{\beta}}, t \in [0, +\infty), (\vartheta, \beta) \in R^+ \times R^+ \right\}$ is a differentiable reliability manifold.

Proof:

The set N is diffeomorphic to the upper half-plane in R^2 , and the entire reliability manifold N is covered by only one atlas consists of only one chart ϕ from open subset U of N onto open subset $\phi(U)$ of R^2 .

From the log-likelihood function $\log(R(t)) = \log \left(e^{-\left(\frac{t}{\vartheta}\right)^{\beta}} \right) = -\left(\frac{t}{\vartheta}\right)^{\beta}$, let $(\vartheta, \beta) = \phi(\log(R(t)))$ is coordinate system. Another coordinate can be defined as follows:

$\mu_1 = E(t) = \vartheta \Gamma\left(\frac{1}{\beta} + 1\right)$, and $\mu_2 = E(t^2) = \vartheta^2 \Gamma\left(\frac{2}{\beta} + 1\right)$, where E denotes the expectation of a random variable.

Then $\phi(\log(R(t))) = (\mu_1, \mu_2)$ be a chart of the same point $R(t)$.

The transition map $T = \varphi \circ \varphi^{-1}: (\vartheta, \beta) \rightarrow (\mu_1, \mu_2) \subset \mathbb{R}^2 \ni \mu = \mu_i(\vartheta, \beta), i = 1, 2$ can be defined by using the Jacobean matrix of the transition map .

The Jacobin matrix of the transition map is

$$J = \begin{bmatrix} \frac{\partial \mu_1}{\partial \vartheta} & \frac{\partial \mu_1}{\partial \beta} \\ \frac{\partial \mu_2}{\partial \vartheta} & \frac{\partial \mu_2}{\partial \beta} \end{bmatrix} = \begin{bmatrix} \Gamma(\frac{1}{\beta} + 1) & \vartheta \Gamma'(\frac{1}{\beta} + 1) \\ 2\vartheta \Gamma(\frac{2}{\beta} + 1) & \vartheta^2 \Gamma'(\frac{2}{\beta} + 1) \end{bmatrix}.$$

Since $\det(J) \neq 0$, then T is diffeomorphism (i.e) it is continuously differentiable of all order .

It's invers T^{-1} is also diffeomorphism , since $T^{-1} = \varphi \circ \varphi^{-1}: (\mu_1, \mu_2) \rightarrow (\vartheta, \beta)$

$$\hat{J} = \begin{bmatrix} \frac{\partial \vartheta}{\partial \mu_1} & \frac{\partial \vartheta}{\partial \mu_2} \\ \frac{\partial \beta}{\partial \mu_1} & \frac{\partial \beta}{\partial \mu_2} \end{bmatrix} = \begin{bmatrix} \frac{\mu_1 \Gamma'(\frac{1}{\beta} + 1) - \Gamma(\frac{1}{\beta} + 1)}{\mu_1^2} & 0 \\ 0 & \frac{1}{\Gamma^{-1}(\frac{\mu_2}{\vartheta^2}) - 1} \Gamma'(\frac{\mu_2}{\vartheta^2}) \end{bmatrix} \neq 0.$$

Since $\det(\hat{J}) \neq 0$ and T^{-1} is continuously differentiable for all (ϑ, β) .

Hence φ, φ^{-1} are C^∞ - smoothly compatible .

The atlas $\{(U_i, \varphi_i)\}$ is C^∞ -atlas and $\{(U_i, \varphi_i)\}$ can be extended to maximal C^∞ -atlas which is smooth structure.

So we get , $N = \left\{ e^{-\left(\frac{t}{\vartheta}\right)^\beta} \right\}$ be smooth manifold of the reliability function with lifetime weibull distribution .

Theorem 3.4 : The Riemannian reliability manifold with weibull lifetime distribution is (N, g_{ij}) where

$$g_{ij} = \begin{bmatrix} \frac{\beta}{\vartheta^2} \Gamma\left(3\beta + \frac{1}{\beta - 1}\right) & \vartheta \Gamma(n + 1) \\ \vartheta \Gamma(n + 1) & \frac{\vartheta^\beta}{\beta} \Gamma\left(\frac{1}{\beta - 1}\right) \end{bmatrix}$$

Proof:

Let g_{ij} be a metric tensor such that

$$g_{ij} = \langle \partial_i, \partial_j \rangle = E_\vartheta [\partial_{\vartheta^i} \log R(t, \vartheta) * \partial_{\vartheta^j} \log R(t, \vartheta)]$$

Since $f(t) = \frac{\beta}{\vartheta} \left(\frac{t}{\vartheta}\right)^{\beta-1} e^{-\left(\frac{t}{\vartheta}\right)^\beta}$ and it's reliability function is $R(t) = e^{-\left(\frac{t}{\vartheta}\right)^\beta}$

So $\log R(t) = -\left(\frac{t}{\vartheta}\right)^\beta$, $\partial_\vartheta \log R(t) = \frac{\beta}{\vartheta} \left(\frac{t}{\vartheta}\right)^\beta$ and $\partial_\beta \log R(t) = -\left(\frac{t}{\vartheta}\right)^\beta \log\left(\frac{t}{\vartheta}\right)$

Then $g_{11}(\vartheta, \beta) = E[\partial_\vartheta \log R(t) \cdot \partial_\vartheta \log R(t)]$

$$\begin{aligned} &= \int_0^\infty \left[\left(\frac{\beta}{\vartheta} \right) \left(\frac{t}{\vartheta} \right)^\beta \right]^2 \cdot \frac{\beta}{\vartheta} \left(\frac{t}{\vartheta} \right)^{\beta-1} e^{-\left(\frac{t}{\vartheta}\right)^\beta} dt \\ &= \int_0^\infty \frac{\beta^2}{\vartheta^2} \left(\frac{t}{\vartheta} \right)^{2\beta} \cdot \frac{\beta}{\vartheta} \left(\frac{t}{\vartheta} \right)^{\beta-1} e^{-\left(\frac{t}{\vartheta}\right)^\beta} dt \\ &= \frac{\beta^3}{\vartheta^3} \int_0^\infty \left(\frac{t}{\vartheta} \right)^{3\beta-1} \cdot e^{-\left(\frac{t}{\vartheta}\right)^\beta} dt \end{aligned}$$

$$g_{11} = \left(\frac{\beta}{\vartheta}\right)^3 \left(\frac{\vartheta}{\beta^2}\right) \Gamma\left(3\beta + \frac{1}{\beta - 1}\right) = \frac{\beta}{\vartheta^2} \Gamma\left(3\beta + \frac{1}{\beta - 1}\right)$$

$$\begin{aligned} g_{22}(\vartheta, \beta) &= E[\partial_{\beta} \log R(t) \cdot \partial_{\beta} \log R(t)] \\ &= \int_0^{\infty} \left(\left(\frac{t}{\vartheta}\right)^{\beta} \ln\left(\frac{\beta}{\vartheta}\right)\right)^2 \cdot \frac{\beta}{\vartheta} \left(\frac{t}{\vartheta}\right)^{\beta-1} e^{-\left(\frac{t}{\vartheta}\right)^{\beta}} dt \\ &= -2 \left(\ln\left(\frac{t}{\vartheta}\right)\right) \cdot e^{-\left(\frac{t}{\vartheta}\right)^{\beta}} \Big|_0^{\infty} + \frac{2}{\vartheta} \int_0^{\infty} \frac{1}{t} e^{-\left(\frac{t}{\vartheta}\right)^{\beta}} dt \\ &= -2 \left(\ln\left(\frac{t}{\vartheta}\right)\right) \cdot e^{-\left(\frac{t}{\vartheta}\right)^{\beta}} \Big|_0^{\infty} + \frac{\vartheta^{\beta}}{\beta} \Gamma\left(\frac{1}{\beta - 1}\right) \end{aligned}$$

$$g_{22} = \frac{\vartheta^{\beta}}{\beta} \Gamma\left(\frac{1}{\beta - 1}\right)$$

$$\begin{aligned} g_{12} &= E[\partial_{\vartheta} \log R(t) \cdot \partial_{\beta} \log R(t)] \\ &= \int_0^{\infty} \frac{\beta}{\vartheta} \left(\frac{t}{\vartheta}\right)^{\beta} \cdot \left(\frac{-t}{\vartheta}\right)^{\beta} \ln \frac{t}{\vartheta} \cdot \frac{\beta}{\vartheta} \left(\frac{t}{\vartheta}\right)^{\beta-1} e^{-\left(\frac{t}{\vartheta}\right)^{\beta}} dt \\ &= \vartheta \cdot \Gamma\left(\frac{1}{\beta - 1}\right) (n + 1) \text{ where } n \text{ is a positive integer} \end{aligned}$$

$$g_{ij} = \begin{bmatrix} \frac{\beta}{\vartheta^2} \Gamma\left(3\beta + \frac{1}{\beta - 1}\right) & \vartheta \Gamma(n + 1) \\ \vartheta \Gamma(n + 1) & \frac{\vartheta^{\beta}}{\beta} \Gamma\left(\frac{1}{\beta - 1}\right) \end{bmatrix}. \text{ The distance between two reliability function with lifetime}$$

Weibull distribution is

$$ds^2 = \frac{\beta}{\vartheta^2} \Gamma\left(3\beta + \frac{1}{\beta - 1}\right) (d\vartheta)^2 + \vartheta \Gamma(n + 1) d\vartheta d\beta + \vartheta \cdot \Gamma(n + 1) d\vartheta d\beta + \frac{\vartheta^{\beta}}{\beta} \Gamma\left(\frac{1}{\beta - 1}\right) d\beta^2$$

Theorem 3.5:- The reliability function $e^{-\left(\frac{t}{\vartheta}\right)^{\beta}}$ is acritical point of its log-likelihood and it is saddle point.

Proof: -

Let $\log R(t) = -\left(\frac{t}{\vartheta}\right)^{\beta}$ be a log-likelihood of the reliability function.

Then the partial derivative with respect to ϑ is

$$\frac{\partial \log R(t)}{\partial \vartheta} = \frac{\beta t^{\beta}}{\vartheta^{\beta+1}}$$

While the partial derivative with respect to β is

$$\frac{\partial \log R(t)}{\partial \beta} = \left(-\ln\left(\frac{t}{\vartheta}\right)\right) \left(\frac{t}{\vartheta}\right)^{\beta}$$

Setting both of these partial derivatives equal to zero, we have

$$\frac{\partial \log R(t)}{\partial \vartheta} = 0 \text{ and } \frac{\partial \log R(t)}{\partial \beta} = 0$$

Now, if $\frac{\partial \log R(t)}{\partial \vartheta} = 0$ we get $\frac{\beta t^{\beta}}{\vartheta^{\beta+1}} = 0$

Then either ϑ or β could be zero but ϑ should not be zero for the function to be defined. So, let's consider the $\beta = 0$

$$\text{If } \frac{\partial \log R(t)}{\partial \beta} = 0 \text{ we get } \left(-\ln\left(\frac{t}{\vartheta}\right)\right)\left(\frac{t}{\vartheta}\right)^\beta = 0$$

This equation implies $\ln\left(\frac{t}{\vartheta}\right) = 0$, leading to $\frac{t}{\vartheta} = 1$ then $t = \vartheta$

So the critical point for $\beta = 0$ is $(t, \beta) = (\vartheta, \beta)$ and for $\beta \neq 0$ the critical point is $(t, \beta) = (\vartheta, \beta)$ since $\varnothing: N \rightarrow R^{+2}$ be a chart.

Then $(\vartheta, \beta) \in R^{+2}$ and we get $\varnothing^{-1}(\vartheta, \beta) = e^{-\left(\frac{t}{\vartheta}\right)^\beta}$.

So the critical point is any Reliability function in the manifold N depend on the parameter (ϑ, β) .

To find the Hessian matrix of the log-likelihood function at the point (ϑ, β) , we need the second-order partial derivatives

$$\frac{\partial^2 \log R(t)}{\partial \vartheta^2} = \frac{-\beta(\beta+1)t^\beta}{\vartheta^{\beta+2}}$$

$$\frac{\partial^2 \log R(t)}{\partial \beta^2} = -\ln\left(\frac{t}{\vartheta}\right)^2 \left(\frac{t}{\vartheta}\right)^\beta$$

$$\frac{\partial^2 \log R(t)}{\partial \vartheta \partial \beta} = \frac{-\beta t^\beta}{\vartheta^{\beta+1}} \ln\left(\frac{t}{\vartheta}\right)$$

$$\text{So, } H = \begin{bmatrix} \frac{-\beta(\beta+1)t^\beta}{\vartheta^{\beta+2}} & \frac{-\beta t^\beta}{\vartheta^{\beta+1}} \ln\left(\frac{t}{\vartheta}\right) \\ \frac{-\beta t^\beta}{\vartheta^{\beta+1}} \ln\left(\frac{t}{\vartheta}\right) & -\ln\left(\frac{t}{\vartheta}\right)^2 \left(\frac{t}{\vartheta}\right)^\beta \end{bmatrix}$$

To find the eigen values we need to solve $\det(H - \vartheta I) = 0$, where I is the identity matrix.

$$\begin{aligned} \det(H - \vartheta I) &= \det \begin{bmatrix} \frac{-\beta(\beta+1)t^\beta}{\vartheta^{\beta+2}} - \vartheta & \frac{-\beta t^\beta}{\vartheta^{\beta+1}} \ln\left(\frac{t}{\vartheta}\right) \\ \frac{-\beta t^\beta}{\vartheta^{\beta+1}} \ln\left(\frac{t}{\vartheta}\right) & \left(-\ln\left(\frac{t}{\vartheta}\right)^2 \left(\frac{t}{\vartheta}\right)^\beta\right) - \vartheta \end{bmatrix} = 0 \\ &= \left(\frac{-\beta(\beta+1)t^\beta}{\vartheta^{\beta+2}} - \vartheta\right) \left[\left(-\ln\left(\frac{t}{\vartheta}\right)^2 \left(\frac{t}{\vartheta}\right)^\beta\right) - \vartheta\right] - \frac{-\beta^2 t^{2\beta} \ln^2\left(\frac{t}{\vartheta}\right)}{\vartheta^{2\beta+2}} = 0 \end{aligned}$$

If $\vartheta = 2$, $\beta = 3$ and $t = 1$, we get

$$\vartheta_1 = 0.052725 \text{ and } \vartheta_2 = -0.532525$$

Sine the eigen values of matrix one positive and one negative, then this configuration does represent a saddle point in the sense that the transformation expands a long one direction (corresponding to the positive eigen value) and contracts a long another direction (corresponding to the negative eigen value).

So the critical point $(\vartheta, \beta) = \varnothing\left(e^{-\left(\frac{t}{\vartheta}\right)^\beta}\right)$ is saddle point.

4. Treatment in a neutrosophic environment

We will discuss the same results by replacing real coefficients and variables by neutrosophic ones.

Theorem 4.1 : Let $N(I) = \left\{ R(t + zI) = e^{-\left(\frac{t+zI}{\vartheta}\right)^\beta}, t + zI \in [0, +\infty), (\vartheta, \beta) \in R^+(I) \times R^+(I) \right\}$ and

$f: R^+(I) \times R^+(I) \rightarrow N(I)$ be one-to-one mapping, such that $g(\vartheta, \beta) = R(t + zI)$ where $R(t + zI)$ is the neutrosophic reliability of Weibull distribution. Then N be four-dimension surface parametrized by $R^+(I) \times R^+(I)$

Proof:

The log-likelihood function is $\log(R(t + zI)) = \log\left(e^{-\left(\frac{t+zI}{\vartheta}\right)^\beta}\right) = -\left(\frac{t+zI}{\vartheta}\right)^\beta$

let $\omega_1(t + zI) = \partial_\vartheta \log R(t + zI) = \frac{\beta}{\vartheta} \left(\frac{t+zI}{\vartheta}\right)^{\beta-1}$ and $\omega_2(t + zI) = \partial_\beta \log R(t + zI)$

$$= -\left(\frac{t+zI}{\vartheta}\right)^\beta \log\left(\frac{t+zI}{\vartheta}\right)$$

Then

$$\det \begin{bmatrix} \omega_1(t + zI) & \omega_2(t + zI) \\ \omega'_1(t + zI) & \omega'_2(t + zI) \end{bmatrix} = \det \begin{bmatrix} \frac{\beta}{\vartheta} \left(\frac{t+zI}{\vartheta}\right)^{\beta-1} & -\left(\frac{t+zI}{\vartheta}\right)^\beta \log\left(\frac{t+zI}{\vartheta}\right) \\ \frac{\beta^2}{\vartheta^2} \left(\frac{t+zI}{\vartheta}\right)^{\beta-2} & \frac{\left(\frac{t+zI}{\vartheta}\right)^\beta (\beta \log\left(\frac{t+zI}{\vartheta}\right) + 1)}{t} \end{bmatrix} =$$

$$\left(\left[\frac{\beta}{\vartheta} \left(\frac{t+zI}{\vartheta}\right)^{\beta-1} \right] \cdot \left[\frac{\left(\frac{t+zI}{\vartheta}\right)^\beta (\beta \log\left(\frac{t+zI}{\vartheta}\right) + 1)}{t} \right] \right) + \left[\left(\frac{t+zI}{\vartheta}\right)^\beta \log\left(\frac{t+zI}{\vartheta}\right) \right] \cdot \left[\frac{\beta^2}{\vartheta^2} \left(\frac{t+zI}{\vartheta}\right)^{\beta-2} \right] \neq 0$$

Then $\forall (\vartheta, \beta) \in R^+(I) \times R^+(I)$ and $\in (0, \infty)$, the set N is a four-dimensional surface parameterized by $R^+(I) \times R^+(I)$

Theorem 4.2:- The graph of $R(t + zI) = e^{-\left(\frac{t+zI}{\vartheta}\right)^\beta}$ is given by the set

$$N = \left\{ \left(\vartheta, \beta, e^{-\left(\frac{t+zI}{\vartheta}\right)^\beta} \right), (\vartheta, \beta) \in R^+(I) \times R^+(I) \right\} \subseteq R^{+3}(I)$$

is a topological 4-Reliability Manifold

Proof: -

let $(R^{+3}(I), \tau)$ be topological space and $N(I)$ is subspace of $R^{+3}(I)$

Define the family τ_N as family of subset of $N(I)$ as follow:

$$\tau_N = \{N(I) \cap V : V \in \tau\}$$

Sine $R^{+3}(I) \in \tau$ and $N(I) \subseteq R^{+3}(I)$, we have $N(I) \cap R^{+3}(I) = N(I)$, so $N(I) \in \tau_N \dots \dots \dots (1)$

Sine $\emptyset \in \tau$ and $\emptyset \subseteq R^{+3}(I)$, then $\emptyset = N(I) \cap \emptyset$ and $\emptyset \in \tau_N \dots \dots \dots (2)$

From (1) and (2) we get $\emptyset, N(I) \in \tau_N$

Let $U_1, U_2 \in \tau_N$, then there exist $V_1, V_2 \in \tau \ni U_1 = N(I) \cap V_1$ and $U_2 = N(I) \cap V_2$. Hence $U_1 \cap U_2 = (N(I) \cap V_1) \cap (N(I) \cap V_2)$

$$= N(I) \cap (V_1 \cap V_2) \in \tau_N$$

So $U_1 \cap U_2 \in \tau_N$.

Let $U_i \in \tau_N, i \in \Lambda$. Then there exist $V_i \in \tau \ni U_i = N(I) \cap V_i, i \in \Lambda$ and

$$\cup_{i \in \Lambda} U_i = \cup_{i \in \Lambda} (N(I) \cap V_i) = N(I) \cap (\cup_{i \in \Lambda} V_i)$$

Hence $\cup_{i \in \Lambda} U_i \in \tau_N$. Therefore τ_N is topological space on $N(I)$

Now, let $\left(\vartheta_1, \beta_1, e^{-\left(\frac{t+zI}{\vartheta_1}\right)^{\beta_1}} \right), \left(\vartheta_2, \beta_2, e^{-\left(\frac{t+zI}{\vartheta_2}\right)^{\beta_2}} \right) \in N(I)$, then there exists,

$$V_1, V_2 \in \tau \ni \left(\vartheta_1, \beta_1, e^{-\left(\frac{t+zI}{\vartheta_1}\right)^{\beta_1}} \right) \in V_1, \left(\vartheta_2, \beta_2, e^{-\left(\frac{t+zI}{\vartheta_2}\right)^{\beta_2}} \right) \in V_2,$$

and $V_1 \cap V_2 = \emptyset$.

Sine $N(I)$ is subspace of $R^{+3}(I)$, then $U_1 = V_1 \cap N(I)$ and $U_2 = V_2 \cap N(I)$ are two disjoint open sets in $N(I)$ and containing $\left(\vartheta_1, \beta_1, e^{-\left(\frac{t+zI}{\vartheta_1}\right)^{\beta_1}}\right), \left(\vartheta_2, \beta_2, e^{-\left(\frac{t+zI}{\vartheta_2}\right)^{\beta_2}}\right)$ respectively. So $(N(I), \tau_N)$ is Hausdorff space

let $B = \left\{ B_r \left(\vartheta, \beta, e^{-\left(\frac{t+zI}{\vartheta}\right)^{\beta}} \right) : \left(\vartheta, \beta, e^{-\left(\frac{t+zI}{\vartheta}\right)^{\beta}} \right) \in Q^3, r > 0, r \in Q \right\}$

is countable basic for $R^{+3}(I)$.

Hence $B_N = \left\{ B_r \left(\vartheta, \beta, e^{-\left(\frac{t+zI}{\vartheta}\right)^{\beta}} \right) \cap N : \left(\vartheta, \beta, e^{-\left(\frac{t+zI}{\vartheta}\right)^{\beta}} \right) \in Q^3, r > 0, r \in Q \right\}$ is a countable basis for the subspace $N(I)$. So $(N(I), \tau_N)$ is second countable.

Let $\emptyset: N(I) \rightarrow R^{+2}(I)$ defined as $\emptyset \left(\vartheta, \beta, e^{-\left(\frac{t+zI}{\vartheta}\right)^{\beta}} \right) = (\vartheta, \beta)$, and

$\left(\vartheta_1, \beta_1, e^{-\left(\frac{t+zI}{\vartheta_1}\right)^{\beta_1}} \right), \left(\vartheta_2, \beta_2, e^{-\left(\frac{t+zI}{\vartheta_2}\right)^{\beta_2}} \right) \in N(I)$

Then if $\emptyset \left(\vartheta_1, \beta_1, e^{-\left(\frac{t+zI}{\vartheta_1}\right)^{\beta_1}} \right) = \emptyset \left(\vartheta_2, \beta_2, e^{-\left(\frac{t+zI}{\vartheta_2}\right)^{\beta_2}} \right)$, we have

$(\vartheta_1, \beta_1) = (\vartheta_2, \beta_2)$, and $\left(\vartheta_1, \beta_1, e^{-\left(\frac{t+zI}{\vartheta_1}\right)^{\beta_1}} \right) = \left(\vartheta_2, \beta_2, e^{-\left(\frac{t+zI}{\vartheta_1}\right)^{\beta_1}} \right)$.

So $\emptyset$ is one-to-one.

Let $(\vartheta, \beta) \in R^{+2}(I)$, then there exist $\left(\vartheta, \beta, e^{-\left(\frac{t+zI}{\vartheta}\right)^{\beta}} \right) \in N(I)$

$\ni \emptyset \left(\vartheta, \beta, e^{-\left(\frac{t+zI}{\vartheta}\right)^{\beta}} \right) = (\vartheta, \beta)$. Hence $\emptyset$ is onto

$N \left(\vartheta, \beta, e^{-\left(\frac{t+zI}{\vartheta}\right)^{\beta_1}} \right) = (\vartheta, \beta)$ and $\emptyset^{-1}(\vartheta, \beta) = \left(\vartheta, \beta, e^{-\left(\frac{t+zI}{\vartheta}\right)^{\beta}} \right)$ both are continuous. Then φ is

homeomorphism from $N(I)$ into $R^{+2}(I)$ so $(N(I), \varphi)$ is a chart

So, $N(I)$ is a locally Euclidean of dimension 2. Finally $N(I)$ is a topological manifold of dimension 2.

Theorem 4.3: The set $N(I) = \left\{ R(t+zI, \vartheta, \beta) : R(t+zI, \vartheta, \beta) = e^{-\left(\frac{t+zI}{\vartheta}\right)^{\beta}}, t+zI \in [0, +\infty), (\vartheta, \beta) \in \right.$

$R^+(I) \times R^+(I) \left. \right\}$ is a differentiable reliability manifold.

Proof:

The set $N(I)$ is diffeomorphic to the upper neutrosophic half-plane in $R^2(I)$, and the entire reliability manifold $N(I)$ is covered by only one atlas consists of only one chart $\emptyset$ from open subset U of $N(I)$ onto open subset $\emptyset(U)$ of $R^2(I)$.

From the log-likelihood function $\log(R(t+zI)) = \log \left(e^{-\left(\frac{t+zI}{\vartheta}\right)^{\beta}} \right) = -\left(\frac{t+zI}{\vartheta}\right)^{\beta}$, let $(\vartheta, \beta) = \emptyset(\log(R(t+zI)))$ is coordinate system. Another coordinate can be defined as follows:

$\mu_1 = E(t+zI) = \vartheta \Gamma\left(\frac{1}{\beta} + 1\right)$, and $\mu_2 = E((t+zI)^2) = \vartheta^2 \Gamma\left(\frac{2}{\beta} + 1\right)$, where E denotes the expectation of a neutrosophic random variable.

Then $\varphi(\log(R(t + zI))) = (\mu_1, \mu_2)$ be a chart of the same point $R(t + zI)$.

The transition map $T = \varphi \circ \varphi^{-1}: (\vartheta, \beta) \rightarrow (\mu_1, \mu_2) \subset R^2(I) \ni \mu = \mu_i(\vartheta, \beta), i = 1, 2$ can be defined by using the Jacobian matrix of the transition map.

The Jacobian matrix of the transition map is

$$J = \begin{bmatrix} \frac{\partial \mu_1}{\partial \vartheta} & \frac{\partial \mu_1}{\partial \beta} \\ \frac{\partial \mu_2}{\partial \vartheta} & \frac{\partial \mu_2}{\partial \beta} \end{bmatrix} = \begin{bmatrix} \Gamma(\frac{1}{\beta} + 1) & \vartheta \Gamma'(\frac{1}{\beta} + 1) \\ 2\vartheta \Gamma(\frac{2}{\beta} + 1) & \vartheta^2 \Gamma'(\frac{2}{\beta} + 1) \end{bmatrix}.$$

Since $\det(J) \neq 0$ and $\text{invertible in the neutrosophic ring } R(I)$, then T is diffeomorphism (i.e) it is continuously differentiable of all order.

Its inverse T^{-1} is also diffeomorphism, since $T^{-1} = \varphi \circ \varphi^{-1}: (\mu_1, \mu_2) \rightarrow (\vartheta, \beta)$

$$\hat{J} = \begin{bmatrix} \frac{\partial \vartheta}{\partial \mu_1} & \frac{\partial \vartheta}{\partial \mu_2} \\ \frac{\partial \beta}{\partial \mu_1} & \frac{\partial \beta}{\partial \mu_2} \end{bmatrix} = \begin{bmatrix} \frac{\mu_1 \Gamma'(\frac{1}{\beta} + 1) - \Gamma(\frac{1}{\beta} + 1)}{\mu_1^2} & 0 \\ 0 & \frac{1}{\Gamma^{-1}(\frac{\mu_2}{\vartheta^2}) - 1} \Gamma'(\frac{\mu_2}{\vartheta^2}) \end{bmatrix} \neq$$

0 and it is an invertible neutrosophic number.

Since $\det(\hat{J}) \neq 0$ and T^{-1} is continuously differentiable for all (ϑ, β) .

Hence φ, φ^{-1} are C^∞ -smoothly compatible.

The atlas $\{(U_i, \varphi_i)\}$ is C^∞ -atlas and $\{(U_i, \varphi_i)\}$ can be extended to maximal C^∞ -atlas which is smooth structure.

So we get, $N(I) = \left\{ e^{-\left(\frac{t+zI}{\vartheta}\right)^\beta} \right\}$ be smooth manifold of the reliability function with lifetime weibull distribution.

Theorem 4.4 : The Riemannian reliability manifold with weibull lifetime distribution is $(N(I), g_{ij})$ where

$$g_{ij} = \begin{bmatrix} \frac{\beta}{\vartheta^2} \Gamma\left(3\beta + \frac{1}{\beta - 1}\right) & \vartheta \Gamma(n + 1) \\ \vartheta \Gamma(n + 1) & \frac{\vartheta^\beta}{\beta} \Gamma\left(\frac{1}{\beta - 1}\right) \end{bmatrix}$$

Proof:

Let g_{ij} be a metric tensor such that

$$g_{ij} = \langle \partial_i, \partial_j \rangle = E_\vartheta [\partial_{\vartheta i} \log R(t + zI, \vartheta) * \partial_{\vartheta j} \log R(t + zI, \vartheta)]$$

Since $f(t + zI) = \frac{\beta}{\vartheta} \left(\frac{t+zI}{\vartheta}\right)^{\beta-1} e^{-\left(\frac{t+zI}{\vartheta}\right)^\beta}$ and its reliability function is $R(t + zI) = e^{-\left(\frac{t+zI}{\vartheta}\right)^\beta}$

So $\log R(t + zI) = -\left(\frac{t+zI}{\vartheta}\right)^\beta$, $\partial_\vartheta \log R(t + zI) = \frac{\beta}{\vartheta} \left(\frac{t+zI}{\vartheta}\right)^{\beta-1}$ and $\partial_\beta \log R(t + zI) = -\left(\frac{t+zI}{\vartheta}\right)^\beta \log\left(\frac{t+zI}{\vartheta}\right)$

Then $g_{11}(\vartheta, \beta) = E[\partial_\vartheta \log R(t + zI) \cdot \partial_\vartheta \log R(t + zI)]$

$$\begin{aligned} &= \int_0^\infty \left[\left(\frac{\beta}{\vartheta} \right) \left(\frac{t+zI}{\vartheta} \right)^{\beta-1} \right]^2 \cdot \frac{\beta}{\vartheta} \left(\frac{t+zI}{\vartheta} \right)^{\beta-1} e^{-\left(\frac{t+zI}{\vartheta}\right)^\beta} d(t + zI) \\ &= \int_0^\infty \frac{\beta^2}{\vartheta^2} \left(\frac{t+zI}{\vartheta} \right)^{2\beta} \cdot \frac{\beta}{\vartheta} \left(\frac{t+zI}{\vartheta} \right)^{\beta-1} e^{-\left(\frac{t+zI}{\vartheta}\right)^\beta} d(t + zI) \end{aligned}$$

$$= \frac{\beta^3}{\vartheta^3} \int_0^\infty \left(\frac{t+zI}{\vartheta} \right)^{3\beta-1} \cdot e^{-\left(\frac{t+zI}{\vartheta} \right)^\beta} d(t+zI)$$

$$g_{11} = \left(\frac{\beta}{\vartheta} \right)^3 \left(\frac{\vartheta}{\beta^2} \right) \Gamma \left(3\beta + \frac{1}{\beta-1} \right) = \frac{\beta}{\vartheta^2} \Gamma \left(3\beta + \frac{1}{\beta-1} \right)$$

$$g_{22}(\vartheta, \beta) = E [\partial_\beta \log R(t+zI) \cdot \partial_\beta \log R(t+zI)]$$

$$= \int_0^\infty \left(\left(\frac{t+zI}{\vartheta} \right)^\beta \ln \left(\frac{\beta}{\vartheta} \right) \right)^2 \cdot \frac{\beta}{\vartheta} \left(\frac{t+zI}{\vartheta} \right)^{\beta-1} e^{-\left(\frac{t+zI}{\vartheta} \right)^\beta} d(t+zI)$$

$$= -2 \left(\ln \left(\frac{t+zI}{\vartheta} \right) \right) \cdot e^{-\left(\frac{t+zI}{\vartheta} \right)^\beta} \Big|_0^\infty + \frac{2}{\vartheta} \int_0^\infty \frac{1}{t} e^{-\left(\frac{t+zI}{\vartheta} \right)^\beta} dt + zI$$

$$= -2 \left(\ln \left(\frac{t+zI}{\vartheta} \right) \right) \cdot e^{-\left(\frac{t+zI}{\vartheta} \right)^\beta} \Big|_0^\infty + \frac{\vartheta^\beta}{\beta} \Gamma \frac{1}{\beta-1}$$

$$g_{22} = \frac{\vartheta^\beta}{\beta} \Gamma \frac{1}{\beta-1}$$

$$g_{12} = E [\partial_\vartheta \log R(t+zI) \cdot \partial_\beta \log R(t+zI)]$$

$$= \int_0^\infty \frac{\beta}{\vartheta} \left(\frac{t+zI}{\vartheta} \right)^\beta \cdot \left(-\frac{(t+zI)}{\vartheta} \right)^\beta \ln \frac{t+zI}{\vartheta} \cdot \frac{\beta}{\vartheta} \left(\frac{t+zI}{\vartheta} \right)^{\beta-1} e^{-\left(\frac{t+zI}{\vartheta} \right)^\beta} d(t+zI)$$

$$= \vartheta \cdot \Gamma \frac{1}{\beta-1} (n+1) \text{ where } n \text{ is a positive integer}$$

$$g_{ij} = \begin{bmatrix} \frac{\beta}{\vartheta^2} \Gamma \left(3\beta + \frac{1}{\beta-1} \right) & \vartheta \Gamma(n+1) \\ \vartheta \Gamma(n+1) & \frac{\vartheta^\beta}{\beta} \Gamma \left(\frac{1}{\beta-1} \right) \end{bmatrix}. \text{ The distance between two reliability function with lifetime}$$

Weibull distribution is

$$ds^2 = \frac{\beta}{\vartheta^2} \Gamma \left(3\beta + \frac{1}{\beta-1} \right) (d\vartheta)^2 + \vartheta \Gamma(n+1) d\vartheta d\beta + \vartheta \cdot \Gamma(n+1) d\vartheta d\beta + \frac{\vartheta^\beta}{\beta} \Gamma \left(\frac{1}{\beta-1} \right) d\beta^2$$

Theorem 3.5:- The reliability function $e^{-\left(\frac{t+zI}{\vartheta} \right)^\beta}$ is acritical point of its log-likelihood and it is saddle point.

Proof: -

Let $\log R(t+zI) = -\left(\frac{t+zI}{\vartheta} \right)^\beta$ be a log-likelihood of the reliability function.

Then the partial derivative with respect to ϑ is

$$\frac{\partial \log R(t+zI)}{\partial \vartheta} = \frac{\beta(t+zI)^\beta}{\vartheta^{\beta+1}}$$

While the partial derivative with respect to β is

$$\frac{\partial \log R(t+zI)}{\partial \beta} = \left(-\ln \left(\frac{t+zI}{\vartheta} \right) \right) \left(\frac{t+zI}{\vartheta} \right)^\beta$$

Setting both of these partial derivatives equal to zero, we have

$$\frac{\partial \log R(t + zI)}{\partial \vartheta} = 0 \text{ and } \frac{\partial \log R(t + zI)}{\partial \beta} = 0$$

Now, if $\frac{\partial \log R(t+zI)}{\partial \vartheta} = 0$ we get $\frac{\beta(t+zI)^\beta}{\vartheta^{\beta+1}} = 0$

Then either ϑ or β could be zero but ϑ should not be zero for the function to be defined. So, let's consider the $\beta = 0$

If $\frac{\partial \log R(t+zI)}{\partial \beta} = 0$ we get $\left(-\ln\left(\frac{t+zI}{\vartheta}\right)\right)\left(\frac{t+zI}{\vartheta}\right)^\beta = 0$

This equation implies $\ln\left(\frac{t+zI}{\vartheta}\right) = 0$, leading to $\frac{t+zI}{\vartheta} = 1 + I$ then $t + zI = \vartheta + I$

So the critical point for $\beta = 0$ is $(t, \beta) = (\vartheta, \beta)$ and for $\beta \neq 0$ the critical point is $(t + zI, \beta) = (\vartheta, \beta)$ since $\phi: N(I) \rightarrow R^{+2}(I)$ be a chart.

Then $(\vartheta, \beta) \in R^{+2}$ and we get $\phi^{-1}(\vartheta, \beta) = e^{-\left(\frac{t+zI}{\vartheta}\right)^\beta}$.

So the critical point is any Reliability function in the manifold N depend on the parameter (ϑ, β) .

To find the Hessian matrix of the log-likelihood function at the point (ϑ, β) , we need the second-order partial derivatives

$$\frac{\partial^2 \log R(t + zI)}{\partial \vartheta^2} = \frac{-\beta(\beta + 1)(t + zI)^\beta}{\vartheta^{\beta+2}}$$

$$\frac{\partial^2 \log R(t + zI)}{\partial \beta^2} = -\ln\left(\frac{t + zI}{\vartheta}\right)^2 \left(\frac{t + zI}{\vartheta}\right)^\beta$$

$$\frac{\partial^2 \log R(t + zI)}{\partial \vartheta \partial \beta} = \frac{-\beta(t + zI)^\beta}{\vartheta^{\beta+1}} \ln\left(\frac{t + zI}{\vartheta}\right)$$

$$\text{So, } H = \begin{bmatrix} \frac{-\beta(\beta + 1)(t + zI)^\beta}{\vartheta^{\beta+2}} & \frac{-\beta t^\beta}{\vartheta^{\beta+1}} \ln\left(\frac{t + zI}{\vartheta}\right) \\ \frac{-\beta(t + zI)^\beta}{\vartheta^{\beta+1}} \ln\left(\frac{t + zI}{\vartheta}\right) & -\ln\left(\frac{t + zI}{\vartheta}\right)^2 \left(\frac{t + zI}{\vartheta}\right)^\beta \end{bmatrix}$$

To find the eigen values we need to solve $\det(H - \vartheta I) = 0$, where I is the identity matrix.

$$\begin{aligned} \det(H - \vartheta I) &= \det \begin{bmatrix} \frac{-\beta(\beta + 1)(t + zI)^\beta}{\vartheta^{\beta+2}} - \vartheta & \frac{-\beta t^\beta}{\vartheta^{\beta+1}} \ln\left(\frac{t + zI}{\vartheta}\right) \\ \frac{-\beta(t + zI)^\beta}{\vartheta^{\beta+1}} \ln\left(\frac{t + zI}{\vartheta}\right) & \left(-\ln\left(\frac{t + zI}{\vartheta}\right)^2 \left(\frac{t + zI}{\vartheta}\right)^\beta\right) - \vartheta \end{bmatrix} = 0 \\ &= \left(\frac{-\beta(\beta + 1)(t + zI)^\beta}{\vartheta^{\beta+2}} - \vartheta\right) \left[\left(-\ln\left(\frac{t + zI}{\vartheta}\right)^2 \left(\frac{t + zI}{\vartheta}\right)^\beta\right) - \vartheta\right] \\ &\quad - \frac{-\beta^2(t + zI)^{2\beta} \ln^2\left(\frac{t + zI}{\vartheta}\right)}{\vartheta^{2\beta+2}} = 0 \end{aligned}$$

If $\vartheta = 2 + 2I$, $\beta = 3 + I$ and $t = 1 + I$, we get

$$\vartheta_1 = 0.052725 + 0.077613I \text{ and } \vartheta_2 = -0.532525 - 0.410314I$$

5. Conclusions

This study has achieved significant results in applying differential geometry concepts to the analysis of reliability functions for the Weibull distribution. The research demonstrated the

possibility of representing the family of reliability functions as a smooth two-dimensional Riemannian manifold, verifying the conditions of a differentiable manifold by proving the existence of an inverse coordinate map and the non-vanishing of the Jacobian determinant. A non-trivial Riemannian metric was derived from the log-likelihood function, which the study confirmed to be positive definite and symmetric, producing non-zero curvatures. The differential-topological analysis revealed that the resulting manifold is orientable and exhibits negative curvature in critical regions, with non-trivial saddle points and distinct tensor patterns on the contact space. This structure was successfully extended to neutrosophic spaces, where key properties such as differentiability and Hausdorff separation were preserved, leading to the construction of a four-dimensional manifold. These results provide an integrated framework for computing geodesic paths between reliability states, studying system evolution as a flow on the manifold, and analyzing stability via the Hessian matrix. This offers new analytical tools in reliability geometry based on the solid mathematical foundations of differential geometry

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Effectiveness Assessment of Brand Microblog Marketing Communication under Trapezoidal Fuzzy Neutrosophic Sets

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Abstract: Brand communication tactics have changed to take use of new media channels, especially microblogging, in the age of digital transformation. It is now crucial to assess the success of brand microblog marketing communication to comprehend customer involvement, message distribution, and brand positioning. With an emphasis on elements like audience engagement, material quality, and brand perception, this study investigates the standards for evaluating the effectiveness of microblog marketing initiatives. The study offers insights into the tactics that are most important for effective communication using a systematic evaluation approach. We use multi-criteria decision making (MCDM) to deal with different criteria. We use the Trapezoidal Fuzzy Neutrosophic Sets (TFNSs) to deal with uncertainty information. The results are meant to help businesses maximize their microblog presence for more effective marketing.

Keywords: Brand Microblog Marketing; Communications; Uncertainty; Trapezoidal Fuzzy Neutrosophic Sets.

1. Introduction

The way businesses engage with their consumers has been completely transformed by the digital revolution, and microblogging platforms have become extremely effective communication tools. Social media sites like Twitter, Weibo, and Threads provide marketers with the chance to interact with customers directly, publish information quickly, and use viral material to spread their message[1], [2]. To make sure that microblog marketing initiatives accomplish the desired communication goals, a methodical assessment of their efficacy is necessary.

Brand microblogs are frequently assessed using metrics including reach, likes, shares, and comments. However, concentrating only on numeric engagement ignores important factors like audience emotional resonance, message clarity, and content relevancy[3], [4]. To properly

evaluate marketing performance, a more thorough assessment model must take into consideration both quantitative and qualitative factors.

One of the most important measures of microblog performance is still the audience engagement rate. Posts with high levels of engagement are more likely to be discussed and shared, increasing brand awareness[5], [6]. Furthermore, audiences' perceptions and reactions to marketing messaging may be greatly influenced by the content's emotional tone and cultural alignment. Effective assessment therefore depends on having a comprehensive knowledge of involvement.

The content strategy that brands use is another important consideration. Posts that are timely, visually appealing, and related to popular subjects tend to do well. A post's efficacy may be further increased by utilizing hashtags strategically, creating a compelling tale, collaborating with influencers, and including interactive components like challenges or polls[7], [8]. Deeper understanding of best practices may be gained by evaluating these content techniques.

Consistency and regularity are also directly related to the success of microblog communication. Relationships between brands and their audience are strengthened by consistent posting schedules and the development of a consistent brand voice[9], [10]. Assessments need to consider how posts' tempo and theme coherence affect customer loyalty and trust.

Sentiment analysis's increasing significance emphasizes the necessity of evaluating both the quantity and quality of interactions with a post[11], [12]. A crucial component of microblog marketing assessment is positive brand sentiment, which correlated with higher customer retention and advocacy. Monitoring sentiment over time yields insightful information about communication tactics.

Another important indicator is conversion rates, which measure how many microblog conversations result in desired consumer actions. In addition to encouraging interaction, successful microblog marketing should provide measurable results like website visits, product queries, or purchases[13], [14]. Therefore, assessing the communication-to-conversion pathway is essential to comprehending marketing success in its whole.

All things considered; a multifaceted assessment framework is required to fully grasp the intricacies of brand microblog marketing. Brands may more accurately evaluate the success of their communication initiatives by using metrics for sentiment, conversion, engagement, content quality, and consistency[15], [16]. To support better-informed, strategic microblog marketing initiatives, this study aims to develop such a framework.

A strategic method for a thorough evaluation of options considering several competing criteria, multi-criteria decision-making (MCDM) has been effectively used for evaluations in the MCDM issues. One crucial contextual factor that might influence the decision-making process for evaluating MCDM issues is uncertainty. However, uncertainty and hesitation, which are prevalent in real-world decision-making situations, are difficult for standard MCDM models to

handle. To get around these restrictions, Zadeh [17] developed the idea of a fuzzy set (FS), where each element is represented by a membership degree with a value ranging from 0 to 1.

The FS views the non-membership degree as a supplement to the membership degree, which is the sole degree it contains. However, in practical issues, this isn't always the case. Additionally, several FS generalizations have been proposed and used for diverse objectives. As an improvement on FS, Atanassov [18] developed the idea of Intuitionistic Fuzzy Set (IFS), which is represented by the degrees of membership and non-membership with. More focus has been placed on this set to address the ambiguity and uncertainty brought on by imperfect information.

There may be instances in which a decision-maker provides an independent rating in a variety of problems. For example, when we ask a decision-maker what they think about a certain phrase. He or she has a 0.7 chance of saying that the statement is accurate, a 0.3 chance of being unsure, and a 0.5 chance of saying that the statement is untrue. Smarandache [19], [20] developed the Neutrosophic Set (NS) theory to address these circumstances, according to which the functions of Truth Membership (TM), Indeterminacy Membership (IM), and Falsity Membership (FM) are independent. In addition to being helpful for handling incomplete information, this set may also be used to handle inconsistent and uncertain information[21], [22].

2. Proposed Approach

We use the trapezoidal fuzzy neutrosophic sets (TFNSs) to deal with uncertainty information in the decision-making process[23], [24]. We show the operations of the trapezoidal fuzzy neutrosophic numbers (TFNNs) such as:

Definition 1

We can define the TFNSs such as

$$A = \{(y, T_A(y), I_A(y), F_A(y)) | y \in A\} \quad (1)$$

$$T_A(y) \subset [0,1], I_A(y) \subset [0,1], F_A(y) \subset [0,1] \quad (2)$$

$$T_A(y) = (t_A^1(y), t_A^2(y), t_A^3(y), t_A^4(y)) : A \rightarrow [0,1] \quad (3)$$

$$I_A(y) = (i_A^1(y), i_A^2(y), i_A^3(y), i_A^4(y)) : A \rightarrow [0,1] \quad (4)$$

$$F_A(y) = (f_A^1(y), f_A^2(y), f_A^3(y), f_A^4(y)) : A \rightarrow [0,1] \quad (5)$$

$$0 \leq t_A^4(y) + i_A^4(y) + f_A^4(y) \leq 3 \quad (6)$$

Definition 2

We can define the TFNNs such as:

$$A = ((d_1, d_2, d_3, d_4), (e_1, e_2, e_3, e_4), (f_1, f_2, f_3, f_4))$$

$$T_A(y) = \begin{pmatrix} \frac{y-d_1}{d_2-d_1} & d_1 \leq y \leq d_2 \\ 1 & d_2 \leq y \leq d_3 \\ \frac{d_4-y}{d_4-d_3} & d_3 \leq y \leq d_4 \\ 0 & \text{otherwise} \end{pmatrix} \quad (7)$$

$$I_A(y) = \begin{pmatrix} \frac{e-y}{e_2-e_1} & e_1 \leq y \leq e_2 \\ 0 & e_2 \leq y \leq e_3 \\ \frac{y-e_3}{e_4-e_3} & e_3 \leq y \leq e_4 \\ 1 & \text{otherwise} \end{pmatrix} \quad (8)$$

$$F_A(y) = \begin{pmatrix} \frac{f_2-y}{f_2-f_1} & f_1 \leq y \leq f_2 \\ 0 & f_2 \leq y \leq f_3 \\ \frac{y-f_3}{f_4-f_3} & f_3 \leq y \leq f_4 \\ 1 & \text{otherwise} \end{pmatrix} \quad (9)$$

Definition 3

Let $A_1 = ((d_1, d_2, d_3, d_4), (e_1, e_2, e_3, e_4), (f_1, f_2, f_3, f_4))$ and $A_2 = ((g_1, g_2, g_3, g_4), (h_1, h_2, h_3, h_4), (k_1, k_2, k_3, k_4))$ be two TFNNs. We can show the operations of TFNNs such as:

$$A_1 \oplus A_2 = \begin{pmatrix} (d_1 + g_1 - d_1 g_1, d_2 + g_2 - d_2 g_2, \\ d_3 + g_3 - d_3 g_3 - d_4 + g_4 - d_4 g_4), \\ (e_1 h_1, e_2 h_2, e_3 h_3, e_4 h_4), \\ (f_1 k_1, f_2 k_2, f_3 k_3, f_4 k_4) \end{pmatrix} \quad (10)$$

$$A_1 \otimes A_2 = \begin{pmatrix} (d_1 g_1, d_2 g_2, d_3 g_3, d_4 g_4), \\ (e_1 + h_1 - e_1 h_1, e_2 + h_2 - e_2 h_2, \\ e_3 + h_3 - e_3 h_3 - e_4 + h_4 - e_4 h_4), \\ (f_1 + k_1 - f_1 k_1, f_2 + k_2 - f_2 k_2, \\ f_3 + k_3 - f_3 k_3 - f_4 + k_4 - f_4 k_4) \end{pmatrix} \quad (11)$$

$$YA_1 = \begin{pmatrix} (1 - (1 - d_1)^Y, 1 - (1 - d_2)^Y, \\ 1 - (1 - d_3)^Y, 1 - (1 - d_4)^Y), \\ (e_1^Y, e_2^Y, e_3^Y, e_4^Y), \\ (f_1^Y, f_2^Y, f_3^Y, f_4^Y) \end{pmatrix} \quad (12)$$

$$A_1^Y = \begin{pmatrix} (d_1^Y, d_2^Y, d_3^Y, d_4^Y), \\ (1 - (1 - e_1)^Y, 1 - (1 - e_2)^Y, \\ 1 - (1 - e_3)^Y, 1 - (1 - e_4)^Y), \\ (1 - (1 - f_1)^Y, 1 - (1 - f_2)^Y, \\ 1 - (1 - f_3)^Y, 1 - (1 - f_4)^Y) \end{pmatrix} \quad (13)$$

We show the steps of the TODIM method to rank alternatives. Create the decision matrix between the criteria and alternatives. Experts use the TFNNs to evaluate the criteria and alternatives. We apply the score function to obtain crisp values. We combine the decision matrices into one matrix.

Compute the criteria weights.

The weights of criteria are computed using the average method.

Compute the normalized decision matrix for positive and negative criteria such as:

$$q_{ij} = \frac{y_{ij}}{\sum_{i=1}^m y_{ij}}; i = 1, \dots, m; j = 1, \dots, n \quad (14)$$

$$q_{ij} = \frac{1/q_{ij}}{\sum_{i=1}^m 1/q_{ij}}; i = 1, \dots, m; j = 1, \dots, n \quad (15)$$

Compute the relative weights

$$W_j^- = \frac{w_j}{\max W} \quad (16)$$

Obtain the dominance degree matrix such as:

$$q_i = q(D_i, D_{i'}) = \sum_{j=1}^n f_i(D_i, D_{i'}) \quad (17)$$

Where the f can be computed such as:

$$f_i(D_i, D_{i'}) = \begin{cases} -\frac{1}{H} \sqrt{\frac{(\sum_{j=1}^n w_j^-) |q_{ij} - q_{ji}|}{w_j^-}} & \text{if } (q_{ij} - q_{ji}) < 0 \\ 0 & \text{if } (q_{ij} - q_{ji}) = 0 \\ \sqrt{\frac{(w_j^-) |q_{ij} - q_{ji}|}{\sum_{j=1}^n w_j^-}} & \text{if } (q_{ij} - q_{ji}) > 0 \end{cases} \quad (18)$$

Where H refers to the attenuation factor for the losses.

Compute the overall dominance matrix.

$$S_i = \frac{f_i - \min f_i}{\max f_i - \min f_i} \quad (19)$$

3. Application

We show the application of the proposed approach to compute the criteria weights and ranking the alternatives. The proposed approach is applied for Effectiveness Evaluation of Brand Microblog Marketing Communication. We use ten criteria and ten alternatives. The criteria are: Audience Engagement Rate, Content Relevance, Message Clarity, Visual Appeal of Posts, Consistency of Posting, Audience Growth Rate, Shareability of Content, Brand Sentiment Score, Conversion Rate from Campaigns, Responsiveness to Audience Feedback. The alternatives are: Influencer Collaboration Campaign, User-Generated Content Promotion, Real-Time Event

Coverage, Hashtag Challenge Campaign, Product Launch Microblog Series, Storytelling and Brand Narrative Posts, Giveaway and Contest Campaigns, Educational or Informative Content Series, Social Issues and Corporate Social Responsibility, Interactive Polls and Surveys Strategy

Table 1. The first decision matrix.

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈	C ₉	C ₁₀
A ₁	((0.7,0.8,0.9,1),(0.0,0.2,0.3),(0.0,0.1,0.2,0.3))	((1,1,1,1),(0.0,0,0),(0.0,0,0))	((0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.2,0.4,0.5,0.7))	((0.7,0.8,0.9,1),(0.0,0.2,0.3),(0.0,0.1,0.2,0.3))	((1,1,1,1),(0.0,0,0),(0.0,0,0))	((1,1,1,1),(0.0,0,0),(0.0,0,0))	((1,1,1,1),(0.0,0,0),(0.0,0,0))	((1,1,1,1),(0.0,0,0),(0.0,0,0))	((0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1),(0.7,0.8,0.9,1))	((0.0,0,0),(1,1,1,1),(1,1,1,1))
A ₂	((0.5,0.6,0.7,0.8),(0.2,0.3,0.4),(0.0,0.2,0.3,0.4))	((0.7,0.8,0.9,1),(0.0,1,0.2,0.3),(0.0,0.1,0.2,0.3))	((0.1,0.2,0.3,0.4),(0.0,0.4,0.9),(0.0,0.4,0.9))	((0.5,0.6,0.7,0.8),(0.2,0.3,0.4),(0.0,0.2,0.3,0.4))	((1,1,1,1),(0.0,0,0),(0.0,0,0))	((0.0,0,0),(1,1,1,1),(1,1,1,1))	((0.1,0.2,0.3,0.4),(0.0,0.4,0.9),(0.0,0.4,0.9))	((0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.2,0.4,0.5,0.7))	((0.5,0.6,0.7,0.8),(0.2,0.3,0.4),(0.0,0.2,0.3,0.4))	((0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.2,0.4,0.5,0.7))
A ₃	((0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.0,0.2,0.3,0.4))	((0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1),(0.7,0.8,0.9,1))	((0.1,0.2,0.3,0.4),(0.0,0.4,0.9),(0.0,0.4,0.9))	((0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.0,0.2,0.3,0.4))	((0.5,0.6,0.7,0.8),(0.2,0.3,0.4),(0.0,0.2,0.3,0.4))	((0.7,0.8,0.9,1),(0.0,0.2,0.3,0.4),(0.0,0.2,0.3,0.4))	((0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.0,0.2,0.3,0.4))	((0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.0,0.2,0.3,0.4))	((1,1,1,1),(0.0,0,0),(0.0,0,0))	((0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.0,0.2,0.3,0.4))
A ₄	((0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1),(0.7,0.8,0.9,1))	((1,1,1,1),(0.0,0,0),(0.0,0,0))	((0.0,0,0),(1,1,1,1),(1,1,1,1))	((0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1),(0.7,0.8,0.9,1))	((0.1,0.2,0.3,0.4),(0.0,0.4,0.9),(0.0,0.4,0.9))	((0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.0,0.2,0.3,0.4))	((0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1),(0.7,0.8,0.9,1))	((0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1),(0.7,0.8,0.9,1))	((1,1,1,1),(0.0,0,0),(0.0,0,0))	((0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1),(0.7,0.8,0.9,1))
A ₅	((0.0,0,0),(1,1,1,1),(1,1,1,1))	((0.7,0.8,0.9,1),(0.0,1,0.2,0.3),(0.0,0.1,0.2,0.3))	((0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.0,0.2,0.3,0.4))	((0.7,0.8,0.9,1),(0.0,1,0.2,0.3),(0.0,0.1,0.2,0.3))	((1,1,1,1),(0.0,0,0),(0.0,0,0))	((0.0,0,0),(1,1,1,1),(1,1,1,1))	((0.0,0,0),(1,1,1,1),(1,1,1,1))	((0.0,0,0),(1,1,1,1),(1,1,1,1))	((0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.0,0.2,0.3,0.4))	((0.0,0,0),(1,1,1,1),(1,1,1,1))
A ₆	((0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1),(0.7,0.8,0.9,1))	((0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.0,0.2,0.3,0.4))	((0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1),(0.7,0.8,0.9,1))	((0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.0,0.2,0.3,0.4))	((0.7,0.8,0.9,1),(0.0,1,0.2,0.3),(0.0,0.1,0.2,0.3))	((0.5,0.6,0.7,0.8),(0.2,0.3,0.4),(0.0,0.2,0.3,0.4))	((0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1),(0.7,0.8,0.9,1))	((0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.0,0.2,0.3,0.4))	((0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.0,0.2,0.3,0.4))	((0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1),(0.7,0.8,0.9,1))
A ₇	((0.1,0.2,0.3,0.4),(0.0,0.4,0.9),(0.0,0.4,0.9))	((0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1),(0.7,0.8,0.9,1))	((0.0,0,0),(1,1,1,1),(1,1,1,1))	((0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1),(0.7,0.8,0.9,1))	((0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.0,0.2,0.3,0.4))	((0.7,0.8,0.9,1),(0.0,1,0.2,0.3),(0.0,0.1,0.2,0.3))	((0.1,0.2,0.3,0.4),(0.0,0.4,0.9),(0.0,0.4,0.9))	((0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1),(0.7,0.8,0.9,1))	((0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1),(0.7,0.8,0.9,1))	((0.1,0.2,0.3,0.4),(0.0,0.4,0.9),(0.0,0.4,0.9))
A ₈	((0.0,0,0),(1,1,1,1),(1,1,1,1))	((1,1,1,1),(0.0,0,0),(0.0,0,0))	((0.7,0.8,0.9,1),(0.0,1,0.2,0.3),(0.0,0.1,0.2,0.3))	((1,1,1,1),(0.0,0,0),(0.0,0,0))	((0.1,0.2,0.3,0.4),(0.0,0.4,0.9),(0.0,0.4,0.9))	((0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1),(0.7,0.8,0.9,1))	((0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.0,0.2,0.3,0.4))	((0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1),(0.7,0.8,0.9,1))	((0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1),(0.7,0.8,0.9,1))	((0.0,0,0),(1,1,1,1),(1,1,1,1))
A ₉	((1,1,1,1),(0.0,0,0),(0.0,0,0))	((0.7,0.8,0.9,1),(0.0,1,0.2,0.3),(0.0,0.1,0.2,0.3))	((1,1,1,1),(0.0,0,0),(0.0,0,0))	((0.7,0.8,0.9,1),(0.0,1,0.2,0.3),(0.0,0.1,0.2,0.3))	((0.5,0.6,0.7,0.8),(0.2,0.3,0.4),(0.0,0.2,0.3,0.4))	((0.1,0.2,0.3,0.4),(0.0,0.4,0.9),(0.0,0.4,0.9))	((0.1,0.2,0.3,0.4),(0.0,0.4,0.9),(0.0,0.4,0.9))	((0.1,0.2,0.3,0.4),(0.0,0.4,0.9),(0.0,0.4,0.9))	((0.0,0,0),(1,1,1,1),(1,1,1,1))	((1,1,1,1),(0.0,0,0),(0.0,0,0))
A ₁₀	((0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1),(0.7,0.8,0.9,1))	((0.1,0.2,0.3,0.4),(0.0,0.4,0.9),(0.0,0.4,0.9))	((0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.0,0.2,0.3,0.4))	((0.5,0.6,0.7,0.8),(0.2,0.3,0.4),(0.0,0.2,0.3,0.4))	((0.7,0.8,0.9,1),(0.0,1,0.2,0.3),(0.0,0.1,0.2,0.3))	((0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1),(0.7,0.8,0.9,1))	((0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1),(0.7,0.8,0.9,1))	((0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1),(0.7,0.8,0.9,1))	((1,1,1,1),(0.0,0,0),(0.0,0,0))	((0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1),(0.7,0.8,0.9,1))

Table 2. The second decision matrix.

C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈	C ₉	C ₁₀
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Table 3. The third decision matrix.

Yao Zhou, Effectiveness Assessment of Brand Microblog Marketing Communication under Trapezoidal Fuzzy Neutrosophic Sets

	(0,0.1,0.2, 0.3))			(0.7,0.8,0. 9,1))	.9),(0.4,0. 6,0.7,0.9))	.7),(0.2,0. 4,0.5,0.7))	),(0,0.2,0. 3,0.4))	(0,0.1,0.2, 0.3))		(0,0.1,0.2, 0.3))
A ₃	((1,1,1,1),(0,0,0,0),(0 ,0,0,0))	((0.7,0.8,0 .9,1),(0,0. 1,0.2,0.3), (0,0.1,0.2, 0.3))	((1,1,1,1),(0,0,0,0),(0 ,0,0,0))	((0.7,0.8,0 .9,1),(0,0. 1,0.2,0.3), (0,0.1,0.2, 0.3))	((1,1,1,1),(0,0,0,0),(0 ,0,0,0))	((0,0,0,0),(1,1,1,1),(1 ,1,1,1))	((0,0.1,0.2 ,0.3),(0.7, 0.8,0.9,1), (0.7,0.8,0. 9,1))	((0.1,0.2,0 .3,0.4),(0. 4,0.6,0.7,0 .9),(0.4,0. 6,0.7,0.9))	((0.3,0.4,0 .5,0.6),(0. 2,0.4,0.5,0 .7),(0.2,0. 4,0.5,0.7))	((0.5,0.6,0 .7,0.8),(0. 0.2,0.3,0.4 ,0.0.2,0. 3,0.4))
A ₄	((0.7,0.8,0 .9,1),(0,0. 1,0.2,0.3), (0,0.1,0.2, 0.3))	((0.5,0.6,0 .7,0.8),(0, 0.2,0.3,0.4 ,0.0.2,0. 3,0.4))	((0.7,0.8,0 .9,1),(0,0. 1,0.2,0.3), (0,0.1,0.2, 0.3))	((1,1,1,1),(0,0,0,0),(0 ,0,0,0))	((0.7,0.8,0 .9,1),(0,0. 1,0.2,0.3), (0,0.1,0.2, 0.3))	((0.5,0.6,0 .7,0.8),(0, 0.2,0.3,0.4 ,0.0.2,0. 3,0.4))	((0.3,0.4,0 .5,0.6),(0. 2,0.4,0.5,0 .7),(0.2,0. 4,0.5,0.7))	((0.1,0.2,0 .3,0.4),(0. 4,0.6,0.7,0 .9),(0.4,0. 6,0.7,0.9))	((0,0.1,0.2 ,0.3),(0.7, 0.8,0.9,1), (0.7,0.8,0. 9,1))	((0,0,0,0),(1,1,1,1),(1 ,1,1,1))
A ₅	((0.5,0.6,0 .7,0.8),(0, 0.2,0.3,0.4 ,0.0.2,0. 3,0.4))	((0.3,0.4,0 .5,0.6),(0. 2,0.4,0.5,0 .7),(0.2,0. 4,0.5,0.7))	((1,1,1,1),(0,0,0,0),(0 ,0,0,0))	((0,0,0,0),(1,1,1,1),(1 ,1,1,1))	((1,1,1,1),(0,0,0,0),(0 ,0,0,0))	((0.7,0.8,0 .9,1),(0,0. 1,0.2,0.3), (0,0.1,0.2, 0.3))	((1,1,1,1),(0,0,0,0),(0 ,0,0,0))	((0.7,0.8,0 .9,1),(0,0. 1,0.2,0.3), (0,0.1,0.2, 0.3))	((0.5,0.6,0 .7,0.8),(0, 0.2,0.3,0.4 ,0.0.2,0. 3,0.4))	((0.3,0.4,0 .5,0.6),(0. 2,0.4,0.5,0 .7),(0.2,0. 4,0.5,0.7))
A ₆	((0.3,0.4,0 .5,0.6),(0. 2,0.4,0.5,0 .7),(0.2,0. 4,0.5,0.7))	((0.1,0.2,0 .3,0.4),(0. 4,0.6,0.7,0 .9),(0.4,0. 6,0.7,0.9))	((0,0,0,0),(1,1,1,1),(1 ,1,1,1))	((0,0.1,0.2 ,0.3),(0.7, 0.8,0.9,1), (0.7,0.8,0. 9,1))	((0.5,0.6,0 .7,0.8),(0, 0.2,0.3,0.4 ,0.0.2,0. 3,0.4))	((0.3,0.4,0 .5,0.6),(0. 2,0.4,0.5,0 .7),(0.2,0. 4,0.5,0.7))	((0.1,0.2,0 .3,0.4),(0. 4,0.6,0.7,0 .9),(0.4,0. 6,0.7,0.9))	((0.1,0.2 ,0.3),(0.7, 0.8,0.9,1), (0.7,0.8,0. 9,1))	((0,0,0,0),(1,1,1,1),(1 ,1,1,1))	((0.1,0.2,0 .3,0.4),(0. 4,0.6,0.7,0 .9),(0.4,0. 6,0.7,0.9))
A ₇	((0.1,0.2,0 .3,0.4),(0. 4,0.6,0.7,0 .9),(0.4,0. 6,0.7,0.9))	((0,0.1,0.2 ,0.3),(0.7, 0.8,0.9,1), (0.7,0.8,0. 9,1))	((0,0.1,0.2 ,0.3),(0.7, 0.8,0.9,1), (0.7,0.8,0. 9,1))	((0.1,0.2,0 .3,0.4),(0. 4,0.6,0.7,0 .9),(0.4,0. 6,0.7,0.9))	((0.3,0.4,0 .5,0.6),(0. 2,0.4,0.5,0 .7),(0.2,0. 4,0.5,0.7))	((0.5,0.6,0 .7,0.8),(0, 0.2,0.3,0.4 ,0.0.2,0. 3,0.4))	((0.7,0.8,0 .9,1),(0,0. 1,0.2,0.3), (0,0.1,0.2, 0.3))	((1,1,1,1),(0,0,0,0),(0 ,0,0,0))	((0.7,0.8,0 .9,1),(0,0. 1,0.2,0.3), (0,0.1,0.2, 0.3))	((0.1,0.2 ,0.3),(0.7, 0.8,0.9,1), (0.7,0.8,0. 9,1))
A ₈	((0,0.1,0.2 ,0.3),(0.7, 0.8,0.9,1), (0.7,0.8,0. 9,1))	((0,0,0,0),(1,1,1,1),(1 ,1,1,1))	((0.1,0.2,0 .3,0.4),(0. 4,0.6,0.7,0 .9),(0.4,0. 6,0.7,0.9))	((0.3,0.4,0 .5,0.6),(0. 2,0.4,0.5,0 .7),(0.2,0. 4,0.5,0.7))	((0.5,0.6,0 .7,0.8),(0, 0.2,0.3,0.4 ,0.0.2,0. 3,0.4))	((0.7,0.8,0 .9,1),(0,0. 1,0.2,0.3), (0,0.1,0.2, 0.3))	((1,1,1,1),(0,0,0,0),(0 ,0,0,0))	((0.7,0.8,0 .9,1),(0,0. 1,0.2,0.3), (0,0.1,0.2, 0.3))	((1,1,1,1),(0,0,0,0),(0 ,0,0,0))	((0,0,0,0),(1,1,1,1),(1 ,1,1,1))
A ₉	((0,0,0,0),(1,1,1,1),(1 ,1,1,1))	((1,1,1,1),(0,0,0,0),(0 ,0,0,0))	((0.7,0.8,0 .9,1),(0,0. 1,0.2,0.3), (0,0.1,0.2, 0.3))	((1,1,1,1),(0,0,0,0),(0 ,0,0,0))	((0.7,0.8,0 .9,1),(0,0. 1,0.2,0.3), (0,0.1,0.2, 0.3))	((0.5,0.6,0 .7,0.8),(0, 0.2,0.3,0.4 ,0.0.2,0. 3,0.4))	((0.3,0.4,0 .5,0.6),(0. 2,0.4,0.5,0 .7),(0.2,0. 4,0.5,0.7))	((0.1,0.2,0 .3,0.4),(0. 4,0.6,0.7,0 .9),(0.4,0. 6,0.7,0.9))	((0,0.1,0.2 ,0.3),(0.7, 0.8,0.9,1), (0.7,0.8,0. 9,1))	((0,0,0,0),(1,1,1,1),(1 ,1,1,1))
A ₁₀	((1,1,1,1),(0,0,0,0),(0 ,0,0,0))	((0.7,0.8,0 .9,1),(0,0. 1,0.2,0.3), (0,0.1,0.2, 0.3))	((1,1,1,1),(0,0,0,0),(0 ,0,0,0))	((0.7,0.8,0 .9,1),(0,0. 1,0.2,0.3), (0,0.1,0.2, 0.3))	((0.5,0.6,0 .7,0.8),(0, 0.2,0.3,0.4 ,0.0.2,0. 3,0.4))	((0.3,0.4,0 .5,0.6),(0. 2,0.4,0.5,0 .7),(0.2,0. 4,0.5,0.7))	((0.1,0.2,0 .3,0.4),(0. 4,0.6,0.7,0 .9),(0.4,0. 6,0.7,0.9))	((0.1,0.2 ,0.3),(0.7, 0.8,0.9,1), (0.7,0.8,0. 9,1))	((0,0,0,0),(1,1,1,1),(1 ,1,1,1))	((1,1,1,1),(0,0,0,0),(0 ,0,0,0))

Three experts create the decision matrix using the TFNNs as shown in Tables 1-3. We apply the score function to obtain crisp values. We combine the decision matrix into a single matrix. We compute the criteria weights using the average method such as: $C1=0.096716059$, $C2=0.106728074$, $C3=0.088606328$, $C4=0.10402483$, $C5=0.131457749$, $C6=0.096015218$, $C7=0.09721666$, $C8=0.097917501$, $C9=0.11123348$, $C10=0.070084101$.

Compute the normalized decision matrix for positive and negative criteria using eqs. (14 and 15) as shown in Fig 1.

Compute the relative weights using eq. (16).

Obtain the dominance degree matrix using eq. (17) as shown in Fig 2. The dominance degree matrix by the relative weights is shown in Fig 3.

Compute the overall dominance matrix using eq. (19) as shown in Fig 4. We rank the alternatives as shown in Fig 5.

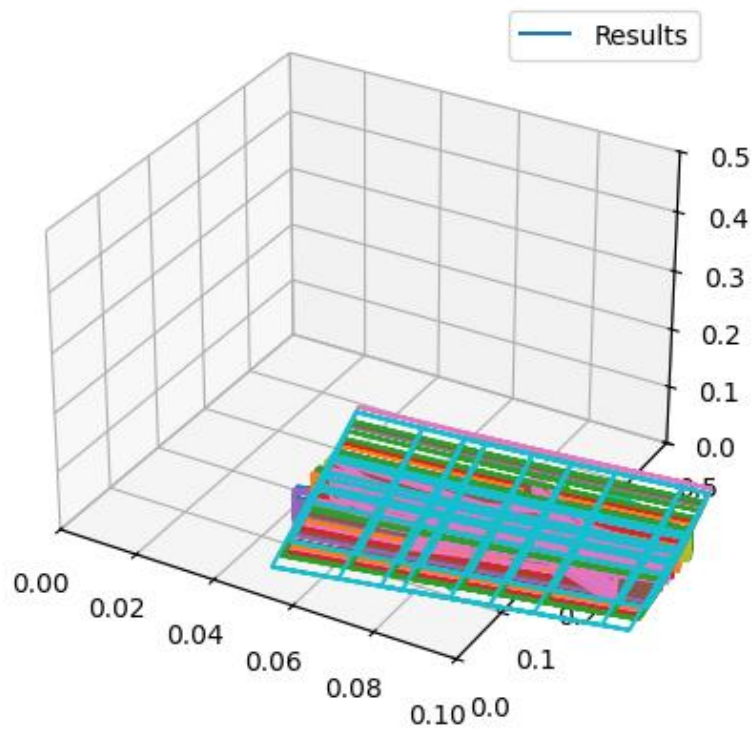


Fig 1. The normalized decision matrix.

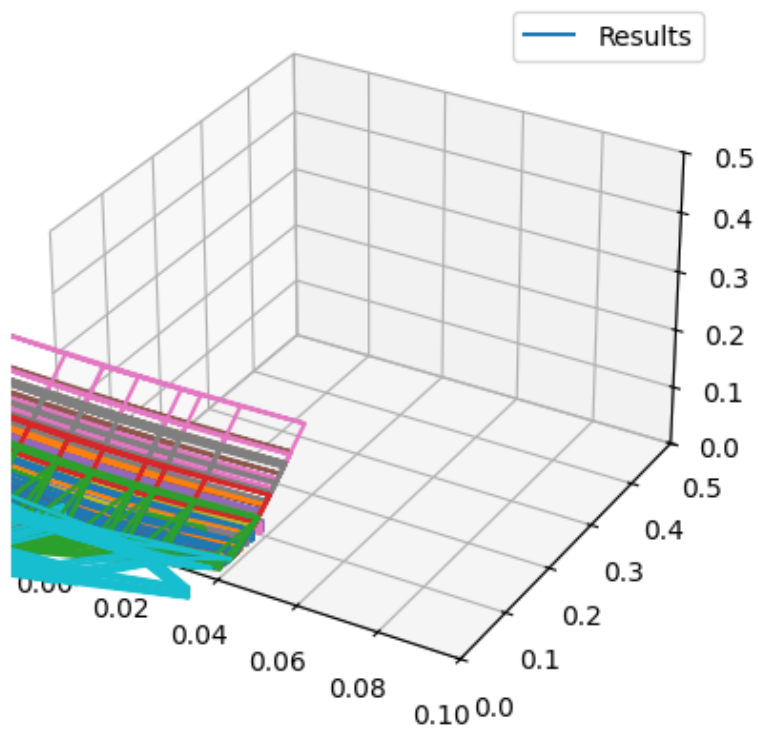


Fig 2-1. The dominance matrix.

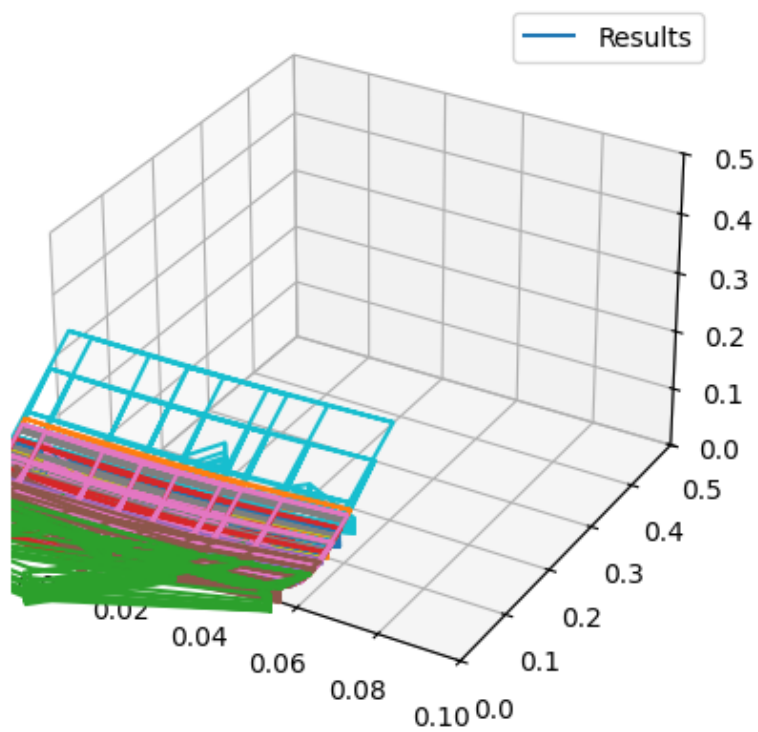


Fig 2-2. The dominance matrix.

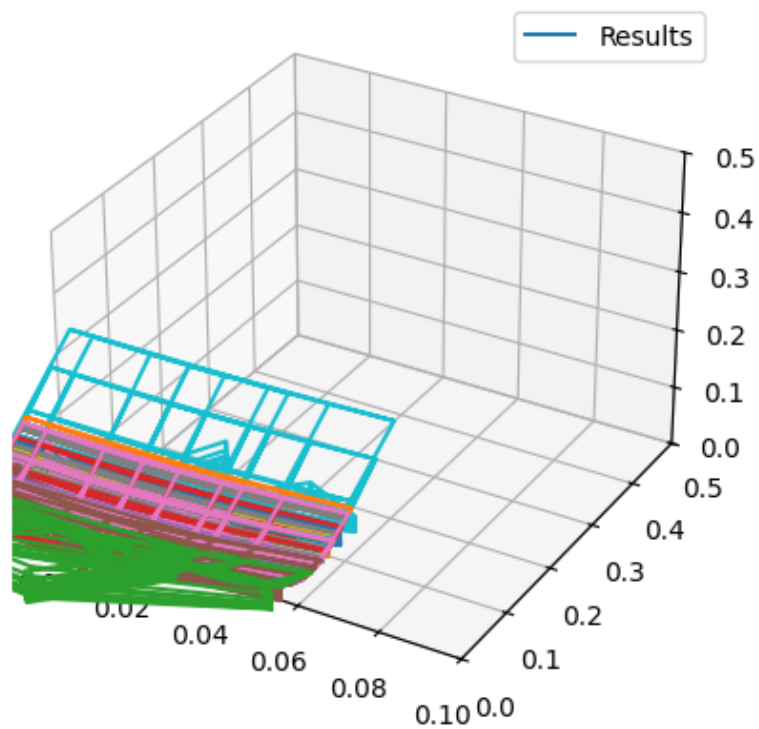


Fig 2-3. The dominance matrix.

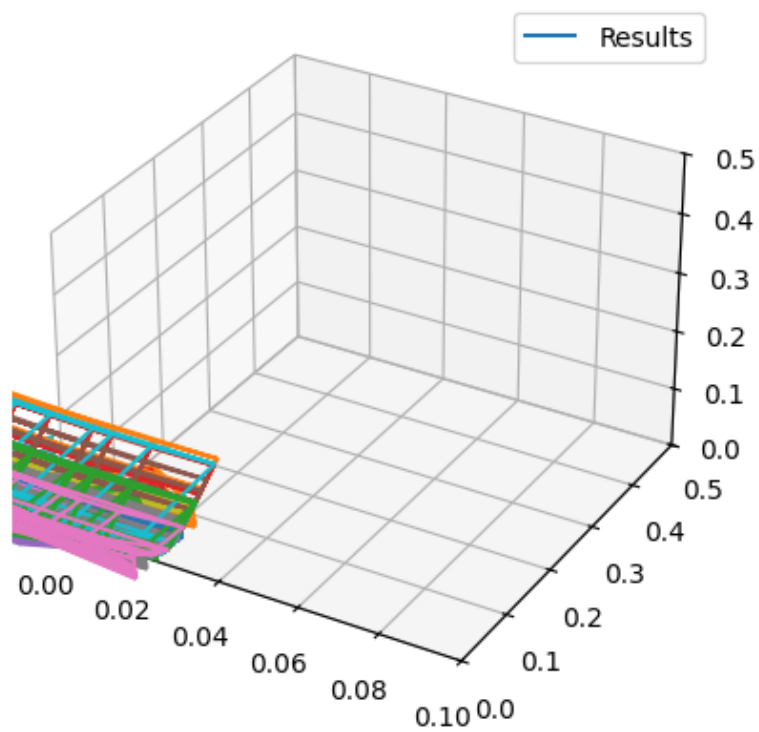


Fig 2-4. The dominance matrix.

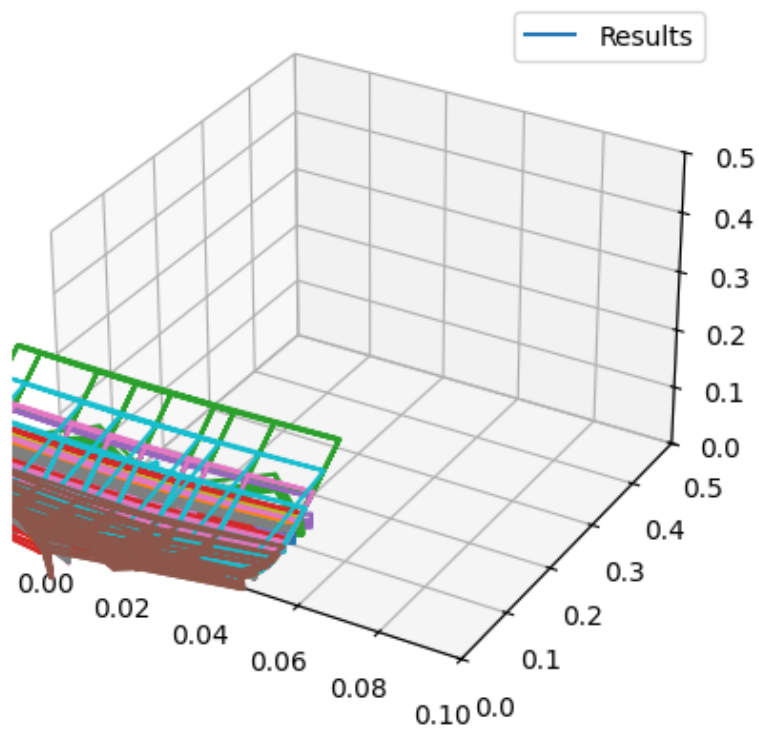


Fig 2-5. The dominance matrix.

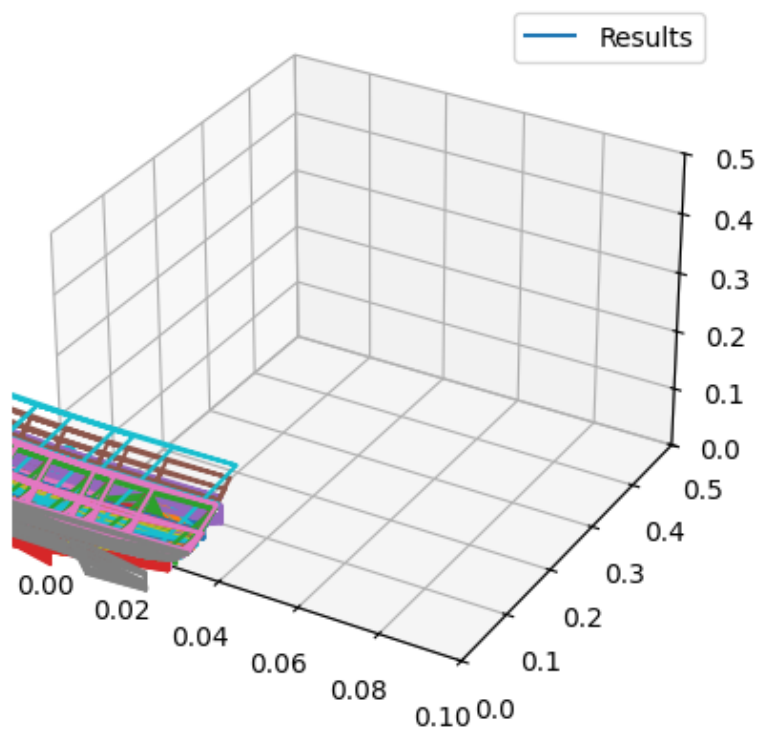


Fig 2-6. The dominance matrix.

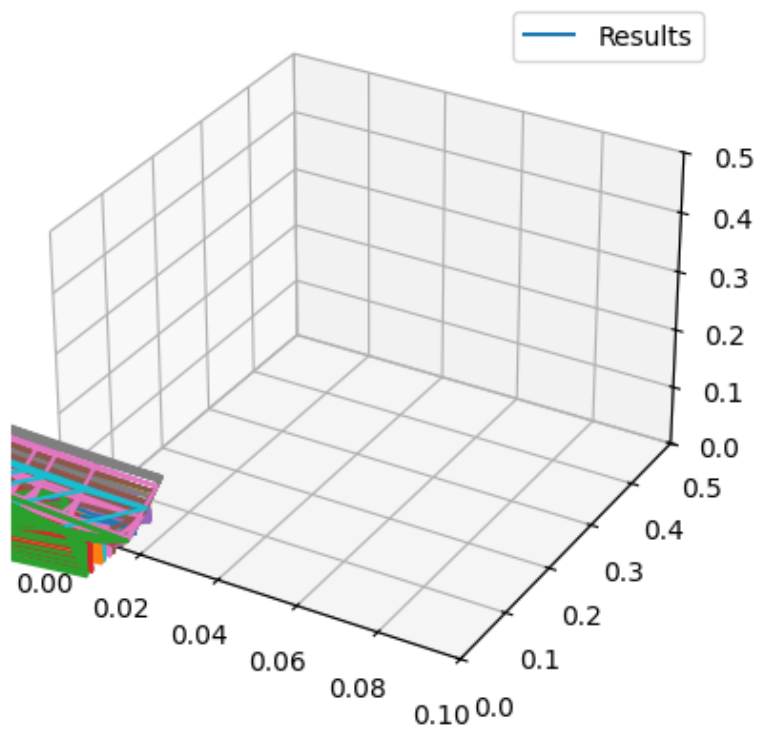


Fig 2-7. The dominance matrix.

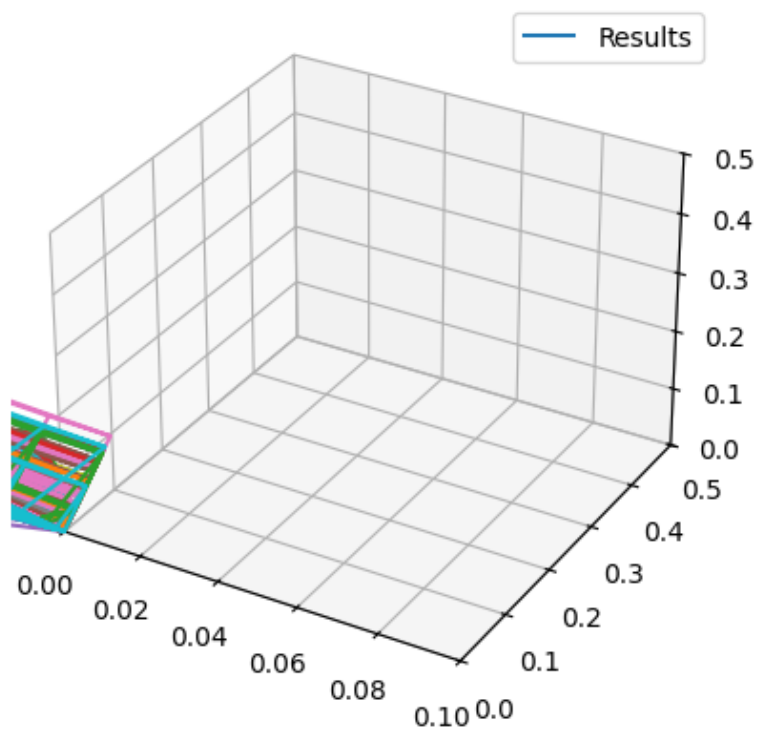


Fig 2-8. The dominance matrix.

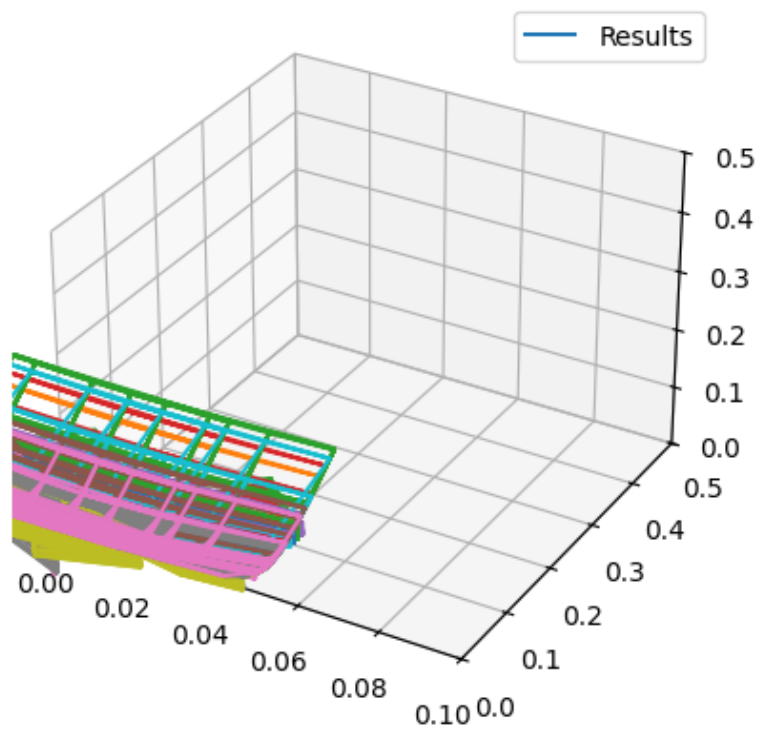


Fig 2-9. The dominance matrix.

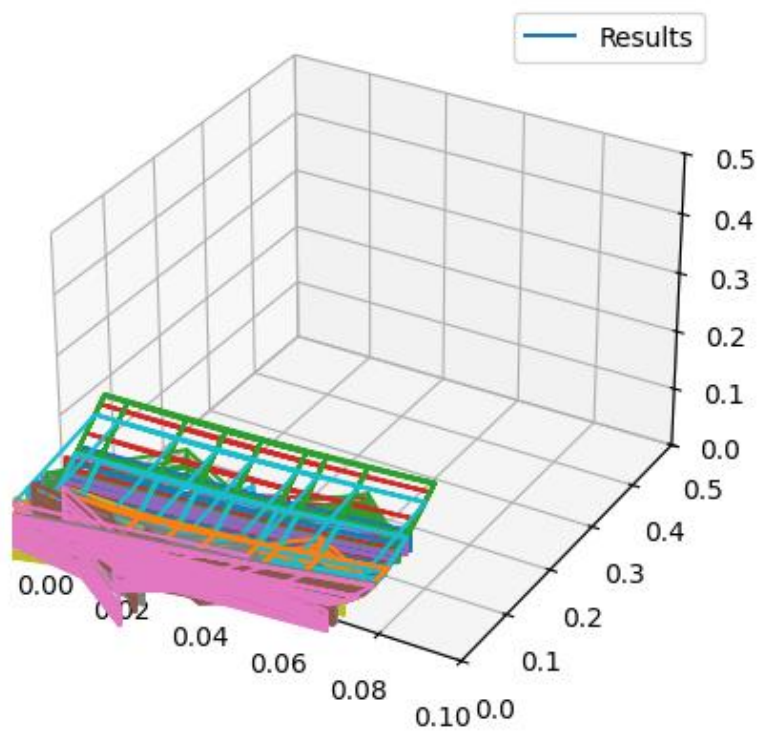


Fig 2-10. The dominance matrix.

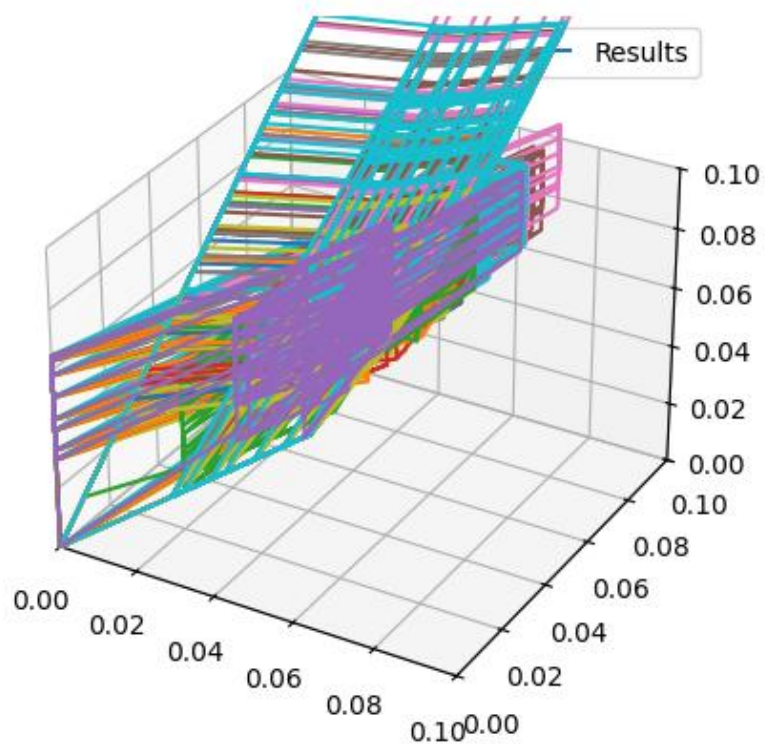


Fig 3-1. The dominance matrix by weights.

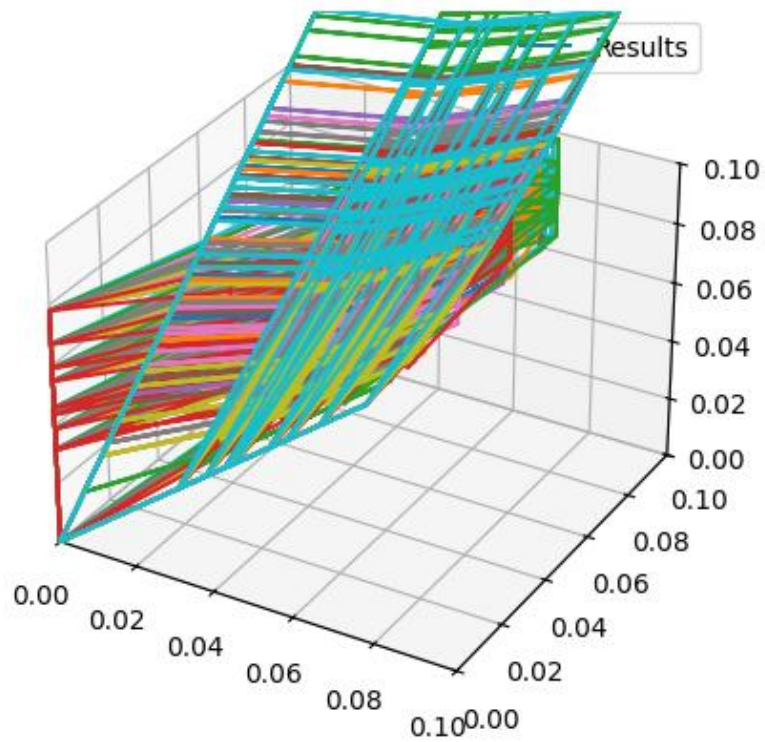


Fig 3-2. The dominance matrix by weights.

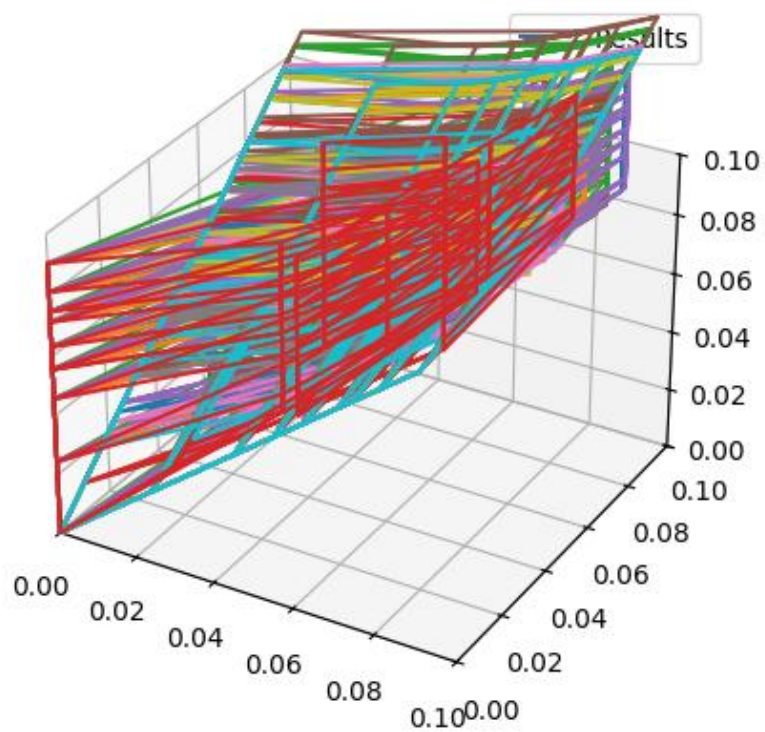


Fig 3-3. The dominance matrix by weights.

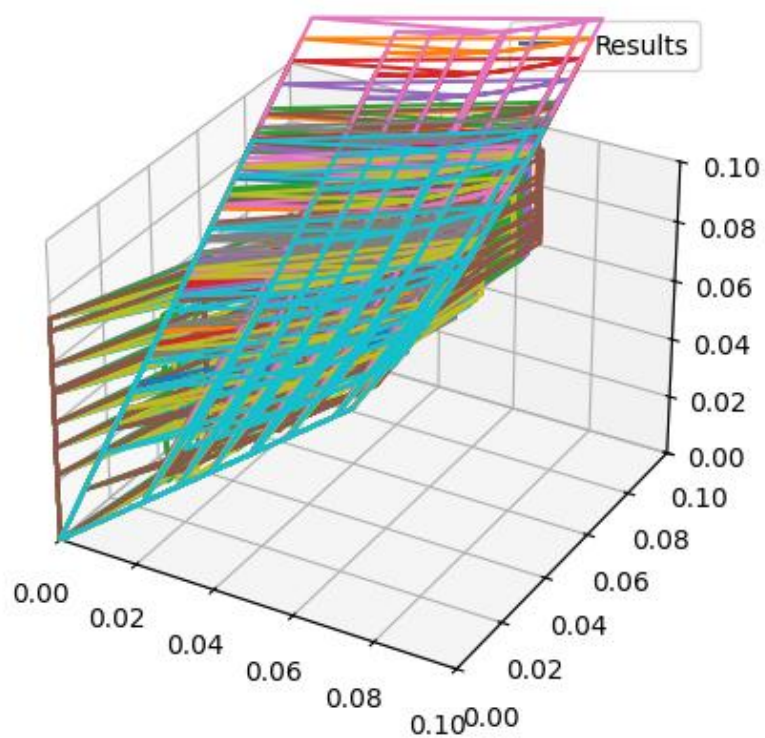


Fig 3-4. The dominance matrix by weights.

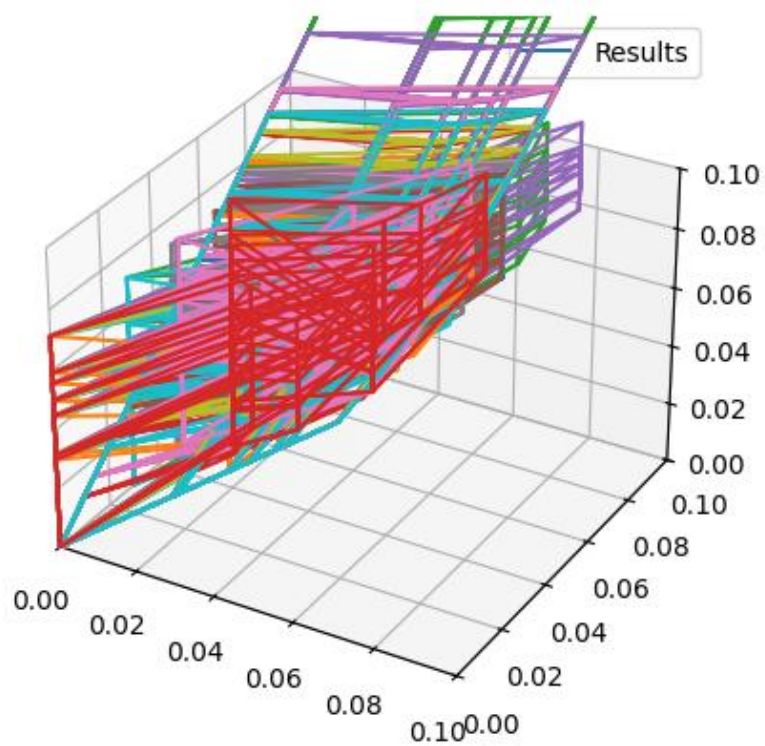


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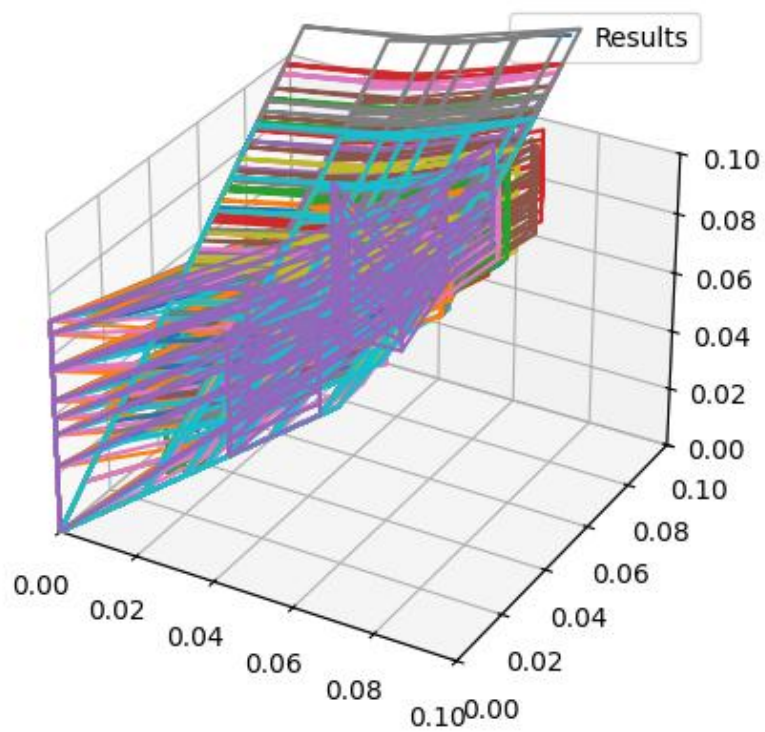


Fig 3-6. The dominance matrix by weights.

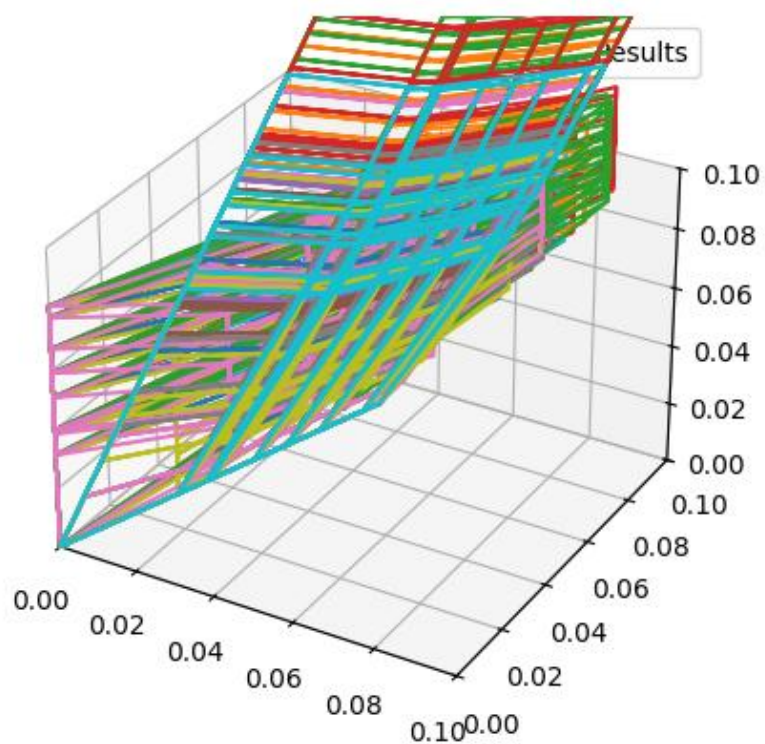


Fig 3-7. The dominance matrix by weights.

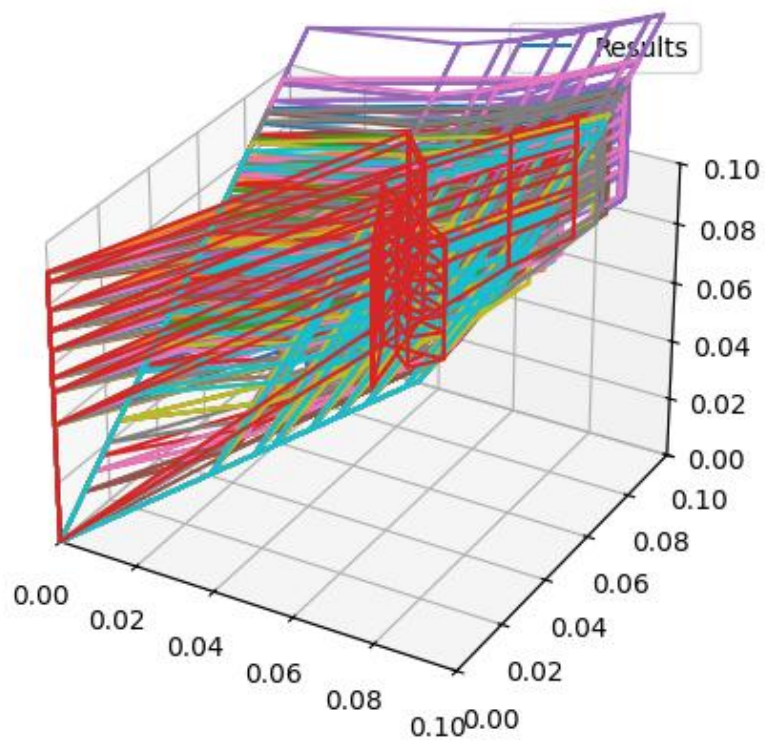


Fig 3-8. The dominance matrix by weights.

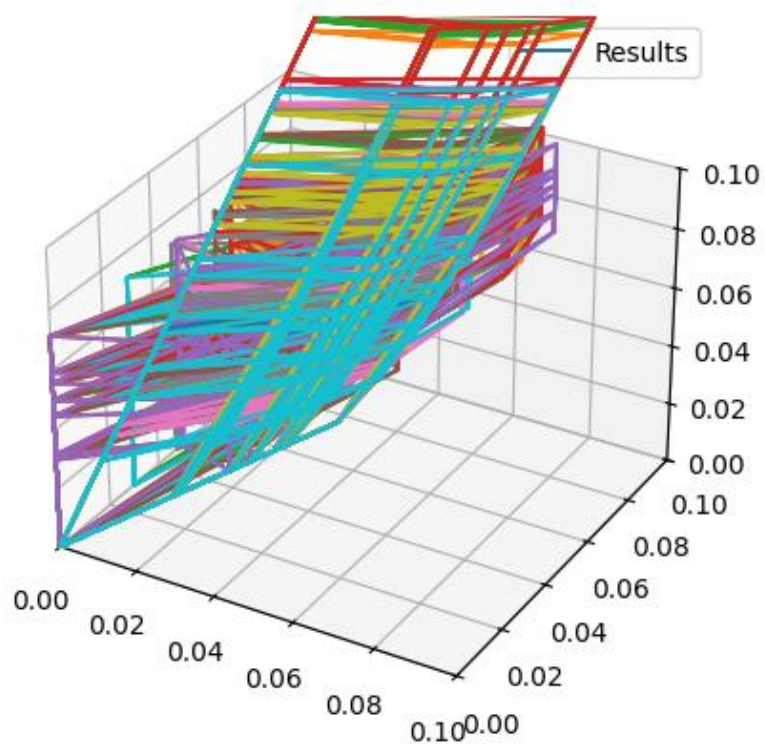


Fig 3-9. The dominance matrix by weights.

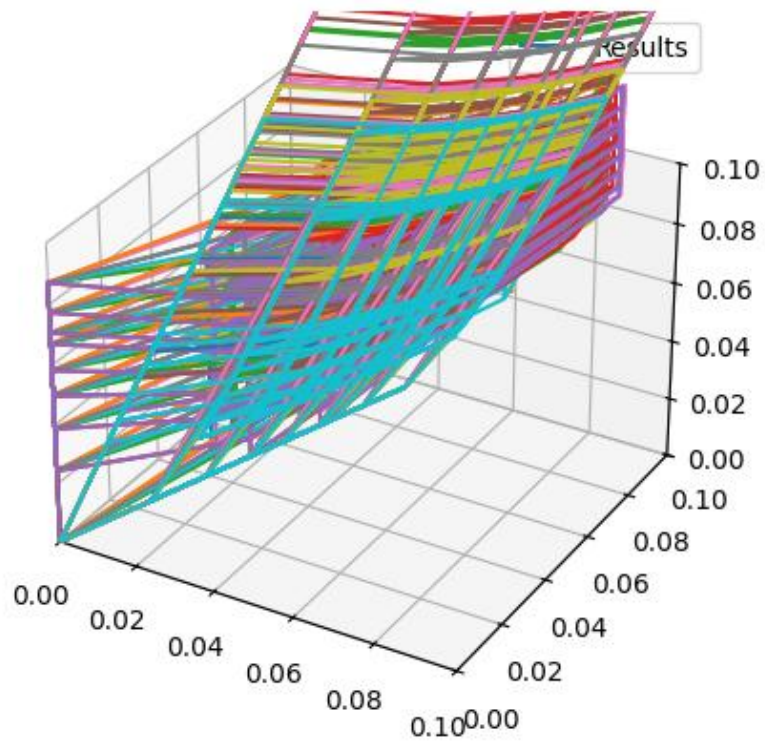


Fig 3-10. The dominance matrix by weights.

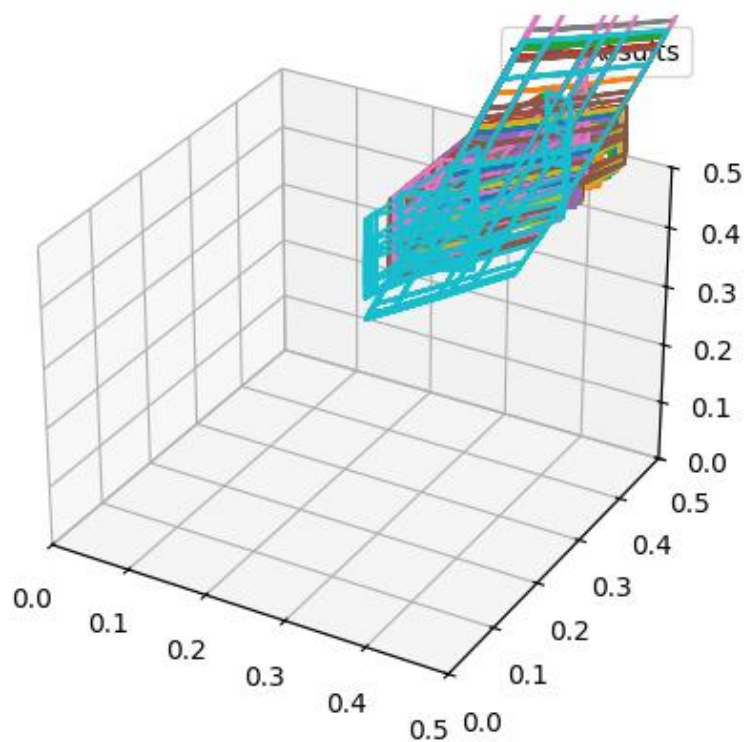


Fig 4. The overall dominance.

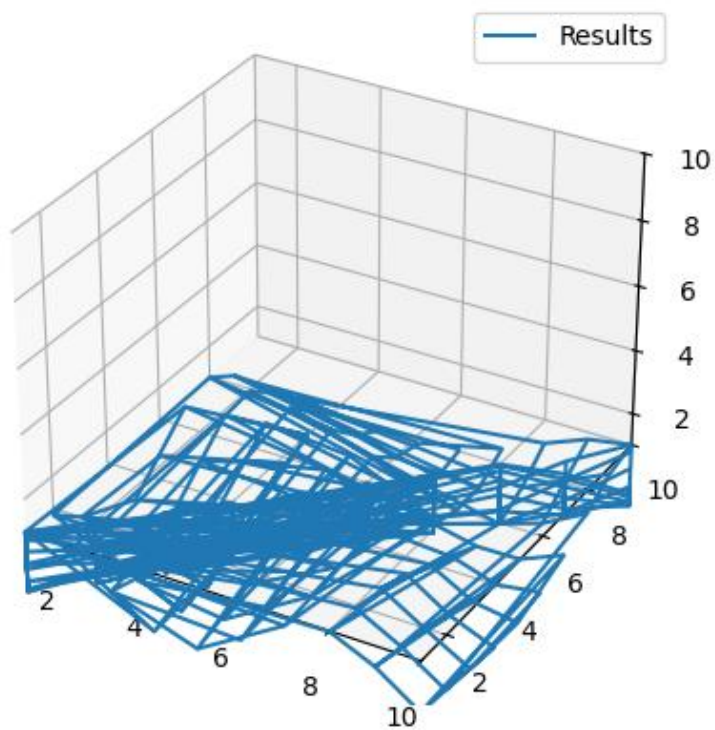


Fig 5. The rank of alternatives.

4. Conclusions

Using microblogging platforms to effectively communicate a brand is a dynamic process that is driven by several interconnected aspects. Beyond superficial engagement numbers, a thorough assessment must consider posting tactics, emotional effect, conversion success, and content relevancy. Brands may make data-driven enhancements and obtain deeper insights into the efficacy of their microblog marketing by implementing a multifaceted approach. For companies looking to maximize their communication tactics in the fiercely competitive and constantly changing digital market, this report provides a solid basis. We used the MCDM methodology such as TODIM methodology to rank alternatives. We used ten criteria and ten alternatives to validate the proposed approach. The results show alternative 1 is the best and alternative 6 is the worst.

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Introduction to Collective Neutrosophy and Collective n-ary Neutrosophy

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Abstract: This paper introduces the novel concepts of Collective Neutrosophy and Collective n-ary Neutrosophy as extensions of Smarandache's foundational theories of Neutrosophy and n-ary Neutrosophy. While classical Neutrosophy generalizes traditional dialectical thinking by incorporating neutralities into the dynamics of opposites, the proposed collective forms address interactions at the group level, where multiple agents and ideas simultaneously engage in opposition, cooperation, or neutrality. Applications in geopolitics, philosophy, corporate ecosystems, and cultural studies are discussed, alongside a theoretical framework for modeling complex relational dynamics. Diagrams illustrating single, collective, and collective n-ary neutrosophic structures are proposed to visualize these concepts.

Keywords: Neutrosophy; n-ary Neutrosophy; Collective Neutrosophy; Dialectics; Neutralities; Opposites; Complex Systems; Conflict Dynamics; Group Theory.

1. Introduction

Smarandache's Neutrosophy, introduced in 1998 [1], and its subsequent extension into n-ary Neutrosophy in 2003 [3], represent significant generalizations of dialectical philosophy. Traditional dialectics, such as Hegelian synthesis and the Yin–Yang model from Chinese philosophy, focus on the tension between two opposites. However, these frameworks largely neglect the role of neutrality or indeterminacy, crucial elements that influence the balance between oppositional forces.

Neutrosophy broadens this view by considering not only the dynamics between opposites but also the influence of neutralities, elements that are neither fully aligned with one side nor the other but that affect the outcome of oppositional interactions. We term this foundational model Single Neutrosophy.

This paper introduces two new theoretical expansions:

- *Collective Neutrosophy*: modeling the dynamics between two opposite groups and their collective neutralities.

- *Collective n-ary Neutrosophy*: further generalizing to interactions among multiple ($n \geq 2$) opposite groups and their respective multiple collective neutralities.

These developments allow for the neutrosophic modeling of highly complex systems characterized by multifaceted relationships between ideas, entities, or actors.

2. Foundations of Single and n-ary Neutrosophy

2.1. Single Neutrosophy

In Single Neutrosophy, the basic unit of analysis is the dynamic between two opposing elements (e.g., two people, two ideas), alongside neutral or indeterminate elements that may sway the dynamic [1-4].

Example: Two individuals are in conflict, while third parties—neutral bystanders—may intervene, supporting one side, mediating, or introducing new dynamics.

2.2. Single n -ary Neutrosophy

n -ary Neutrosophy extends this model to n opposing elements ($n \geq 2$), considering interactions and neutralities between each possible pair.

Example: In a multiparty political system, several parties compete simultaneously, with shifting alliances and neutral stances influencing the overall balance of power.

3. Collective Neutrosophy

Collective Neutrosophy advances the neutrosophic model from individual to group dynamics. It studies the opposition and cooperation between groups of elements, along with the collective neutralities or collective indeterminacies that may emerge between or within these groups.

3.1. Structure

- *Groups:* Sets of elements (individuals, ideas, organizations) that share a general alignment.
- *Intergroup Dynamics:* Interactions between entire groups.
- *Intragroup Dynamics:* Internal oppositions within each group.
- *Collective Neutralities:* Neutral groups or forces that influence the intergroup dynamics.

3.2. Applications

- *Geopolitical Conflicts:* Conflicts between nation-states, where neutral countries (e.g., Switzerland in WWII) mediate or subtly influence the balance.
- *Philosophical Systems:* Oppositions between schools of thought (e.g., materialism vs. idealism) and the influence of interdisciplinary or neutral intellectual movements.
- *Business Ecosystems:* Rivalry between corporate groups, moderated by regulatory bodies, open-source communities, or independent developers.

4. Collective n -ary Neutrosophy

Collective n -ary Neutrosophy generalizes Collective Neutrosophy to n groups ($n \geq 2$), where complex, multi-faceted interactions are observed.

4.1. Structure

- *n Groups of Opposites:* $\{G_1, G_2, \dots, G_n\}$
- *Intergroup Dynamics:* Competitive, cooperative, or neutral relationships between any pair of groups.
- *Collective Neutralities:* Entities or groups that interact neutrally or mediate among multiple groups simultaneously.

4.2. Applications

- *International Alliances:* NATO, BRICS, ASEAN, and other coalitions interacting with neutral states or organizations like the UN.
- *Multi-platform Technological Ecosystems:* Interaction between ecosystems like Apple, Google, Microsoft, with neutral cross-platform developers or regulatory influences shaping market dynamics.
- *Cultural Movements:* Interaction among multiple art or literary movements, where certain neutral movements (e.g., postmodernism) mediate between classical conflicts.

5. Diagrams and Visual Representations

Several diagrams can aid in understanding these complex relationships:

5.1. Single Neutrosophy Diagram

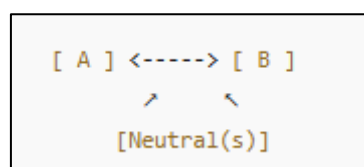


Figure 1. Two nodes (A and B) connected by a dynamic arrow, with neutral elements (N) influencing the arrow's balance. A and B are two opposites (e.g., two people, two ideas); neutral(s) can influence either side or mediate.

5.2. Single n-ary Neutrosophy Network

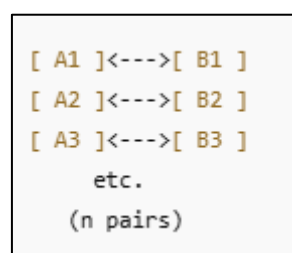


Figure 2.1. Several independent oppositional pairs; multiple nodes ($A_1, A_2, \dots, A_n$) with pairwise connections; neutral nodes positioned between each connected pair.

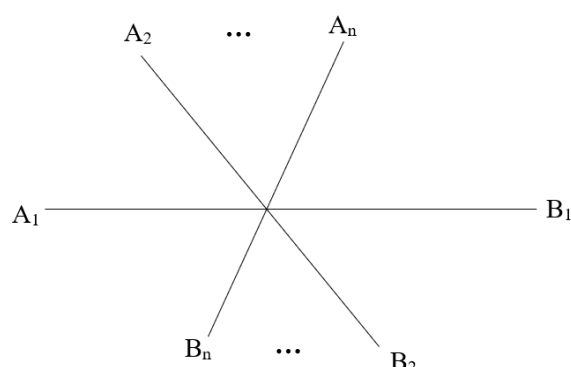


Figure 2.2. Single n-ary Neutrosophy Network.

5.3. Collective Neutrosophy

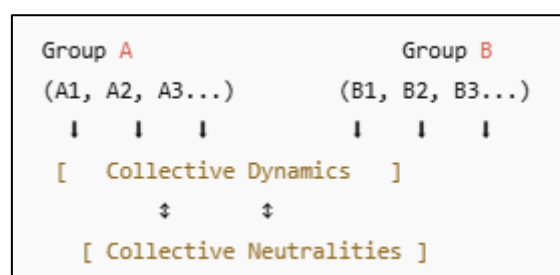


Figure 3.1. Two groups of nodes (Group A, Group B) with internal and external connections; collective neutralities as intermediary nodes between groups.

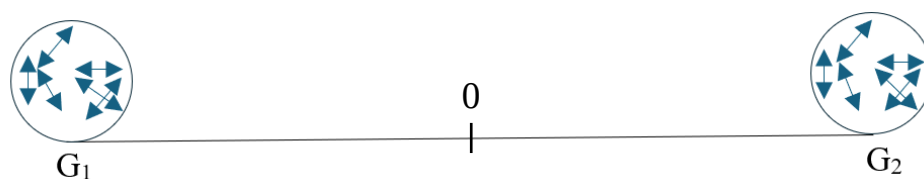


Figure 3.2. Collective Neutrosophy.

5.4. Collective n -ary Neutrosophy Network

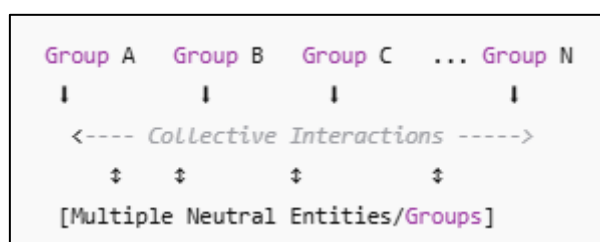


Figure 4.1. Multiple groups with cross-interactions and neutral clusters linking multiple groups.

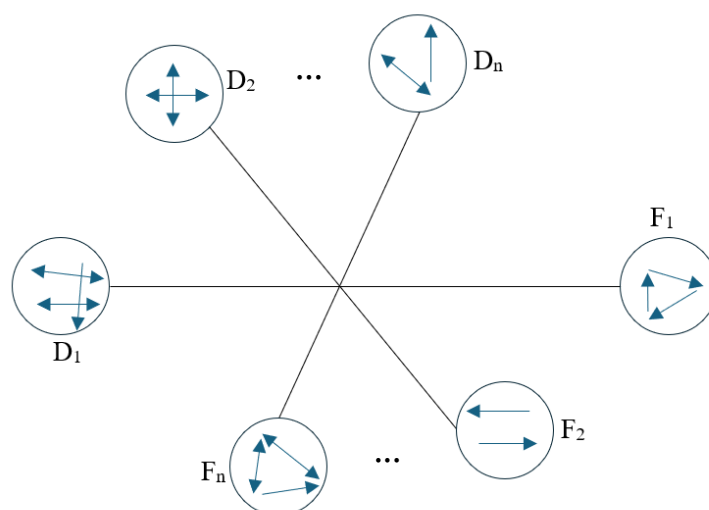


Figure 4.2. Collective n -ary Neutrosophy Network.

6. A Short Comparative Analysis: EU vs. MERCOSUR through Collective Neutrosophy and Collective n -ary Neutrosophy

6.1. Basic Structure

EU is political and economic union of 27 countries with deep integration (common market, customs union, shared policies, and in some cases, common currency - the Euro).

MERCOSUR is a South American trade bloc (Argentina, Brazil, Paraguay, Uruguay, Venezuela - suspended) aiming primarily at free trade and fluid movement of goods, people, currency.

Each bloc is a collective made of multiple nations (not individuals), thus fitting Collective Neutrosophy.

Internal dynamics among member states = internal single neutrosophic tensions.

6.2. Dynamics of Opposites and Neutralities

Aspect	European Union (EU)	MERCOSUR
Internal Opposites	Differences between northern vs. southern economies; national sovereignties vs. federalism; euro-zone vs. non-eurozone members.	Brazil vs. Argentina economic competition; agricultural vs. industrial interests; political instability vs. democratic consolidation.
Neutralities	Neutral countries (e.g., Austria in military matters); mediators like Germany in conflicts; EU Commission trying to balance East-West tensions.	Paraguay and Uruguay often act as balancing powers between Brazil and Argentina; external neutral actors (e.g., Bolivia trying to mediate trade terms).
External Opposites	EU vs. US/China in global trade; EU vs. Russia geopolitically.	MERCOSUR vs. Pacific Alliance (competing South American trade blocs); MERCOSUR vs. external trade blocs (like the EU itself during free-trade talks).

Neutrosophically, each market isn't just a binary tension but has multiple tensions, neutral mediations, and indeterminacies.

6.3. n-ary Collective Neutrosophic Dynamics

In EU:

- Germany–France–Italy–Poland–Spain: major economic players, each having different priorities.

- Each pair (Germany–Italy, France–Poland, etc.) can have tensions, cooperation, and neutrals.

In MERCOSUR:

- Brazil–Argentina–Paraguay–Uruguay–Venezuela (suspended).

- Similar: Brazil–Argentina rivalry, Paraguay–Uruguay as mediators, Venezuela's instability adding indeterminacy.

6.4. Strength of Neutrality Zones

Bloc	Internal Neutral Zones	External Neutral Interactions
EU	Countries like Austria, Finland; neutral platforms like European Parliament debates.	EFTA (European Free Trade Association) as semi-neutral economic players.
MERCOSUR	Uruguay and Paraguay trying to mediate disputes.	Chile and Bolivia acting as external neutral or semi-associated countries.

6.5. Remarks

- Collective Neutralities help stabilize each market — but the degree of stabilization differs.
- EU shows stronger structured neutrality mechanisms (formal mediation processes).
- MERCOSUR's neutralities are more informal and sometimes fragile.

6.6. Summary Interpretation

Concept	EU	MERCOSUR
Collective Dynamics	Highly institutionalized	More political, informal
Internal Neutrosophy	Complex; multilayered (economy, policy, sovereignty)	Primarily economic with some political overlaps
n-ary Dynamics	>5 strong member interactions	3-5 strong, others weaker
Neutralities	Formal, legally enforced (e.g., treaties)	More ad hoc, based on diplomacy
Indeterminacies	Brexit, Eurozone crises, East-West divergence	Political instability (Venezuela), economic crises

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An Empirical Study on the Quality of Industry-Linked Education in Vocational Colleges: Double-Framed TreeSoft Set Framework

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Abstract: Integrating industry-linked education into vocational college courses has been a key tactic for improving workforce preparedness in recent years. This study examines the extent and efficacy of industrial cooperation in vocational education contexts, emphasizing the effects of such alliances on innovation, employability, and skill acquisition. The study identifies obstacles that must be overcome and emphasizes important elements that lead to effective integration through empirical investigation. This study uses the multi-criteria decision making (MCDM) to deal with criteria and alternatives. The MAIRCA method is used to rank the alternatives. We use the Double-Framed TreeSoft Set to divide the main criteria into a tree. The findings are intended to provide useful information to industry partners, educators, and legislators who are trying to improve the caliber of vocational education.

Keywords: Double-Framed TreeSoft Set; Quality of Industry; Quality of Education; Vocational Colleges.

1. Introduction

In the past, vocational education has been essential in bridging the gap between academic education and the demands of the job market. Technology breakthroughs have caused the global economy to change quickly, which has increased the need for vocational training to better match business demands. Because of this congruence, graduates are guaranteed to have applicable, useful abilities that they can use right away in real-world situations[1], [2].

Through internships, cooperative education initiatives, and industry-driven projects, students at vocational institutions interact directly with companies through industry-linked education, which is acknowledged as a transformational strategy. These programs foster soft skills like communication, collaboration, and flexibility in addition to technical competence, all of which are critical for succeeding in the fast-paced industries of today.

The quality of industry-linked education implementation varies greatly throughout universities, despite its acknowledged significance. While some universities suffer with a lack of business relationships or antiquated training approaches, others have effectively incorporated experiential education into their programs. This discrepancy emphasizes the necessity of empirical research that methodically evaluates the efficacy of these kinds of partnerships[3], [4].

This study aims to assess the quality of industry-linked education from several angles, such as graduate employability, student happiness, faculty involvement with industry, and curricular relevance. It looks at how successfully vocational schools integrate real-world business practices into their instruction and how these practices affect the education results of their students[5], [6].

Keeping up mutually beneficial and long-lasting relationships between industry and vocational colleges is one of the primary issues noted. While universities must strike a balance between these demands and more general educational objectives, industries are frequently focused with short-term skill shortages. Designing successful, long-lasting partnerships requires an understanding of this equilibrium[7], [8].

This study's empirical methodology includes both quantitative and qualitative techniques, such as faculty interviews, student surveys, and input from industry partners. This multi-stakeholder viewpoint illuminates areas that require development and offers a comprehensive assessment of the condition of industry-linked education today[9].

According to preliminary research, there are still gaps in areas like curriculum innovation, regular industry engagement, and student competency evaluation, even though many vocational institutions have made great progress in incorporating industry collaborations. It will take both deliberate actions and continuous evaluation systems to close these gaps.

Finally, by providing evidence-based suggestions, this study hopes to add to the larger conversation on vocational education reform. In addition to improving individual career possibilities, raising the standard of industry-linked education would make the workforce more competitive and adaptable overall in the face of swift advancements in technology and the economy[10], [11].

2. Double-Framed TreeSoft Set

Smarandache created new types of soft set[12], [13], [14]s. We show the operations of the Double-Framed TreeSoft Set (DFTSS). DFTSS can extend the concept of the TreeSoft set to divide the criteria into tree by two frames[15], [16].

Let U be a universal set, $Tree(A)$ be a hierarchical citron tree constructed from a criterion set $A = \{A_1, A_2, A_3, A_4, A_5, A_6, \dots, A_n\}$. $P(Tree(A))$ is the power set of all nodes in the $Tree(A)$.

DFTSS is a triple can be defined as:

$$D_1, D_2; Tree(A) \quad (1)$$

$$D_1: P(Tree(A) \rightarrow P(U)) \quad (2)$$

$$D_2: P(Tree(A) \rightarrow P(U)) \quad (3)$$

$X \sqsubseteq Tree(A), D_1(X), D_2(X)$ introduces two frames positive and negative.

We show the steps of the MAIRCA. Let experts create the decision matrix by their operations. We combine their opinions into a single matrix.

Determine the criteria weights by the average method. The criteria weights effect on the decision-making ranking.

Compute the components $Q_{p_{ij}}$ of the theoretical rating matrix

$$Q_{p_{ij}} = x_{ij} * W_j \quad (4)$$

Where x_{ij} refers to the opinion of experts and decision makers.

Compute the real rating matrix

$$R_{ij} = Q_{p_{ij}} \left(\frac{x_{ij} - \min x_i}{\max x_i - \min x_i} \right) \quad (5)$$

$$R_{ij} = Q_{p_{ij}} \left(\frac{x_{ij} - \max x_i}{\min x_i - \max x_i} \right) \quad (6)$$

Compute the total gap matrix

$$G_{ij} = Q_{p_{ij}} - R_{ij} \quad (7)$$

Compute the value of each alternative.

$$V_i = \sum_{j=1}^m G_{ij} \quad (8)$$

Rank the alternatives.

3. Results and Discussion

This section shows the results of the proposed approach. We use four main criteria and ten alternatives. We divide four main criteria into a tree. In each brace we compute the criteria weights.

C1: Curriculum Alignment with Industry Needs

- C1.1: Industry-Relevant Course Content (High, Medium, Low)
 - C1.1.1: Updated syllabus → High
 - C1.1.2: Practical skill components → Medium
 - C1.1.3: Certification and licensing integration → High

- C1.2: Collaboration in Curriculum Design (Strong, Moderate, Weak)
 - C1.2.1: Industry advisory boards → Strong
 - C1.2.2: Joint curriculum reviews → Moderate
 - C1.2.3: Industry-led course modules → Strong
- C1.3: Technology and Equipment Matching Industry Standards (Excellent, Good, Fair)
 - C1.3.1: Access to latest equipment → Excellent
 - C1.3.2: Digital education tools → Good
 - C1.3.3: Simulation environments → Good

C2: Practical Training Opportunities

- C2.1: Internship and Apprenticeship Programs (Comprehensive, Adequate, Limited)
 - C2.1.1: Duration and intensity → Comprehensive
 - C2.1.2: Relevance to coursework → Adequate
 - C2.1.3: Supervision and feedback → Comprehensive
- C2.2: On-the-Job Training Modules (Structured, Semi-structured, Unstructured)
 - C2.2.1: Structured training phases → Structured
 - C2.2.2: Education objectives clarity → Structured
 - C2.2.3: Performance evaluation systems → Semi-structured
- C2.3: Participation in Real Industry Projects (Frequent, Occasional, Rare)
 - C2.3.1: Industry-sponsored case studies → Frequent
 - C2.3.2: Capstone project integration → Frequent
 - C2.3.3: Field visits and observations → Occasional

C3: Faculty-Industry Engagement

- C3.1: Faculty Internships in Industry (Regular, Occasional, Rare)
 - C3.1.1: Frequency of industry immersion → Regular
 - C3.1.2: Duration of placements → Regular
 - C3.1.3: Application of new knowledge to teaching → Regular
- C3.2: Joint Research and Development Projects (Active, Moderate, Minimal)

- C3.2.1: Industry-cofunded research → Active
 - C3.2.2: Product development collaborations → Moderate
 - C3.2.3: Publication and patents outcomes → Minimal
- C3.3: Professional Development through Industry Certifications (Extensive, Moderate, Limited)
 - C3.3.1: Number of certifications attained → Extensive
 - C3.3.2: Training workshops attended → Moderate
 - C3.3.3: Integration of certification topics into classes → Extensive

C4: Graduate Employment and Satisfaction

- C4.1: Employment Rate in Related Fields (High, Moderate, Low)
 - C4.1.1: Job placement ratio → High
 - C4.1.2: Time to first employment → Moderate
 - C4.1.3: Alignment with studied discipline → High
- C4.2: Employer Satisfaction with Graduates (Excellent, Good, Fair)
 - C4.2.1: Technical skill competence → Excellent
 - C4.2.2: Soft skills and communication → Good
 - C4.2.3: Adaptability to workplace dynamics → Good
- C4.3: Graduate Self-Assessment (Positive, Neutral, Negative)
 - C4.3.1: Satisfaction with training → Positive
 - C4.3.2: Career progression tracking → Positive
 - C4.3.3: Perceived skill gaps → Neutral

Three experts rate the criteria and alternatives. We combine the opinions of the experts into a single matrix. We compute the criteria weights by the average method such as: $C1=0.251891874$, $C2=0.229720641$, $C3=0.281808625$, $C4=0.23657886$. In each branch, we compute the criteria weights.

In the first branch, we combine the decision matrix into a single matrix as shown in Fig 1. Then we show the criteria weights as shown in Fig 2.

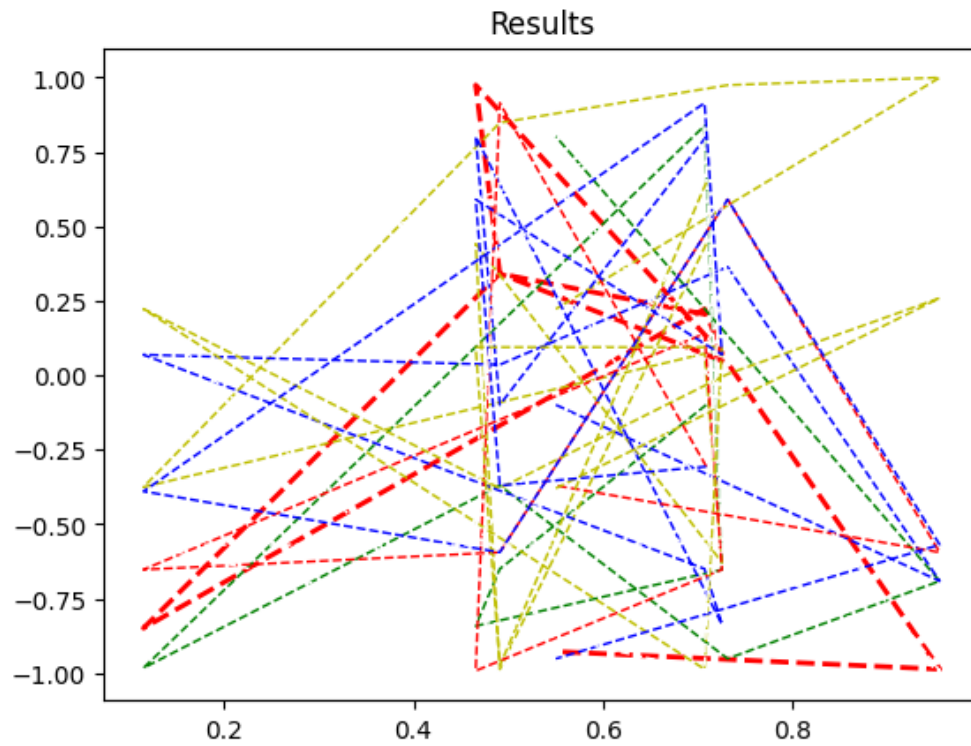


Figure 1. The combine decision matrix.

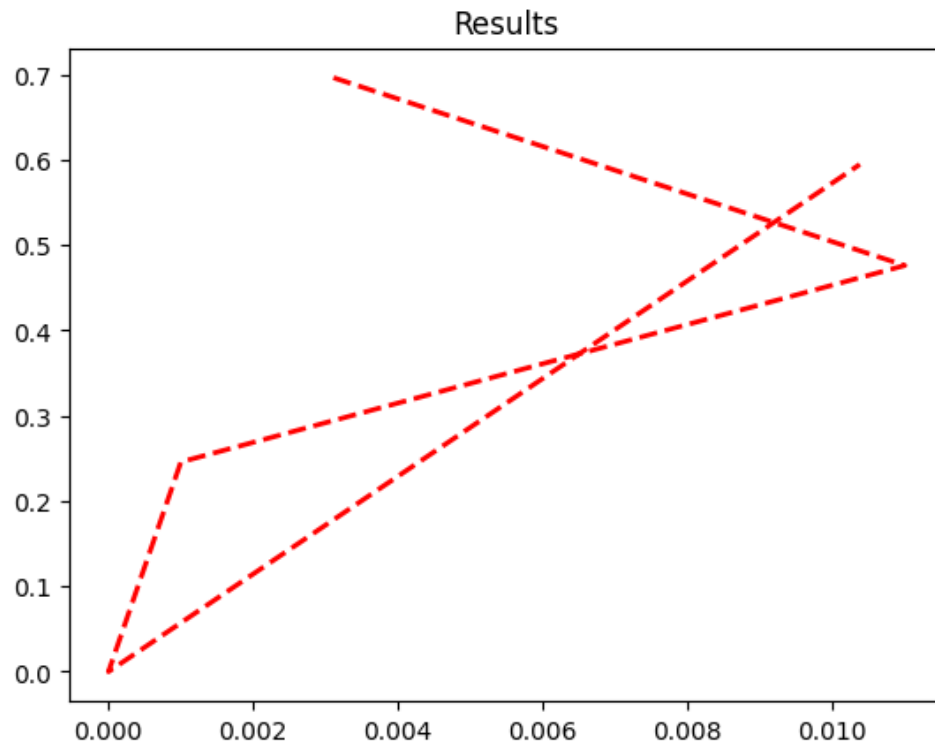


Figure 2. The criteria weights.

In the second branch, we combine the decision matrix into a single matrix as shown in Fig 3. Then we show the criteria weights as shown in Fig 4.

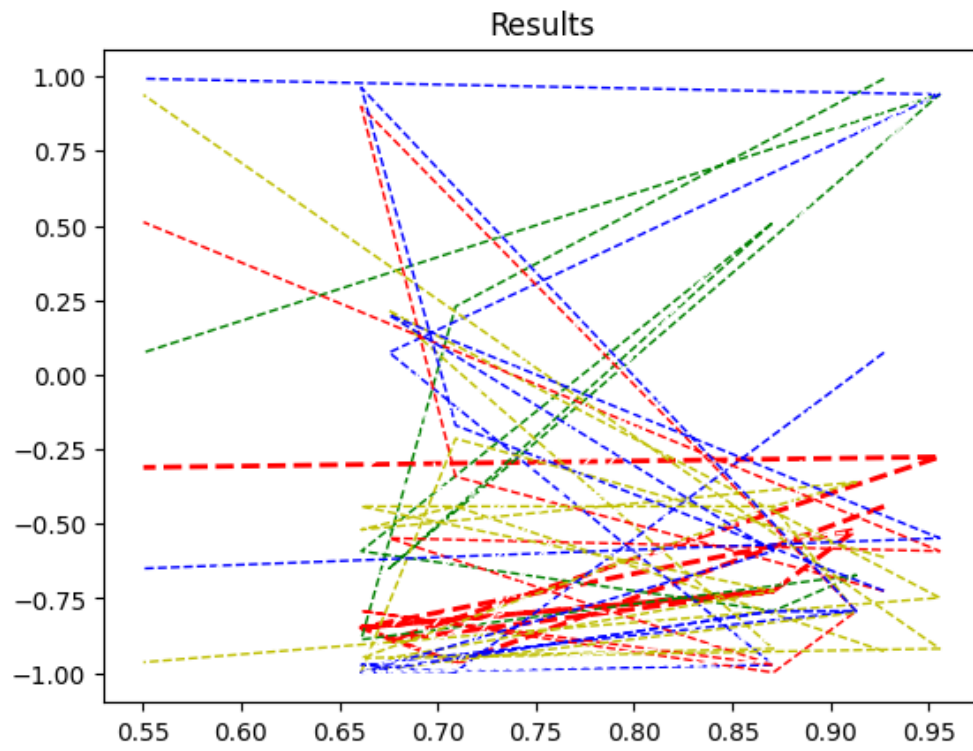


Figure 3. The combine decision matrix.

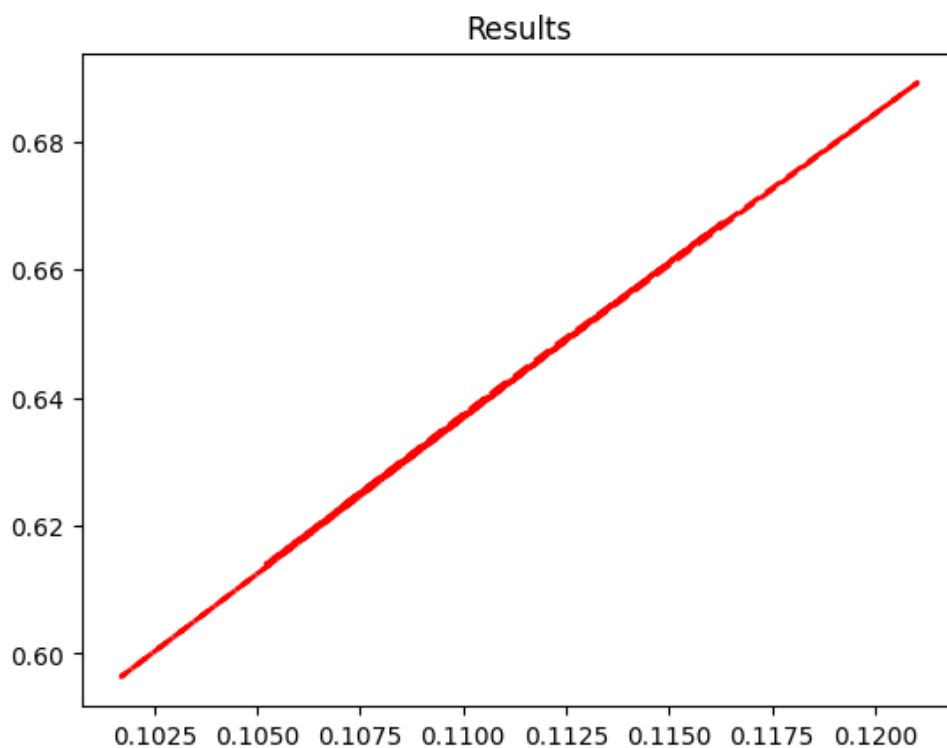


Figure 4. The criteria weights.

In the third branch, we combine the decision matrix into a single matrix as shown in Fig 5. Then we show the criteria weights as shown in Fig 6.

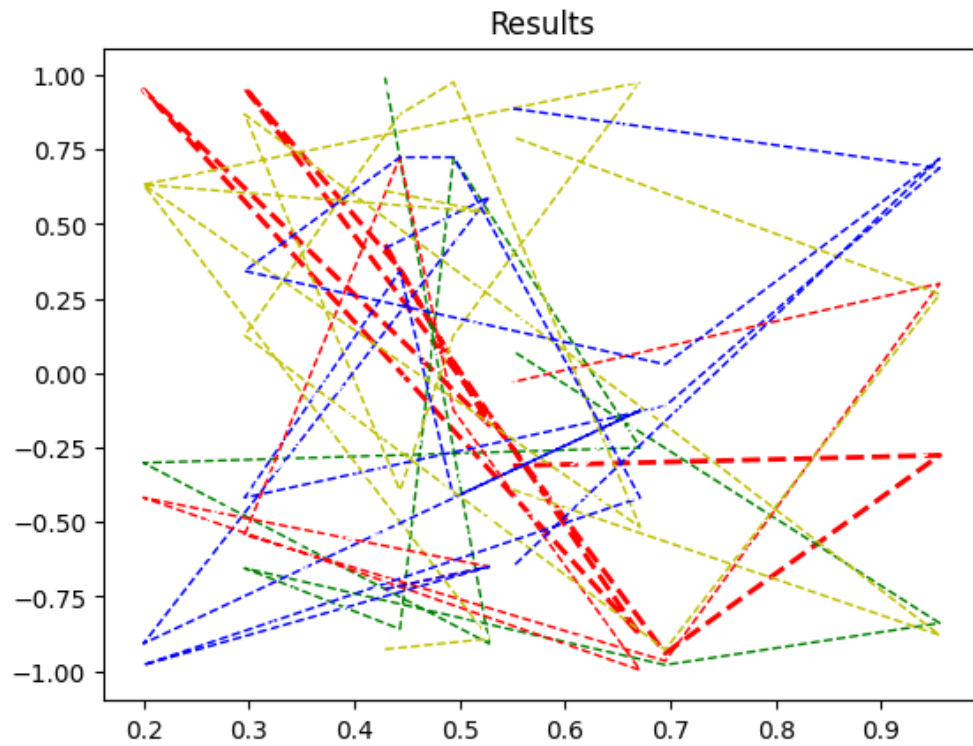


Figure 5. The combine decision matrix.

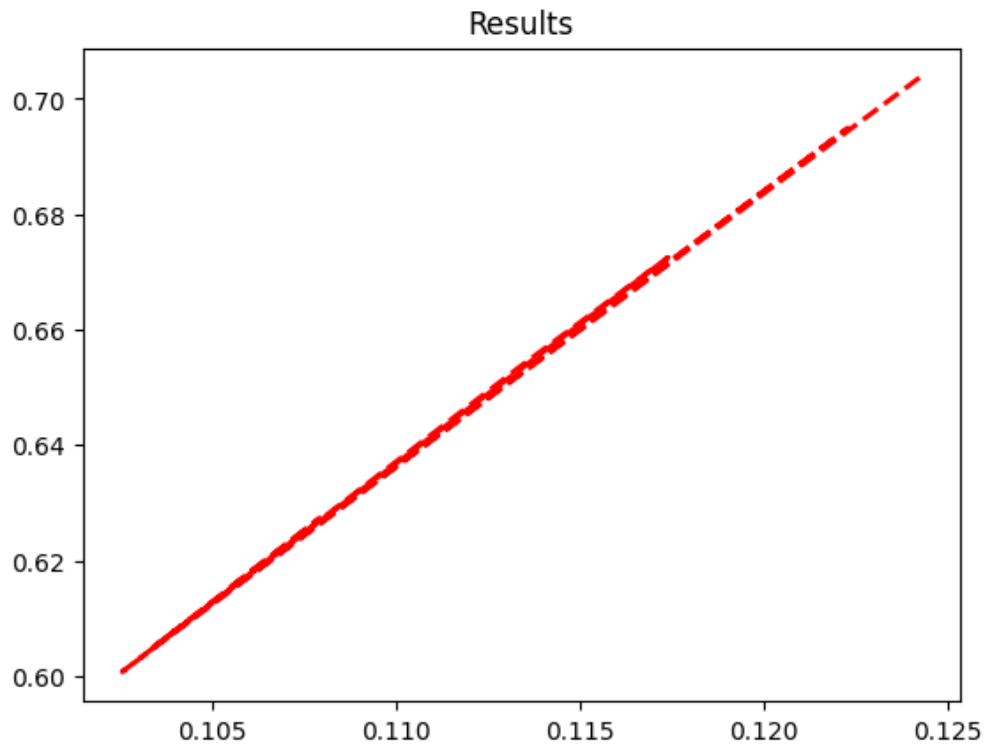


Figure 6. The criteria weights.

In the fourth branch, we combine the decision matrix into a single matrix as shown in Fig 7. Then we show the criteria weights as shown in Fig 8.

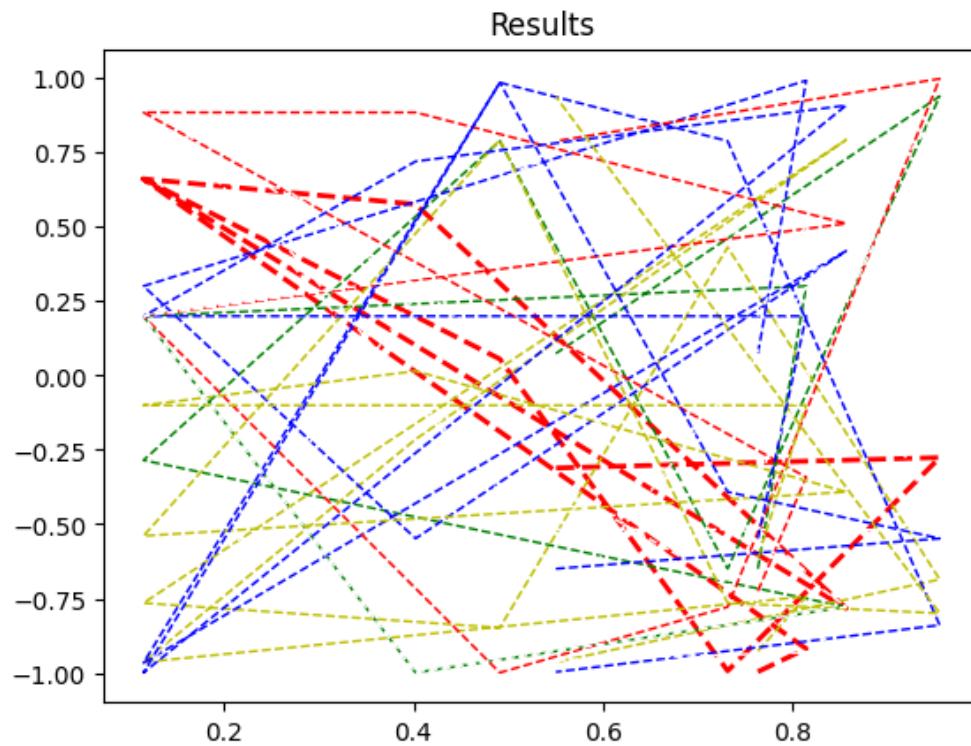


Figure 7. The combine decision matrix.

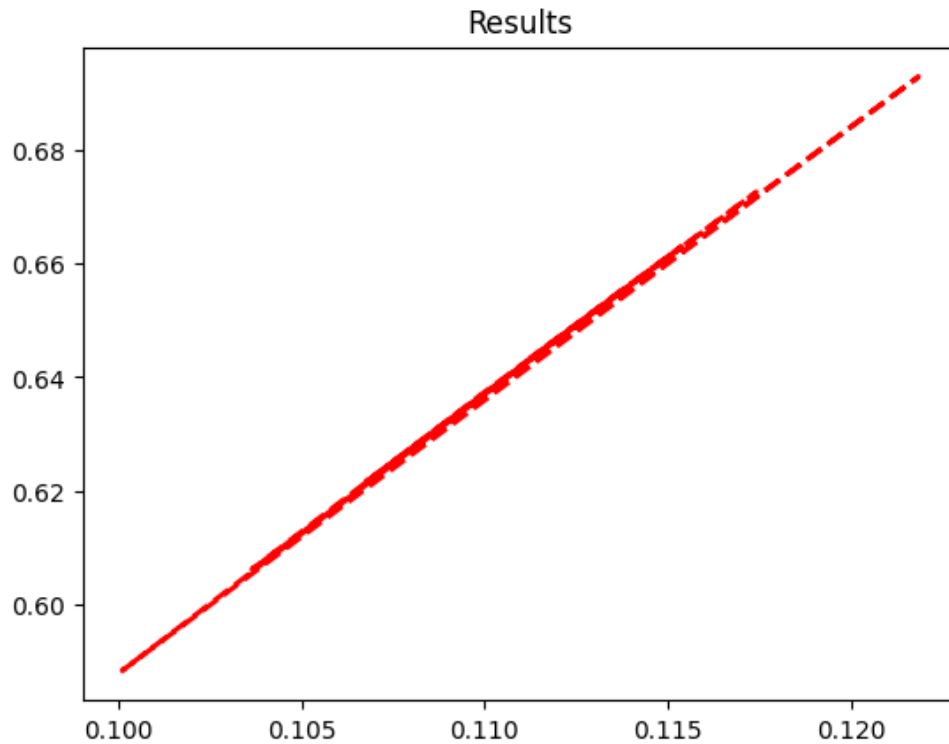


Figure 8. The criteria weights.

We obtain the global criteria weights such as:

- C1-1-1 0.0271 → Moderately important, ensuring course content is regularly refreshed.
- C1-1-2: 0.0247 → Slightly lower importance, but still essential for hands-on training.
- C1-1-3: 0.0303 → High importance, indicating strong emphasis on industry-recognized qualifications.
- C1-2-1: 0.0254 → Moderate value, showing engagement with industry in course design.
- C1-2-2: 0.0288 → Higher weight, stressing the need for continuous industry feedback.
- C1-2-3 0.0305 → Very important, highlighting industry professionals directly influencing education.
- C1-3-1: 0.0271 → Moderately high priority for keeping technical training up to date.
- C1-3-2 0.0292 → Important for integrating technology into education.
- C1-3-3: 0.0288 → Reflects a significant focus on realistic education experiences.
- C2-1-1: 0.0266 → Valuable for ensuring deep, meaningful work experience.

- C2-1-2: 0.0234 → Slightly less important but necessary to tie internships to studies.
- C2-1-3: 0.0267 → Moderate, emphasizing structured mentoring during internships.
- C2-2-1: 0.0242 → Important for guiding students during fieldwork.
- C2-2-2: 0.0258 → Moderately necessary for effective on-the-job training.
- C2-2-3: 0.0278 → High priority to track student education outcomes.
- C2-3-1: 0.0246 → Moderate focus, linking real-world problems to education.
- C2-3-2: 0.0252 → Necessary for deepening applied education.
- C2-3-3: 0.0255 → Helpful but a bit lower in emphasis compared to core projects.
- C3-1-1: 0.0350 → Highest among all, extremely crucial for upskilling faculty.
- C3-1-2: 0.0291 → Important but slightly less so than frequency.
- C3-1-3: 0.0345 → Very significant, ensuring fresh industry practices are taught.
- C3-2-1: 0.0289 → Important for promoting innovation through research.
- C3-2-2: 0.0319 → Fairly high importance, supporting active industry-driven R&D.
- C3-2-3: 0.0331 → Highly important, rewarding practical research achievements.
- C3-3-1: 0.0291 → Valuable, reflects ongoing faculty competence.
- C3-3-2: 0.0304 → Important to continually refresh skills.
- C3-3-3: 0.0297 → High value, making courses more career-relevant.
- C4-1-1: 0.0269 → Important for evaluating the success of education programs.
- C4-1-2: 0.0237 → Somewhat important, faster employment reflects better preparation.
- C4-1-3: 0.0288 → Strongly emphasized, linking training directly to careers.
- C4-2-1: 0.0245 → Needed to measure whether practical skills match jobs.
- C4-2-2: 0.0260 → Slightly higher focus on critical non-technical capabilities.
- C4-2-3: 0.0278 → Strong focus on graduates adjusting to real-world conditions.
- C4-3-1: 0.0253 → Important, relates to self-perceived readiness.
- C4-3-2: 0.0273 → Moderately high, showing long-term effectiveness.
- C4-3-3: 0.0263 → Valuable to identify areas for improvement.

We rank alternatives based on the criteria weights. We use DFTSS to split the alternatives into two frames such as:

$$D_1: \{A_1, A_2, A_3, A_4, A_5\}$$

$$D_2: \{A_6, A_7, A_8, A_9, A_{10}\}$$

In the first frame,

Compute the components $Q_{p_{ij}}$ of the theoretical rating matrix using eq. (4) as shown in Figure 9. Compute the real rating matrix using eq. (5 and 6) as shown in Figure 10. Compute the total gap matrix using eq. (7) as shown in Figure 11. Compute the value of each alternative using eq. (8) as Rank the alternatives as shown in Fig 12.

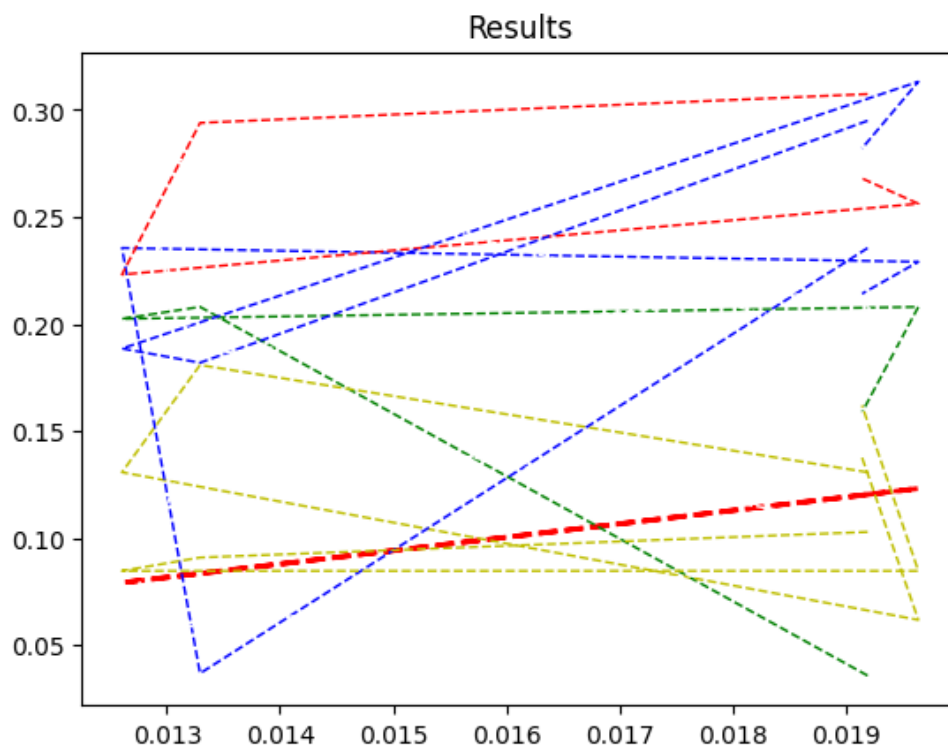


Fig 9. The theoretical rating matrix.

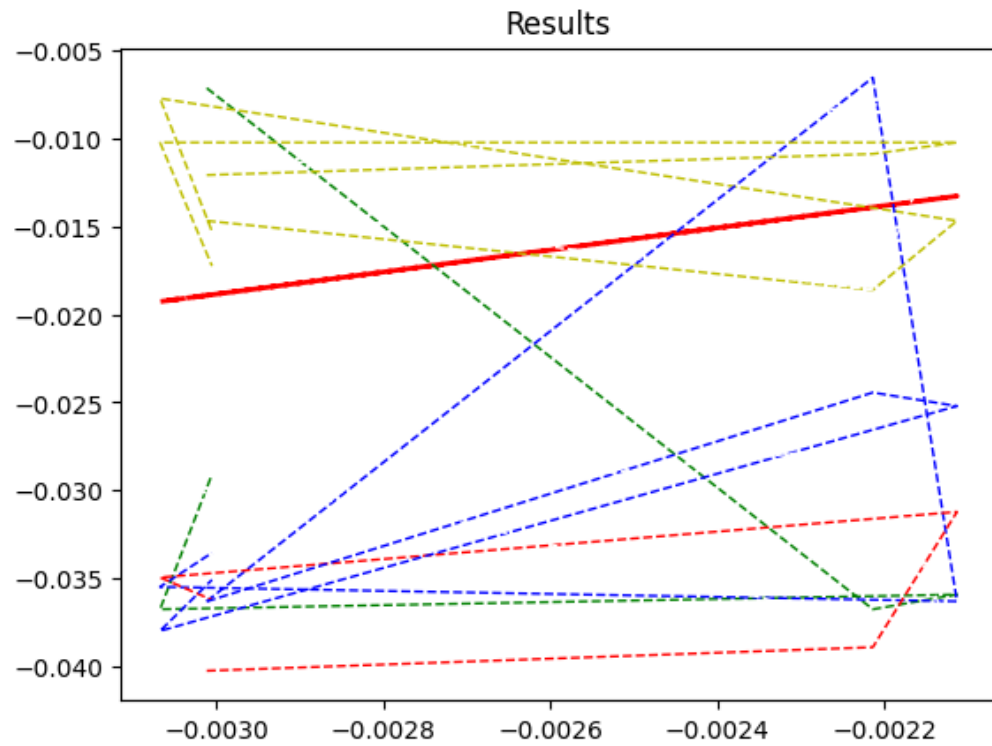


Fig 10. The rating matrix.

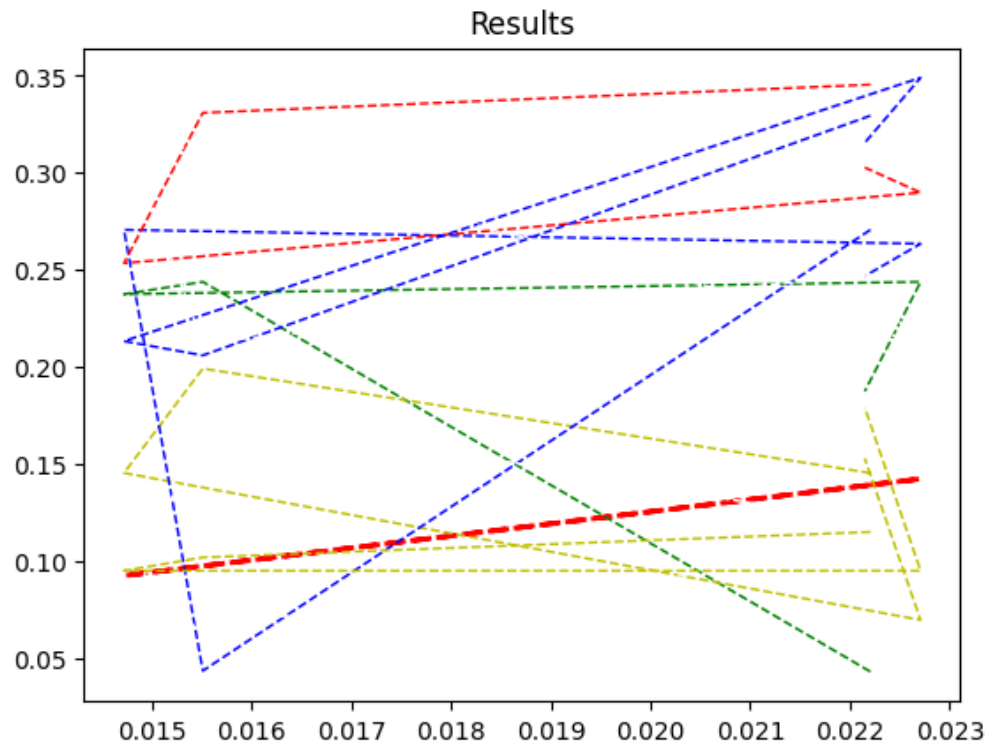


Fig 11. The total gap matrix.

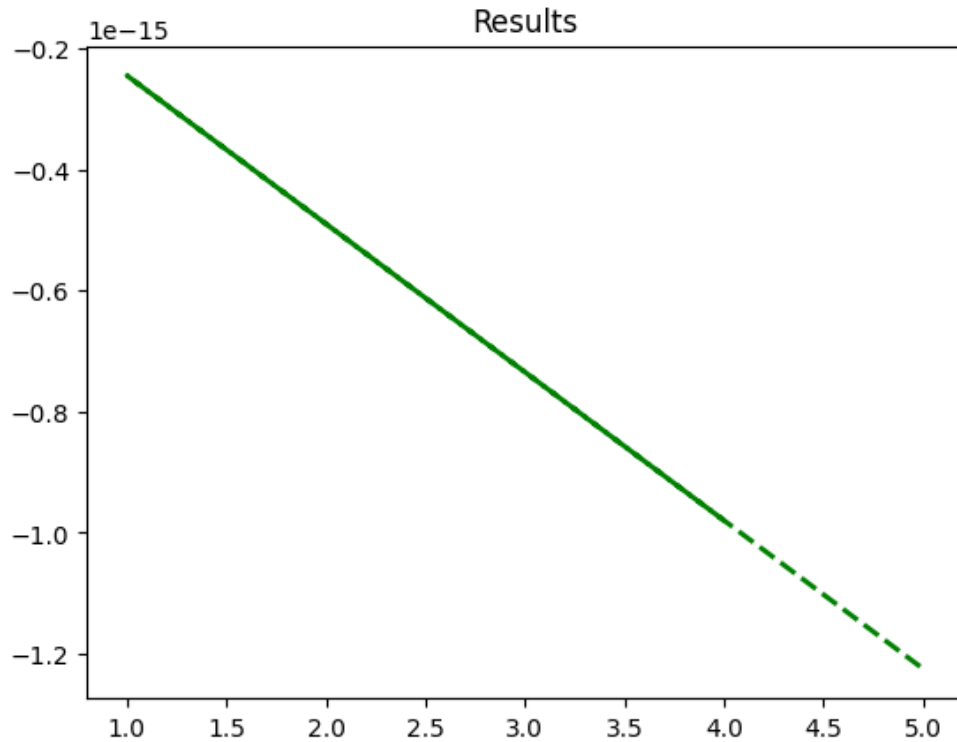


Fig 12. The ranks of alternative.

In the second frame,

Compute the components $Q_{p_{ij}}$ of the theoretical rating matrix using eq. (4) as shown in Figure 13. Compute the real rating matrix using eq. (5 and 6) as shown in Figure 14. Compute the total gap matrix using eq. (7) as shown in Figure 15. Compute the value of each alternative using eq. (8) as Rank the alternatives as shown in Fig 16.

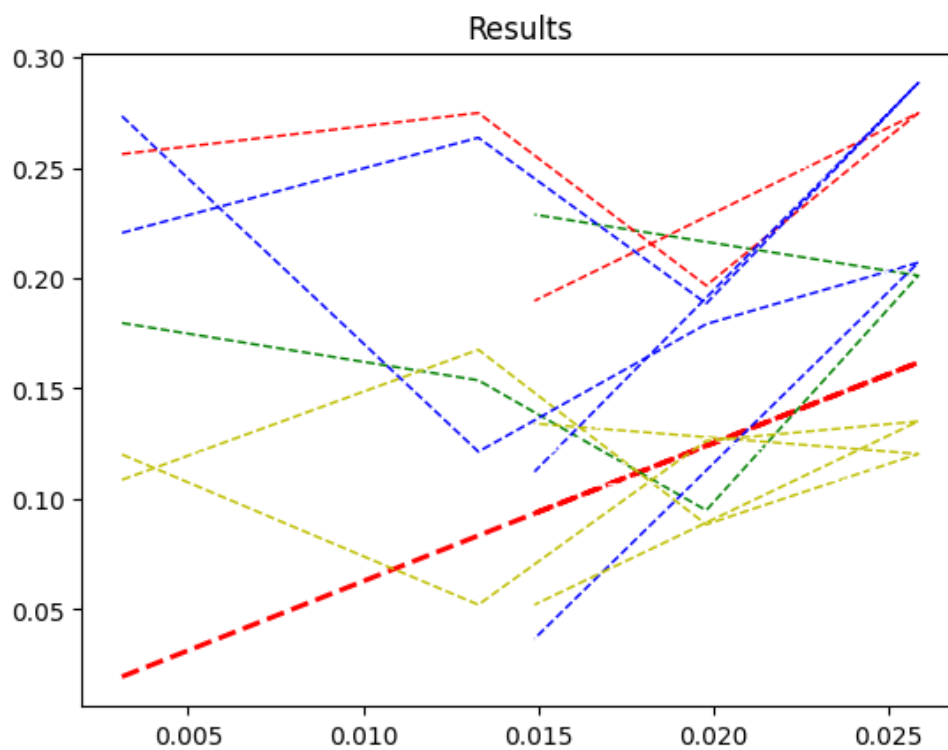


Fig 3. The theoretical rating matrix.

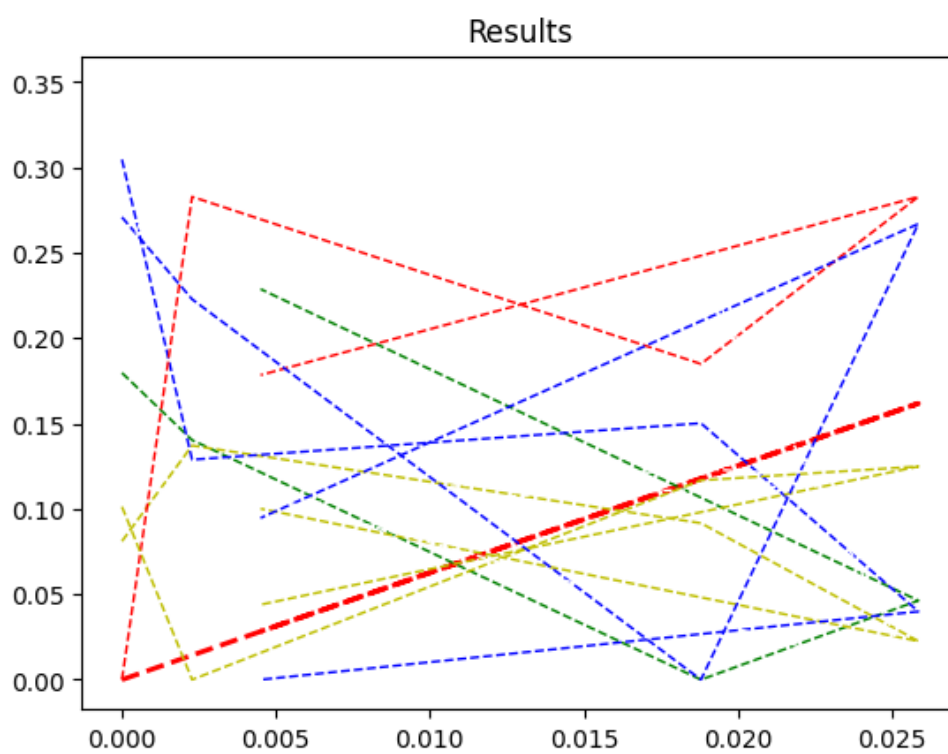


Fig 14. The rating matrix.

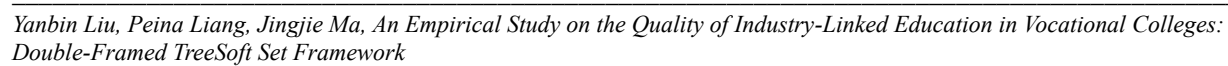


Fig 16. The ranks of alternative.

4. Conclusions

To adequately prepare students for the demands of contemporary companies, vocational institutions must provide high-quality industry-linked education. This empirical study emphasizes how companies and educational institutions must work together more closely to develop education experiences that are more dynamic, powerful, and relevant. Vocational colleges may enhance their strategies and produce more competent graduates and pleased industry partners by recognizing their strengths and areas for development. Double Framed TreeSoft Set is used in this study to divide alternatives to lower and higher. We use four main criteria and ten alternatives. In each main criterion, we compute the criteria weights and obtain the global weights. As industries change, future studies should keep an eye on these partnerships to make sure vocational education is adaptable, creative, and successful.

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Inverse Neutrosophic Planar Graphs and its application in optimizing the expenditure in inter-cropping.

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Abstract: Though Neutrosophic graphs have high impact on dealing with problems of indeterminacy, it may not directly address all aspects of edge membership degree constraints such as the truth membership degree of all edge is greater than or equal to the minimum of the membership degrees of its nodes is a specific structural constraint that may not be the case with the general framework of Neutrosophic graphs. This paper introduces a novel concept in the realm of Neutrosophic graphs defining a new graph variation called “Inverse Neutrosophic Graph”. This idea has the opportunity to enhance the field of Neutrosophic graph and provide new perspective into handling ambiguity in complex systems. Additionally, the basic properties and algebraic operations are studied in Inverse Neutrosophic graph. Moreover, we define Inverse Neutrosophic multi-graph and planarity in this paper. The related theories that establish a Neutrosophic Multi-graph’s strong (weak) planarity and score functions are then introduced. In our final section, we demonstrate how planarity in Inverse Neutrosophic Multi-graphs can be used to make decisions about how to minimize the expense of managing crops in inter cropping.

Keywords: Inverse Neutrosophic graphs, Inverse Neutrosophic (multi)graphs, planarity, decision making.

1.Introduction

Neutrosophic graphs are a new, emerging discipline area in graph theory and fuzzy logic, which falls under the larger theme of neutrosophy. Neutrosophy, initiated by Florentin Smarandach, is a generalization of classical logic that brings in the perception of indeterminacy and uncertainty, which provides models for complex and real-world systems. Neutrosophy introduced by Florentin Smarandache in the late 1990s, expands the classical ideas of true, false, and indeterminacy. It emphasizes the study of indeterminate and uncertain phenomena more than binary logic. In a neutrosophic graph, each edge or vertex is associated with a triplet of membership functions representing truth, indeterminacy, and falsity. Early research on neutrosophic graphs was primarily concerning on the extension of the classical graph theory concepts, such as connectivity and paths, to the neutrosophic framework. These studies were designed to define basic properties and operations within neutrosophic graphs, including the concept of neutrosophic distance and centrality. Also advanced studies were developed by focusing on algorithm for shortest path, minimum spanning tree and other graph related theories into neutrosophic theory. Furthermore, hybrid modelling was established by incorporating fuzzy or probabilistic graphs in neutrosophy structure to enhance the modelling of complex structure.

Arindam Dey and co-authors explores various types of operations on neutrosophic graph [2], Naz delves into the operations on Single valued neutrosophic graphs along with its application [4]. Zeng S; Shoaib M and co-authors, reviewed Certain Properties of Single-Valued Neutrosophic Graph with Application in Food and Agriculture Organization [11]. Akram analyses the planarity concept in single valued neutrosophic

graph and presented their exciting properties [1]. The Limitations in neutrosophic planar graph were unveiled by Rupkumar Mahapatra, Sovan Samanta & Madhumangal Pal through the investigation of generalized neutrosophic planar graphs by determining the overall planarity score including truth, falsity and indeterminacy. Also, their properties were analysed and effective utilisation of GNPG is explored with a real time applications. The Forgotten Topological Index (ToI) and the Edge Forgotten Index (EFI) that provide deeper insight for quantifying uncertainty and the quality of connections in a neutrosophic graph along with its application were explore by Vetrivel, G., Mullai, M., & Buvaneshwari, R, moreover these authors present a comprehensive evaluation that showcases the transition from fuzzy logic to neutrosophic logic in graph theory. [16,17]

The perception of Inverse Neutrosophic mixed graph were brought forward by Thempaavai and Antony crispin sweety that deals with the scenario, where truth membership degrees of the edges are $\geq$ to the minimum truth membership degrees of the associated vertices, and where the false membership and indeterminacy values of the edges are $\leq$ to the maximum false membership and indeterminacy values of the corresponding vertices. incorporating directed and undirected edges together[15]

The article probes deeper into the concept of Inverse Neutrosophic graph and provider in-depth perspective on

- Commencing with an introductory section that likely explains the motivation behind Inverse NG that is debuted as the solution for the scenario possessing inverse relationships.
- Exploring various mathematical concepts related to Inverse NG, such as complement, degrees, subgraphs and various operations and associated theorems within this framework. These concepts are clarified and demonstrated using examples.
- Familiarizing the concept of multigraphs, planarity, strong/weak planarity within the context of Inverse NG and its application that offers a significant insight.

2. Preliminary

Within this segment, the basic definitions are introduced.

DEFINITION 2.1

A fuzzy graph $G(\sigma, \mu)$ is an ordered pair, where σ signifies the fuzzy subset of vertices and μ signifies edge set that is a fuzzy relation on σ .

DEFINITION 2.2

A graph $G^* = [V, \sigma(T_1, I_1, F_1), \mu(T_2, I_2, F_2)]$ is triplet that is said to be Neutrosophic graph, where $\sigma(T_1, I_1, F_1)$ signifies the neutrosophic subset of vertices and $\mu(T_2, I_2, F_2)$ signifies edge set that is a neutrosophic connection on $\sigma(T_1, I_1, F_1)$.

DEFINITION 2.3

The Inverse fuzzy graph containing a non-void set $V, E \subseteq V \times V$ and $G = \{V, E, \mu, \tau\}$, where $\mu: V \rightarrow [0,1]$, $\tau: E \rightarrow [0,1]$ such that $\tau(ab) \geq \mu(a) \wedge \mu(b)$, $\forall ab \in E$ where μ signifies membership value of vertices and τ signifies membership value of the edges.

DEFINITION 2.4

A Neutrosophic multi graph, the triplet $G^* = [V, \sigma (T_1, I_1, F_1), \mu (T_{2n}, I_{2n}, F_{2n})]$ where

$V = \{v_1, v_2, v_3, \dots, v_n\}$ such that $T_1: V \rightarrow [0,1]$, $I_1: V \rightarrow [0,1]$ and $F_1: V \rightarrow [0,1]$ and $T_{2n}: V \times V \rightarrow [0,1]$, $I_{2n}: V \times V \rightarrow [0,1]$ and $F_{2n}: V \times V \rightarrow [0,1]$, $n = 1, 2, \dots, m \mid (v_i, v_j) \in V \times V \}$ such that

$$T_{2n}(v_i, v_j) \leq \min \{T_1(v_i), T_1(v_j)\}$$

$$I_{2n}(v_i, v_j) \leq \min \{I_1(v_i), I_1(v_j)\}$$

$$F_{2n}(v_i, v_j) \leq \max \{F_1(v_i), F_1(v_j)\}$$

and $0 \leq T_i(v) + I_i(v) + F_i(v) \leq 3 \quad T_{2n}(v_i, v_j) + I_{2n}(v_i, v_j) + F_{2n}(v_i, v_j) \leq 3 \quad \forall v_i \in V \quad (i = 1, 2, 3, \dots, n)$

here, (T_1, I_1, F_1) signify the degree of truth membership function, indeterminacy membership function, and falsity membership function of the vertex $v_1 \in V$ correspondingly and $T_{2n}(v_i, v_j)$, $I_{2n}(v_i, v_j)$ and $F_{2n}(v_i, v_j)$ signify the degree of truth-membership function, indeterminacy-membership function, and falsity-membership function of the edge (v_i, v_j) correspondingly.

DEFINITION 2.5

A graph is termed to be planar if it can be embedded in the plane in a manner that no edges intersect, except at their vertices.

3. INVERSE NEUTROSOPHIC GRAPH

Within this segment the Inverse Neutrosophic Graph is presented. Initially the Inspiration behind this work is given, along with some preliminary concepts are analyzed.

3.1 Inspiration behind this work

Despite numerous real time application of Neutrosophic graphs, they are unable to address many problems where graphs are employed for its representation. For the case, according to mixed cropping in agricultural, when we cultivate two crops X and Y together in the same field minimizes the expenditure and increases profitability over cultivating the crops individually, consequently Neutrosophic graphs is inefficient in addressing this scenario. In general, according to probability theory for mutually exclusive events, the probability of their union is equal to the sum of the probabilities of each individual event. whereas for independent events the probability of union of two events is equal to the sum of probabilities of each individual events from which the probabilities of its intersection are subtracted. Additionally, the probabilities of intersection of two events are $\leq$ to minimum of individual events. Also, the probability of union of two events is $\geq$ to maximum of individual events that is greater than equal to minimum of two individual events in both the scenarios. Considering the event X to be the vertices of the graph, the true, indeterminacy and false membership values for each vertex can be assigned such that membership values can be equal to the probability of the associated events. more over the edge between two vertices X and Y represents the event happening $X \cup Y$, the truth membership of edge will be $\geq$ to minimum of the

membership degrees of its vertices, indeterminacy and false membership degree of edge will be $\leq$ to maximum of the membership degrees of its vertices. To deal with the issues aforementioned, we propose the Inverse Neutrosophic Graph to handle the problem where traditional graphs fall short. We evaluate the properties of Inverse NG and examine the planarity in Inverse NG to determine the effectiveness and developed the relates theorems. Finally explore the application for this model.

3.2. Frame work of Inverse NG

DEFINITION 3.1

Let $\mathbb{V}$ be a finite vertex set and $\mathbb{E}$ be a finite edge set contained in $\mathbb{V} \times \mathbb{V}$. Then $\zeta = [\mathbb{V}, \mathbb{E}, (\chi_T, \chi_I, \chi_F), (\varepsilon_T, \varepsilon_I, \varepsilon_F)]$ is said to be an Inverse Neutrosophic Graph, were $(\varepsilon_T, \varepsilon_I, \varepsilon_F): \mathbb{V} \rightarrow [0, 1]$, $(\chi_T, \chi_I, \chi_F): \mathbb{E} \rightarrow [0, 1]$ such that,

$$\chi_T(mn) \geq \varepsilon_T(m) \wedge \varepsilon_T(n), \forall mn \in \mathbb{E}$$

$$\chi_I(mn) \leq \varepsilon_I(m) \vee \varepsilon_I(n), \forall mn \in \mathbb{E}$$

$$\chi_F(mn) \leq \varepsilon_F(m) \vee \varepsilon_F(n), \forall mn \in \mathbb{E}$$

thereabouts $(\varepsilon_T, \varepsilon_I, \varepsilon_F)$ denotes the true – membership degree of vertices, indeterminacy – membership degree of vertices and false membership degree of vertices consequently, (χ_T, χ_I, χ_F) denotes the true-membership degree of edges, indeterminacy - membership degree of edges and false membership degree of edges consequently.

Example 3.1

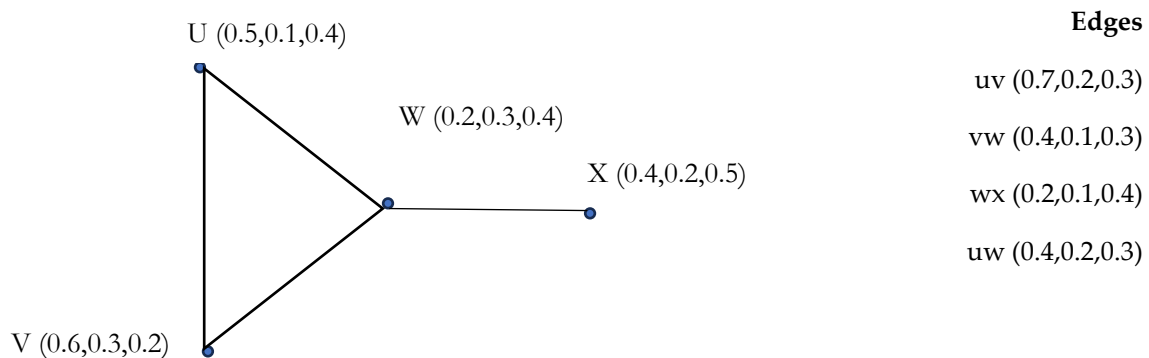


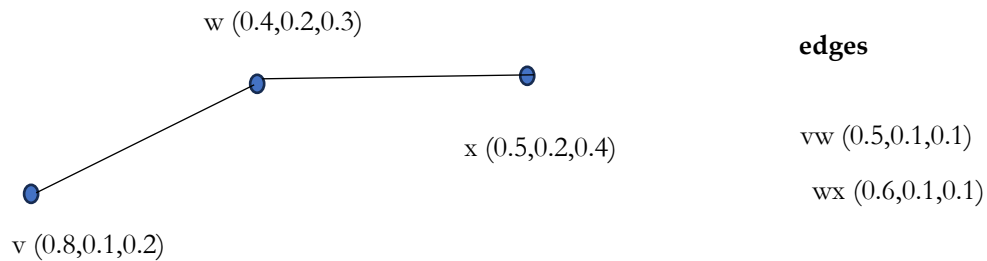
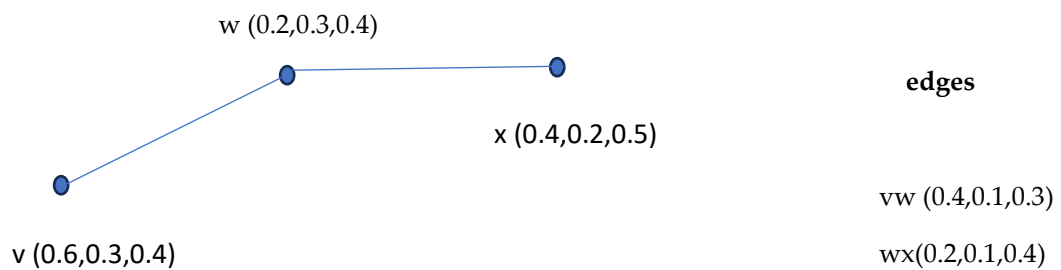
Figure 1.

DEFINITION 3.2

Assume $\zeta' = [\mathbb{V}', \mathbb{E}', (\chi'_T, \chi'_I, \chi'_F), (\varepsilon'_T, \varepsilon'_I, \varepsilon'_F)]$ and $\zeta'' = [\mathbb{V}'', \mathbb{E}'', (\chi''_T, \chi''_I, \chi''_F), (\varepsilon''_T, \varepsilon''_I, \varepsilon''_F)]$ be two Inverse NG. Then ζ'' is termed as the partial Inverse Neutrosophic subgraph, if $\mathbb{V}'' \subseteq \mathbb{V}'$, where $\chi''_{Ti} \geq \chi'_{Ti}$, $\chi''_{Ii} \leq \chi'_{Ii}$ and $\chi''_{Fi} \leq \chi'_{Fi}$, for all $m_i \in \mathbb{V}''$. ζ'' is termed as an Inverse Neutrosophic subgraph, if

$\mathbb{V}'' \subseteq \mathbb{V}'$, where $\chi''_{Tij} = \chi'_{Tij}$, $\chi''_{Iij} = \chi'_{Iij}$ and $\chi''_{Fij} = \chi'_{Fij}$ for all $(m_i, n_j) \in \mathbb{E}''$.

Corollary: It's obvious that Inverse Neutrosophic subgraph is partial Inverse NG but reverse is not factual generally.

Example 3.2**Figure 2.a****Figure 2.b.****DEFINITION 3.3**

Assume $\zeta = [\mathbb{V}, \mathbb{E}, (\chi_T, \chi_I, \chi_F), (\varepsilon_T, \varepsilon_I, \varepsilon_F)]$ is an Inverse NG, subsequently

(i) The vertex degree of any vertex $m \in \mathbb{V}$ is signified by $\deg(m)$ and described as

$$\deg(m) = (\deg_T(m), \deg_I(m), \deg_F(m)) \text{ where}$$

$$\deg_T(m) = \sum_{mn \in \mathbb{E}} \chi_T(mn),$$

$$\deg_I(m) = \sum_{mn \in \mathbb{E}} \chi_I(mn),$$

$$\deg_F(m) = \sum_{mn \in \mathbb{E}} \chi_F(mn).$$

(ii) The order of ζ is signified by $O(\zeta)$ and well-defined as

$$O(\zeta) = \sum_{m \in \mathbb{V}} (\varepsilon_T(m), \varepsilon_I(m), \varepsilon_F(m)).$$

(iii) The **size** of ζ is signified by $S(\zeta)$ and well-defined as $S = \sum_{mn \in \mathbb{E}} (\chi_T(mn), \chi_I(mn), \chi_F(mn))$

Theorem 3.1

Assume that ζ is an Inverse NG. subsequently $\sum_{m \in \mathbb{V}} \deg(m) = 2 \sum_{mn \in \mathbb{E}} (\chi_T(mn), \chi_I(mn), \chi_F(mn))$

Proof: straight forward

Example 3.3

Consider the Inverse NG ζ as in Figure 3.

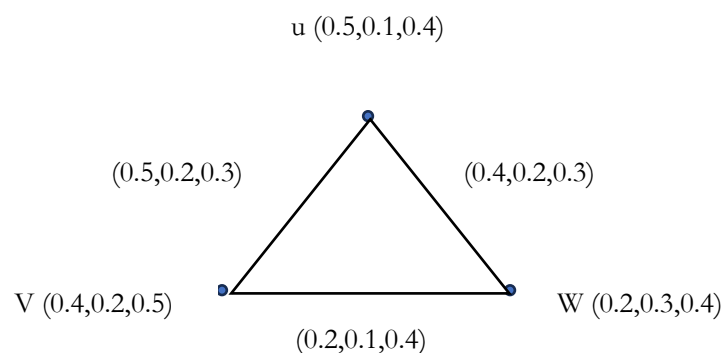


Figure.3

The degree of vertices of ζ , $\deg(u) = (0.9, 0.4, 0.6)$, $\deg(v) = (0.7, 0.3, 0.7)$, $\deg(w) = (0.6, 0.3, 0.7)$

The order of ζ , $O(\zeta) = (1.1, 0.6, 1.3)$ and the size of ζ , $S(\zeta) = (1.1, 0.5, 1)$.

Also, $\sum_{m \in V} \deg(m) = \deg(u) + \deg(v) + \deg(w) = (0.9, 0.4, 0.6) + (0.7, 0.3, 0.7) + (0.6, 0.3, 0.7) = (2.2, 1.2) = 2 \sum_{mn \in E} \chi_T(mn), \chi_I(mn), \chi_F(mn) = 2(1.1, 0.5, 1)$.

DEFINITION 3.4

Assume $\zeta = [V, E, (\chi_T, \chi_I, \chi_F), (\varepsilon_T, \varepsilon_I, \varepsilon_F)]$ is an Inverse NG, subsequently ζ is said to have stable vertex if for any $mn \in E$, $(\varepsilon_T, \varepsilon_I, \varepsilon_F)(m) = (\varepsilon_T, \varepsilon_I, \varepsilon_F)(n)$ and to have stable edge if for any $mn \in E$, $(\chi_T, \chi_I, \chi_F)(mn) = (\chi_T, \chi_I, \chi_F)(uv)$ and said to be stable if ζ has both stable vertex and stable edge.

Example 3.4

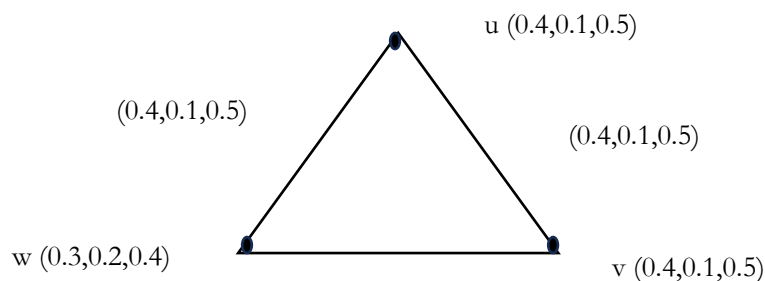


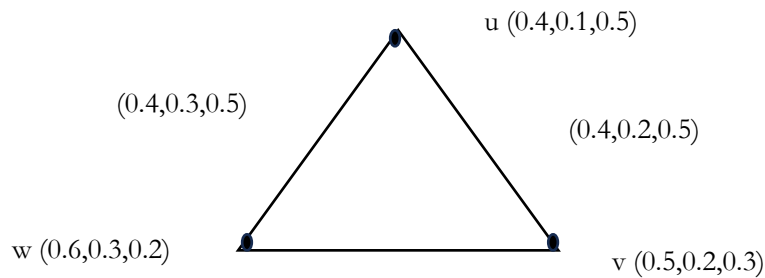
Figure 4.

In the above figure, vertex u and v are said to be a stable vertex and edges uv & uw are said to be stable edges.

DEFINITION 3.5

Assume $\zeta = [V, E, (\chi_T, \chi_I, \chi_F), (\varepsilon_T, \varepsilon_I, \varepsilon_F)]$ is an Inverse NG, subsequently **complete** Inverse NG is defined as

$$\begin{aligned}\chi_T(mn) &= \varepsilon_T(m) \wedge \varepsilon_T(n), \forall mn \in E \\ \chi_I(mn) &= \varepsilon_I(m) \vee \varepsilon_I(n), \forall mn \in E \\ \chi_F(mn) &= \varepsilon_F(m) \vee \varepsilon_F(n), \forall mn \in E\end{aligned}$$

Example 3.5**Figure 5.****4. Operations on Inverse NG.**

We explore several operations involving Inverse NG, such as union, intersection and cartesian product.

DEFINITION 4.1

Assume $\zeta' = [\mathbb{V}', \mathbb{E}', (\chi'_T, \chi'_I, \chi'_F), (\varepsilon'_t, \varepsilon'_I, \varepsilon'_F)]$ and $\zeta'' = [\mathbb{V}'', \mathbb{E}'', (\chi''_T, \chi''_I, \chi''_F), (\varepsilon''_t, \varepsilon''_I, \varepsilon''_F)]$ be two ING. Subsequently, we denote and define the union of Inverse NG ζ' and ζ'' as

$$\zeta' \cup \zeta'' = [(\mathbb{V}' \cup \mathbb{V}''), (\mathbb{E}' \cup \mathbb{E}''), (\chi'_T \cup \chi''_T, \chi'_I \cup \chi''_I, \chi'_F \cup \chi''_F), (\varepsilon'_t \cup \varepsilon''_t, \varepsilon'_I \cup \varepsilon''_I, \varepsilon'_F \cup \varepsilon''_F)]$$

$$\varepsilon'_T \cup \varepsilon''_T(m) = \begin{cases} \varepsilon'_T(m) & \text{when } m \in \mathbb{V}' \text{ and } m \notin \mathbb{V}'' \\ \varepsilon''_T(m) & \text{when } m \in \mathbb{V}'' \text{ and } m \notin \mathbb{V}' \\ \varepsilon'_T(m) \vee \varepsilon''_T(m) & \text{when } m \in \mathbb{V}' \cap \mathbb{V}'' \end{cases}$$

$$\varepsilon'_I \cup \varepsilon''_I(m) = \begin{cases} \varepsilon'_I(m) & \text{when } m \in \mathbb{V}' \text{ and } m \notin \mathbb{V}'' \\ \varepsilon''_I(m) & \text{when } m \in \mathbb{V}'' \text{ and } m \notin \mathbb{V}' \\ \varepsilon'_I(m) \wedge \varepsilon''_I(m) & \text{when } m \in \mathbb{V}' \cap \mathbb{V}'' \end{cases}$$

$$\varepsilon'_F \cup \varepsilon''_F(m) = \begin{cases} \varepsilon'_F(m) & \text{when } m \in \mathbb{V}' \text{ and } m \notin \mathbb{V}'' \\ \varepsilon''_F(m) & \text{when } m \in \mathbb{V}'' \text{ and } m \notin \mathbb{V}' \\ \varepsilon'_F(m) \wedge \varepsilon''_F(m) & \text{when } m \in \mathbb{V}' \cap \mathbb{V}'' \end{cases}$$

$$\chi'_T \cup \chi''_T(mn) = \begin{cases} \chi'_T(mn) & \text{when } mn \in \mathbb{E}' \text{ and } mn \notin \mathbb{E}'' \\ \chi''_T(mn) & \text{when } mn \in \mathbb{E}'' \text{ and } mn \notin \mathbb{E}' \\ \chi'_T(mn) \vee \chi''_T(mn) & \text{when } mn \in \mathbb{E}' \cap \mathbb{E}'' \end{cases}$$

$$\chi'_I \cup \chi''_I(mn) = \begin{cases} \chi'_I(mn) & \text{when } mn \in \mathbb{E}' \text{ and } mn \notin \mathbb{E}'' \\ \chi''_I(mn) & \text{when } mn \in \mathbb{E}'' \text{ and } mn \notin \mathbb{E}' \\ \chi'_I(mn) \wedge \chi''_I(mn) & \text{when } mn \in \mathbb{E}' \cap \mathbb{E}'' \end{cases}$$

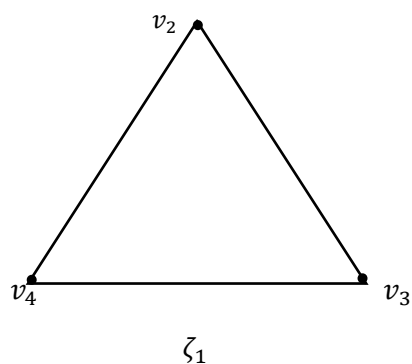
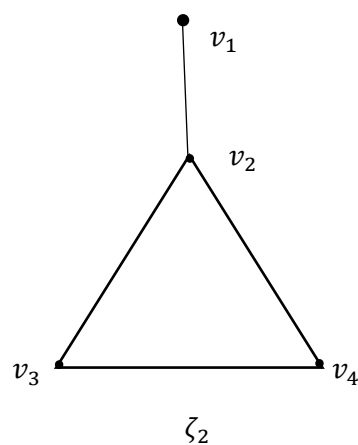
$$\chi'_F \cup \chi''_F(mn) = \begin{cases} \chi'_F(mn) & \text{when } mn \in \mathbb{E}' \text{ and } mn \notin \mathbb{E}'' \\ \chi''_F(mn) & \text{when } mn \in \mathbb{E}'' \text{ and } mn \notin \mathbb{E}' \\ \chi'_F(mn) \wedge \chi''_F(mn) & \text{when } mn \in \mathbb{E}' \cap \mathbb{E}'' \end{cases}$$

Example 4.1**Table 1:** Two Inverse Neutrosophic graphs ζ_1 and ζ_2 as in **Figure 6.a**

Vertices of ζ_1	Edges of ζ_1	Vertices of ζ_2	Edges of ζ_2
$v_2(0.1,0.2,0.1)$	$v_2v_3(0.4,0.1,0.1)$	$v_1(0.5,0.1,0.4)$	$v_1v_2(0.7,0.2,0.2)$
$v_3(0.5,0.2,0.4)$	$v_4v_3(0.3,0,0.2)$	$v_2(0.6,0.3,0.2)$	$v_2v_3(0.8,0.1,0.3)$
$v_4(0.1,0.2,0.2)$	$v_2v_4(0.2,0.1,0.1)$	$v_3(0.2,0.3,0.4)$	$v_4v_3(0.3,0.1,0.3)$
		$v_4(0.4,0.2,0.5)$	$v_2v_4(0.5,0.1,0.4)$

Table 2: The union of two Inverse Neutrosophic graph is **Figure 6.b**

Vertices of $\zeta_1 \cup \zeta_2$	Edges of $\zeta_1 \cup \zeta_2$
$v_1(0.5,0.1,0.4)$	$v_1v_2(0.7,0.2,0.2)$
$v_2(0.6,0.2,0.1)$	$v_2v_3(0.8,0.1,0.1)$
$v_3(0.5,0.2,0.4)$	$v_4v_3(0.3,0,0.2)$
$v_4(0.4,0.2,0.2)$	$v_2v_4(0.5,0.1,0.1)$

**Figure 6.a****Figure 6.b**

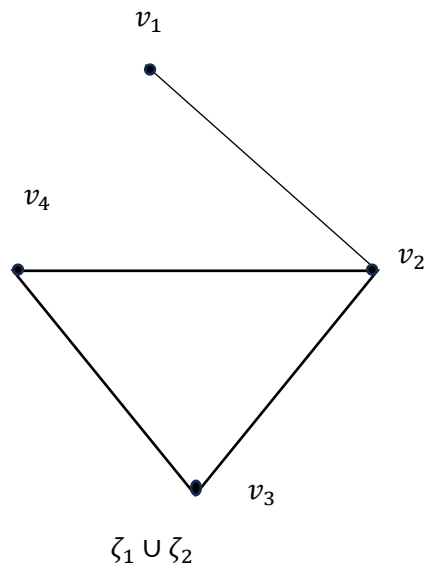


Figure 6.c

From the above example it is evident that union of two inverse Neutrosophic graph is not an inverse Neutrosophic graph. Therefore, a new operation called quasi – Union is introduced which ensures that resulting graph maintains the characteristics of Inverse Neutrosophic graph.

DEFINITION 4.1

Assume $\zeta' = [\mathbb{V}', \mathbb{E}', (\chi'_T, \chi'_I, \chi'_F), (\varepsilon'_t, \varepsilon'_I, \varepsilon'_F)]$ and $\zeta'' = [\mathbb{V}'', \mathbb{E}'', (\chi''_T, \chi''_I, \chi''_F), (\varepsilon''_t, \varepsilon''_I, \varepsilon''_F)]$ be two ING. Subsequently, we denote and define the quasi-union of Inverse NG ζ' and ζ'' as

$$\zeta' \cup \zeta'' = [(\mathbb{V}' \cup \mathbb{V}''), (\mathbb{E}' \cup \mathbb{E}''), (\chi'_T \cup \chi''_T), (\chi'_I \cup \chi''_I), (\chi'_F \cup \chi''_F), (\varepsilon'_t \cup \varepsilon''_t), (\varepsilon'_I \cup \varepsilon''_I), (\varepsilon'_F \cup \varepsilon''_F)]$$

$$\varepsilon'_T \cup \varepsilon''_T(m) = \begin{cases} \varepsilon'_T(m) & \text{when } m \in \mathbb{V}' \text{ and } m \notin \mathbb{V}'' \\ \varepsilon''_T(m) & \text{when } m \in \mathbb{V}'' \text{ and } m \notin \mathbb{V}' \\ \varepsilon'_T(m) \wedge \varepsilon''_T(m) & \text{when } m \in \mathbb{V}' \cap \mathbb{V}'' \end{cases}$$

$$\varepsilon'_I \cup \varepsilon''_I(m) = \begin{cases} \varepsilon'_I(m) & \text{when } m \in \mathbb{V}' \text{ and } m \notin \mathbb{V}'' \\ \varepsilon''_I(m) & \text{when } m \in \mathbb{V}'' \text{ and } m \notin \mathbb{V}' \\ \varepsilon'_I(m) \vee \varepsilon''_I(m) & \text{when } m \in \mathbb{V}' \cap \mathbb{V}'' \end{cases}$$

$$\varepsilon'_F \cup \varepsilon''_F(m) = \begin{cases} \varepsilon'_F(m) & \text{when } m \in \mathbb{V}' \text{ and } m \notin \mathbb{V}'' \\ \varepsilon''_F(m) & \text{when } m \in \mathbb{V}'' \text{ and } m \notin \mathbb{V}' \\ \varepsilon'_F(m) \vee \varepsilon''_F(m) & \text{when } m \in \mathbb{V}' \cap \mathbb{V}'' \end{cases}$$

$$\chi'_T \cup \chi''_T(mn) = \begin{cases} \chi'_T(mn) & \text{when } mn \in \mathbb{E}' \text{ and } mn \notin \mathbb{E}'' \\ \chi''_T(mn) & \text{when } mn \in \mathbb{E}'' \text{ and } mn \notin \mathbb{E}' \\ \chi'_T(mn) \vee \chi''_T(mn) & \text{when } mn \in \mathbb{E}' \cap \mathbb{E}'' \end{cases}$$

$$\chi'_I \cup \chi''_I(mn) = \begin{cases} \chi'_I(mn) & \text{when } mn \in \mathbb{E}' \text{ and } mn \notin \mathbb{E}'' \\ \chi''_I(mn) & \text{when } mn \in \mathbb{E}'' \text{ and } mn \notin \mathbb{E}' \\ \chi'_I(mn) \wedge \chi''_I(mn) & \text{when } mn \in \mathbb{E}' \cap \mathbb{E}'' \end{cases}$$

$$\chi'_F \cup \chi''_F(mn) = \begin{cases} \chi'_F(mn) & \text{when } mn \in \mathbb{E}' \text{ and } mn \notin \mathbb{E}'' \\ \chi''_F(mn) & \text{when } mn \in \mathbb{E}'' \text{ and } mn \notin \mathbb{E}' \\ \chi'_F(mn) \wedge \chi''_F(mn) & \text{when } mn \in \mathbb{E}' \cap \mathbb{E}'' \end{cases}$$

Theorem 4.1

The Quasi - Union of every two ING is an ING.

Proof:

Consider ζ' and ζ'' be two Inverse Neutrosophic graphs.

For every $mn \in \mathbb{E}' \cup \mathbb{E}''$, one of these three instances occur.

Instance I:

Assume, $mn \in \mathbb{E}'$ and $mn \notin \mathbb{E}''$, Thus

$$(i) \quad \chi_T(mn) = \chi'_T(mn) \geq \chi'_T(m) \wedge \chi'_T(n)$$

given that, $mn \in \mathbb{E}'$ and $mn \notin \mathbb{E}''$ therefore $m \in \mathbb{V}'$ and $m \notin \mathbb{V}''$ or $m \in \mathbb{V}' \cap \mathbb{V}''$

(i.e.) $\varepsilon_T(m) = \varepsilon'_T(m)$ or $\varepsilon_T(m) = \varepsilon'_T(m) \wedge \varepsilon''_T(m)$, thus $\varepsilon'_T(m) \geq \varepsilon_T(m)$.

Furthermore $\varepsilon'_T(n) \geq \varepsilon_T(n)$.

Therefore $\chi_T(mn) = \chi'_T(mn) \geq \chi'_T(m) \wedge \chi'_T(n) \geq \chi_T(m) \wedge \chi_T(n)$.

$$(ii) \quad \chi_I(mn) = \chi'_I(mn) \leq \chi'_I(m) \vee \chi'_I(n)$$

given that, $mn \in \mathbb{E}'$ and $mn \notin \mathbb{E}''$ therefore $m \in \mathbb{V}'$ and $m \notin \mathbb{V}''$ or $m \in \mathbb{V}' \cap \mathbb{V}''$

(i.e.) $\varepsilon_I(m) = \varepsilon'_I(m)$ or $\varepsilon_I(m) = \varepsilon'_I(m) \vee \varepsilon''_I(m)$, thus $\varepsilon'_I(m) \leq \varepsilon_I(m)$.

Furthermore $\varepsilon'_I(n) \leq \varepsilon_I(n)$.

Therefore $\chi_I(mn) = \chi'_I(mn) \leq \chi'_I(m) \vee \chi'_I(n) \leq \chi_I(m) \vee \chi_I(n)$

$$(iii) \quad \chi_F(mn) = \chi'_F(mn) \leq \chi'_F(m) \vee \chi'_F(n)$$

given that, $mn \in \mathbb{E}'$ and $mn \notin \mathbb{E}''$ therefore $m \in \mathbb{V}'$ and $m \notin \mathbb{V}''$ or $m \in \mathbb{V}' \cap \mathbb{V}''$

(i.e.) $\varepsilon_F(m) = \varepsilon'_F(m)$ or $\varepsilon_F(m) = \varepsilon'_F(m) \vee \varepsilon''_F(m)$, thus $\varepsilon'_F(m) \leq \varepsilon_F(m)$.

Furthermore $\varepsilon'_F(n) \leq \varepsilon_F(n)$.

Therefore $\chi_F(mn) = \chi'_F(mn) \leq \chi'_F(m) \vee \chi'_F(n) \leq \chi_F(m) \vee \chi_F(n)$

Instance II:

Assume, $mn \in \mathbb{E}''$ and $mn \notin \mathbb{E}'$, Thus in similar way we obtain,

$$\chi_T(mn) \geq \chi_T(m) \wedge \chi_T(n), \chi_I(mn) \leq \chi_I(m) \vee \chi_I(n), \chi_F(mn) \leq \chi_F(m) \vee \chi_F(n)$$

Instance III:

Assume $mn \in \mathbb{E}' \cap \mathbb{E}''$, Thus

$$(i) \quad \chi_T(mn) = \chi'_T(mn) \vee \chi''_T(mn), \text{ where } m, n \in \mathbb{V}' \cap \mathbb{V}''$$

(i.e.) $\varepsilon_T(m) = \varepsilon'_T(m) \wedge \varepsilon''_T(m)$ and $\varepsilon_T(n) = \varepsilon'_T(n) \wedge \varepsilon''_T(n)$.

Therefore, $\chi_T(mn) = \chi'_T(mn) \vee \chi''_T(mn) \geq \{(\varepsilon'_T(m) \wedge \varepsilon'_T(n)) \vee (\varepsilon''_T(m) \wedge \varepsilon''_T(n))\}$

$$\geq \chi_T(m) \vee \chi_T(n).$$

$$(ii) \chi_I(mn) = \chi'_I(mn) \wedge \chi''_I(mn), \text{ where } m, n \in \mathbb{V}' \cap \mathbb{V}''$$

$$(i.e.) \varepsilon_I(m) = \varepsilon'_I(m) \vee \varepsilon''_I(m) \text{ and } \varepsilon_I(n) = \varepsilon'_I(n) \vee \varepsilon''_I(n).$$

$$\text{Therefore, } \chi_I(mn) = \chi'_I(mn) \wedge \chi''_I(mn) \leq \{(\varepsilon'_I(m) \vee \varepsilon'_I(n)) \wedge (\varepsilon''_I(m) \vee \varepsilon''_I(n))\}$$

$$\leq \chi_I(m) \wedge \chi_I(n).$$

$$(iii) \chi_F(mn) = \chi'_F(mn) \wedge \chi''_F(mn), \text{ where } m, n \in \mathbb{V}' \cap \mathbb{V}''$$

$$(i.e.) \varepsilon_F(m) = \varepsilon'_F(m) \vee \varepsilon''_F(m) \text{ and } \varepsilon_F(n) = \varepsilon'_F(n) \vee \varepsilon''_F(n).$$

$$\text{Therefore, } \chi_F(mn) = \chi'_F(mn) \wedge \chi''_F(mn) \leq \{(\varepsilon'_F(m) \vee \varepsilon'_F(n)) \wedge (\varepsilon''_F(m) \vee \varepsilon''_F(n))\}$$

$$\leq \chi_F(m) \wedge \chi_F(n).$$

Hence, The Quasi - Union of ζ' and ζ'' is an Inverse Neutrosophic Graph.

Hence the proof

Example 4.2

Considering two Inverse Neutrosophic Graph in **Figure 6.a & 6.b**,

The quasi union of ζ_1 and ζ_2 is

Table 3: Quasi – union of ζ_1 and ζ_2

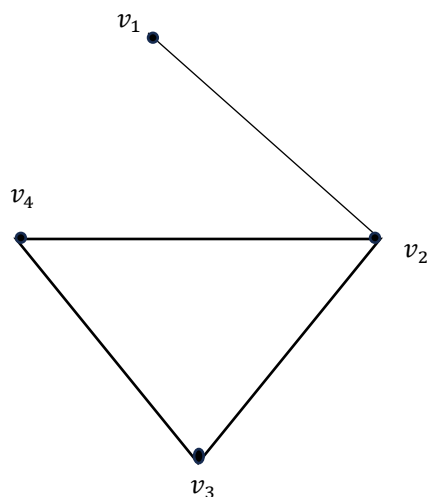


Figure 7

Vertices of $\zeta_1 \cup \zeta_2$	Edges of $\zeta_1 \cup \zeta_2$
$v_1(0.5,0.1,0.4)$	$v_1v_2(0.7,0.2,0.2)$
$v_2(0.1,0.3,0.2)$	$v_2v_3(0.8,0.1,0.1)$
$v_3(0.2,0.3,0.4)$	$v_4v_3(0.3,0,0.2)$
$v_4(0.1,0.2,0.5)$	$v_2v_4(0.5,0.1,0.1)$

DEFINITION 4.3

Assume $\zeta' = [\mathbb{V}', \mathbb{E}', (\chi'_T, \chi'_I, \chi'_F), (\varepsilon'_t, \varepsilon'_I, \varepsilon'_F)]$ and $\zeta'' = [\mathbb{V}'', \mathbb{E}'', (\chi''_T, \chi''_I, \chi''_F), (\varepsilon''_t, \varepsilon''_I, \varepsilon''_F)]$ be two ING. Subsequently, we denote and define the intersection of Inverse NG ζ' and ζ'' as

$$\zeta' \cap \zeta'' = [(\mathbb{V}' \cap \mathbb{V}''), (\mathbb{E}' \cap \mathbb{E}''), (\chi'_T \cap \chi''_T), (\chi'_I \cap \chi''_I), (\chi'_F \cap \chi''_F), (\varepsilon'_t \cap \varepsilon''_t), (\varepsilon'_I \cap \varepsilon''_I), (\varepsilon'_F \cap \varepsilon''_F)]$$

$$\varepsilon'_T \cap \varepsilon''_T(m) = \varepsilon'_T(m) \wedge \varepsilon''_T(m) \quad \forall m \in \mathbb{V}' \cap \mathbb{V}''$$

$$\varepsilon'_I \cap \varepsilon''_I(m) = \varepsilon'_I(m) \vee \varepsilon''_I(m) \quad \forall m \in \mathbb{V}' \cap \mathbb{V}''$$

$$\varepsilon'_F \cap \varepsilon''_F(m) = \varepsilon'_F(m) \vee \varepsilon''_F(m) \quad \forall m \in \mathbb{V}' \cap \mathbb{V}''$$

$$\begin{aligned}\chi'_T \cap \chi''_T(mn) &= \chi'_T(mn) \wedge \chi''_T(mn) \text{ when } mn \in \mathbb{E}' \cap \mathbb{E}'' \\ \chi'_I \cap \chi''_I(mn) &= \chi'_I(mn) \vee \chi''_I(mn) \text{ when } mn \in \mathbb{E}' \cap \mathbb{E}'' \\ \chi'_F \cap \chi''_F(mn) &= \chi'_F(mn) \vee \chi''_F(mn) \text{ when } mn \in \mathbb{E}' \cap \mathbb{E}''\end{aligned}$$

Theorem 4.2

The Intersection of any two ING is an ING.

Proof:

Consider ζ' and ζ'' be two Inverse Neutrosophic graphs, whereby $\mathbb{V}' \cap \mathbb{V}''$ is a filled set.

Then for all $mn \in \mathbb{E}' \cap \mathbb{E}''$,

$$(i) \quad \chi'_T(mn) \geq \varepsilon'_T(m) \wedge \varepsilon'_T(n) \text{ and } \chi'_T(mn) \geq \varepsilon''_T(m) \wedge \varepsilon''_T(n)$$

Therefore,

$$\begin{aligned}\chi_T(mn) &= \chi'_T(mn) \wedge \chi''_T(mn) \\ &\geq \{ \varepsilon'_T(m) \wedge \varepsilon'_T(n) \} \wedge \{ \varepsilon''_T(m) \wedge \varepsilon''_T(n) \} \\ &\geq \chi_T(m) \wedge \chi_T(n).\end{aligned}$$

$$(ii) \quad \chi'_I(mn) \leq \varepsilon'_I(m) \vee \varepsilon'_I(n) \text{ and } \chi'_I(mn) \leq \varepsilon''_I(m) \vee \varepsilon''_I(n)$$

Therefore,

$$\begin{aligned}\chi_I(mn) &= \chi'_I(mn) \vee \chi''_I(mn) \\ &\leq \{ \varepsilon'_I(m) \vee \varepsilon'_I(n) \} \vee \{ \varepsilon''_I(m) \vee \varepsilon''_I(n) \} \\ &\leq \chi_I(m) \vee \chi_I(n).\end{aligned}$$

$$(iii) \quad \chi'_F(mn) \leq \varepsilon'_F(m) \vee \varepsilon'_F(n) \text{ and } \chi'_F(mn) \leq \varepsilon''_F(m) \vee \varepsilon''_F(n)$$

Therefore,

$$\begin{aligned}\chi_F(mn) &= \chi'_F(mn) \vee \chi''_F(mn) \\ &\leq \{ \varepsilon'_F(m) \vee \varepsilon'_F(n) \} \vee \{ \varepsilon''_F(m) \vee \varepsilon''_F(n) \} \\ &\leq \chi_F(m) \vee \chi_F(n).\end{aligned}$$

Consequently, $\zeta = \zeta' \cap \zeta''$ is an Inverse Neutrosophic Graph.

Hence the proof

DEFINITION 4.3

Assume $\zeta' = [\mathbb{V}', \mathbb{E}', (\chi'_T, \chi'_I, \chi'_F), (\varepsilon'_T, \varepsilon'_I, \varepsilon'_F)]$ and $\zeta'' = [\mathbb{V}'', \mathbb{E}'', (\chi''_T, \chi''_I, \chi''_F), (\varepsilon''_T, \varepsilon''_I, \varepsilon''_F)]$ be two ING. Subsequently, we denote and define the cartesian product of Inverse NG ζ' and ζ'' as

$$\begin{aligned}\zeta' \times \zeta'' &= [(\mathbb{V}' \times \mathbb{V}''), (\mathbb{E}' \times \mathbb{E}''), (\chi'_T \times \chi''_T), (\chi'_I \times \chi''_I), (\chi'_F \times \chi''_F), (\varepsilon'_T \times \varepsilon''_T), \\ &(\varepsilon'_I \times \varepsilon''_I), (\varepsilon'_F \times \varepsilon''_F)]\end{aligned}$$

$$\varepsilon_T(mn) = \varepsilon'_T(m) \wedge \varepsilon''_T(n),$$

$$\chi_T\{(m'n')(m''n'')\} = \varepsilon'_T(m) \wedge \chi''_T(n'n''), \chi_T\{(m'n)(m''n'')\} = \chi'_T(m'm'') \wedge \varepsilon''_T(n)$$

$$\begin{aligned}
\varepsilon_I(m\ n) &= \varepsilon'_I(m) \vee \varepsilon''_I(n), \\
\chi_I\{(m\ n')(m\ n'')\} &= \varepsilon'_I(m) \vee \chi''_I(n'n''), \chi_I\{(m'n)(m''n)\} = \chi'_I(m'm'') \vee \varepsilon''_I(n) \\
\varepsilon_F(m\ n) &= \varepsilon'_F(m) \vee \varepsilon''_F(n), \\
\chi_F\{(m\ n')(m\ n'')\} &= \varepsilon'_F(m) \vee \chi''_F(n'n''), \chi_F\{(m'n)(m''n)\} = \chi'_F(m'm'') \vee \varepsilon''_F(n)
\end{aligned}$$

Theorem 4.3

The cartesian product of any two ING is also an ING.

Proof:

Consider ζ' and ζ'' be two Inverse Neutrosophic graphs, the cartesian product of two ING possess two categories of edges

(i) $(m\ n')(m\ n'')$ that fulfills $\chi_T\{(m\ n')(m\ n'')\} = \varepsilon'_T(m) \wedge \chi''_T(n'n'')$.

In this category, Given that, $\chi''_T(n'n'') \geq \varepsilon''_T(n') \wedge \varepsilon''_T(n'')$.

Thus, $\varepsilon'_T(m) \wedge \chi''_T(n'n'') \geq \varepsilon'_T(m) \wedge [\varepsilon''_T(n') \wedge \varepsilon''_T(n'')]$.

$$= \varepsilon_T(m\ n') \wedge \varepsilon_T(m\ n'')$$

Therefore, $\chi_T\{(m\ n')(m\ n'')\} \geq \varepsilon_T(m\ n') \wedge \varepsilon_T(m\ n'')$

(ii) $(m\ n')(m\ n'')$ that fulfills $\chi_I\{(m\ n')(m\ n'')\} = \varepsilon'_I(m) \vee \chi''_I(n'n'')$.

Given that, $\chi''_I(n'n'') \leq \varepsilon''_I(n') \vee \varepsilon''_I(n'')$.

Thus, $\varepsilon'_I(m) \vee \chi''_I(n'n'') \leq \varepsilon'_I(m) \vee [\varepsilon''_I(n') \vee \varepsilon''_I(n'')]$.

$$= \varepsilon_I(m\ n') \vee \varepsilon_I(m\ n'')$$

Therefore, $\chi_I\{(m\ n')(m\ n'')\} \leq \varepsilon_I(m\ n') \vee \varepsilon_I(m\ n'')$

(ii) $(m\ n')(m\ n'')$ that fulfills $\chi_F\{(m\ n')(m\ n'')\} = \varepsilon'_F(m) \vee \chi''_F(n'n'')$.

Given that, $\chi''_F(n'n'') \leq \varepsilon''_F(n') \vee \varepsilon''_F(n'')$.

Thus, $\varepsilon'_F(m) \vee \chi''_F(n'n'') \leq \varepsilon'_F(m) \vee [\varepsilon''_F(n') \vee \varepsilon''_F(n'')]$.

$$= \varepsilon_F(m\ n') \vee \varepsilon_F(m\ n'')$$

Therefore, $\chi_F\{(m\ n')(m\ n'')\} \leq \varepsilon_F(m\ n') \vee \varepsilon_F(m\ n'')$

b. $(m'n)(m''n)$ that fulfills $\chi_T[(m'n)(m''n)] = \chi'(m'm'') \wedge \varepsilon''_T(n)$

In this category, by using the same approach, we can demonstrate

$$\chi_T\{(m'n)(m''n)\} \geq \varepsilon_T(m'n) \wedge \varepsilon_T(m''n)$$

$$\chi_I\{(m'n)(m''n)\} \leq \varepsilon_I(m'n) \vee \varepsilon_I(m''n)$$

$$\chi_F\{(m'n)(m''n)\} \leq \varepsilon_F(m'n) \vee \varepsilon_F(m''n)$$

Consequently, $\zeta = \zeta' \times \zeta''$ is an Inverse Neutrosophic Graph.

Hence the proof

5. Inverse Neutrosophic Multigraphs

DEFINITION 5.1

Let $\mathbb{V}$ is a non-void set, $(\varepsilon_T, \varepsilon_I, \varepsilon_F): \mathbb{V} \rightarrow [0,1]$ is Neutrosophic Set and $\mathbb{E} = \{(x,y), [(\chi_{Ti}, \chi_{Ii}, \chi_{Fi}) (x,y)], i=1,2,\dots,m \mid (x,y) \in \mathbb{V} \times \mathbb{V}\}$ be a Neutrosophic multiset of $\mathbb{V} \times \mathbb{V}$ such that $\chi_{Ti}(xy) \geq \varepsilon_T(x) \wedge \varepsilon_T(y)$, $\chi_{Ii}(xy) \leq \varepsilon_I(x) \vee \varepsilon_I(y)$, $\chi_{Fi}(xy) \leq \varepsilon_F(x) \vee \varepsilon_F(y)$ for $i=1,2,\dots,m$ where $m = \text{maximum } \{i : \chi_{Ti}(xy), \chi_{Ii}(xy), \chi_{Fi}(xy) \neq 0\}$. Then $\zeta = (\{\mathbb{V}, \mathbb{E}, (\varepsilon_T, \varepsilon_I, \varepsilon_F), (\chi_{Ti}, \chi_{Ii}, \chi_{Fi})\})$ is called as Inverse Neutrosophic Multi graphs where $\varepsilon_T(x)$, $\varepsilon_I(x)$, $\varepsilon_F(x)$ signify true membership, indeterminacy membership and false membership of vertices x and $\chi_{Ti}(xy)$, $\chi_{Ii}(xy)$, $\chi_{Fi}(xy)$ signifies the true membership, indeterminacy membership and false membership of edges (xy) within ζ correspondingly.

6. Inverse Neutrosophic Multigraphs and its planarity.

Within this segment, the planarity in terms of some key related concepts, such as interconnecting value and inverse Neutrosophic planarity values are defined.

Let $\zeta = (\{\mathbb{V}, \mathbb{E}, (\varepsilon_T, \varepsilon_I, \varepsilon_F), (\chi_{Ti}, \chi_{Ii}, \chi_{Fi})\})$ be an Inverse Neutrosophic Multi-graph and a specific geometric illustration it has two Inverse Neutrosophic edges $((x,y), (\chi_{Ti}, \chi_{Ii}, \chi_{Fi}) (x,y))$ and $((p,q), (\chi_{Tj}, \chi_{Ij}, \chi_{Fj}) (p,q))$ which are interconnected by a single point P, where i, j are constant integers. In Neutrosophic sense, if at least one of $((x,y), (\chi_{Ti}, \chi_{Ii}, \chi_{Fi}) (x,y))$ and $((p,q), (\chi_{Tj}, \chi_{Ij}, \chi_{Fj}) (p,q))$ is close to zero, then the overlapping won't be significant in the illustration. However if both of $((x,y), (\chi_{Ti}, \chi_{Ii}, \chi_{Fi}) (x,y))$ and $((p,q), (\chi_{Tj}, \chi_{Ij}, \chi_{Fj}) (p,q))$ are close to one, then the overlapping will be considered significant in the illustration.

Building on the concept, let us describe the interconnecting value and Inverse Neutrosophic planarity values.

DEFINITION 6.1

Consider $\zeta = (\{\mathbb{V}, \mathbb{E}, (\varepsilon_T, \varepsilon_I, \varepsilon_F), (\chi_{Ti}, \chi_{Ii}, \chi_{Fi})\})$ to be an Inverse Neutrosophic multi-graph possessing a certain geometric illustration which has the interconnecting Point S among inverse Neutrosophic edges $((x,y), (\chi_{Ti}, \chi_{Ii}, \chi_{Fi}) (x,y))$ and $((p,q), (\chi_{Tj}, \chi_{Ij}, \chi_{Fj}) (p,q))$, Then the value of intersecting edges at the point S is defined by

$$S = (S_T, S_I, S_F) = \left(\frac{\chi_{Ti}(x,y) + \chi_{Tj}(p,q)}{2}, \frac{\chi_{Ii}(x,y) + \chi_{Ij}(p,q)}{2}, \frac{\chi_{Fi}(x,y) + \chi_{Fj}(p,q)}{2} \right)$$

If the number of interconnecting points in an Inverse Neutrosophic multi-graph rises, then planarity value declines. Also, if the value of interconnecting points declines, then planarity value rises. Therefore, IP is inversely related to the planarity value. Based on this concept, the idea of planarity value of an Inverse Neutrosophic multi-graph is defined.

DEFINITION 6.2

Let $\zeta = (\{\mathbb{V}, \mathbb{E}, (\varepsilon_T, \varepsilon_I, \varepsilon_F), (\chi_{Ti}, \chi_{Ii}, \chi_{Fi})\})$ be an Inverse Neutrosophic multi-graph with a specific geometric illustration, which has n intersection Points $S_1, S_2, S_3 \dots S_n$. Then the Inverse Neutrosophic planarity value of S' of this geometric illustration of χ is well-defined by

$$S' = \frac{(2+S'_T - S'_I - S'_F)}{3}$$

Where $S'_T = \frac{1}{1+\{S_{T1}+S_{T2}+\dots+S_{Tn}\}}$, $S'_I = \frac{S_{I1}+S_{I2}+\dots+S_{In}}{n}$, $S'_F = \frac{S_{F1}+S_{F2}+\dots+S_{Fn}}{n}$

It is evident that $0 \leq S' \leq 1$ and we assert that each geometric illustration of Inverse Neutrosophic multi-graph is planar by specific planarity value S' .

Examples 6.1

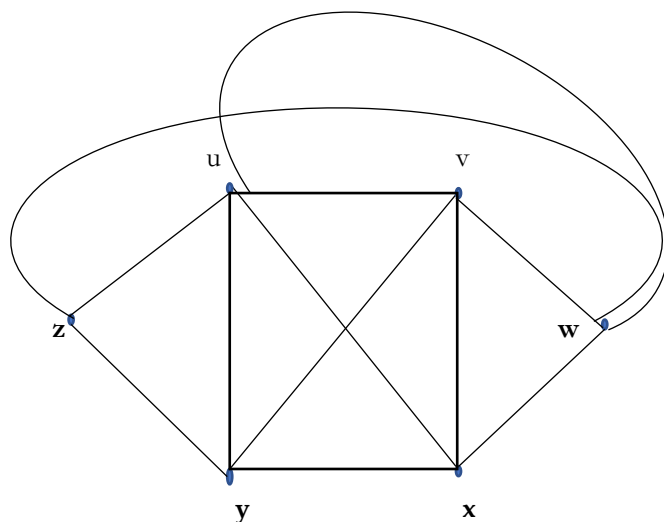


Figure. 7

Table 4: Vertices & Edges of Figure. 7

Vertices	Edges
u (0.6,0.6,0.4)	uz(0.8,0.4,0.2)
v (0.8,0.7,0.2)	uw(0.7,0.5,0.3)
w (0.3,0.4,0.6)	zw(0.5,0.1,0.2)
x (0.6,0.1,0.7)	xy (0.5,0.6,0.3)
y (0.5,0.8,0.4)	yz(0.5,0.4,0.1)
z (0.7,0.5,0.3)	ux(0.6,0.3,0.2)
	vy(0.7,0.5,0.1)

The geometric illustration of ζ has 2 intersection points, S_1 and S_2 .

The intersection value of edges at the point S_1 and S_2 is

$$S_{T1} = \frac{0.6+0.7}{2} = 0.65, S_{I1} = \frac{0.3+0.5}{2} = 0.4, S_{F1} = \frac{0.2+0.1}{2} = 0.15,$$

$$S_{T2} = \frac{0.8+0.7}{2} = 0.75, S_{I2} = \frac{0.5+0.4}{2} = 0.45, S_{F2} = \frac{0.3+0.2}{2} = 0.25.$$

$$S'_T = 0.42, S'_I = 0.41, S'_F = 0.2,$$

Therefore, the planarity value S' of this geometric illustration of ζ is $S' = 0.6$.

Lemma 6.1

Provided that the degree of planarity of an ING be (S'_T, S'_I, S'_F) , consequently $S'_T + S'_I + S'_F \leq 3$.

Proof.

Let ζ be an ING. Therefore $0 \leq \chi_T(xy) \leq 1$, $0 \leq \chi_I(xy) \leq 1$, $0 \leq \chi_F(xy) \leq 1$.

Hence, the score of connection amid two edges are $0 \leq S_T \leq 1$, $0 \leq S_I \leq 1$, $0 \leq S_F \leq 1$.

Consequently, the degree of planarity value $0 \leq S'_T \leq 1$, $0 \leq S'_I \leq 1$, $0 \leq S'_F \leq 1$.

Therefore $S'_T + S'_I + S'_F \leq 3$.

2. Provided that the score of planarity value of an ING is S' consequently $0 \leq S' \leq 1$.

Proof.

Let S' be the score of planarity value of an ING, therefore $S' = \frac{(2+S'_T-S'_I-S'_F)}{3}$ and $0 \leq S'_T \leq 1, 0 \leq S'_I \leq 1, 0 \leq S'_F \leq 1$. Furthermore, it is noted that the value of S' is max if the value of S'_T is max and S'_I, S'_F are minimal.

So, the max value of S' is $S' = \frac{(2+1-0-0)}{3} = 1$.

Again, the value of S' is minimal. if S'_T is minimal. and S'_I, S'_F are max. So, the minimal. value of S' is $S' = \frac{(2+0-1-1)}{3} = 0$. Then $0 \leq S' \leq 1$.

Theorem 6.1

Each Inverse neutrosophic multi-graph ζ with the geometric illustration having n intersection points

$$\frac{1}{1+n} \leq S' \leq \frac{1}{1+\alpha n}$$

Where $\alpha = \min \{(\varepsilon_T, \varepsilon_I, \varepsilon_F)(x) : x \in \mathbb{V}\}$.

Proof:

In accordance to Definition 6.2:

$$S' = \frac{(2+S'_T - S'_I - S'_F)}{3}$$

Wherein, $S'_{Ti} = \frac{1}{1+\{S_{T1}+S_{T2}+\dots+S_{Tn}\}}$, $S'_{Ii} = \frac{S_{I1}+S_{I2}+\dots+S_{In}}{n}$, $S'_{Fi} = \frac{S_{F1}+S_{F2}+\dots+S_{Fn}}{n}$

and in accordance to the definition 6.1 and 6.2,

$$(S_T, S_I, S_F) = \left(\frac{\chi_{Ti}(x,y)+\chi_{Tj}(p,q)}{2}, \frac{\chi_{Ii}(x,y)+\chi_{Ij}(p,q)}{2}, \frac{\chi_{Fi}(x,y)+\chi_{Fj}(p,q)}{2} \right)$$

and lemmas, we attain $(S'_{Ti}, S'_{Ii}, S'_{Fi}) \leq 1$

for $i = \{1, 2, \dots, n\}$, we attain $\{S_{T1} + S_{T2} + \dots + S_{Tn} \leq n$ and via lemma, $S' = \frac{(2+S'_T-S'_I-S'_F)}{3} \geq \frac{1}{1+n}$

conversely,

$\chi_T(xy) \geq \varepsilon_T(x) \wedge \varepsilon_T(y)$, $\chi_I(xy) \leq \varepsilon_I(x) \vee \varepsilon_I(y)$, $\chi_F(xy) \leq \varepsilon_F(x) \vee \varepsilon_F(y)$ and for each inverse Neutrosophic edge.

As $\alpha = \min \{(\varepsilon_T, \varepsilon_I, \varepsilon_F)(x) : x \in \mathbb{V}\}$. and we attain $\chi_{Ti}(x, y) \geq \alpha$ so, $(S'_{Ti}, S'_{Ii}, S'_{Fi}) \geq 1$

for $i = \{1, 2, \dots, n\}$, we attain $\{S_{T1} + S_{T2} + \dots + S_{Tn} \geq n\alpha$ and via lemma so

$$S' = \frac{(2+S'_T-S'_I-S'_F)}{3} \leq \frac{1}{1+\alpha n}.$$

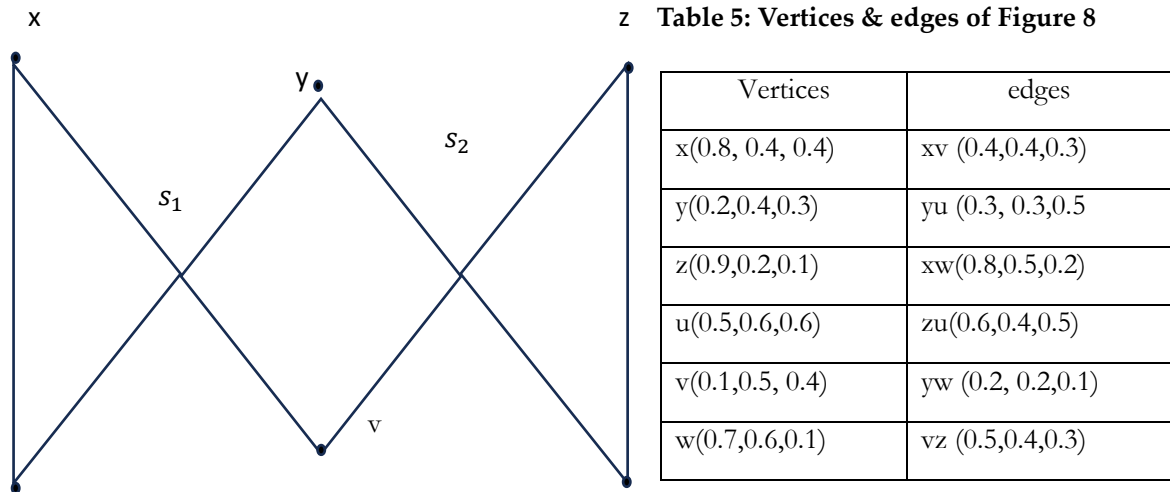
DEFINITION 6.3:

Consider $\zeta = (\{\mathbb{V}, \mathbb{E}, (\varepsilon_T, \varepsilon_I, \varepsilon_F), (\chi_{Ti}, \chi_{Ii}, \chi_{Fi})\}$ to be an Inverse Neutrosophic Multi-graphs with a specific geometric illustration having n intersecting points $S_1, S_1, \dots, S_n$ so that m of these interconnecting points fulfills $I_{S_i} \geq 0.5$, for $i = 1, 2, \dots, m$. Therefore, this geometric illustration of ζ is termed as inverse **Neutrosophic strong planar multi-graph** $S > 0.5$ and $m < \frac{n}{2}$.

This indicates that a geometric illustration of ζ is an inverse Neutrosophic strong planar multi-graph, provided the inverse Neutrosophic planarity value is superior than 0.5 and the interconnecting points that fulfil $I_S \geq 0.5$ are fewer than half of the total interconnecting points n. Alternatively, this geometric illustration of ζ is termed as inverse **Neutrosophic weak planar multi-graph**.

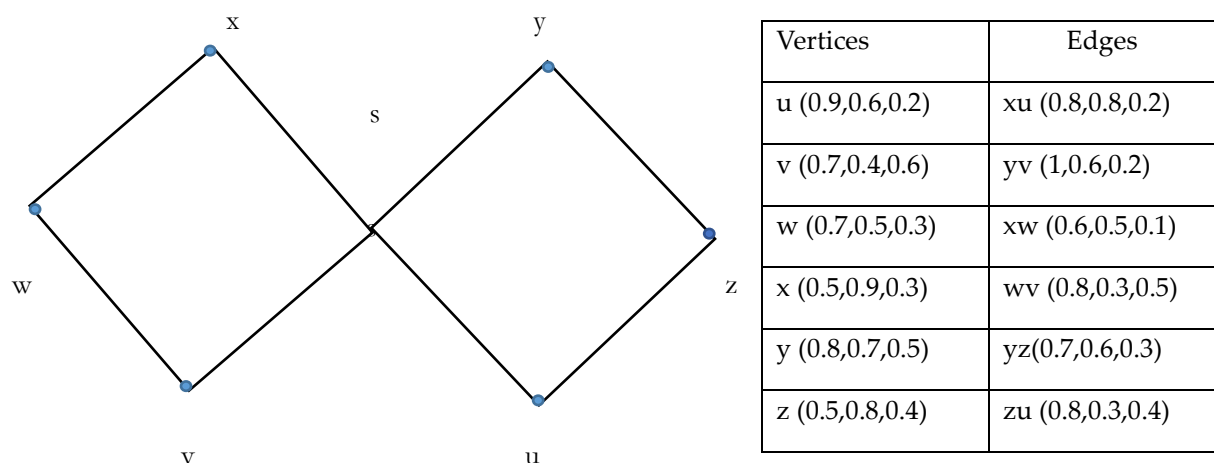
Example 6.2

Consider $\zeta = (\{V, E, (\varepsilon_T, \varepsilon_I, \varepsilon_F), (\chi_{Ti}, \chi_{Ii}, \chi_{Fi})\})$ to be an Inverse Neutrosophic Multi-graph with a specific geometric illustration as depicted in the figure as follows,

**Figure. 8**

Consequently, this geometric representation of ζ has two intersecting points, S_1 and S_2 . The interconnecting value at the location S_1 is $S_{T_1} = 0.3, S_{I_1} = 0.3, S_{F_1} = 0.2, S_{T_2} = 0.4, S_{I_2} = 0.35, S_{F_2} = 0.4. S'_T = 0.588, S'_I = 0.325, S'_F = 0.3$, therefore $S' = 0.654$. Hence, this geometric illustration from ζ is **strong planar**.

By drawing the Inverse Neutrosophic multi-graph ζ in alternative geometric illustration presented within Figure beneath,

**Figure. 9**

Consequently, the above geometric illustration of ζ has one intersecting point S (0.8, 0.7, 0.2), thus the planarity value of the intersecting edge S is $S'_T = 0.56, S'_I = 1, S'_F = 0.7$, therefore $S' = 0.5$.

Hence aforementioned geometric representation from ζ is **weak planar**.

Remark: In the Inverse Neutrosophic graph the circumstances is distinct, since as per the application, a particular number of intersection point is sometime needed to resolve our issue. In the inverse Neutrosophic graph, our focus is not on finding a geometric illustration devoid of intersecting points. Instead, based on the problem requirements, we identify the required number of intersecting points and aim to determine a geometric illustration that meets our condition. Additionally, we attempt to minimize the danger associated with these intersecting points (lowest $\{I_{S_1}\}$) and maximize the inverse planarity value (S'), as will be demonstrated in the application.

Example 6.3

The geometric illustration of ζ that is depicted in Figure 8, having 2 intersecting points such that S_1 and S_2 , $S'_T = 0.42$, $S'_I = 0.41$, $S'_F = 0.2$ and $S' = 0.6$. Despite $P' > 0.5$, the condition $m < \frac{n}{2}$ fails to be met.

Therefore, the aforementioned geometric illustration of ζ is **weak planar**.

7. Application: Decision making in mixed cropping using Inverse Neutrosophic graph:

Mixed cropping or inter cropping is an agricultural practice that involves two or more crops are grown together in the same field. This approach integrates different farming activities to maximize resource use, improve productivity, and enhance sustainability. Mixed cropping offers numerous advantages in terms of resource efficiency, economic stability, and environmental benefits. However, it also presents challenges related to management complexity and economic risks. Successful mixed cropping relies on effective planning, resource management, and integration of sustainable practices to optimize the benefits of combining different crop production.

For instance, combining of crops can be done with minimizing expenditure that becomes more significant. Each crop is considered as a vertex of a graph as expenditure is considered as significant in our study, we assign a membership degree representing its resource management expenditure. This membership degree can be regularized by dividing every expected expenditure for each crops with average total expenditure, the expenditure is varying on certain circumstances, the expected expenditure for resource management of the particular crops is considered as truth membership, there is a chance of high level of uncertainty about the exact expenditure, possibly due to complex management, pest and disease management & economic and market risk which is considered as the indeterminacy membership and There may be chance that the stated expenditure of is inaccurate or misleading which is considered as false membership.

Accordingly, provided that the expenditure (that incorporates labor expenditure, crop maintenance charge) for different crops w, x, y, z is $\{3200, 2000, 2500, 4000\}$ also the satisfactory factor of the expected expenditure is $\{90\%, 80\%, 70\%, 85\%\}$

The Membership values are denoted as

Truth membership value (TMV) = Ratio of estimated expenditure to average total expenditure (provided that the estimated expenditure is greater than average total expenditure, then TMV is considered as 1.

Indeterminacy Membership Value (IMV) = 20 % of the estimated expenditure.

FMV = considering the satisfactory factor False membership of a vertex is 1- satisfactory.

$$(\varepsilon_T, \varepsilon_I, \varepsilon_F)(w) = (1, 0.2, 0.1)$$

$$(\varepsilon_T, \varepsilon_I, \varepsilon_F)(x) = (0.7, 0.14, 0.2)$$

$$(\varepsilon_T, \varepsilon_I, \varepsilon_F)(y) = (0.8, 0.16, 0.3)$$

$$(\varepsilon_T, \varepsilon_I, \varepsilon_F)(z) = (1, 0.2, 0.15)$$

Our objective is combining of crops so that we can allocate crop in one field. To evaluate our options, we construct an inverse Neutrosophic graph for each situation. In this graph, we connect vertices representing combining of crops with edges that intersect at a point P, representing the same farm.

For n farms (particular area), each graph will feature n intersecting points, each representing a farm the particular area. The vertices connected by intersecting edges denote the two crops in same area field of the farm.

Each graph consists of four vertices (each crop), two edges (illustrates combining of crops), and one intersecting point (illustrates the same field of the farm). This configuration forms an inverse Neutrosophic graph where each edge is allocated a membership degree, indeterminacy degree and false degree representing the required expenditure of maintaining crops in the same field of the farm. To ensure accuracy, these membership degrees are normalized by considering average of the total maintenance expenditure. It's crucial to note that each edge's membership degree (illustrates combining of crops) will inevitably exceed the expenditure of maintaining crops separately. Optimal planning involves selecting pairs of crops that can be combined in same field of the form that leads to minimization of maintenance and maximizing the production and profit. So, to estimate the best combination, the study of planarity in every possible inverse neutrosophic graph is sufficient and to select the one which is strong planar that has the greatest inverse neutrosophic planarity value P' . If all the selections are weak planar, then it is sufficient to choose the one with the greatest inverse Neutrosophic planarity value P' .

In our instance we have 3 possible options. First option, to combine crop w and crop x in the same field of the fam, crop y and crop z in the other field of the farm.

We signify this option by the inverse Neutrosophic graph ζ_1 revealed in the Figure 10 (a)

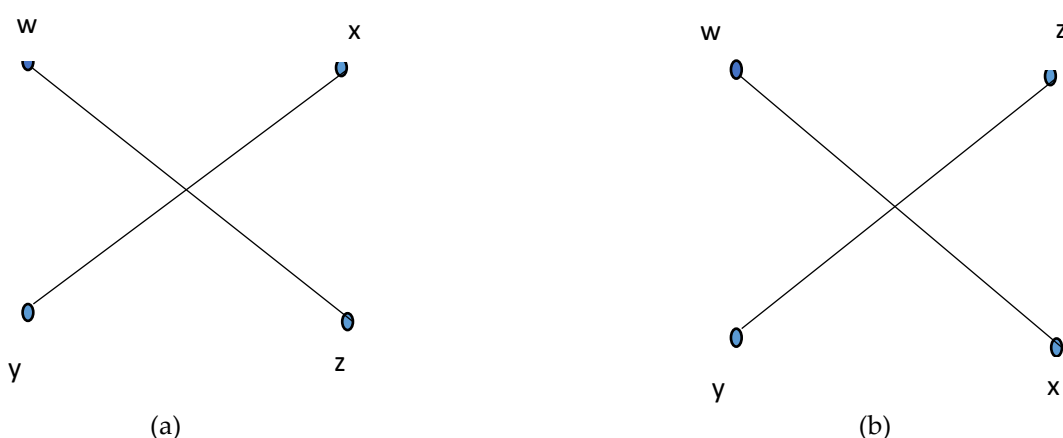


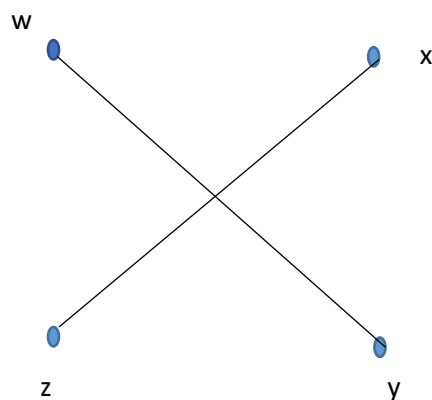
Figure 10.

Second option, to combine crop w and z in the particular field of the farm and crops x and y in another field of the farm.

We signify this option by the Neutrosophic fuzzy graph ζ_2 revealed in Figure 10 (b)

Third option, to combine crop w and x in particular field of the farm and crops z and y in another field of the farm.

We signify this option by the Neutrosophic fuzzy graph ζ_3 revealed in Figure 10 (c).



(c)

To determine the optimal option, the calculation of the Inverse Neutrosophic planarity value for every option are required.

Within ζ_1 , we obtain $S'_T = 0.54$, $S'_I = 0.05$, $S'_F = 0.075$ and $S' = 0.8$ indicating that ζ_1 is strong planar.

Within ζ_2 , we obtain $S'_T = 0.5$, $S'_I = 0.075$, $S'_F = 0.12$ and $S' = 0.79$ indicating that ζ_2 is strong planar.

Within ζ_3 , we obtain $S'_T = 0.57$, $S'_I = 0.1$, $S'_F = 0.1$ and $S' = 0.7$ indicating that ζ_3 is strong planar.

As a result, the Inverse Neutrosophic strong planar graph exhibiting highest Inverse Neutrosophic planarity value serves as the optimal option, considering that the expenditure will be minimized. It means that the First choice is the optimal one in our instance.

8. Conclusion:

The core intension of developing Inverse Neutrosophic graph was to provide an enhanced and flexible framework interpreting multifaced system where uncertainty and the directionality of interactions are significant. Explores both the similarities and differences between Inverse Neutrosophic graph and Neutrosophic graphs. Then, perception of Inverse Neutrosophic Multi-graphs and defined the notation for the planarity of such structures using intersecting values and inverse Neutrosophic planarity values. Several theorems were presented to establish extremes for the inverse neutrosophic planarity value. Furthermore, we defined strong and weak planarity and applications related to these concepts. The significant crop combinations for intercropping are identified using the notion of planarity in Inverse Neutrosophic Graphs that minimizes the expenditure in maintenance of the crops. Looking ahead, future research will focus on further exploring planarity and investigating the concepts of faces and duality in inverse Neutrosophic multi-graphs.

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The authors affirms that they possess no competing interests.

Authors Contribution

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IndermSoft Set for Artificial Intelligence Solutions for Teaching Competition Plans Assessment in Colleges

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Abstract: Artificial Intelligence (AI) is becoming a key factor in transforming conventional educational institutions in the age of digital transformation. The review and assessment of college teaching competition plans, a field that has hitherto relied on human judgment, is one of the newer uses. By utilizing AI, assessments might become more transparent, data-driven, and uniform. This study investigates how AI-based solutions might improve feedback quality, streamline the assessment process, and assist teachers in honing their pedagogical approaches. Our goal is to provide an effective framework for assessing teaching competition strategies by examining contemporary AI approaches and how they have been modified for use in educational settings. We use the IndermSoft Set to solve the indeterminacy in the criteria values. We use the multi-criteria decision making (MCDM) method to show the ranks of alternatives. We use the COBRA method as a MCDM to rank the alternatives.

Keywords: IndermSoft Set; Artificial Intelligence; Teaching Competition Plans; Education; Colleges.

1. Introduction

New possibilities for enhancing teaching methods at all levels, especially in higher education, have been brought about by the development of AI technology. College teaching contests provide a forum for showcasing cutting-edge teaching strategies, but their assessment systems frequently have issues with scalability, consistency, and fairness. One of the most important steps toward a more methodical and objective review procedure is the use of AI techniques for evaluating these contests[1], [2]. Conventional techniques for assessing teaching competition plans mostly depend on expert panels that use arbitrary criteria, which might result in discrepancies. The results may

vary depending on the assessors' own biases, experience, and criteria for making judgments[3]. By evaluating lesson plans using established criteria, AI provides an option that guarantees a more impartial and repeatable evaluation. The capacity of AI to manage intricate and multifaceted evaluation tasks is one of its most important contributions to this discipline. Submissions to teaching competitions may be examined from a variety of angles, including originality, instructional design, engagement strategy, and learning results, using machine learning techniques, natural language processing (NLP), and predictive analytics[4], [5]. The time and effort required for comprehensive examinations can be significantly decreased by these capabilities.

Real-time feedback production is another benefit of using AI into teaching plan assessment. AI systems, in contrast to manual review, can quickly pinpoint areas of strength and development and provide thorough recommendations. In addition to offering participants practical insights, this kind of dynamic feedback promotes a culture of ongoing pedagogical practice development.

But there are obstacles to overcome when integrating AI technology into educational assessment. It is important to carefully address issues like data privacy, model openness, and the danger of relying too much on automated evaluations[6], [6]. A hybrid approach is necessary for effective integration, one in which AI enhances human judgment rather than takes its place, guaranteeing that subtle facets of high-quality instruction are not missed.

The creation of thorough evaluation criteria is another essential component in the implementation of AI-based assessment systems. The criteria must be clear, reflective of many teaching philosophies, and adaptable to various educational environments for AI models to function efficiently. To create these frameworks, educators and technologists must work together.

Promising outcomes are shown by successful case studies from organizations that have tested AI-based evaluation systems[7], [8]. AI can revolutionize the assessment of teaching competition plans, as evidenced by increased evaluation accuracy, participant happiness, and richer feedback content. These early adopters offer insightful information for broader college deployment.

This study offers a methodical strategy for implementing AI solutions for competition plan evaluation instruction in colleges considering these advancements[9], [10]. We hope to provide a useful resource for organizations wishing to update and improve their teaching evaluation systems by examining different AI models, assessing their efficacy, and describing workable implementation options.

2. IndtermSoft Set Model

Between 2018-2024 Smarandache [<https://fs.unm.edu/TSS/>] introduced six new types of soft sets: HyperSoft Set, IndtermSoft Set, IndtermHyperSoft Set, SuperHyperSoft Set, TreeSoft Set, ForestSoft Set[11], [12], [13], [14], [15]. This study uses the IndtermSoft set to solve the uncertainty in the criteria values. The IndtermSoft set is used with the MCDM approach to

compute the criteria weights and ranking the alternatives. Smarandache developed different sets[11], [12].

K is a non-empty subset of Y , Y is a discourse universe, and $P(K)$ is the powerset of K . Suppose that D is a set of its values and that a is an attribute. D function $F: D \rightarrow (K)$ pertaining to the values of one or more attributes is known as an IndermSoft Set (Function); set A contains some ambiguity[13], [14].

$P(K)$ has some indeterminacy. Alternatively, if at least one attribute value $v \in D$, then $F(v) =$ indeterminate (uncertain, ambiguous, or not unique). or any mix of those three situations.

An IndermSoft Set is a soft set that contains a given quantity of indeterminate (ambiguous, uncertain, alternate, conflicting) data or techniques, according to Smarandache.

We show the steps of the COBRA methodology to rank the alternatives.

The decision matrix is developed by experts and decision makers. Let experts evaluate the criteria and alternatives. We integrate these criteria and alternatives into a single matrix.

Compute the criteria weights.

The weights of criteria are computed using the average method.

Normalize the decision matrix.

$$A_{ij} = \frac{r_{ij}}{\max r_{ij}} \quad (1)$$

Compute the weighted decision matrix.

$$H_{ij} = W_j A_{ij} \quad (2)$$

Compute the distance values from ideal solutions

$$D_j = \max H_{ij} ; D_j = \min H_{ij} \text{ for positive criteria} \quad (3)$$

$$A_j = \min q_{ij} ; B_j = \max q_{ij} \text{ for positive criteria} \quad (4)$$

$$Q_j = \frac{\sum_{i=1}^m H_{ij}}{m} \quad (5)$$

Determine the distance to positive and cost criteria

$$T(D_j) = \sqrt{\sum_{j=1}^n (D_j - H_{ij})^2} \quad (6)$$

$$T(A_j) = \sqrt{\sum_{j=1}^n (A_j - H_{ij})^2} \quad (7)$$

$$T(Q_j) = \sqrt{\sum_{j=1}^n (Q_j - H_{ij})^2} \quad (8)$$

Determine the final distance

$$T_i = \frac{T(D_j) - T(A_j) - T(Q_j)}{3} \quad (9)$$

Order the alternatives

3. Application

We implemented the steps of the proposed approach for Artificial Intelligence Solutions for Teaching Competition Plans Assessment in Colleges. We use eight criteria and seven alternatives. This study has indeterminacy in the first and second criterion. So the IndtermSoft set is used to solve this indeterminacy. We have seven plans such as: Integrative Learning Project Plan, AI-Enhanced Interactive Lecture Strategy, Flipped Classroom Model for STEM Subjects, Gamification-Based Language Learning Plan, Problem-Based Learning with IoT Applications, Cross-Disciplinary Collaborative Workshop Series, Data-Driven Personalized Learning Paths Design. We have eight criteria such as:

- Innovation and Creativity {High, Medium, Low}
- Alignment with Educational Objectives {Excellent, Good, Fair}
- Feasibility of Implementation {Practical, Moderate, Challenging}
- Technological Integration {High, Medium, Low}
- Pedagogical Soundness {Strong, Adequate, Weak}
- Scalability Across Courses {Highly Scalable, Partially Scalable, Limited}
- Assessment Methods and Tools {Comprehensive, Sufficient, Basic}
- Student Engagement Potential {High, Medium, Low}

We select the criteria values such as:

- $C1: \{High, Medium, Low\}$
- $C2: \{Excellent, Good, Fair\}$
- $C3: \{Practical\}$
- $C4: \{High\}$
- $C5: \{Strong\}$
- $C6: \{Highly Scalable\}$
- $C7: \{Comprehensive\}$
- $C8: \{High\}$

So, we divide the criteria values into different sets such as:

- ⊙ Set 1: $C1: \{High\}$, $C2: \{Excellent\}$, $C3: \{Practical\}$, $C4: \{High\}$, $C5: \{Strong\}$, $C6: \{Highly Scalable\}$, $C7: \{Comprehensive\}$, $C8: \{High\}$.
- ⊙ Set 2: $C1: \{High, Medium\}$, $C2: \{Excellent\}$, $C3: \{Practical\}$, $C4: \{High\}$, $C5: \{Strong\}$, $C6: \{Highly Scalable\}$, $C7: \{Comprehensive\}$, $C8: \{High\}$.
- ⊙ Set 3: $C1: \{High, Medium, and Low\}$, $C2: \{Excellent\}$, $C3: \{Practical\}$, $C4: \{High\}$, $C5: \{Strong\}$, $C6: \{Highly Scalable\}$, $C7: \{Comprehensive\}$, $C8: \{High\}$.

- ⊙ Set 4: C1: { High }, C2: {Excellent}, C3: {Practical}, C4: {High}, C5: {Strong}, C6: {Highly Scalable}, C7: {Comprehensive}, C8: {High}.
- ⊙ Set 5: C1: { High }, C2: { Excellent, Good}, C3: {Practical}, C4: {High}, C5: {Strong}, C6: {Highly Scalable}, C7: {Comprehensive}, C8: {High}.
- ⊙ Set 6: C1: { High }, C2: { Excellent, Good, Fair }, C3: {Practical}, C4: {High}, C5: {Strong}, C6: {Highly Scalable}, C7: {Comprehensive}, C8: {High}.

Let three experts evaluate the criteria and alternatives to build the decision matrix. We combine these value into a single matrix. We compute the criteria weights such as:

- $C1 = 0.121142278$
- $C2 = 0.116289651$
- $C3 = 0.110285161$
- $C4 = 0.129811627$
- $C5 = 0.133473736$
- $C6 = 0.127446054$
- $C7 = 0.131094911$
- $C8 = 0.130456582$

Set 1 results described such as:

Normalize the decision matrix using eq.(1) as shown in Fig 1.

Compute the weighted decision matrix using eq. (2) as shown in Fig 2.

Compute the distance values from ideal solutions using eqs. (3,4, and 5).

Determine the distance to positive and cost criteria using eqs. (6, 7, and 8) as shown in Figs 3,4, and 5.

Determine the final distance using eq. (9).

Order the alternatives.

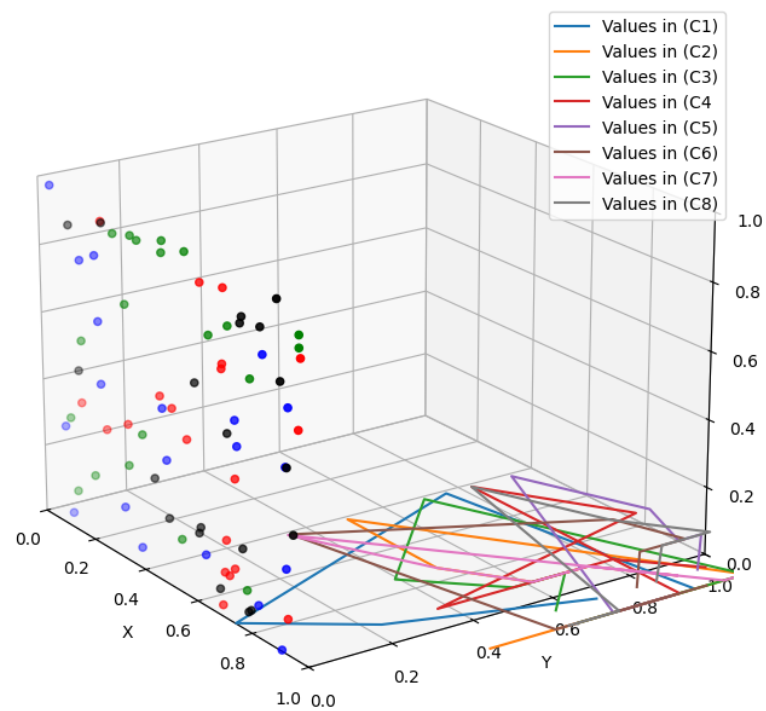


Fig 1. The normalized decision matrix.

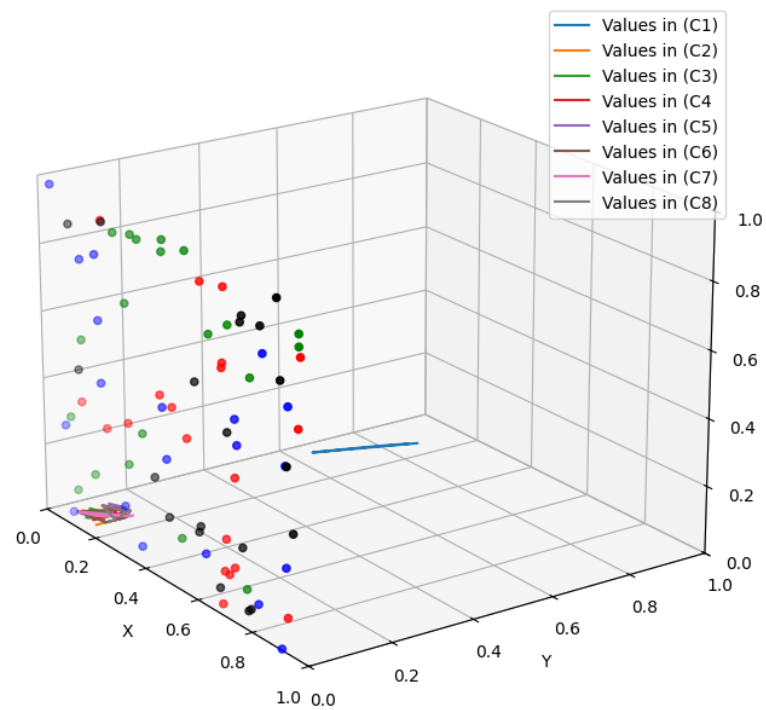
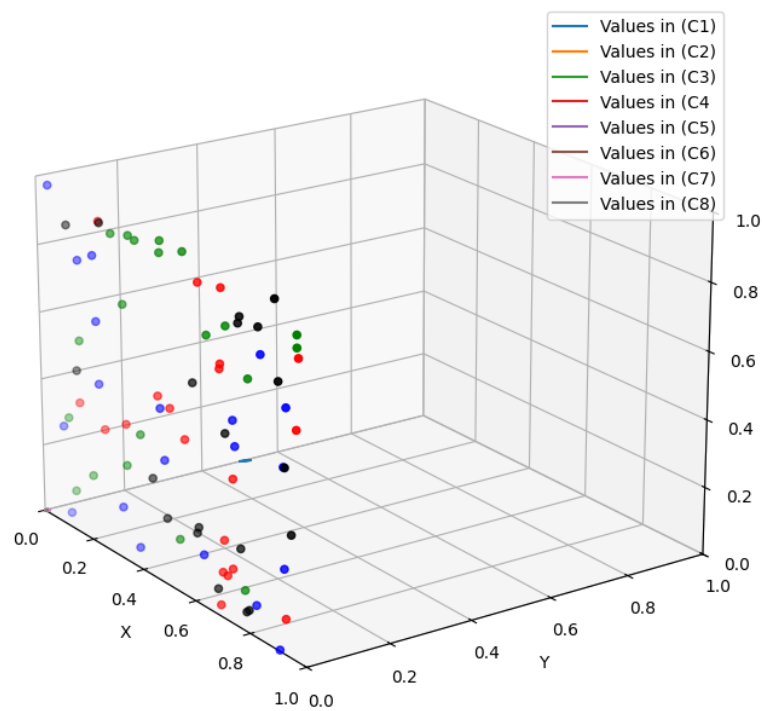
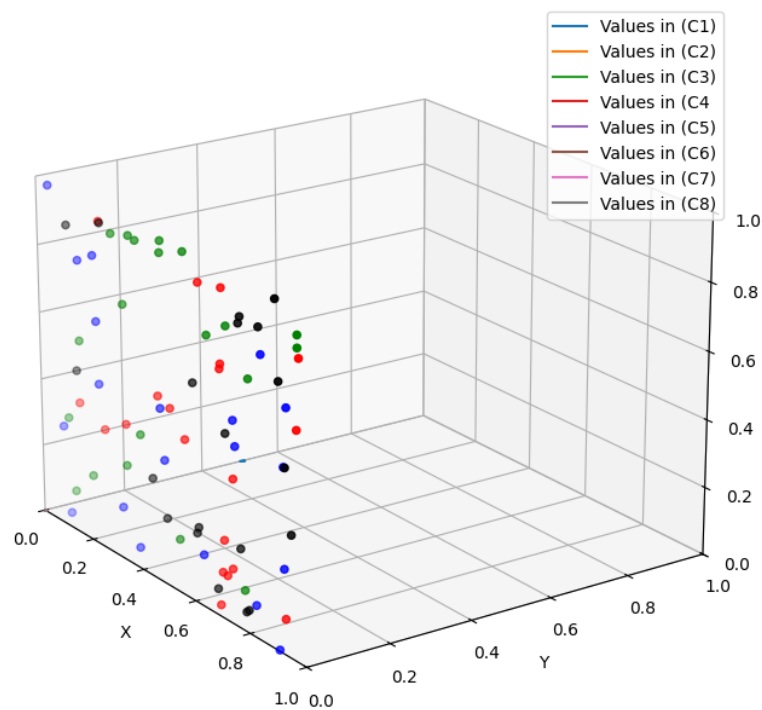


Fig 2. The weighted decision matrix.

Fig 3. The values of $T(D_j)$.Fig 4. The values of $T(A_j)$.

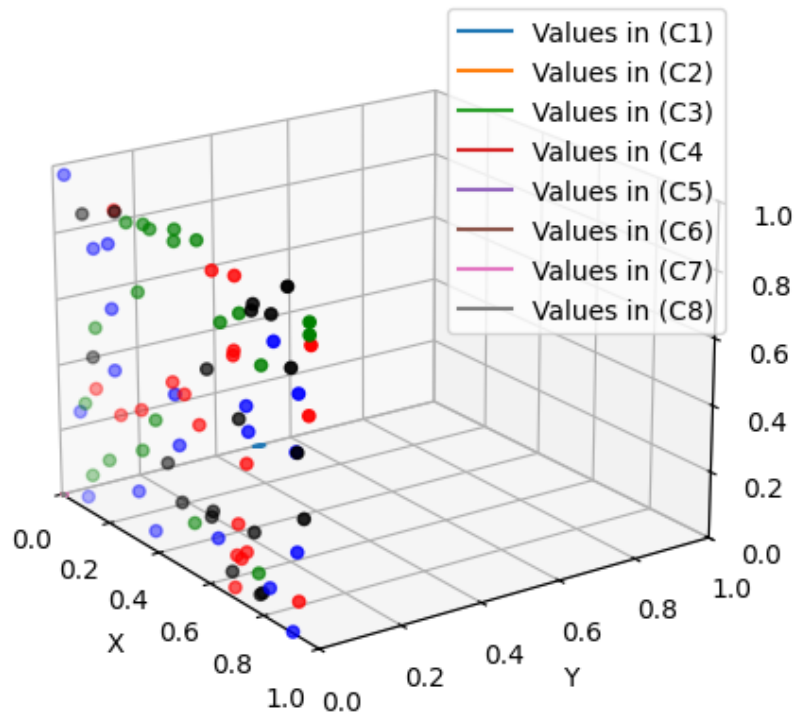


Fig 5. The values of $T(Q_j)$.

Set 2 results described such as:

Normalize the decision matrix using eq.(1) as shown in Fig 6.

Compute the weighted decision matrix using eq. (2) as shown in Fig 7.

Compute the distance values from ideal solutions using eqs. (3,4, and 5).

Determine the distance to positive and cost criteria using eqs. (6, 7, and 8) as shown in Figs 8,9, and 10.

Determine the final distance using eq. (9).

Order the alternatives.

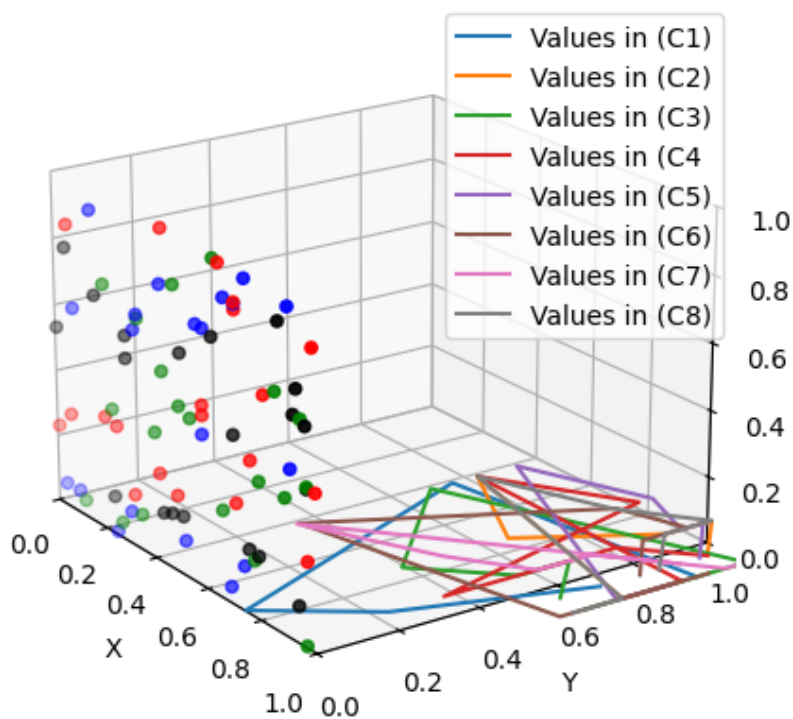


Fig 6. The normalized decision matrix.

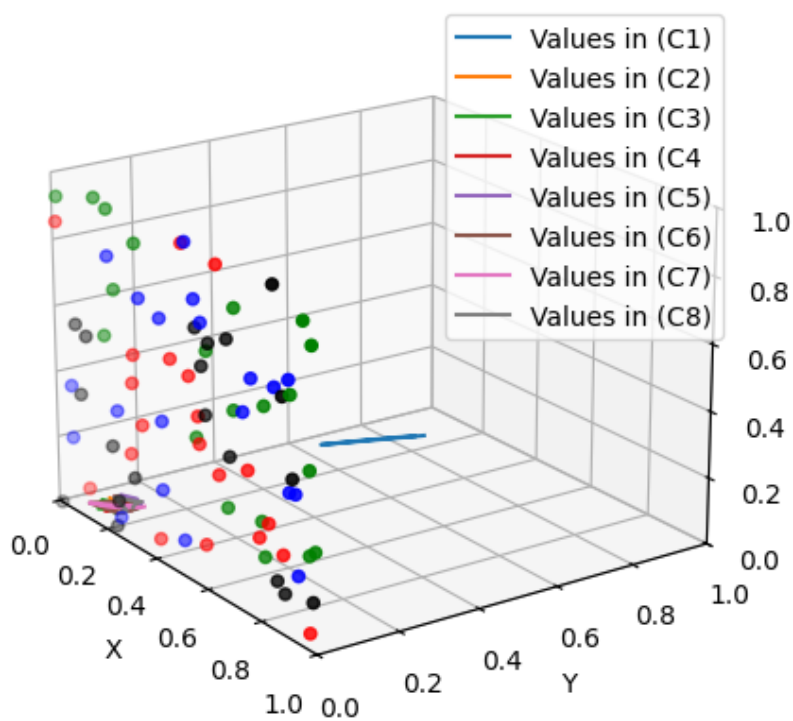
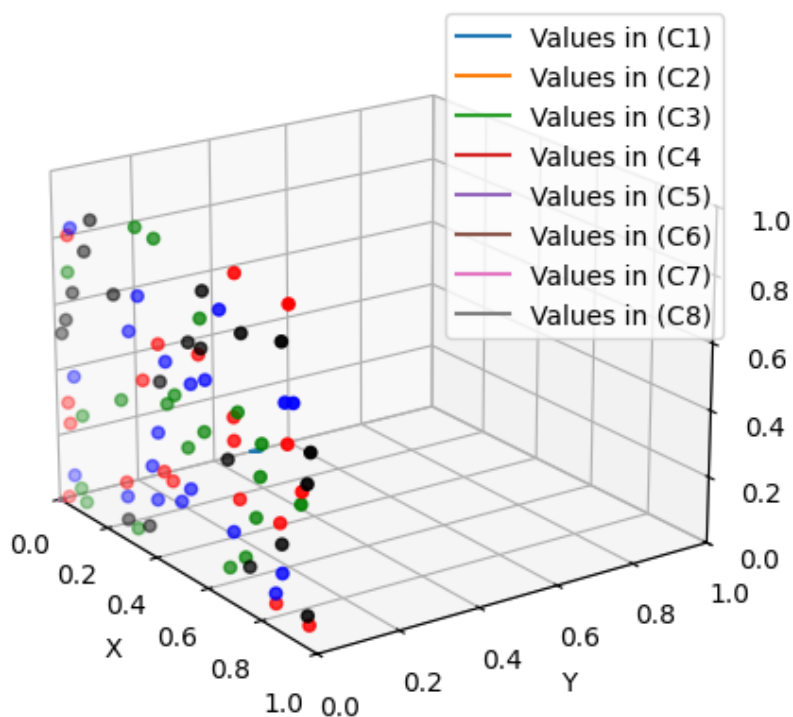
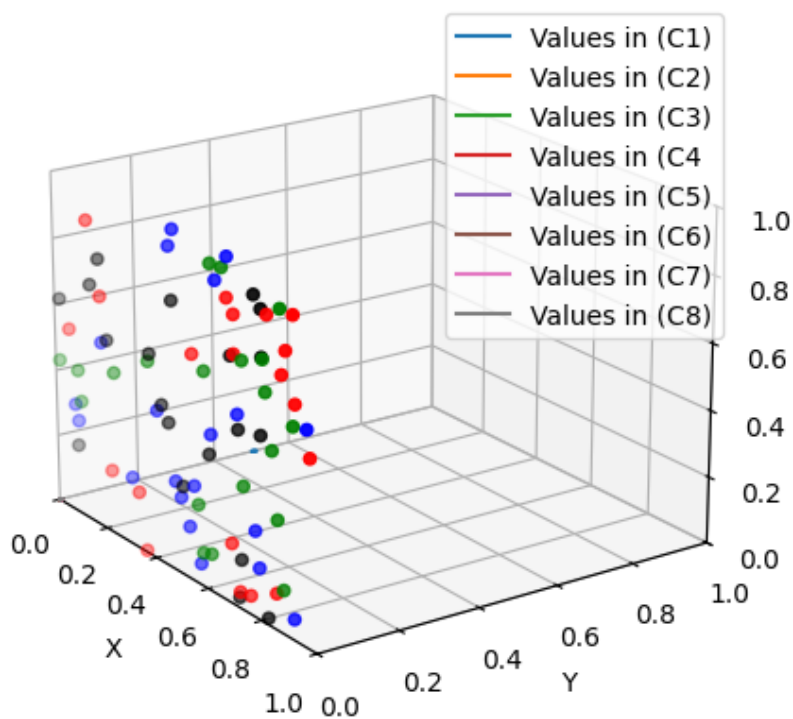


Fig 7. The weighted decision matrix.

Fig 8. The values of $T(D_j)$.Fig 9. The values of $T(A_j)$.

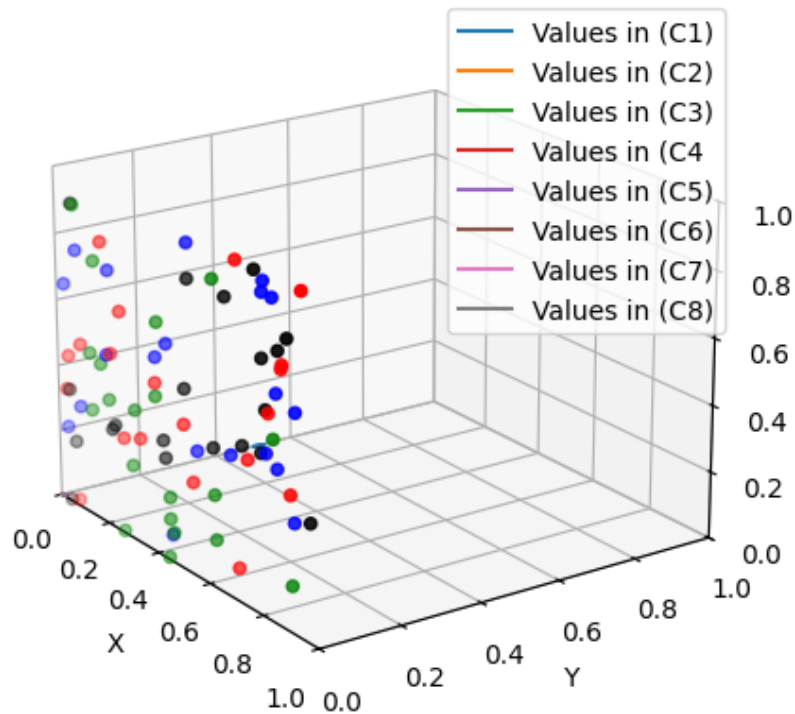


Fig 10. The values of $T(Q_j)$.

Set 3 results described such as:

Normalize the decision matrix using eq.(1) as shown in Fig 11.

Compute the weighted decision matrix using eq. (2) as shown in Fig 12.

Compute the distance values from ideal solutions using eqs. (3,4, and 5).

Determine the distance to positive and cost criteria using eqs. (6, 7, and 8) as shown in Figs 13,14, and 15.

Determine the final distance using eq. (9).

Order the alternatives.

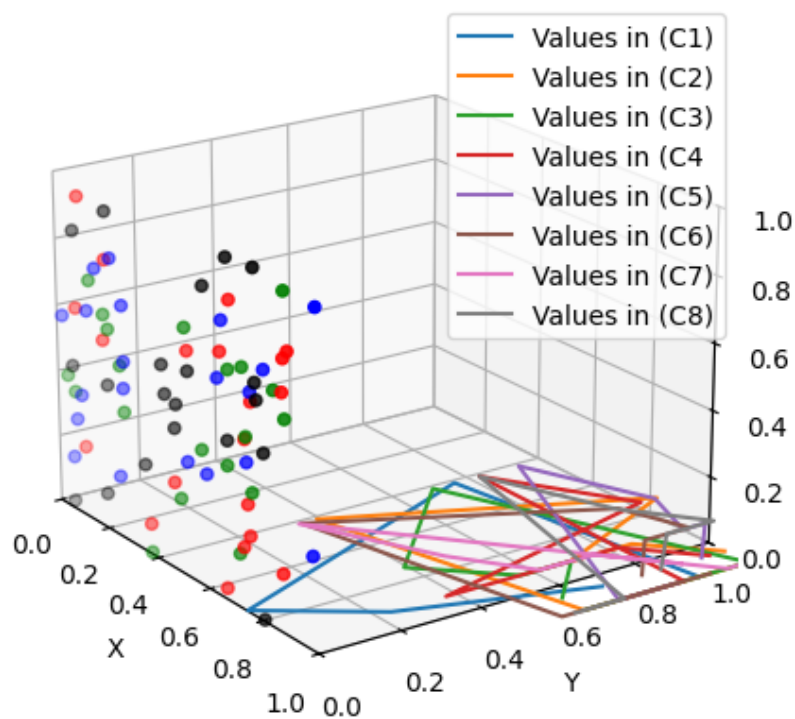


Fig 11. The normalized decision matrix.

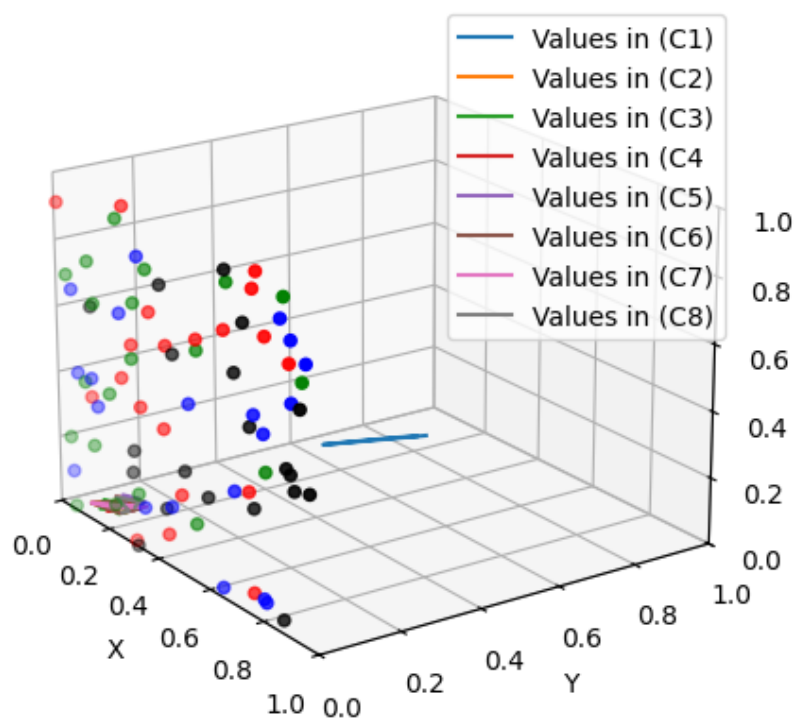
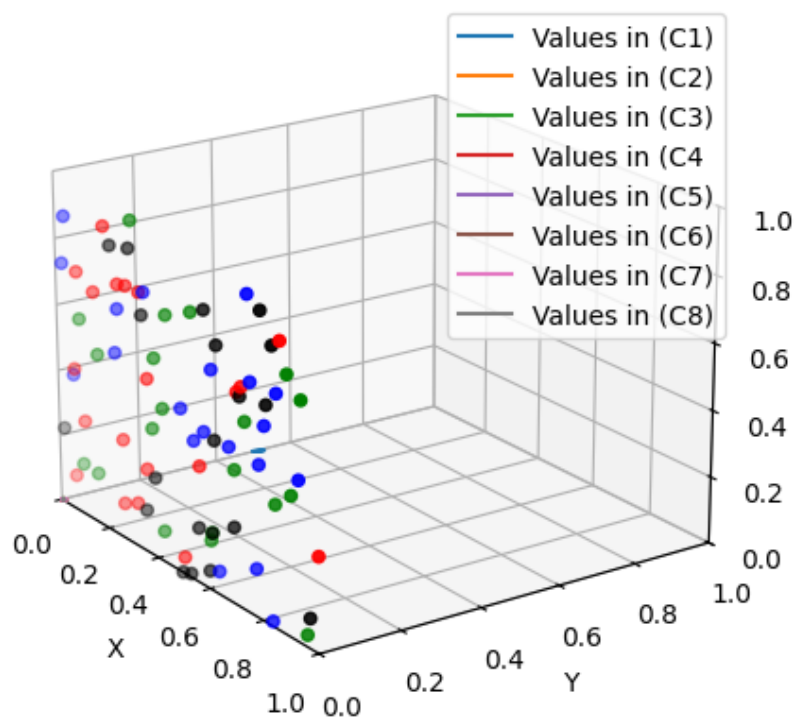
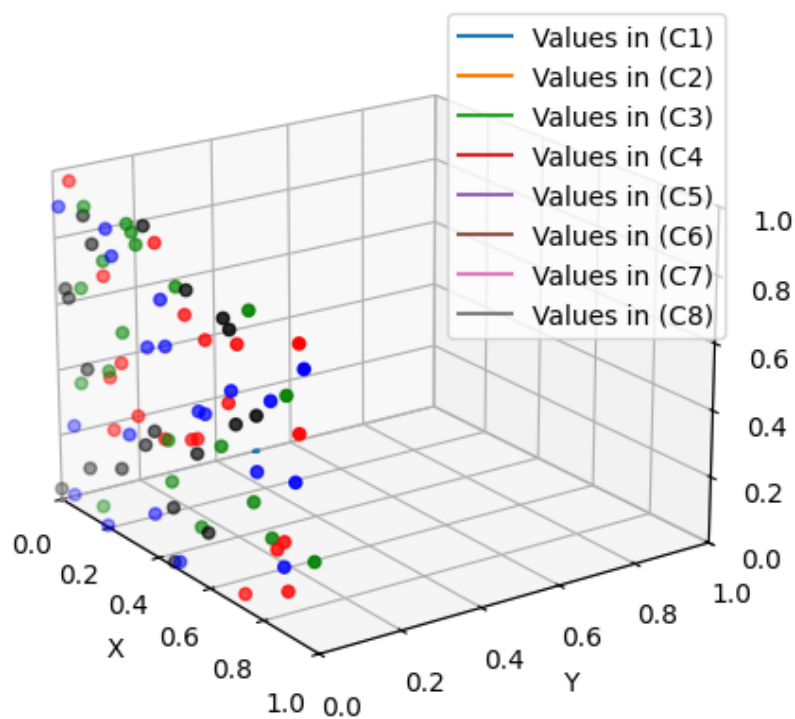


Fig 12. The weighted decision matrix.

Fig 13. The values of $T(D_j)$.Fig 14. The values of $T(A_j)$.

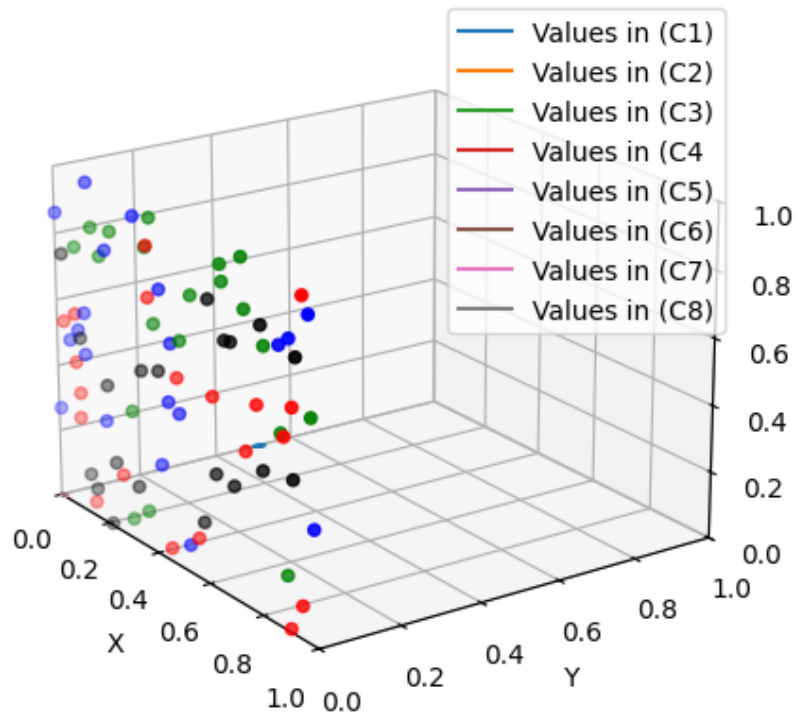


Fig 15. The values of $T(Q_j)$.

Set 4 results described such as:

Normalize the decision matrix using eq.(1) as shown in Fig 16.

Compute the weighted decision matrix using eq. (2) as shown in Fig 17.

Compute the distance values from ideal solutions using eqs. (3,4, and 5).

Determine the distance to positive and cost criteria using eqs. (6, 7, and 8) as shown in Figs 18,19, and 20.

Determine the final distance using eq. (9).

Order the alternatives.

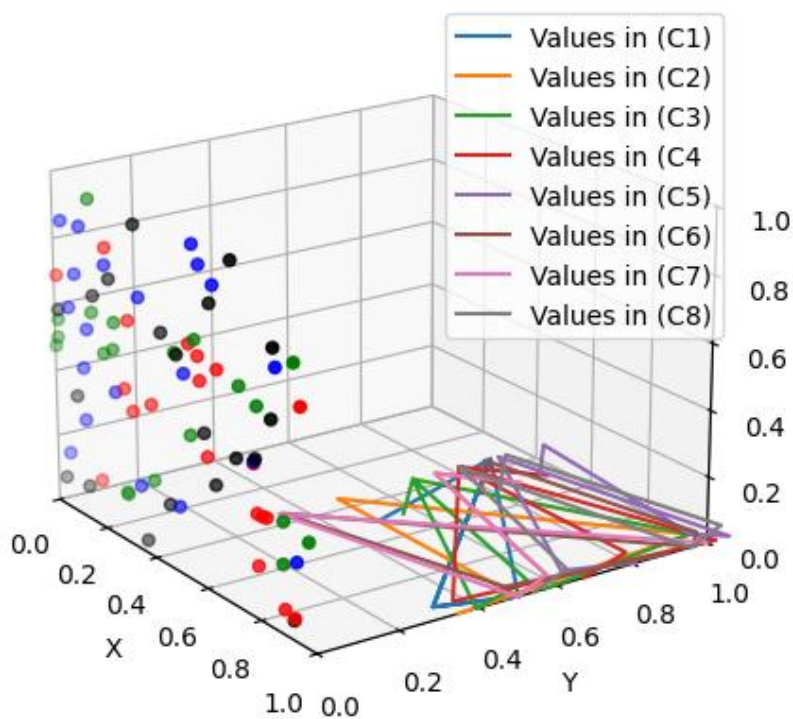


Fig 16. The normalized decision matrix.

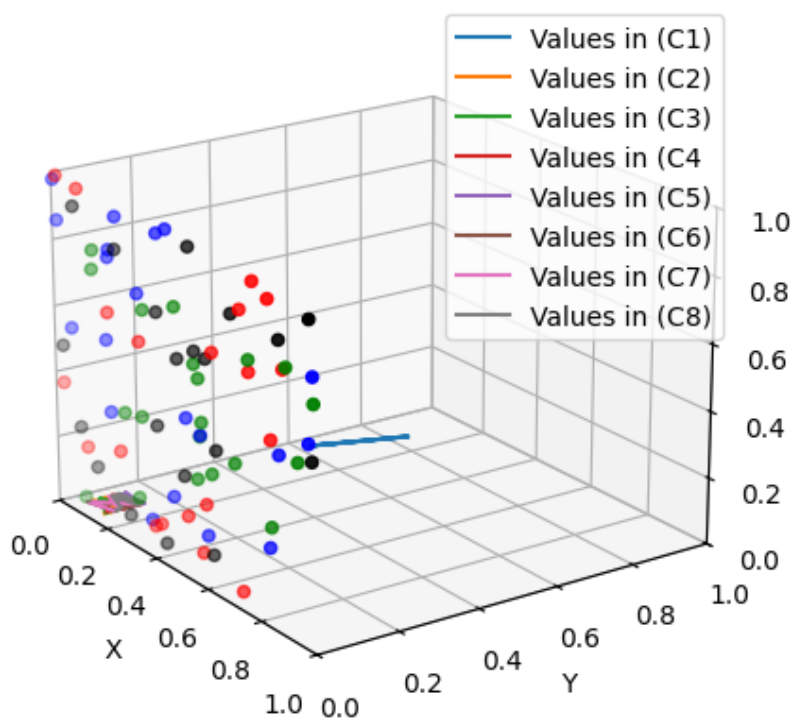
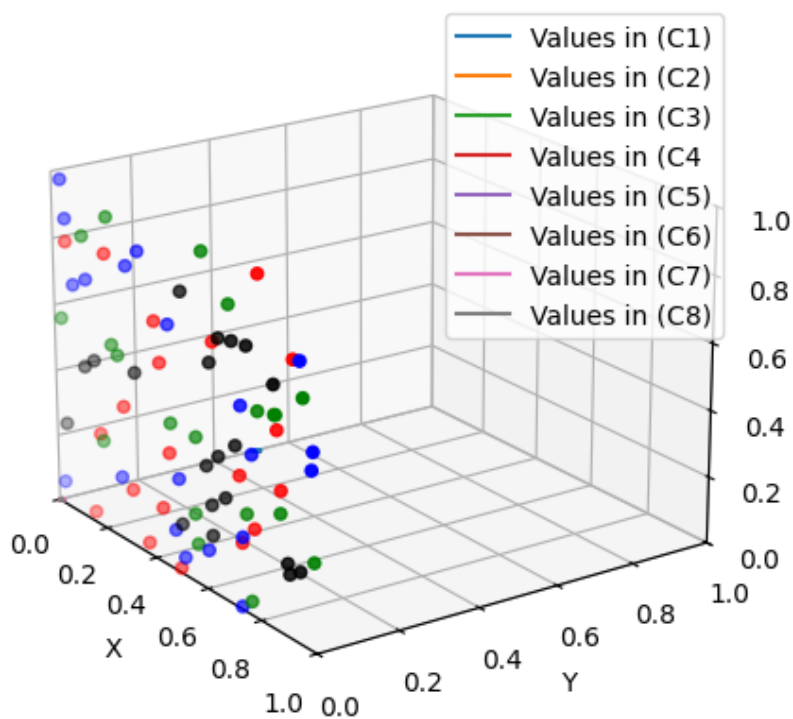
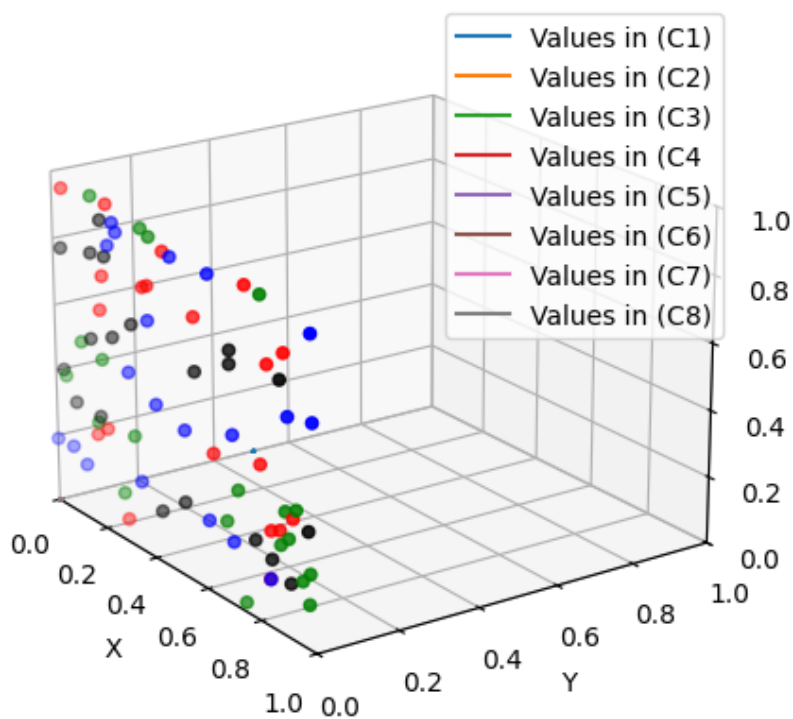


Fig 17. The weighted decision matrix.

Fig 18. The values of $T(D_j)$.Fig 19. The values of $T(A_j)$.

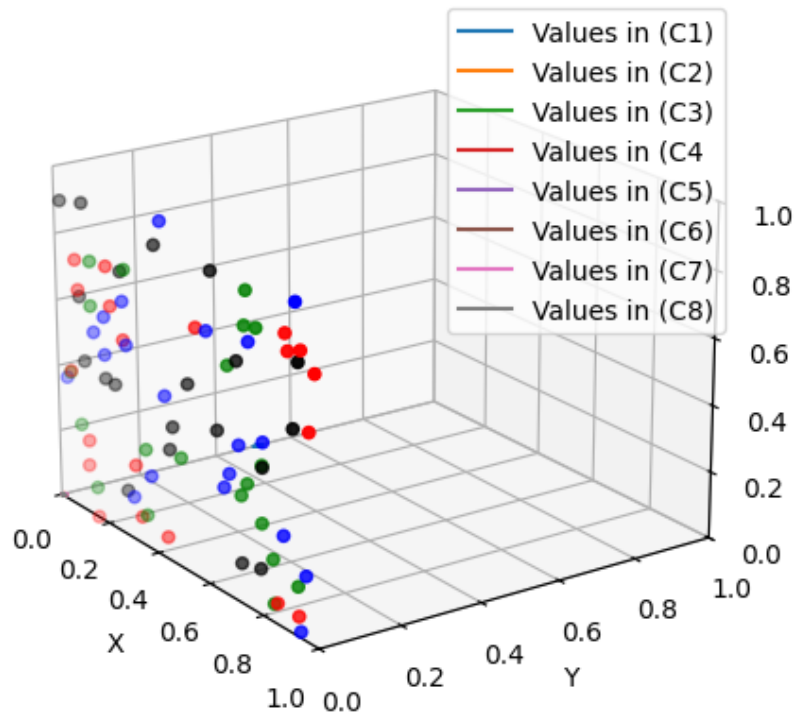


Fig 20. The values of $T(Q_j)$.

Set 5 results described such as:

Normalize the decision matrix using eq.(1) as shown in Fig 21.

Compute the weighted decision matrix using eq. (2) as shown in Fig 22.

Compute the distance values from ideal solutions using eqs. (3,4, and 5).

Determine the distance to positive and cost criteria using eqs. (6, 7, and 8) as shown in Figs 23,24, and 25.

Determine the final distance using eq. (9).

Order the alternatives.

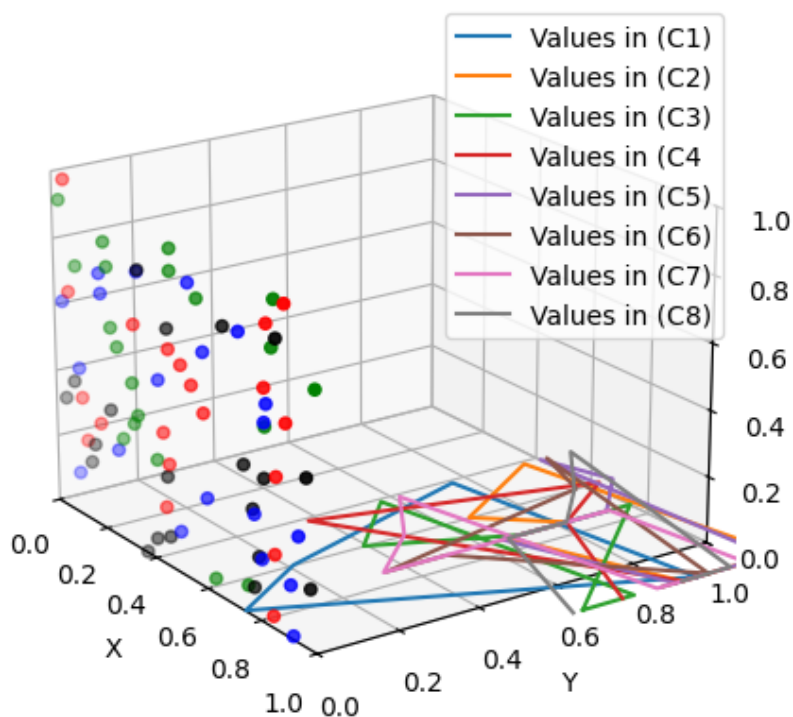


Fig 21. The normalized decision matrix.

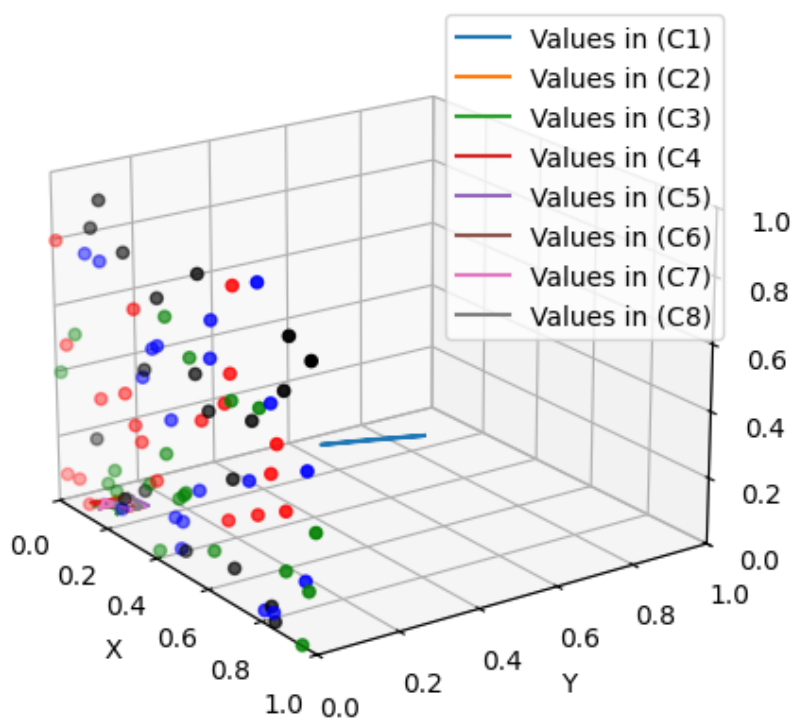
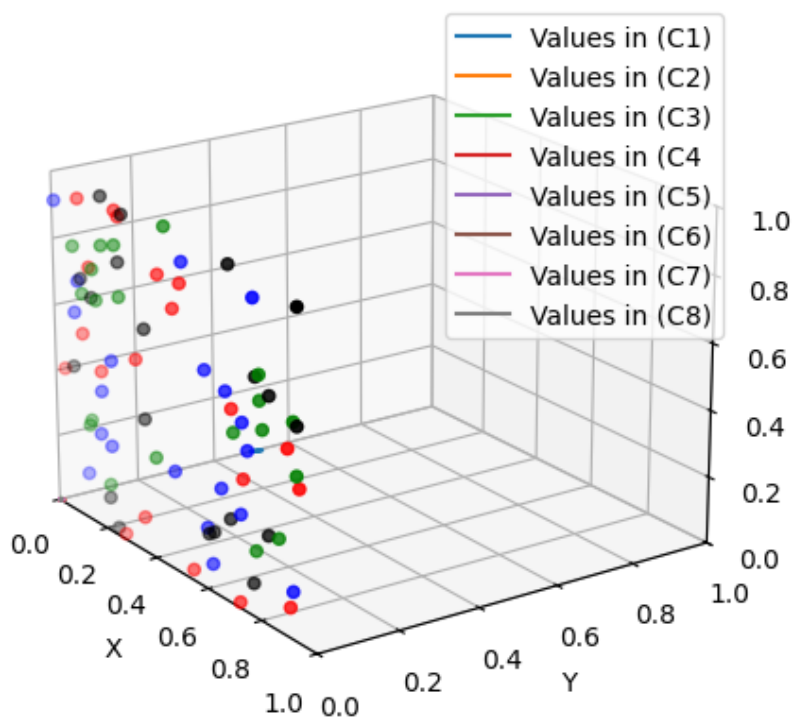
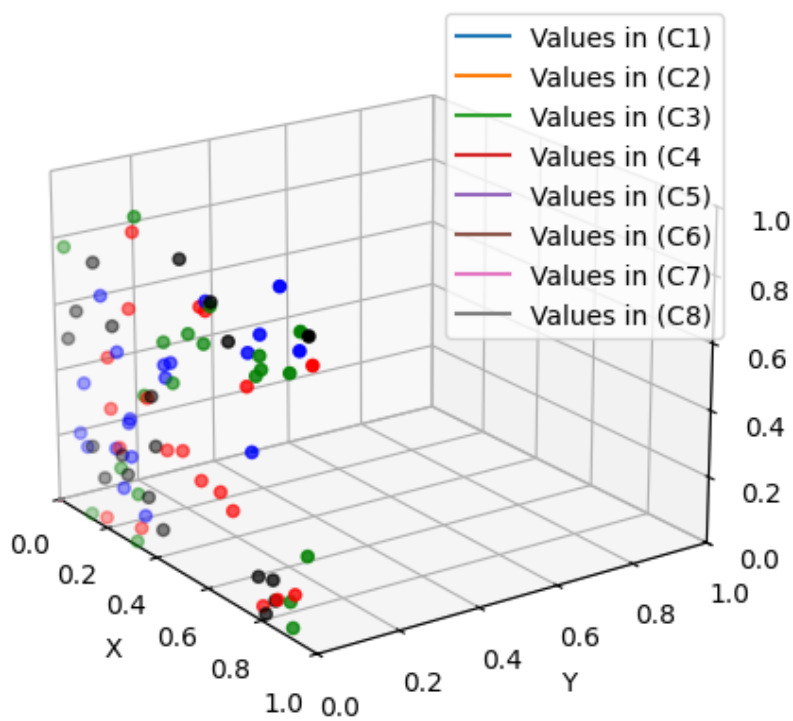


Fig 22. The weighted decision matrix.

Fig 23. The values of $T(D_j)$.Fig 24. The values of $T(A_j)$.

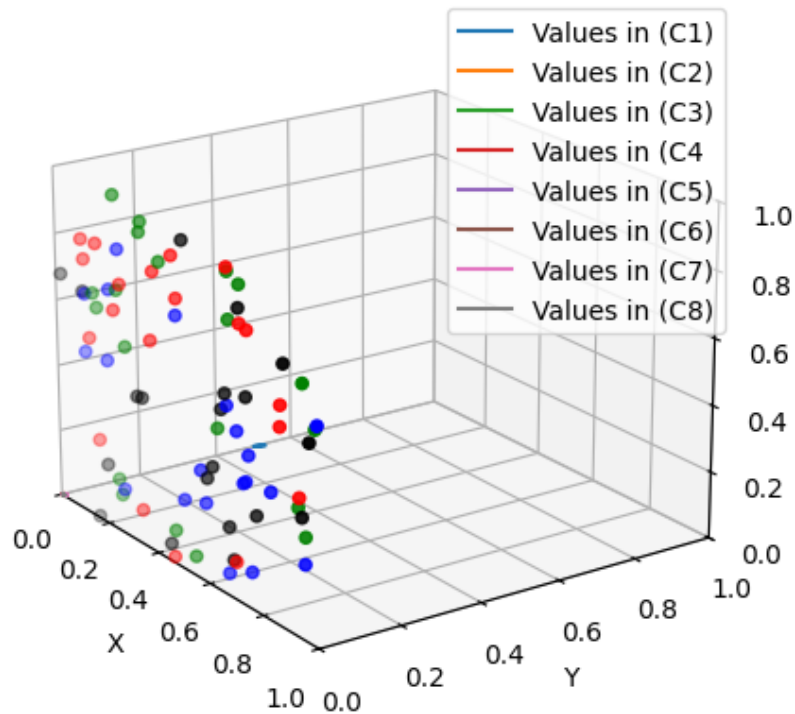


Fig 25. The values of $T(Q_j)$.

Set 6 results described such as:

Normalize the decision matrix using eq.(1) as shown in Fig 26.

Compute the weighted decision matrix using eq. (2) as shown in Fig 27.

Compute the distance values from ideal solutions using eqs. (3,4, and 5).

Determine the distance to positive and cost criteria using eqs. (6, 7, and 8) as shown in Figs 28,29, and 30.

Determine the final distance using eq. (9).

Order the alternatives.

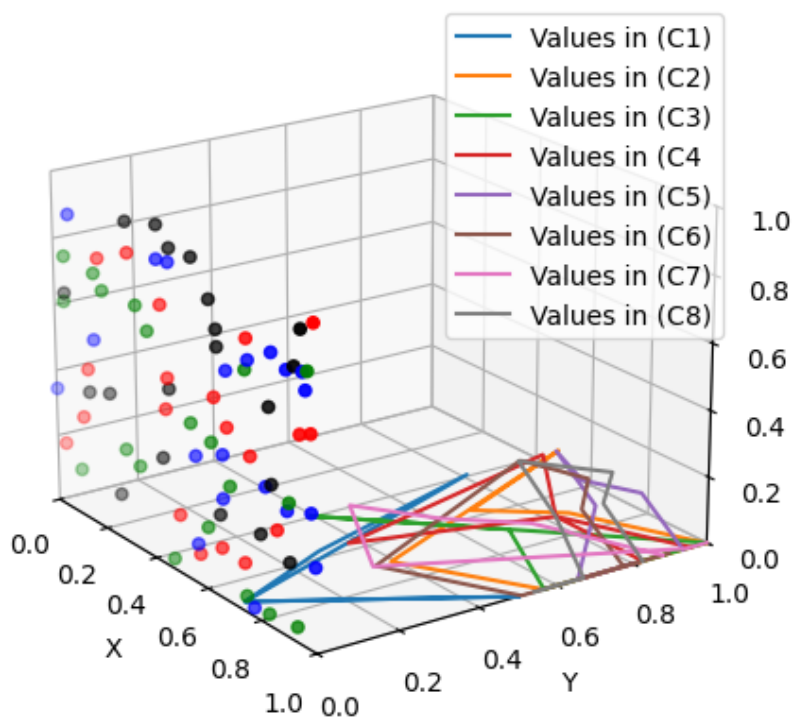


Fig 26. The normalized decision matrix.

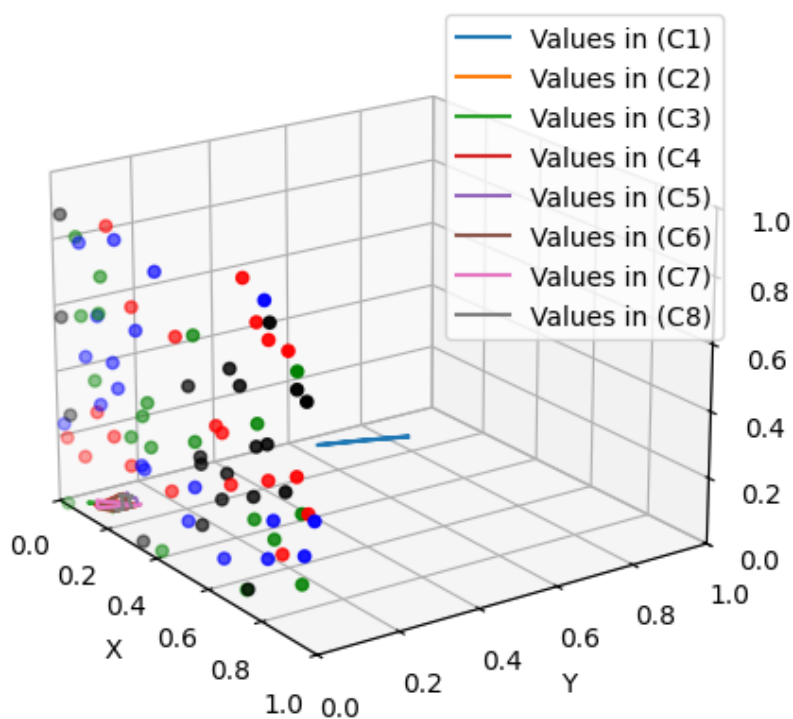
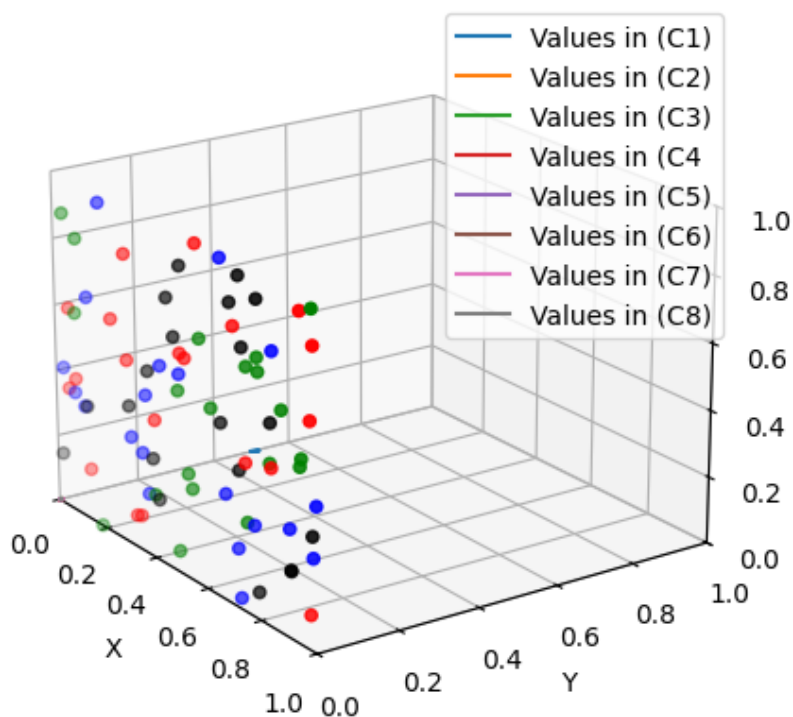
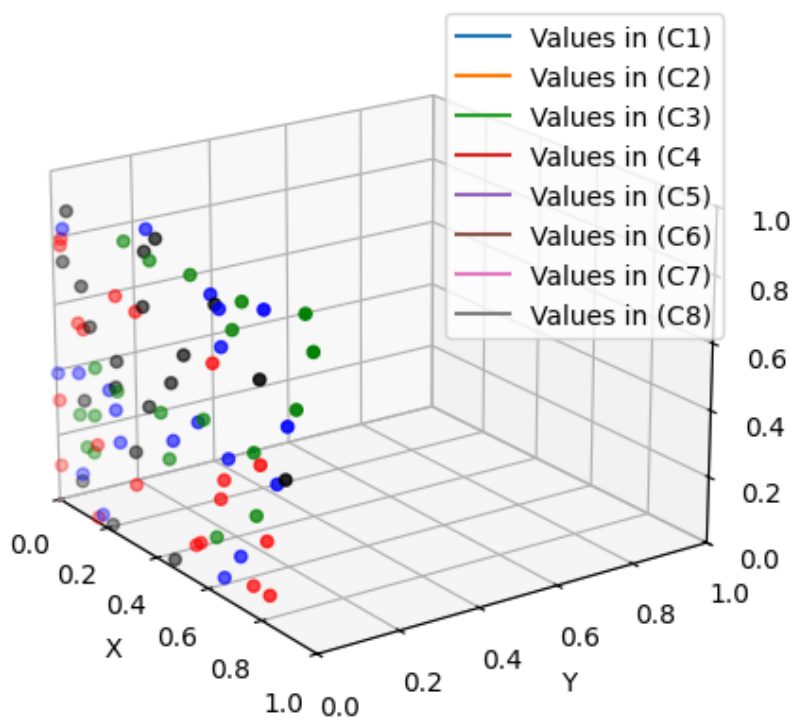


Fig 27. The weighted decision matrix.

Fig 28. The values of $T(D_j)$.Fig 29. The values of $T(A_j)$.

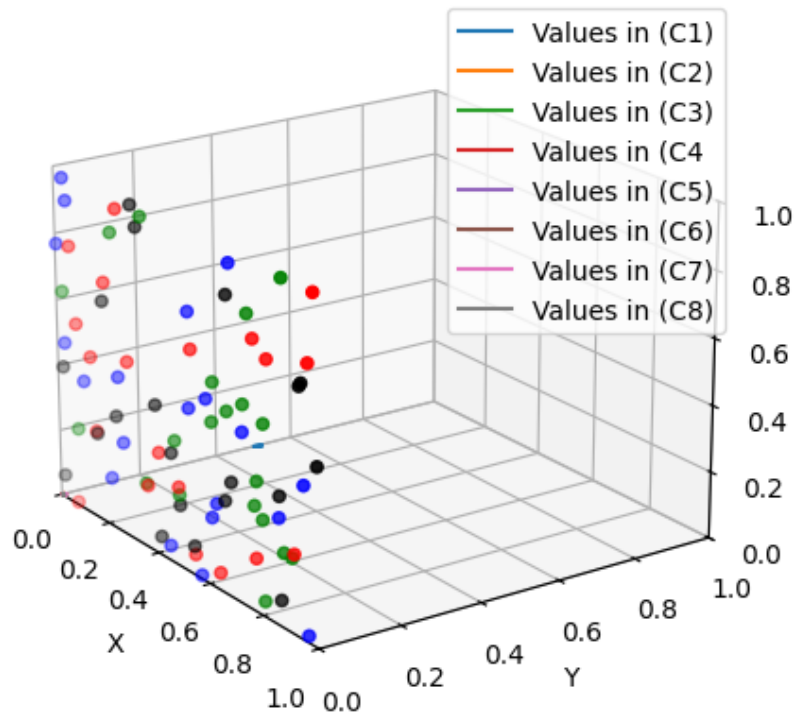


Fig 30. The values of $T(Q_j)$.

We obtain the final ranks of alternatives in each set such as:

Set 1

- Highest Performer: A6 (Rank 1 - lowest), A7 (2 - better).
- Mid Performers: A4 (3), A5 (4), A3 (5).
- Lower Performers: A1 (6), A2 (7 - highest).
- A2 is the strongest (Rank 7); A6 is the weakest.

Set 2

- Highest Performer: A6 (1), A7 (2).
- Mid Performers: A1 (3), A4 (4), A5 (5).
- Lower Performers: A3 (6), A2 (7).
- A2 again is the strongest performer; A6 struggles.

Set 3

- Highest Performer: A6 (1), A7 (2).
- Mid Performers: A5 (3), A1 (4), A4 (5).
- Lower Performers: A2 (6), A3 (7).
- A3 is the best (Rank 7); A6 is the weakest.

Set 4

- Highest Performer: A7 (1), A6 (2).
- Mid Performers: A3 (3), A5 (4), A4 (5).
- Lower Performers: A1 (6), A2 (7).
- A2 shows the highest performance (7); A7 is at the lowest (1).

Set 5

- Highest Performer: A6 (1), A7 (2).
- Mid Performers: A3 (3), A1 (4).
- Lower Performers: A2 (5), A4 (6), A5 (7).
- A5 shows the best result (7); A6 is the lowest.

Set 6

- Highest Performer: A6 (1), A7 (2).
- Mid Performers: A1 (3), A5 (4), A3 (5).
- Lower Performers: A2 (6), A4 (7).
- A4 is the best performer (7); A6 the worst (1).

4. Conclusions

The evaluation of college teaching competition strategies might be revolutionized by artificial intelligence. AI may overcome many of the long-standing drawbacks of conventional approaches by introducing objectivity, scalability, and data-driven insights into the review process. However, ethical issues, system openness, and the complementing function of human assessors must all be carefully considered. AI-driven assessment tools will probably play a key role in initiatives to foster excellent instruction and encourage academic achievement as education develops in tandem with technology. We use the IndtermSoft Set to solve the indeterminacy in the criteria values. This study has eight criteria and seven alternatives. The COBRA method is used to rank the alternatives.

Acknowledgment

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Application of Neutrosophic SuperHyperSoft Sets in MADM

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Abstract: In this work, we introduce the concept of Neutrosophic SuperHyperSoft Sets (NSHSS) by considering a universe of discourse and disjoint attribute structures, and develop fundamental operations including addition, multiplication, scalar multiplication, and scalar exponentiation. A novel Neutrosophic SuperHyperSoft Weighted Aggregation (NSHSSWA) operator is proposed, and a new entropy measure is defined to quantify uncertainty, with its theoretical properties validated. Building on this, we propose a Multi-Attribute Decision-Making (MADM) framework for selecting optimal mobile phones by combining NSHSS with entropy-based weighting. Five primary criteria- operational efficiency, power endurance, imaging capability, cost, and robustness are evaluated using power sets and neutrosophic modelling to capture uncertainty in user preferences. Subjective linguistic evaluations are mapped to neutrosophic values, and aggregated using the NSHSSWA operator. Among 162 propositions constructed, the top alternatives are identified and ranked using a neutrosophic score function. The proposed methodology offers a comprehensive and reliable tool for guiding consumers toward selecting high quality mobile phones based on personalized needs, while providing a strong foundation for future uncertain information modelling and decision support.

Keywords: Neutrosophic SuperHyperSoft Sets, SuperHyperSoft Sets, Neutrosophic Soft Sets, Neutrosophic Sets.

1. Introduction and Preliminaries

Decision-making is in general a complex task and requires the frameworks capable of dealing with ambiguity, uncertainty and a lack of knowledge. After being introduced by Smarandache in 1999, Neutrosophic Sets (NSs) offer a mathematically sound method to express indeterminacy with three independent parameters: truth, indeterminacy and falsehood. NSs are especially very useful in modelling complex situations involving overlapping and contradiction information due to this extension of IFs. The concept of Soft Sets (SSs) was introduced by Molodtsov in 1999. They offer a flexible mathematical framework for managing uncertainty without the need for additional elements like membership grades or probabilities. SSs have been found very useful in several decision-making problems due to their possibility of effective expression and interpretation of qualitative and quantitative data. By coupling neutrosophic logic with soft sets, Maji et al. (2003) introduced Neutrosophic Soft Sets (NSSs), which extend the decision-making capabilities by apprehend the varying degrees of truth, indeterminacy and falsity. Abbas with his colleagues (2020) instigated the concept of HyperSoft Sets (HSSs) to deal with multi-attribute problems having a hierarchical data structure. The HSSs generalize SSs by introducing

sub-attributes and enable more extensive analysis of complex systems.

Later, the extension of neutrosophic concepts into Hyper Soft Sets (HSSs) led to the creation of Neutrosophic HyperSoft Sets (NHSSs), which combined the advantages of Hyper Soft Sets and Neutrosophic Sets to simultaneously address uncertainty and granularity (Saeed et al., 2020). The latest development in soft set theory is the 2023 foundation by Smarandache of SuperHyperSoft Sets (SHSSs). Compared to HSSs, SHSSs have greater representational power due to the incorporation of multi-dimensional analysis and higher levels of granularity. The Neutrosophic extension, NSHSSs, further strengthens the framework's ability to represent ambiguity, uncertainty and interdependencies among criteria (Smarandache, 2024). The present paper is an attempt to apply NSHSSs to Multi-Attribute Decision Making, with a view to showing how their strong representational capabilities can be used in solving real-life decision-making problems.

Definition 1.1 [7]: Consider X is universe of discourse, $\mathfrak{N}(X)$ the power set of X and $\ell_1, \ell_2, \dots, \ell_n$ ($n \geq 1$) distinct attributes with disjoint corresponding sets $\wp_1, \wp_2, \dots, \wp_n$. Consider $\mathfrak{N}(\wp_i)$ denote the powerset of $\wp_i$ for $i = 1, 2, \dots, n$. A SuperHyperSoft Set (SHSS) over X is defined as the pair $(\gamma, \mathfrak{N}(\wp_1) \times \mathfrak{N}(\wp_2) \times \dots \times \mathfrak{N}(\wp_n))$ where $\gamma : \mathfrak{N}(\wp_1) \times \mathfrak{N}(\wp_2) \times \dots \times \mathfrak{N}(\wp_n) \rightarrow \mathfrak{N}(X)$.

Definition 1.2 [7,8]: Consider X is universe of discourse, $\mathfrak{N}(X)$ the powerset of X and $\ell_1, \ell_2, \dots, \ell_n$ ($n \geq 1$) distinct attributes with disjoint corresponding sets $\wp_1, \wp_2, \dots, \wp_n$. Let $\mathfrak{N}(\wp_i)$ denote the powerset of $\wp_i$, for $i = 1, 2, \dots, n$. A Neutrosophic SuperHyperSoft Set (NSHSS) over X is defined as the pair $(\gamma, \mathfrak{N}(\wp_1) \times \mathfrak{N}(\wp_2) \times \dots \times \mathfrak{N}(\wp_n))$ where $\gamma : \mathfrak{N}(\wp_1) \times \mathfrak{N}(\wp_2) \times \dots \times \mathfrak{N}(\wp_n) \rightarrow \mathfrak{N}(X)$ and $\gamma = \{b, < a, \mathcal{T}_{\gamma(b)}(a), \mathcal{I}_{\gamma(b)}(a), \mathcal{F}_{\gamma(b)}(a) > : a \in X, b \in \mathfrak{N}(\wp_1) \times \mathfrak{N}(\wp_2) \times \dots \times \mathfrak{N}(\wp_n)\}$. Here $\mathcal{T}_{\gamma(b)}(a), \mathcal{I}_{\gamma(b)}(a), \mathcal{F}_{\gamma(b)}(a) : X \rightarrow [0,1]$ designate the membership, indeterminacy and non-membership degrees of $x \in X$ for every y , satisfying: $0 \leq \mathcal{T}_{\gamma(b)}(a) + \mathcal{I}_{\gamma(b)}(a) + \mathcal{F}_{\gamma(b)}(a) \leq 3$.

Definition 1.3 [8]: Consider X is universe of discourse, $\mathfrak{N}(X)$ the powerset of X and $\ell_1, \ell_2, \dots, \ell_n$ ($n \geq 1$) distinct attributes with disjoint corresponding sets $\wp_1, \wp_2, \dots, \wp_n$. Let $\mathfrak{N}(\wp_i)$ denote the powerset of $\wp_i$, for $i = 1, 2, \dots, n$. Let γ, δ be two NSHSS over X is defined as the pair $(\gamma, \mathfrak{N}(\wp_1) \times \mathfrak{N}(\wp_2) \times \dots \times \mathfrak{N}(\wp_n))$ and $(\delta, \mathfrak{N}(\wp_1) \times \mathfrak{N}(\wp_2) \times \dots \times \mathfrak{N}(\wp_n))$ where $\gamma, \delta : \mathfrak{N}(\wp_1) \times \mathfrak{N}(\wp_2) \times \dots \times \mathfrak{N}(\wp_n) \rightarrow \mathfrak{N}(X)$ and $\gamma = \{b, < x, \mathcal{T}_{\gamma(b)}(a), \mathcal{I}_{\gamma(b)}(a), \mathcal{F}_{\gamma(b)}(a) > : a \in X, b \in \mathfrak{N}(\wp_1) \times \mathfrak{N}(\wp_2) \times \dots \times \mathfrak{N}(\wp_n)\}$ $\delta = \{b, < c, \mathcal{T}_{\delta(b)}(c), \mathcal{I}_{\delta(b)}(c), \mathcal{F}_{\delta(b)}(c) > : c \in X, b \in \mathfrak{N}(\wp_1) \times \mathfrak{N}(\wp_2) \times \dots \times \mathfrak{N}(\wp_n)\}$.

Then the basic operators are defined as

1. $\gamma \leq \delta = < \mathcal{T}_{\gamma(b)}(a) \leq \mathcal{T}_{\delta(b)}(c), \mathcal{I}_{\gamma(b)}(a) \leq \mathcal{I}_{\delta(b)}(c), \mathcal{F}_{\gamma(b)}(a) \geq \mathcal{F}_{\delta(b)}(c) >.$
2. $\gamma = < \mathcal{T}_{\gamma(b)}(a) = 0, \mathcal{I}_{\gamma(b)}(a) = 0, \mathcal{F}_{\gamma(b)}(a) = 0 >.$
3. $\gamma^c = < \mathcal{F}_{\gamma(b)}(a), \mathcal{I}_{\gamma(b)}(a), \mathcal{T}_{\gamma(b)}(a) >.$
4. $\gamma \cup \delta = < \max\{\mathcal{T}_{\gamma(b)}(a), \mathcal{T}_{\delta(b)}(c)\}, \frac{\mathcal{I}_{\gamma(b)}(a) + \mathcal{I}_{\delta(b)}(c)}{2}, \min\{\mathcal{F}_{\gamma(b)}(a), \mathcal{F}_{\delta(b)}(c)\} >.$
5. $\gamma \cap \delta = < \min\{\mathcal{T}_{\gamma(b)}(a), \mathcal{T}_{\delta(b)}(c)\}, \frac{\mathcal{I}_{\gamma(b)}(a) + \mathcal{I}_{\delta(b)}(c)}{2}, \max\{\mathcal{F}_{\gamma(b)}(a), \mathcal{F}_{\delta(b)}(c)\} >.$

2. Operators in Neutrosophic SuperHypersoft Sets

Definition 2.1: Consider X is universe of discourse, $\aleph(X)$ the powerset of X and $\ell_1, \ell_2, \dots, \ell_n$ ($n > 1$) distinct attributes with disjoint corresponding sets $\wp_1, \wp_2, \dots, \wp_n$. Let $\aleph(\wp_i)$ denote the powerset of $\wp_i$ for $i = 1, 2, \dots, n$.

Let γ, δ be two NSHSS over X is defined as the pair $(\gamma, \aleph(\wp_1) \times \aleph(\wp_2) \times \dots \times \aleph(\wp_n))$ and

$(\delta, \aleph(\wp_1) \times \aleph(\wp_2) \times \dots \times \aleph(\wp_n))$ where $\gamma, \delta : \aleph(\wp_1) \times \aleph(\wp_2) \times \dots \times \aleph(\wp_n) \rightarrow \aleph(X)$ and

$\gamma = \{b, < a, \mathcal{T}_{\gamma(b)}(a), \mathcal{I}_{\gamma(b)}(a), \mathcal{F}_{\gamma(b)}(a) : a \in X, b \in \aleph(\wp_1) \times \aleph(\wp_2) \times \dots \times \aleph(\wp_n)\}$

$\delta = \{b, < c, \mathcal{T}_{\delta(b)}(c), \mathcal{I}_{\delta(b)}(c), \mathcal{F}_{\delta(b)}(c) : c \in X, b \in \aleph(\wp_1) \times \aleph(\wp_2) \times \dots \times \aleph(\wp_n)\}$. Then the basic operators are defined as,

1. $\gamma \oplus \delta = \langle \mathcal{T}_{\gamma(b)}(a) + \mathcal{T}_{\delta(b)}(c) - \mathcal{T}_{\gamma(b)}(a) \cdot \mathcal{T}_{\delta(b)}(c), \mathcal{I}_{\gamma(b)}(a) \cdot \mathcal{I}_{\delta(b)}(c), \mathcal{F}_{\gamma(b)}(a) \cdot \mathcal{F}_{\delta(b)}(c) \rangle$.
2. $\gamma \otimes \delta = \langle \mathcal{T}_{\gamma(b)}(a) \cdot \mathcal{T}_{\delta(b)}(c), \mathcal{I}_{\gamma(b)}(a) + \mathcal{I}_{\delta(b)}(c) - \mathcal{I}_{\gamma(b)}(a) \cdot \mathcal{I}_{\delta(b)}(c), \mathcal{F}_{\gamma(b)}(a) + \mathcal{F}_{\delta(b)}(c) - \mathcal{F}_{\gamma(b)}(a) \cdot \mathcal{F}_{\delta(b)}(c) \rangle$.
3. $\lambda \cdot \gamma = \langle 1 - (1 - \mathcal{T}_{\gamma(b)}(a))^\lambda, (\mathcal{I}_{\gamma(b)}(a))^\lambda, (\mathcal{F}_{\gamma(b)}(a))^\lambda \rangle$, where $\lambda > 0$.
4. $\gamma^\lambda = \langle (\mathcal{T}_{\gamma(b)}(a))^\lambda, 1 - (1 - \mathcal{I}_{\gamma(b)}(a))^\lambda, 1 - (1 - \mathcal{F}_{\gamma(b)}(a))^\lambda \rangle$, where $\lambda > 0$.

Theorem 2.2: Let $\gamma_i = (\mathcal{T}_{\gamma(b)}(a_i), \mathcal{I}_{\gamma(b)}(a_i), \mathcal{F}_{\gamma(b)}(a_i))$, $i = 1, 2, 3, \dots, n$ be a collection of NSHSS, then Neutrosophic SuperHyperSoft Weighted Aggregation (NSHSSWA) operator value is also a NSHSS and $NSHSSWA(\gamma_1, \gamma_2, \gamma_3 \dots \gamma_n) =$

$$(1 - \prod_{i,j=1}^n (1 - \mathcal{T}_{\gamma(b)}(a_i))^{w_j}, \prod_{i,j=1}^n (\mathcal{I}_{\gamma(b)}(a_i))^{w_j}, \prod_{i,j=1}^n (\mathcal{F}_{\gamma(b)}(a_i))^{w_j}) \quad \dots \dots \dots (1)$$

where $w = (w_1, w_2, w_3, \dots, w_n)^T$ be the weight vector of γ_j ($j = 1, 2, 3, \dots, n$) and $w_j > 0$, $\sum_{j=1}^n w_j = 1$.

Proof. Prove this theorem by applying mathematical induction on n .

If $n = 2$, we have $NSHSSWA(\gamma_1, \gamma_2) = w_1 \gamma_1 \oplus w_2 \gamma_2$

By Definition 2.1, we can see that both $w_1 \gamma_1$ and $w_2 \gamma_2$ are NSHSS and the value of $w_1 \gamma_1 \oplus w_2 \gamma_2$ is also a NSHSS. From the operational laws of NSHSS, we have

$$w_1 \gamma_1 = (1 - (1 - \mathcal{T}_{\gamma(b)}(a_1))^{w_1}, (\mathcal{I}_{\gamma(b)}(a_1))^{w_1}, (\mathcal{F}_{\gamma(b)}(a_1))^{w_1});$$

$$w_2 \gamma_2 = (1 - (1 - \mathcal{T}_{\gamma(b)}(a_2))^{w_2}, (\mathcal{I}_{\gamma(b)}(a_2))^{w_2}, (\mathcal{F}_{\gamma(b)}(a_2))^{w_2})$$

Then, $NSHSSWA(\gamma_1, \gamma_2) = w_1 \gamma_1 \oplus w_2 \gamma_2$

$$= \langle 2 - (1 - \mathcal{T}_{\gamma(b)}(a_1))^{w_1} - (1 - \mathcal{T}_{\gamma(b)}(a_2))^{w_2} - (1 - (1 - \mathcal{T}_{\gamma(b)}(a_1))^{w_1})(1 - (1 - \mathcal{T}_{\gamma(b)}(a_2))^{w_2}),$$

$$(\mathcal{I}_{\gamma(b)}(a_1))^{w_1} (\mathcal{I}_{\gamma(b)}(a_2))^{w_2}, (\mathcal{F}_{\gamma(b)}(a_1))^{w_1} (\mathcal{F}_{\gamma(b)}(a_2))^{w_2} \rangle$$

$$= \langle 1 - (1 - \mathcal{T}_{\gamma(b)}(a_1))^{w_1} (1 - \mathcal{T}_{\gamma(b)}(a_2))^{w_2}, (\mathcal{I}_{\gamma(b)}(a_1))^{w_1} (\mathcal{I}_{\gamma(b)}(a_2))^{w_2}, (\mathcal{F}_{\gamma(b)}(a_1))^{w_1} (\mathcal{F}_{\gamma(b)}(a_2))^{w_2} \rangle.$$

If $n = k$, then Equation (1) holds, i.e.,

$$NSHSSWA(\gamma_1, \gamma_2, \gamma_3 \dots \gamma_n) = w_1 \gamma_1 \oplus w_2 \gamma_2 \dots \dots \dots \oplus w_k \gamma_k$$

$$= \langle 1 - \prod_{j=1}^k (1 - \mathcal{T}_{\gamma(b)}(a_i))^{w_j}, \prod_{j=1}^k (\mathcal{I}_{\gamma(b)}(a_i))^{w_j}, \prod_{j=1}^k (\mathcal{F}_{\gamma(b)}(a_i))^{w_j} \rangle$$

and the aggregated value is a NSHSS, then when $n = k + 1$, by the operational laws of NSHSS, we have

$$NSHSSWA(\gamma_1, \gamma_2, \gamma_3 \dots \gamma_{k+1}) = w_1 \gamma_1 \oplus w_2 \gamma_2 \dots \dots \dots \oplus w_k \gamma_k \oplus w_{k+1} \gamma_{k+1}$$

$$= \langle 1 - \prod_{j=1}^k (1 - \mathcal{T}_{\gamma(b)}(a_i))^{w_j} + (1 - (1 - \mathcal{T}_{\gamma(b)}(a_{k+1}))^{w_{k+1}}) - (1 - \prod_{j=1}^k (1 - \mathcal{T}_{\gamma(b)}(a_i))^{w_j}) (1 -$$

$$(1 - \mathcal{T}_{\gamma(b)}(a_{k+1}))^{w_{k+1}}), \prod_{j=1}^{k+1} (\mathcal{I}_{\gamma(b)}(a_i))^{w_j}, \prod_{j=1}^{k+1} (\mathcal{F}_{\gamma(b)}(a_i))^{w_j} \rangle$$

$$= \langle 1 - \prod_{j=1}^{k+1} (1 - \mathcal{T}_{\gamma(b)}(a_i))^{w_j}, \prod_{j=1}^{k+1} (\mathcal{I}_{\gamma(b)}(a_i))^{w_j}, \prod_{j=1}^{k+1} (\mathcal{F}_{\gamma(b)}(a_i))^{w_j} \rangle.$$

By which aggregated value is also a NSHSS, Therefore, when $n = k + 1$, Equation (1) holds. Thus, by steps 1 and 2, we recognize that Equation (1) sustain for all n .

Definition 2.3. Let $\gamma_i = \{(\mathcal{J}_{\gamma(b)}(a_i), \mathcal{I}_{\gamma(b)}(a_i), \mathcal{F}_{\gamma(b)}(a_i)) | a_i \in X\}$ be a NSHSS set on X . Then the entropy of γ_i is defined as,

$$E(\gamma_i) = 1 - \frac{1}{2m} \sum_{i=1}^m ((\mathcal{J}_{\gamma(b)}(a_i) + \mathcal{I}_{\gamma(b)}(a_i)) | (2\mathcal{F}_{\gamma(b)}(a_i) - 1)| + (\mathcal{J}_{\gamma(b)}(a_i) + \mathcal{F}_{\gamma(b)}(a_i)) | (2\mathcal{I}_{\gamma(b)}(a_i) - 1)| + (\mathcal{F}_{\gamma(b)}(a_i) + \mathcal{I}_{\gamma(b)}(a_i)) | (2\mathcal{J}_{\gamma(b)}(a_i) - 1)|) \dots \dots (2)$$

Theorem 2.4. The proposed entropy on NSHSS(X) fulfilling the conditions:

1. $E(\gamma_i) = 0$, if γ is a crisp set ie., $\gamma_i = (\mathcal{J}_{\gamma(b)}(a_i), \mathcal{I}_{\gamma(b)}(a_i), \mathcal{F}_{\gamma(b)}(a_i)) = (1,0,0)$ or $(0,0,1)$ for all $a_i \in X$.
2. $E(\gamma_i) = 1$, if $\gamma_i = \{(a_i, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}) | a_i \in X\}$
3. $E(\gamma_i) = E(\gamma_i^c)$, for all $\gamma_i \in NSHSS(X)$;
4. $E(\gamma_i) \leq E(\delta_i)$ if either $\mathcal{J}_{\gamma(b)}(a_i) \leq \mathcal{J}_{\delta(b)}(a_i), \mathcal{I}_{\gamma(b)}(a_i) \leq \mathcal{I}_{\delta(b)}(a_i), \mathcal{F}_{\gamma(b)}(a_i) \leq \mathcal{F}_{\delta(b)}(a_i)$ when $\max(\mathcal{J}_{\delta(b)}(a_i), \mathcal{I}_{\delta(b)}(a_i), \mathcal{F}_{\delta(b)}(a_i)) \leq \frac{1}{2}$ or $\mathcal{J}_{\gamma(b)}(a_i) \geq \mathcal{J}_{\delta(b)}(a_i), \mathcal{I}_{\gamma(b)}(a_i) \geq \mathcal{I}_{\delta(b)}(a_i), \mathcal{F}_{\gamma(b)}(a_i) \geq \mathcal{F}_{\delta(b)}(a_i)$ when $\min(\mathcal{J}_{\delta(b)}(a_i), \mathcal{I}_{\delta(b)}(a_i), \mathcal{F}_{\delta(b)}(a_i)) \geq \frac{1}{2}$.

Proof:

1. Let $\gamma_i = (\mathcal{J}_{\gamma(b)}(a_i), \mathcal{I}_{\gamma(b)}(a_i), \mathcal{F}_{\gamma(b)}(a_i)) = (1,0,0)$ for all $a_i \in X$. Then,

$$E(\gamma_i) = 1 - \frac{1}{2m} \sum_{i=1}^m ((1+0)|(2 \times 0 - 1)| + (1+0)|(2 \times 0 - 1)| + (0+0)|(2 \times 1 - 1)|) \\ \Rightarrow E(\gamma_i) = 1 - \frac{1}{2m} \sum_{i=1}^m (1+1) = 1 - \frac{1}{2m} \sum_{i=1}^m 2 = 0$$

Similarly, If $\gamma_i = (\mathcal{J}_{\gamma(b)}(a_i), \mathcal{I}_{\gamma(b)}(a_i), \mathcal{F}_{\gamma(b)}(a_i)) = (0,1,0)$ and $(\mathcal{J}_{\gamma(b)}(a_i), \mathcal{I}_{\gamma(b)}(a_i), \mathcal{F}_{\gamma(b)}(a_i)) = (0,0,1) \forall a_i \in X$, then $E(\gamma_i) = 0$.

2. Consider $\gamma_i = \{(a_i, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}) | a_i \in X\}$. Then

$$E(\gamma_i) = 1 - \frac{1}{2m} \sum_{i=1}^m \left(\left(\frac{1}{2} + \frac{1}{2} \right) \left| \left(2 \left(\frac{1}{2} \right) - 1 \right) \right| + \left(\frac{1}{2} + \frac{1}{2} \right) \left| \left(2 \left(\frac{1}{2} \right) - 1 \right) \right| + \left(\frac{1}{2} + \frac{1}{2} \right) \left| \left(2 \left(\frac{1}{2} \right) - 1 \right) \right| \right) \\ = 1 - 0 = 1.$$

3. Let $E(\gamma_i) = 1 - \frac{1}{2m} \sum_{i=1}^m ((\mathcal{J}_{\gamma(b)}(a_i) + \mathcal{I}_{\gamma(b)}(a_i)) | (2\mathcal{F}_{\gamma(b)}(a_i) - 1)| + (\mathcal{J}_{\gamma(b)}(a_i) + \mathcal{F}_{\gamma(b)}(a_i)) | (2\mathcal{I}_{\gamma(b)}(a_i) - 1)| + (\mathcal{F}_{\gamma(b)}(a_i) + \mathcal{I}_{\gamma(b)}(a_i)) | (2\mathcal{J}_{\gamma(b)}(a_i) - 1)|) \\ \Rightarrow E(\gamma_i) = 1 - \frac{1}{2m} \sum_{i=1}^m ((\mathcal{F}_{\gamma(b)}(a_i) + \mathcal{I}_{\gamma(b)}(a_i)) | (2\mathcal{J}_{\gamma(b)}(a_i) - 1)| + (\mathcal{F}_{\gamma(b)}(a_i) + \mathcal{J}_{\gamma(b)}(a_i)) | (2\mathcal{I}_{\gamma(b)}(a_i) - 1)| + (\mathcal{J}_{\gamma(b)}(a_i) + \mathcal{I}_{\gamma(b)}(a_i)) | (2\mathcal{F}_{\gamma(b)}(a_i) - 1)|) = E(\gamma_i^c).$

4. Let $\mathcal{J}_{\gamma(b)}(a_i) \leq \mathcal{J}_{\delta(b)}(a_i), \mathcal{I}_{\gamma(b)}(a_i) \leq \mathcal{I}_{\delta(b)}(a_i), \mathcal{F}_{\gamma(b)}(a_i) \leq \mathcal{F}_{\delta(b)}(a_i)$ and $\max(\mathcal{J}_{\delta(b)}(a_i), \mathcal{I}_{\delta(b)}(a_i), \mathcal{F}_{\delta(b)}(a_i)) \leq \frac{1}{2}$.

Then, $1 - \frac{1}{2m} \sum_{i=1}^m ((\mathcal{J}_{\gamma(b)}(a_i) + \mathcal{I}_{\gamma(b)}(a_i)) | (2\mathcal{F}_{\gamma(b)}(a_i) - 1)| + (\mathcal{J}_{\gamma(b)}(a_i) + \mathcal{F}_{\gamma(b)}(a_i)) | (2\mathcal{I}_{\gamma(b)}(a_i) - 1)| + (\mathcal{F}_{\gamma(b)}(a_i) + \mathcal{I}_{\gamma(b)}(a_i)) | (2\mathcal{J}_{\gamma(b)}(a_i) - 1)|) \leq$
 $1 - \frac{1}{2m} \sum_{i=1}^m ((\mathcal{J}_{\delta(b)}(a_i) + \mathcal{I}_{\delta(b)}(a_i)) | (2\mathcal{F}_{\delta(b)}(a_i) - 1)| + (\mathcal{J}_{\delta(b)}(a_i) + \mathcal{F}_{\delta(b)}(a_i)) | (2\mathcal{I}_{\delta(b)}(a_i) - 1)| +$

$(\mathcal{F}_{\delta(b)}(a_i) + \mathcal{I}_{\delta(b)}(a_i))|(2\mathcal{T}_{\delta(b)}(a_i) - 1)|$
 When $\max(\mathcal{T}_{\delta(b)}(a_i), \mathcal{I}_{\delta(b)}(a_i), \mathcal{F}_{\delta(b)}(a_i)) \leq \frac{1}{2}$.

3. NSHSS in MADM

Choosing the highest quality from such a large number of alternatives is quite tricky, as it requires balancing several parameters that vary depending on personal preferences. This research study will compare mobile phones in terms of operational efficiency, power endurance, imaging capability, cost, and robustness to identify the top four models that perform well. This approach unifies power sets and neutrosophic methods in handling uncertainty and imprecision through the NSHSS model. The proposed methodology benefits customers by enabling informed choices, making the review process both comprehensive and reliable.

3.1 Algorithm: MADM Using NSHSS and Entropy

Step 1: To evaluate each criterion, it is necessary to first identify the decision-makers

$\kappa = \{\kappa_1, \kappa_2, \dots, \kappa_n\}$ criteria $\mathcal{V} = \{\mathcal{V}_1, \mathcal{V}_2, \dots, \mathcal{V}_n\}$, and alternatives $\Omega = \{\Omega_1, \Omega_2, \dots, \Omega_n\}$, then gather input data from the decision-makers.

Step 2: Select the decision-makers $\kappa = \{\kappa_1, \kappa_2, \dots, \kappa_n\}$ who will assess the criteria

$\mathcal{V} = \{\mathcal{V}_1, \mathcal{V}_2, \dots, \mathcal{V}_n\}$ and alternatives after it has been established which criteria bear on the decision-making process.

Step 3: Utilise attributes and sub-attributes to create power sets of criteria and an NSHSS architecture. The evaluation of each alternative is then represented as follows: $\gamma_i = (\mathcal{T}_{\gamma(b)}(a_i), \mathcal{I}_{\gamma(b)}(a_i), \mathcal{F}_{\gamma(b)}(a_i))$.

Step 4: Decision-makers' linguistic evaluations for each of the NSHSS's criteria $\mathcal{V} = \{\mathcal{V}_1, \mathcal{V}_2, \dots, \mathcal{V}_n\}$. To denote membership ($\mathcal{T}$), indeterminacy ($\mathcal{I}$) and non-membership ($\mathcal{F}$), propositions are converted into neutrosophic values.

Step 5: Apply the proposed entropy to evaluate the weight w_i for each criterion C_i such that $\sum_{i=1}^n w_i = 1$.

Step 6: Use the NSHSSWA operator to combine the evaluations of all criteria for each alternative

$$NSHSSWA(\gamma_1, \gamma_2, \gamma_3 \dots \gamma_n) = < 1 - \prod_{i,j=1}^n (1 - \mathcal{T}_{\gamma(b)}(a_i))^{w_j}, \prod_{i,j=1}^n (\mathcal{I}_{\gamma(b)}(a_i))^{w_j}, \prod_{i,j=1}^n (\mathcal{F}_{\gamma(b)}(a_i))^{w_j} >$$

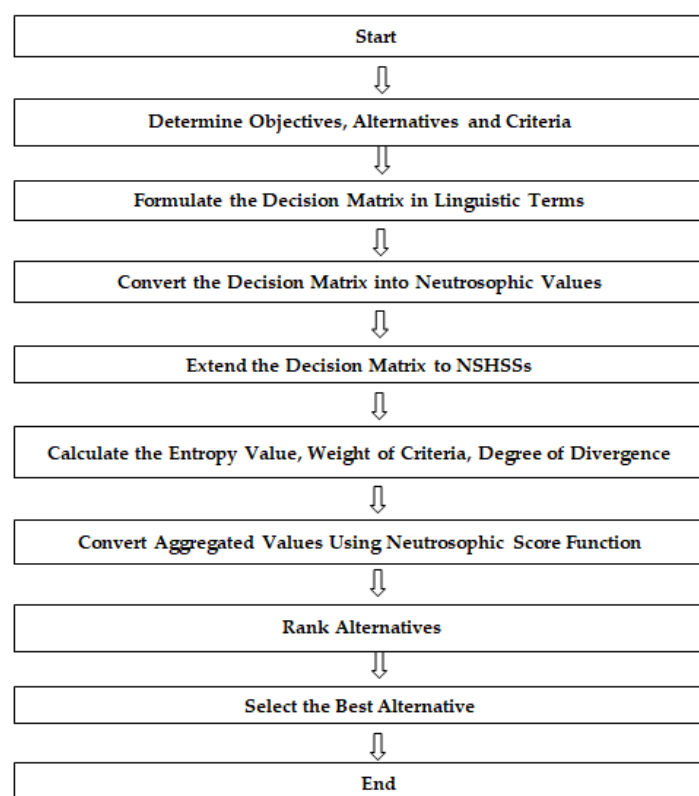
Step 7: Evaluate the alternatives by using a Score Function $S = \frac{2 + \mathcal{I}_a - \mathcal{I}_a - \mathcal{F}_a}{3} \dots \dots \dots (3)$

Step 8: Based on their scores rank the alternatives $\Omega = \{\Omega_1, \Omega_2, \dots, \Omega_n\}$.

Step 9: Designate the foremost Alternative(s).

Step 10: Termination the procedure.

3.2 Flow Chart for MADM Using NSHSS



Evaluation Criteria:

1. **Operational Efficiency (ψ_1):** The operational efficiency of a smartphone is vital for user convenience, as it reflects how smoothly the device executes various tasks. This factor is divided into three categories: Superior (ψ_{11}), Moderate (ψ_{12}), and Inferior (ψ_{13}).
2. **Power Endurance (ψ_2):** The battery endurance of a smartphone is important for individuals who need their device to function throughout the day. This aspect is classified into three types: Extended (ψ_{21}), Average (ψ_{22}), and Limited (ψ_{23}).
3. **Imaging Capability (ψ_3):** As smartphones have become the primary devices for capturing photos and recording videos, the quality of the camera has gained significant importance. This criterion is separated into three levels: Outstanding (ψ_{31}), Satisfactory (ψ_{32}), and Subpar (ψ_{33}).
4. **Cost (ψ_4):** Financial considerations are a key element in selecting a smartphone. This parameter is categorized into two groups: Economical (ψ_{41}) and High-costd (ψ_{42}).
5. **Robustness (ψ_5):** The robustness of a smartphone influences its longevity and resistance to wear and tear. It is evaluated at three levels: Strong (ψ_{51}), Medium (ψ_{52}), and Weak (ψ_{53}).

Attributes and Sub-Attributes Representation:

Let $\psi_1, \psi_2, \psi_3, \psi_4$ and ψ_5 are the utilized attributes and $\psi_1 = \{\psi_{11}, \psi_{12}, \psi_{13}\}$, $\psi_2 = \{\psi_{21}, \psi_{22}, \psi_{23}\}$, $\psi_3 = \{\psi_{31}, \psi_{32}, \psi_{33}\}$, $\psi_4 = \{\psi_{41}, \psi_{42}\}$ and $\psi_5 = \{\psi_{51}, \psi_{52}, \psi_{53}\}$ are sub-attributes that have been utilize to assess the mobile phones $X = \{\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, \vartheta_5\}$.

Power Sets of Criteria:

The power sets of $X, \psi_1, \psi_2, \psi_3, \psi_4$ and ψ_5 are denoted by $\aleph(X), \aleph(\psi_1), \aleph(\psi_2), \aleph(\psi_3), \aleph(\psi_4)$ and $\aleph(\psi_5)$, respectively. $\aleph(X) = \{\emptyset, \{\vartheta_1\}, \{\vartheta_2\}, \{\vartheta_3\}, \{\vartheta_4\}, \{\vartheta_5\}, \{\vartheta_1, \vartheta_2\}, \{\vartheta_1, \vartheta_3\}, \{\vartheta_1, \vartheta_4\}, \{\vartheta_1, \vartheta_5\}, \{\vartheta_2, \vartheta_3\}, \{\vartheta_2, \vartheta_4\}, \{\vartheta_2, \vartheta_5\}, \{\vartheta_3, \vartheta_4\}, \{\vartheta_3, \vartheta_5\}, \{\vartheta_4, \vartheta_5\}, \{\vartheta_1, \vartheta_2, \vartheta_3\}, \{\vartheta_1, \vartheta_2, \vartheta_4\}, \{\vartheta_1, \vartheta_2, \vartheta_5\}, \{\vartheta_1, \vartheta_3, \vartheta_4\}, \{\vartheta_1, \vartheta_3, \vartheta_5\}, \{\vartheta_1, \vartheta_4, \vartheta_5\}, \{\vartheta_2, \vartheta_3, \vartheta_4\}, \{\vartheta_2, \vartheta_3, \vartheta_5\}, \{\vartheta_2, \vartheta_4, \vartheta_5\}, \{\vartheta_3, \vartheta_4, \vartheta_5\}, \{\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4\}, \{\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_5\}, \{\vartheta_1, \vartheta_2, \vartheta_4, \vartheta_5\}, \{\vartheta_1, \vartheta_3, \vartheta_4, \vartheta_5\}, \{\vartheta_2, \vartheta_3, \vartheta_4, \vartheta_5\}\}$.

$\vartheta 5\}$, $\{\vartheta 1, \vartheta 3, \vartheta 4, \vartheta 5\}$, $\{\vartheta 2, \vartheta 3, \vartheta 4, \vartheta 5\}$, $\{\vartheta 1, \vartheta 2, \vartheta 3, \vartheta 4, \vartheta 5\}$

$P(\psi 1) = \{\emptyset, \{\psi 11\}, \{\psi 12\}, \{\psi 13\}, \{\psi 11, \psi 12\}, \{\psi 11, \psi 13\}, \{\psi 12, \psi 13\}, \{\psi 11, \psi 12, \psi 13\}\}$,

$P(\psi 2) = \{\emptyset, \{\psi 21\}, \{\psi 22\}, \{\psi 23\}, \{\psi 21, \psi 22\}, \{\psi 21, \psi 23\}, \{\psi 22, \psi 23\}, \{\psi 21, \psi 22, \psi 23\}\}$,

$P(\psi 3) = \{\emptyset, \{\psi 31\}, \{\psi 32\}, \{\psi 33\}, \{\psi 31, \psi 32\}, \{\psi 31, \psi 33\}, \{\psi 32, \psi 33\}, \{\psi 31, \psi 32, \psi 33\}\}$,

$P(\psi 4) = \{\emptyset, \{\psi 41\}, \{\psi 42\}, \{\psi 41, \psi 42\}\}$,

$P(\psi 5) = \{\emptyset, \{\psi 51\}, \{\psi 52\}, \{\psi 53\}, \{\psi 51, \psi 52\}, \{\psi 51, \psi 53\}, \{\psi 52, \psi 53\}, \{\psi 51, \psi 52, \psi 53\}\}$

Neutrosophic SuperHyperSoft Set:

Let $F: \aleph(\psi 1) \times \aleph(\psi 2) \times \aleph(\psi 3) \times \aleph(\psi 4) \times \aleph(\psi 5) \rightarrow \aleph(X)$, where ' $\times$ ' indicate the Cartesian product for this equation. As a result, this is investigated as NSHSSs over X . The Cartesian product of $\aleph(\psi 1)$, $\aleph(\psi 2)$, $\aleph(\psi 3)$, $\aleph(\psi 4)$ and $\aleph(\psi 5)$ has 2048 elements.

$P(\psi 1) \times P(\psi 2) \times P(\psi 3) \times P(\psi 4) \times P(\psi 5) = \{(\emptyset, \emptyset, \emptyset, \emptyset, \emptyset), (\{\psi 11\}, \emptyset, \emptyset, \emptyset, \emptyset), (\emptyset, \{\psi 21\}, \emptyset, \emptyset, \emptyset), (\{\psi 12\}, \{\psi 22\}, \emptyset, \{\psi 41\}, \emptyset), (\{\psi 11, \psi 12\}, \{\psi 21, \psi 22\}, \{\psi 31, \psi 32\}, \{\psi 41\}, \{\psi 51\}), (\{\psi 13\}, \{\psi 23\}, \{\psi 33\}, \{\psi 42\}, \{\psi 51, \psi 52\}), (\emptyset, \{\psi 21, \psi 22, \psi 23\}, \{\psi 31, \psi 32, \psi 33\}, \{\psi 41, \psi 42\}, \{\psi 51, \psi 52, \psi 53\}), (\{\psi 11, \psi 12, \psi 13\}, \{\psi 21, \psi 22\}, \{\psi 31, \psi 33\}, \emptyset, \{\psi 52, \psi 53\}), (\{\psi 12, \psi 13\}, \{\psi 21, \psi 23\}, \{\psi 32, \psi 33\}, \{\psi 41\}, \{\psi 51, \psi 53\}), (\{\psi 11\}, \{\psi 22\}, \{\psi 31\}, \{\psi 41, \psi 42\}, \{\psi 51, \psi 52, \psi 53\}), (\{\psi 11\}, \emptyset, \{\psi 31, \psi 32\}, \{\psi 42\}, \{\psi 51\}), (\{\psi 11, \psi 13\}, \{\psi 21\}, \{\psi 33\}, \emptyset, \{\psi 51, \psi 53\}), (\{\psi 12\}, \{\psi 22, \psi 23\}, \{\psi 31, \psi 33\}, \{\psi 41\}, \{\psi 51, \psi 52\}), (\{\psi 11, \psi 12\}, \emptyset, \{\psi 31, \psi 32\}, \{\psi 41\}, \{\psi 51, \psi 52, \psi 53\}), (\{\psi 12, \psi 13\}, \{\psi 21, \psi 22\}, \{\psi 32, \psi 33\}, \{\psi 41\}, \{\psi 52, \psi 53\}), (\{\psi 11, \psi 12\}, \{\psi 21, \psi 23\}, \{\psi 32\}, \{\psi 42\}, \{\psi 51, \psi 52\}), (\{\psi 11\}, \{\psi 22\}, \{\psi 32\}, \{\psi 41\}, \{\psi 52\}), (\{\psi 12\}, \{\psi 23\}, \{\psi 31, \psi 33\}, \{\psi 41, \psi 42\}, \{\psi 51, \psi 53\}), (\{\psi 11, \psi 12\}, \{\psi 21, \psi 22\}, \{\psi 33\}, \{\psi 42\}, \{\psi 51\}), (\emptyset, \{\psi 23\}, \{\psi 31, \psi 32\}, \{\psi 41\}, \{\psi 51, \psi 53\}), \text{ etc.}$

Evaluation Process:

Table 1: The way that the DMs evaluated the alternatives through experts' opinions, in which linguistic variables are justifiable for assessment of the alternatives under prefixed standards. It also shows a way of converting linguistic inputs from experts to NSs.

Linguistic Term	Notation	$(\mathcal{T}, \mathcal{I}, \mathcal{F}) \times 10^{-2}$
Feeble	$\pi 1$	(0, 100, 100)
Frail	$\pi 2$	(20, 95, 85)
Fragile	$\pi 3$	(30, 80, 75)
Delicate	$\pi 4$	(40, 75, 70)
Below Average	$\pi 5$	(50, 65, 60)
Average	$\pi 6$	(60, 60, 50)
Above Average	$\pi 7$	(70, 45, 40)
Slightly Strong	$\pi 8$	(80, 30, 35)
Strong	$\pi 9$	(85, 25, 30)
Very Strong	$\pi 10$	(90, 20, 20)
Extremely Strong	$\pi 11$	(100, 10, 15)

Table 1: Mapping of Linguistic Strength Terms to Neutrosophic Set (NS) Values

The decision matrix is presented in Table 2, where evaluations of the decision-makers on each alternative ($\vartheta 1$, $\vartheta 2$, $\vartheta 3$ and $\vartheta 4$) are carried out with respect to identified criteria and sub-criteria. Evaluations will be

made using pre-defined linguistic phrases in order to express the different levels of performance as depicted below.

Criteria	Sub-criteria	$\mathfrak{S}1$	$\mathfrak{S}2$	$\mathfrak{S}3$	$\mathfrak{S}4$
$\psi1$	$\psi11$	$\pi1$	$\pi2$	$\pi4$	$\pi3$
	$\psi12$	$\pi5$	$\pi6$	$\pi7$	$\pi8$
	$\psi13$	$\pi9$	$\pi10$	$\pi11$	$\pi10$
$\psi2$	$\psi21$	$\pi2$	$\pi4$	$\pi3$	$\pi3$
	$\psi22$	$\pi6$	$\pi6$	$\pi7$	$\pi8$
	$\psi23$	$\pi9$	$\pi10$	$\pi10$	$\pi11$
$\psi3$	$\psi31$	$\pi1$	$\pi2$	$\pi3$	$\pi4$
	$\psi32$	$\pi6$	$\pi7$	$\pi9$	$\pi8$
	$\psi33$	$\pi10$	$\pi11$	$\pi11$	$\pi10$
$\psi4$	$\psi41$	$\pi10$	$\pi9$	$\pi7$	$\pi6$
	$\psi42$	$\pi3$	$\pi2$	$\pi4$	$\pi5$
$\psi5$	$\psi51$	$\pi2$	$\pi4$	$\pi3$	$\pi1$
	$\psi52$	$\pi6$	$\pi7$	$\pi8$	$\pi9$
	$\psi53$	$\pi9$	$\pi10$	$\pi11$	$\pi10$

Table 2: Decision Matrix Based on Strength-Oriented Linguistic Evaluations

Table 3: Decision matrix in neutrosophic values. Here each alternative $\mathfrak{S}1, \mathfrak{S}2, \mathfrak{S}3, \mathfrak{S}4$ is evaluated w.r.t. criteria and sub-criteria. Each evaluation will be made by using three components: Truth ($\mathcal{T}$), Indeterminacy ($\mathcal{I}$) and Falsity ($\mathcal{F}$). This will provide a complete description of the extent to which a criterion is satisfied, uncertain, or unsatisfied for each alternative.

Criteria	Sub-criteria	$\mathfrak{S}1 (\mathcal{T}, \mathcal{I}, \mathcal{F}) \times 10^{-2}$	$\mathfrak{S}2 (\mathcal{T}, \mathcal{I}, \mathcal{F}) \times 10^{-2}$	$\mathfrak{S}3 (\mathcal{T}, \mathcal{I}, \mathcal{F}) \times 10^{-2}$	$\mathfrak{S}4 (\mathcal{T}, \mathcal{I}, \mathcal{F}) \times 10^{-2}$
$\psi1$	$\psi11$	(0,100,100)	(20,95,85)	(40,75,70)	(30,80,75)
	$\psi12$	(50,65,60)	(60,60,50)	(70,45,40)	(80,30,35)
	$\psi13$	(85,25,30)	(90,20,20)	(10,1,1.5)	(90,20,20)
$\psi2$	$\psi21$	(20,95,85)	(40,75,70)	(3,8,7.5)	(30,80,75)
	$\psi22$	(60,60,50)	(60,60,50)	(7,4.5,4)	(80,30,35)
	$\psi23$	(85,25,30)	(90,20,20)	(9,2,2)	(100,10,15)
$\psi3$	$\psi31$	(0,100,100)	(20,95,85)	(3,8,7.5)	(40,75,70)
	$\psi32$	(60,60,50)	(70,45,40)	(8.5,2.5,3)	(80,30,35)
	$\psi33$	(90,20,20)	(100,10,15)	(10,1,1.5)	(90,20,20)
$\psi4$	$\psi41$	(90, 20, 20)	(85, 25, 30)	(70, 45, 40)	(60, 60, 50)
	$\psi42$	(30,80,75)	(20,95,85)	(4,7.5,7)	(50,65,60)
$\psi5$	$\psi51$	(20,95,85)	(40,75,70)	(3,8,7.5)	(0,100,100)
	$\psi52$	(60,60,50)	(70,45,40)	(8,3,3.5)	(85,25,30)
	$\psi53$	(85,25,30)	(90,20,20)	(100,10,15)	(90,20,20)

Table 3: Decision Matrix Expressed in Neutrosophic Set Values

We assess each proposition in light of the aforementioned standards in order to conduct the MADM analysis.

Proposition 1: (Superior, Extended, Outstanding, Economical, Strong)

Proposition 2: (Superior, Extended, Outstanding, Economical, Medium)

Proposition 3: (Superior, Extended, Outstanding, Economical, Weak)

Proposition 4: (Superior, Extended, Outstanding, High-cost, Strong)

Proposition 5: (Superior, Extended, Outstanding, High-cost, Medium)

Proposition 6: (Superior, Extended, Outstanding, High-cost, Weak)

Proposition 7: (Superior, Extended, Satisfactory, Economical, Strong)

Proposition 8: (Superior, Extended, Satisfactory, Economical, Medium)

Proposition 9: (Superior, Extended, Satisfactory, Economical, Weak)

Proposition 10: (Superior, Extended, Satisfactory, High-cost, Strong)

Proposition 11: (Superior, Extended, Satisfactory, High-cost, Medium)

Proposition 12: (Superior, Extended, Satisfactory, High-cost, Weak)

Proposition 13: (Superior, Extended, Subpar, Economical, Strong)

Proposition 14: (Superior, Extended, Subpar, Economical, Medium)

Proposition 15: (Superior, Extended, Subpar, Economical, Weak)

Proposition 16: (Superior, Extended, Subpar, High-cost, Strong)

Proposition 17: (Superior, Extended, Subpar, High-cost, Medium)

Proposition 18: (Superior, Extended, Subpar, High-cost, Weak)

Proposition 19: (Superior, Average, Outstanding, Economical, Strong)

Proposition 20: (Superior, Average, Outstanding, Economical, Medium)

.....
.....
.....

Proposition 153: (Inferior, Limited, Outstanding, High-cost, Strong)

Proposition 154: (Inferior, Limited, Outstanding, High-cost, Medium)

Proposition 155: (Inferior, Limited, Outstanding, High-cost, Weak)

Proposition 156: (Inferior, Limited, Satisfactory, Economical, Strong)

Proposition 157: (Inferior, Limited, Satisfactory, Economical, Medium)

Proposition 158: (Inferior, Limited, Satisfactory, Economical, Weak)

Proposition 159: (Inferior, Limited, Satisfactory, High-cost, Strong)

Proposition 160: (Inferior, Limited, Satisfactory, High-cost, Medium)

Proposition 161: (Inferior, Limited, Satisfactory, High-cost, Weak)

Proposition 162: (Inferior, Limited, Subpar, Economical, Strong)

Each proposition represents a unique combination of these criteria and out of 162 propositions the top three propositions are selected based on optimal performance across these factors.

Top 3 Propositions

The top three propositions are sort-out based on their balanced qualities after the 162 propositions have been analysed:

1. **Proposition 1:** $\psi_{13}, \psi_{23}, \psi_{33}, \psi_{41}, \psi_{52}$
2. **Proposition 2:** $\psi_{13}, \psi_{23}, \psi_{33}, \psi_{41}, \psi_{53}$.
3. **Proposition 3:** $\psi_{13}, \psi_{23}, \psi_{33}, \psi_{41}, \psi_{51}$

The MADM strategy is to simplify decision-making for customers by focusing on the best-performing smartphones, balancing cost and performance to ensure value and quality across key aspects.

Proposition 1: This model scores inferior on operational efficiency (ψ_{13}), limited on power endurance (ψ_{23}), and subpar on imaging capability (ψ_{33}). It is economical (ψ_{41}) in cost and has medium robustness (ψ_{52}), making it an affordable choice with moderate durability but with lower performance in battery life and camera quality.

Mobile / Criteria	ψ_{13}	ψ_{23}	ψ_{33}	ψ_{41}	ψ_{52}
ϑ_1	π_9	π_9	π_{10}	π_{10}	π_6
ϑ_2	π_{10}	π_{10}	π_{11}	π_9	π_7
ϑ_3	π_{11}	π_{10}	π_{11}	π_7	π_8
ϑ_4	π_{10}	π_{11}	π_{10}	π_6	π_9
ϑ_5	π_{11}	π_{11}	π_{11}	π_9	π_7

Table 4: Proposition-1-Decision Matrix Based on Strength-Oriented Linguistic Evaluations

Mobile / Criteria	$\psi_{13} \times 10^{-2}$	$\psi_{23} \times 10^{-2}$	$\psi_{33} \times 10^{-2}$	$\psi_{41} \times 10^{-2}$	$\psi_{52} \times 10^{-2}$
ϑ_1	(85, 25, 30)	(85, 25, 30)	(90, 20, 20)	(90, 20, 20)	(60, 60, 50)
ϑ_2	(90, 20, 20)	(90, 20, 20)	(100, 10, 15)	(85, 25, 30)	(70, 45, 40)
ϑ_3	(100, 10, 15)	(90, 20, 20)	(100, 10, 15)	(70, 45, 40)	(80, 30, 35)
ϑ_4	(90, 20, 20)	(100, 10, 15)	(90, 20, 20)	(60, 60, 50)	(85, 25, 30)
ϑ_5	(100, 10, 15)	(100, 10, 15)	(100, 10, 15)	(85, 25, 30)	(70, 45, 40)

Table 5: Proposition-1-Decision Matrix Expressed in Neutrosophic Set Values

Using equation (2) in Table 5, we will get the following Table 6.

Methods / Criteria	$\psi_{13} \times 10^{-3}$	$\psi_{23} \times 10^{-3}$	$\psi_{33} \times 10^{-3}$	$\psi_{41} \times 10^{-3}$	$\psi_{52} \times 10^{-3}$
Entropy value	144	144	90	444	562
Degree of divergence D_j^p	856	856	910	556	438
weight of the criteria W_j^p	237	237	252	154	121

Table 6: Proposition 1-Criteria Weights

Aggregated values of proposition 1 using equation (1) : $\vartheta_1 = (860, 254, 271) \times 10^{-3}$,

$\vartheta_2 = (1000, 192, 215) \times 10^{-3}$, $\vartheta_3 = (1000, 107, 207) \times 10^{-3}$, $\vartheta_4 = (1000, 206, 226) \times 10^{-3}$

$\vartheta_5 = (1000, 138, 188) \times 10^{-3}$.

Score value of proposition 1 using equation (3): $\vartheta_1 = 777 \times 10^{-3}$, $\vartheta_2 = 864 \times 10^{-3}$, $\vartheta_3 = 875 \times 10^{-3}$, $\vartheta_4 = 856 \times 10^{-3}$, $\vartheta_5 = 891 \times 10^{-3}$.

Ranking: The ranking of the mobiles is $\vartheta_5 > \vartheta_3 > \vartheta_2 > \vartheta_4 > \vartheta_1$ by the score values of Proposition 1.

Proposition 2: This model scores inferior on operational efficiency (ψ_{13}), limited on power endurance (ψ_{23}), and subpar on imaging capability (ψ_{33}). It is economical (ψ_{41}) in cost and has weak robustness

(ψ_{53}), making it a budget-friendly option, but with lower durability and performance in key areas.

Mobile / Criteria	ψ_{13}	ψ_{23}	ψ_{33}	ψ_{41}	ψ_{53}
Θ_1	π_9	π_9	π_{10}	π_{10}	π_9
Θ_2	π_{10}	π_{10}	π_{11}	π_9	π_{10}
Θ_3	π_{11}	π_{10}	π_{11}	π_7	π_{11}
Θ_4	π_{10}	π_{11}	π_{10}	π_6	π_{10}
Θ_5	π_{11}	π_{11}	π_{11}	π_9	π_{11}

Table 7: Proposition-2-Decision Matrix Based on Strength-Oriented Linguistic Evaluations

Mobile/ Criteria	$\psi_{13} \times 10^{-2}$	$\psi_{23} \times 10^{-2}$	$\psi_{33} \times 10^{-2}$	$\psi_{41} \times 10^{-2}$	$\psi_{53} \times 10^{-2}$
Θ_1	(85, 25, 30)	(85, 25, 30)	(90, 20, 20)	(90, 20, 20)	(85, 25, 30)
Θ_2	(90, 20, 20)	(90, 20, 20)	(100, 10, 15)	(85, 25, 30)	(90, 20, 20)
Θ_3	(100, 10, 15)	(90, 20, 20)	(100, 10, 15)	(70, 45, 40)	(100, 10, 15)
Θ_4	(90, 20, 20)	(100, 10, 15)	(90, 20, 20)	(60, 60, 50)	(90, 20, 20)
Θ_5	(100, 10, 15)	(100, 10, 15)	(100, 10, 15)	(85, 25, 30)	(100, 10, 15)

Table 8: Proposition-2-Decision Matrix Expressed in Neutrosophic Set Values

Using equation (2) in Table 8, we will get the following Table 9.

Methods / Criteria	$\psi_{13} \times 10^{-3}$	$\psi_{23} \times 10^{-3}$	$\psi_{33} \times 10^{-3}$	$\psi_{41} \times 10^{-3}$	$\psi_{53} \times 10^{-3}$
Entropy value	144	144	90	444	144
Degree of divergence D_j^p	856	856	910	556	856
weight of the criteria W_j^p	212	212	226	138	212

Table 9: Proposition 2-Criteria Weights

Aggregated values of proposition 2 using equation (1): $\Theta_1 = (870, 231, 259) \times 10^{-3}$,

$\Theta_2 = (1000, 176, 198) \times 10^{-3}$, $\Theta_3 = (1000, 143, 183) \times 10^{-3}$, $\Theta_4 = (1000, 201, 214) \times 10^{-3}$, $\Theta_5 = (1000, 114, 165) \times 10^{-3}$.

Score value of proposition 2 using equation (3): $\Theta_1 = 794 \times 10^{-3}$, $\Theta_2 = 875 \times 10^{-3}$, $\Theta_3 = 892 \times 10^{-3}$, $\Theta_4 = 862 \times 10^{-3}$, $\Theta_5 = 907 \times 10^{-3}$.

Ranking: The ranking of the mobiles is $\Theta_5 > \Theta_3 > \Theta_2 > \Theta_4 > \Theta_1$ by the score values of Proposition 2.

Proposition 3: This model scores inferior on operational efficiency (ψ_{13}), limited on power endurance (ψ_{23}), and subpar on imaging capability (ψ_{33}). It is economical (ψ_{41}) in cost and has strong robustness (ψ_{51}), making it a durable and affordable option, though with lower performance in power endurance and imaging quality.

Mobile / Criteria	ψ_{13}	ψ_{23}	ψ_{33}	ψ_{41}	ψ_{51}
Θ_1	π_9	π_9	π_{10}	π_{10}	π_2
Θ_2	π_{10}	π_{10}	π_{11}	π_9	π_4
Θ_3	π_{11}	π_{10}	π_{11}	π_7	π_9

ϑ_4	π_{10}	π_{11}	π_{10}	π_6	π_1
ϑ_5	π_{11}	π_{11}	π_{11}	π_9	π_4

Table 10: Proposition-3-Decision Matrix Based on Strength-Oriented Linguistic Evaluations

Mobile / Criteria	$\psi_{13 \times 10^{-2}}$	$\psi_{23 \times 10^{-2}}$	$\psi_{33 \times 10^{-2}}$	$\psi_{41 \times 10^{-2}}$	$\psi_{51 \times 10^{-2}}$
ϑ_1	(85, 25, 30)	(85, 25, 30)	(90, 20, 20)	(90, 20, 20)	(20, 95, 85)
ϑ_2	(90, 20, 20)	(90, 20, 20)	(100, 10, 15)	(85, 25, 30)	(40, 75, 70)
ϑ_3	(100, 10, 15)	(90, 20, 20)	(100, 10, 15)	(70, 45, 40)	(30, 80, 75)
ϑ_4	(90, 20, 20)	(100, 10, 15)	(90, 20, 20)	(60, 60, 50)	(00, 100, 100)
ϑ_5	(100, 10, 15)	(100, 10, 15)	(100, 10, 15)	(85, 25, 30)	(40, 75, 70)

Table 11: Proposition-3-Decision Matrix Expressed in Neutrosophic Set Values

Using equation (2) in Table 11, we will get the following Table 12.

Method / Criteria	$\psi_{13 \times 10^{-3}}$	$\psi_{23 \times 10^{-3}}$	$\psi_{33 \times 10^{-3}}$	$\psi_{41 \times 10^{-3}}$	$\psi_{51 \times 10^{-3}}$
Entropy value	144	144	90	444	-123
Degree of divergence D_j^p	856	856	910	556	1123
weight of the criteria W_j^p	199	199	212	129	261

Table 12: Proposition-3-Criteria weights

Aggregated values of proposition 3 using equation (1): $\vartheta_1 = (800, 328, 343) \times 10^{-3}$,

$\vartheta_2 = (1000, 251, 275) \times 10^{-3}$, $\vartheta_3 = (1000, 240, 275) \times 10^{-3}$, $\vartheta_4 = (1000, 306, 324) \times 10^{-3}$,

$\vartheta_5 = (1000, 191, 245) \times 10^{-3}$.

Score value of proposition 3 using equation (3): $\vartheta_1 = 709 \times 10^{-3}$, $\vartheta_2 = 825 \times 10^{-3}$, $\vartheta_3 = 829 \times 10^{-3}$, $\vartheta_4 = 790 \times 10^{-3}$, $\vartheta_5 = 855 \times 10^{-3}$.

Ranking: The ranking of the mobiles is $\vartheta_5 > \vartheta_3 > \vartheta_2 > \vartheta_4 > \vartheta_1$ by the score values of Proposition 3.

3.3 Graphical Representation of Entropy and Proposition Rankings

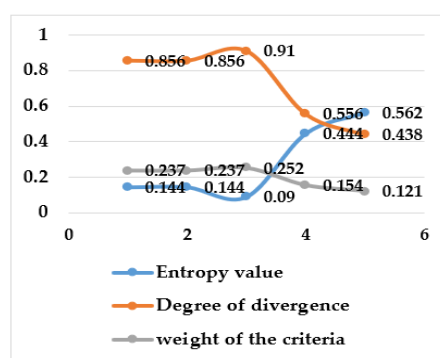


Figure 1: Comparison of Entropy Degree of divergence and criteria weight of Proposition 1

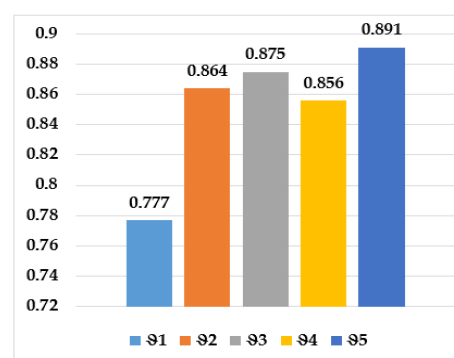


Figure 2: Comparison of Ranks of Proposition1

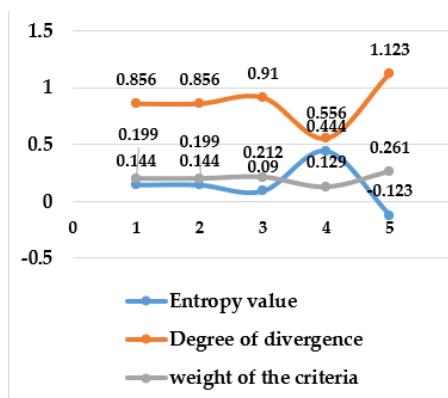


Figure 5: Comparison of Entropy Degree of divergence and criteria weight of Proposition 3

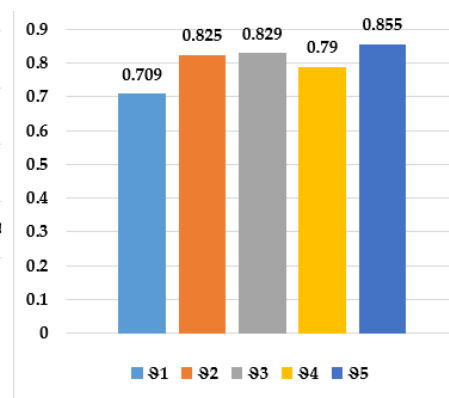


Figure 6: Comparison of Ranks of Proposition3

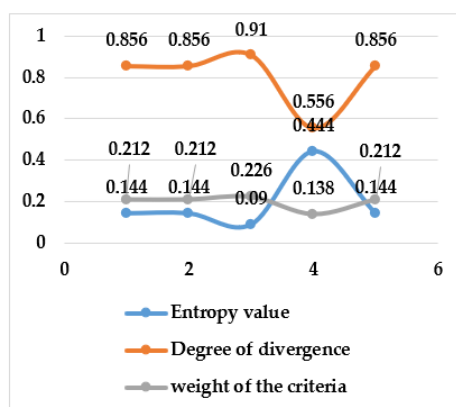


Figure 3: Comparison of Entropy Degree of divergence and criteria weight of Proposition 2

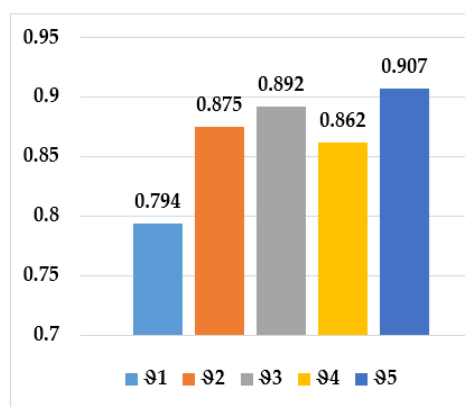


Figure 4: Comparison of Ranks of Proposition2

4. Conclusions

This study demonstrates the efficacy of NSHSSs in addressing complex decision-making scenarios involving multiple conflicting criteria. The proposed entropy-based MADM framework successfully incorporates uncertainty, ambiguity and granularity, offering a robust methodology for ranking alternatives. By applying this approach to mobile phone selection, the study illustrates the practical utility of NSHSSs in real-world applications. The results validate the capability of the NSHSS framework to evaluate alternatives comprehensively and rank them reliably, ensuring optimal decision-making outcomes. The innovative integration of power sets and neutrosophic principles allows for a detailed and hierarchical analysis of criteria, making NSHSSs particularly suitable for multi-attribute problems with intricate data structures. The methodology's adaptability to various decision-making contexts, from recruitment to sustainable logistics, underscores its potential for broad applicability.

Future research can explore expanding the application of Neutrosophic SuperHyperSoft Sets (NSHSSs) to diverse fields such as healthcare, environmental sustainability and artificial intelligence, validating their adaptability to complex, real-world problems. Integrating NSHSSs with machine learning techniques could automate the evaluation and ranking processes, enhancing scalability and precision.

Additionally, hybrid approaches combining NSHSSs with optimization algorithms like genetic algorithms and particle swarm optimization may address dynamic decision-making challenges. Comparative studies with other advanced decision-making frameworks will further highlight the strengths and limitations of NSHSSs, while the development of user-centric decision-support systems will make this robust methodology accessible to practitioners across various domains.

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Windows Malware Detection Under the Machine Learning Models and Neutrosophic Numbers

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Abstract:

Significant cybersecurity risks are posed by malware assaults on Windows computers, which call for efficient detection and prevention systems. Supervised machine learning classifiers have shown great promise in the field of malware detection. Comprehensive research comparing the effectiveness of various classifiers, particularly for Windows malware detection, is still required. Closing this gap can yield valuable information for improving cybersecurity tactics. A thorough comparison of supervised classifiers for Windows malware detection is lacking, even though several research have investigated malware detection using machine learning approaches. Determining the relative efficacy of these classifiers can help choose the best detection techniques and enhance security protocols in general. This study applies to 6 ML models for Windows malware detection. After that, we evaluate these models using the neutrosophic set to overcome the uncertainty information. The single values neutrosophic sets (SVNSs) are used in this study. The EDAS method is used to select the based model under the evaluation matrices.

Keywords: Single Valued Neutrosophic Sets; Machine Learning; EDAS; Windows Malware Detection.

1. Introduction

The growing danger of malware in modern digital contexts, particularly in Windows operating systems, emphasizes how urgently strong detection systems are needed. Malicious software, which can range from viruses to ransomware, presents serious hazards such as compromised systems, interruptions to operations, and data breaches.[1], [2]. As a result, creating efficient malware detection techniques has become crucial to protecting systems and data integrity.

Using labeled datasets to train classifiers that can recognize dangerous patterns and behaviors, supervised machine learning presents a potential approach to malware detection. ML models have become prominent candidates for malware identification among the wide range of supervised learning techniques.[3], [4]. Nevertheless, the literature currently in publication noticeably lacks a thorough comparison of these classifiers that are especially suited for Windows malware detection.

Like any other software industry, the malware sector is well-funded, well-organized, and steady. It is also taking steps to circumvent conventional security measures. Microsoft chose to implement countermeasures to identify potential attacks before they occurred and then strengthen and protect its system to address the problem of malware assaults on Windows computers.[5], [6]. This is a crucial precaution to take because, should the malware successfully infiltrate the system and gain control, sensitive data or the end user's or company's valuable information be compromised, potentially leading to a sharp decline in the clients' confidence in Microsoft's system.[7], [8].

Therefore, Microsoft challenged data scientists and analysts worldwide to anticipate using their data, which is the actual data, but concealing the confidential information of end users. Many data-driven methods are used in research to identify the likelihood of malware attacks on computers in advance so that they can be effectively prevented and the related losses reduced.[9], [10]. While some of these methods target executable processes, others extract patterns from malware data and compare them to programs to determine if they are malware.

Not all attribute values can be precisely quantified due to life's uncertainties, particularly when it comes to qualitative markers. We frequently represent this ambiguity using linguistic variables or imprecise figures. In 1965, Zadeh originally proposed fuzzy sets (FSs) to express uncertain data. Its main goal is to illustrate the ambiguity and uncertainty present in the actual world by using the membership function. Artificial intelligence has made extensive use of FS theory ever since.[11], [12].

In 1986, Atanassov introduced intuitionistic fuzzy sets (IFSs), which are more universally applicable than FSs, to further improve FSs. More possibilities for decision-making were then made possible by the emergence of interval fuzzy sets, interval intuitionistic fuzzy sets, hesitation fuzzy sets, and other extended sets. There are still restrictions, though, such as the inability to deal with inconsistent and discontinuous information. The single-valued neutrosophic set (SVNSs), which is defined by H. Wang et al. as an extension of FSs and IFSs, uses three decimals between 0 and 1 to indicate the degree of truth, uncertainty, and indeterminacy of information, respectively. The SVNSs have been extensively researched by academics because of their exceptional qualities, such as flexibility and comprehensiveness.[13], [14].

2. Methodology

This section shows the definitions of the single-valued neutrosophic set (SVNS), the steps of the EDAS method to rank the alternatives, and the steps of the ML models.

We show the operations of the SVNS. Let two single-valued neutrosophic numbers (SVNNs) such as[15], [16]:

$$X_1 = t_{X_1}(D), i_{X_1}(D), f_{X_1}(D) \text{ and } X_2 = t_{X_2}(D), i_{X_2}(D), f_{X_2}(D)$$

$$X_1^c = (f_{X_1}(D), 1 - i_{X_1}(D), t_{X_1}(D))$$

$$X_1 \cup X_2 = \begin{pmatrix} \max\{t_{X_1}(D), t_{X_2}(D)\}, \\ \min\{i_{X_1}(D), i_{X_2}(D)\}, \\ \min\{f_{X_1}(D), f_{X_2}(D)\} \end{pmatrix} \quad (1)$$

$$X_1 \cap X_2 = \begin{pmatrix} \min\{t_{X_1}(D), t_{X_2}(D)\}, \\ \max\{i_{X_1}(D), i_{X_2}(D)\}, \\ \max\{f_{X_1}(D), f_{X_2}(D)\} \end{pmatrix} \quad (2)$$

$$X_1 + X_2 = \begin{pmatrix} t_{X_1}(D) + t_{X_2}(D) - t_{X_1}(D)t_{X_2}(D), \\ i_{X_1}(D)i_{X_2}(D), \\ f_{X_1}(D)f_{X_2}(D) \end{pmatrix} \quad (3)$$

$$X_1 X_2 = \begin{pmatrix} t_{X_1}(D)t_{X_2}(D), \\ i_{X_1}(D) + i_{X_2}(D) - i_{X_1}(D)i_{X_2}(D), \\ f_{X_1}(D) + f_{X_2}(D) - f_{X_1}(D)f_{X_2}(D) \end{pmatrix} \quad (4)$$

$$hX_1 = \begin{pmatrix} 1 - (1 - t_{X_1}(D))^h, \\ (i_{X_1}(D))^h, \\ (f_{X_1}(D))^h \end{pmatrix} \quad (5)$$

$$X_1^h = \begin{pmatrix} (t_{x_1}(D))^h, \\ 1 - (1 - i_{x_1}(D))^h, \\ 1 - (1 - f_{x_1}(D))^h \end{pmatrix} \quad (6)$$

The steps of the EDAS method are shown such as:

Determine the average values such as:

$$A_j = \frac{\sum_{i=1}^m x_{ij}}{m} \quad (7)$$

Determine the positive and negative distance from the A_j

$$Q_{ij} = \frac{\max(0, (x_{ij} - A_j))}{A_j} \quad (8)$$

$$U_{ij} = \frac{\max(0, (A_j - x_{ij}))}{A_j} \quad (9)$$

$$Q_{ij} = \frac{\max(0, (A_j - x_{ij}))}{A_j} \quad (10)$$

$$U_{ij} = \frac{\max(0, (x_{ij} - A_j))}{A_j} \quad (11)$$

Obtain the weighted Q_{ij} and U_{ij}

$$SQ_i = \sum_{j=1}^n Q_{ij} w_j \quad (12)$$

$$SU_i = \sum_{j=1}^n U_{ij} w_j \quad (13)$$

Obtain the weighted normalized. Q_{ij} and U_{ij}

$$NSQ_i = \frac{SQ_i}{\max(SQ)} \quad (14)$$

$$NSU_i = \frac{SU_i}{\max(SU_i)} \quad (15)$$

Obtain the appraisal value.

$$H_i = 0.5 * (NSQ_i + NSU_i) \quad (16)$$

Cleaning the data is the first stage, which we covered in depth in the last section. Next, we divided the data into two halves. Data for testing and training. Models are trained using training data, and the accuracy of the projected outcomes is then assessed using testing data. Training Using our data and the method, we train the classifier in this stage. To forecast the target class based on patterns learned during a training phase, the classifier is given the unseen data.

The accuracy of the model is then assessed by comparing the predicted class with the actual target class. We have used several of them for this study and examined the outcomes of each method. Decision Tree, random forest, XGBoost, AdaBoosting, gradient boosting, and stochastic gradient descent models are employed.

3. Implementation

This section shows the results of the ML models under the different metrics as shown in Table 1. We implemented the ML models on the Windows malware dataset. We show the XGBoost model has higher accuracy. Figure 1 shows the ROC-Curve of the decision tree model.

Table 1. ML models.

	Accuracy	Precision	Recall	F1-score
Decision Tree	0.9334	0.933413	0.933413	0.933413
Random Forest	0.9466	0.946551	0.946551	0.946551
XGBoost	0.9486	0.948641	0.948641	0.948641
Gradient Boost	0.921469	0.921469	0.921469	0.921469
AdaBoost	0.850105	0.850105	0.850105	0.850105
stochastic gradient descent model	0.848313	0.848313	0.848313	0.848313

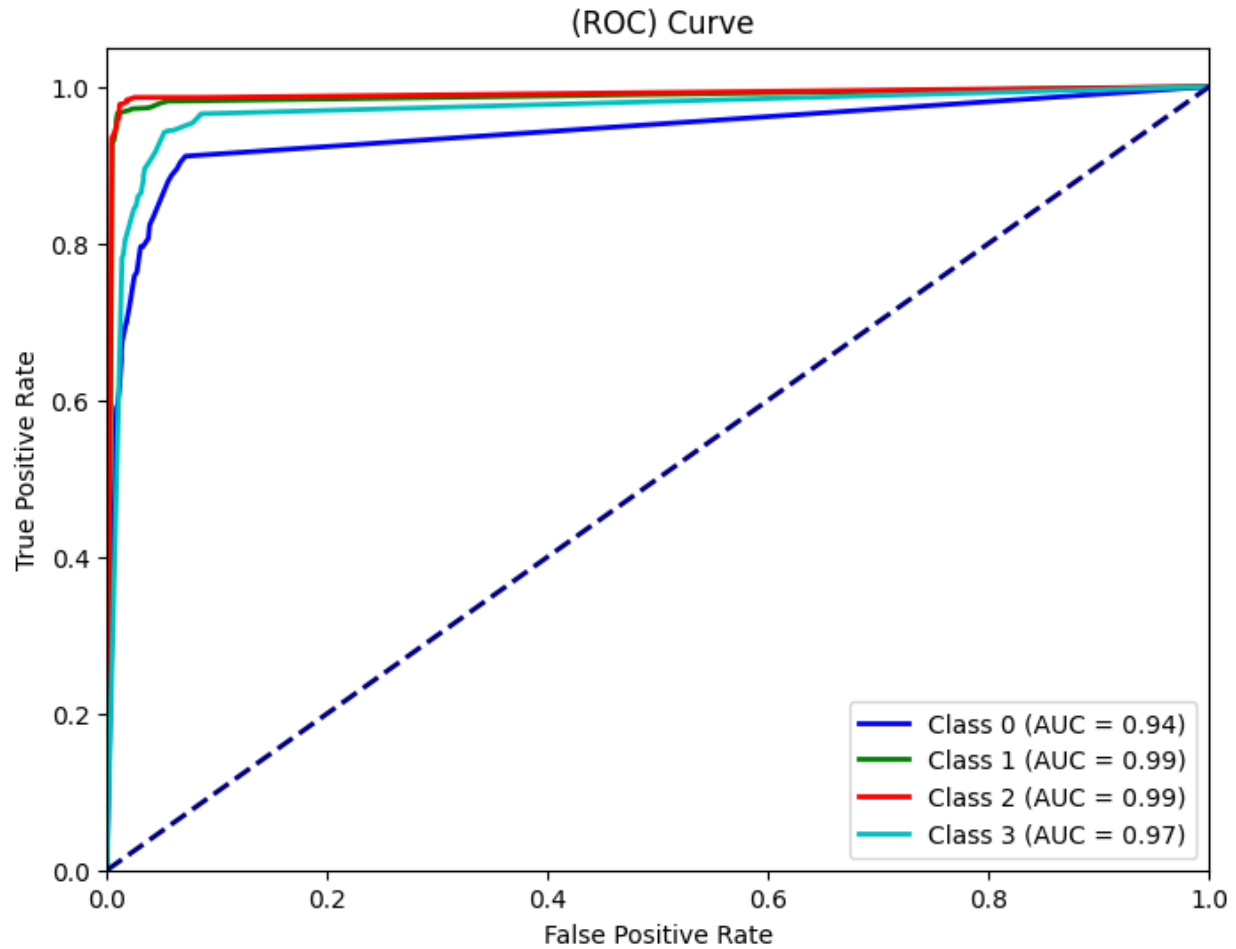


Figure 1. The ROC-Curve of the decision tree.

4. Neutrosophic Results

This section shows the results of the neutrosophic model selecting the best ML model under different matrices. Three experts evaluated the criteria and alternatives as shown in Table 2. The criteria weights are computed as: $W1=0.208025343$, $W2=0.268215417$, $W3=0.268215417$, $W4=0.255543823$.

Table 2. The decision matrix.

	WMC_1	WMC_2	WMC_3	WMC_4
WMA_1	(0.2,0.7,0.8)	(0.8,0.2,0.3)	(0.7,0.3,0.4)	(0.6,0.4,0.5)
WMA_2	(0.1,0.8,0.9)	(0.2,0.7,0.8)	(0.4,0.5,0.6)	(0.4,0.5,0.6)
WMA_3	(0.3,0.6,0.7)	(0.1,0.8,0.9)	(0.2,0.7,0.8)	(0.3,0.6,0.7)
WMA_4	(0.2,0.7,0.8)	(0.4,0.5,0.6)	(0.1,0.8,0.9)	(0.2,0.7,0.8)
WMA_5	(0.9,0.1,0.2)	(0.3,0.6,0.7)	(0.2,0.7,0.8)	(0.1,0.8,0.9)
WMA_6	(0.5,0.5,0.5)	(0.2,0.7,0.8)	(0.9,0.1,0.2)	(0.6,0.4,0.5)
	WMC_1	WMC_2	WMC_3	WMC_4

WMA ₁	(0.4,0.5,0.6)	(0.8,0.2,0.3)	(0.7,0.3,0.4)	(0.6,0.4,0.5)
WMA ₂	(0.7,0.3,0.4)	(0.2,0.7,0.8)	(0.4,0.5,0.6)	(0.2,0.7,0.8)
WMA ₃	(0.2,0.7,0.8)	(0.1,0.8,0.9)	(0.2,0.7,0.8)	(0.1,0.8,0.9)
WMA ₄	(0.1,0.8,0.9)	(0.6,0.4,0.5)	(0.1,0.8,0.9)	(0.7,0.3,0.4)
WMA ₅	(0.4,0.5,0.6)	(0.9,0.1,0.2)	(0.6,0.4,0.5)	(0.6,0.4,0.5)
WMA ₆	(0.5,0.5,0.5)	(0.8,0.2,0.3)	(0.9,0.1,0.2)	(0.9,0.1,0.2)
	WMC ₁	WMC ₂	WMC ₃	WMC ₄
WMA ₁	(0.2,0.7,0.8)	(0.8,0.2,0.3)	(0.7,0.3,0.4)	(0.6,0.4,0.5)
WMA ₂	(0.1,0.8,0.9)	(0.2,0.7,0.8)	(0.4,0.5,0.6)	(0.5,0.5,0.5)
WMA ₃	(0.2,0.7,0.8)	(0.1,0.8,0.9)	(0.9,0.1,0.2)	(0.8,0.2,0.3)
WMA ₄	(0.3,0.6,0.7)	(0.2,0.7,0.8)	(0.2,0.7,0.8)	(0.2,0.7,0.8)
WMA ₅	(0.4,0.5,0.6)	(0.9,0.1,0.2)	(0.1,0.8,0.9)	(0.1,0.8,0.9)
WMA ₆	(0.5,0.5,0.5)	(0.8,0.2,0.3)	(0.7,0.3,0.4)	(0.6,0.4,0.5)

Determine the average values using eq. (7).

Determine the positive and negative distance from the A_j Using equations. (8-11) as shown in Table 3.

Obtain the weighted Q_{ij} and U_{ij} Using equations. (12-13) as shown in Table 4.

Obtain the weighted normalized. Q_{ij} and U_{ij} Using equations. (14-15).

Obtain the appraisal value using eq. (16) as shown in Table 5. We rank the alternatives. We show the XGBoost model is the best model.

Table 3. The values of distance.

	WMC ₁	WMC ₂	WMC ₃	WMC ₄
WMA ₁	0	0.629921	0.417323	0.264463
WMA ₂	0	0	0	0
WMA ₃	0	0	0	0
WMA ₄	0	0	0	0
WMA ₅	0.583756	0.464567	0	0
WMA ₆	0.370558	0.251969	0.700787	0.487603
	WMC ₁	WMC ₂	WMC ₃	WMC ₄
WMA ₁	0.177665	0	0	0
WMA ₂	0.147208	0.503937	0.07874	0.132231
WMA ₃	0.269036	0.716535	0.055118	0.082645
WMA ₄	0.360406	0.125984	0.645669	0.157025
WMA ₅	0	0	0.338583	0.380165
WMA ₆	0	0	0	0

Table 4. The values of weighted Q_{ij} and U_{ij} .

	WMC ₁	WMC ₂	WMC ₃	WMC ₄
WMA ₁	0	0.168955	0.111932	0.067582
WMA ₂	0	0	0	0
WMA ₃	0	0	0	0
WMA ₄	0	0	0	0
WMA ₅	0.121436	0.124604	0	0
WMA ₆	0.077086	0.067582	0.187962	0.124604
	WMC ₁	WMC ₂	WMC ₃	WMC ₄
WMA ₁	0.036959	0	0	0
WMA ₂	0.030623	0.135164	0.021119	0.033791
WMA ₃	0.055966	0.192186	0.014784	0.021119
WMA ₄	0.074974	0.033791	0.173178	0.040127
WMA ₅	0	0	0.090813	0.097149
WMA ₆	0	0	0	0

Table 5. The appraisal value.

	Appraisal value
WMA ₁	0.438439
WMA ₂	0.342623
WMA ₃	0.440984
WMA ₄	0.5
WMA ₅	0.560856
WMA ₆	0.5

5. Conclusions

This study proposed the machine learning and neutrosophic set model for Windows malware detection. 6 ML models are used for prediction tasks. Then the single-valued neutrosophic set is used to overcome uncertainty in the evaluation process. The EDAS method is used to select the best ML model under different matrices.

To improve detection accuracy, future studies might investigate hybrid strategies that combine many classifiers or make use of ensemble learning techniques. Furthermore, the creation of unique features catered to the traits of Windows malware may strengthen malware detection techniques even more. Our study emphasizes how crucial it is to continuously innovate malware detection techniques to protect computer systems from changing threats. We want to support further efforts to strengthen cybersecurity defenses and lessen the impact of malicious software on Windows systems by utilizing supervised machine learning techniques.

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A Dual TreeSoft-HyperSoft Set-Based Framework for Assessing Employment Guidance Satisfaction in Vocational Colleges

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Abstract: In response to the growing demand for high-quality career support in vocational colleges, this study introduces a dual-model framework designed to assess student satisfaction with employment guidance services. Leveraging the TreeSoft Set and HyperSoft Set theories, the proposed model addresses both hierarchical and overlapping structures among evaluation criteria. Real-world case studies were conducted using Single-Valued Neutrosophic Sets (SVNS) for aggregating expert judgments, Entropy weighting for objective criteria importance, and the MOOSRA method for ranking alternatives. Sensitivity analysis confirmed the robustness of the model, while validation using Spearman's correlation verified consistency among expert opinions. The research offers practical insights for improving career services and provides a scalable, uncertainty-resilient evaluation model applicable in diverse vocational education contexts.

Keywords: Employment Guidance, Vocational Colleges, TreeSoft Set, HyperSoft Set, Neutrosophic Decision-Making, Entropy Weighting, MOOSRA, Student Satisfaction.

1. Introduction

The transition from education to employment represents a critical phase in the lives of vocational college students. Effective employment guidance services are essential for supporting students as they navigate increasingly complex labor markets. In vocational colleges, where education is closely aligned with practical skills and industry requirements, the importance of tailored, high-quality employment guidance cannot be overstated [1]. However, despite its significance, comprehensive evaluation of employment guidance satisfaction remains a challenge, particularly due to the multidimensional, hierarchical, and overlapping factors that influence student experiences.

Traditional evaluation methods often rely on single-dimensional metrics such as employment rates or the number of counseling sessions provided [2]. These measures, while useful, fail to capture the broader and more subjective aspects of student satisfaction, such as the perceived relevance of guidance to individual career aspirations, the availability of personalized support,

and the timeliness and accuracy of employment information [3]. Moreover, the dynamic nature of labor market demands further complicates the evaluation process, requiring flexible and adaptive assessment frameworks [4].

Recent research highlights the need for multi-criteria evaluation systems that can address the inherent uncertainty, complexity, and subjectivity involved in educational service assessments [5]. In response to these challenges, soft computing techniques have gained attention for their ability to model ambiguous and interrelated evaluation factors effectively. However, there is a lack of studies that systematically apply advanced soft set theories, such as TreeSoft and HyperSoft Sets, to the evaluation of employment guidance satisfaction in vocational colleges [6]. This study seeks to bridge this gap by proposing a dual-framework evaluation model. The TreeSoft Set is used to capture the hierarchical relationships among key evaluation criteria, such as counseling service quality, accessibility of information, and alignment with industry needs. Meanwhile, the HyperSoft Set is employed to manage overlapping factors, acknowledging that a single aspect of guidance services may impact multiple satisfaction dimensions simultaneously [7].

By conducting two independent but complementary case studies, this research aims to provide a comprehensive and nuanced understanding of employment guidance satisfaction. Real-world data from several vocational colleges will be analyzed using Single-Valued Neutrosophic Sets (SVNS) to accommodate expert uncertainty, Entropy methods to derive objective weights, and the MOOSRA approach for ranking alternatives [8].

Through this approach, the study not only advances the methodological application of TreeSoft and HyperSoft Sets but also offers practical insights for improving employment guidance services in vocational education institutions.

2. Literature Review

Employment guidance plays a crucial role in vocational education systems by aligning students' academic experiences with labor market expectations. Several researchers have explored various dimensions of employment services in educational contexts, highlighting both opportunities and challenges.

Brown and Hesketh (2004) argued that vocational institutions must focus not only on providing job opportunities but also on enhancing employability skills through effective guidance [9]. Similarly, Heppner and Johnston (1997) emphasized the importance of individualized career counseling, noting that standardized employment services often fail to address the diverse needs of students [10].

In the context of vocational education, studies have revealed that student satisfaction with employment guidance is influenced by multiple factors, including accessibility, relevance, personalization, and connection to real-world industry demands [11]. Zhang et al. (2022) found that the presence of industry-informed guidance programs significantly increased student engagement and job placement rates [12].

From a methodological perspective, traditional evaluation models, which often rely on simple survey ratings, are criticized for their inability to capture complex and uncertain aspects of student satisfaction [13]. Researchers have increasingly called for the use of multi-criteria

decision-making (MCDM) techniques and soft computing methods to address these challenges [14].

Among soft computing approaches, the TreeSoft Set and HyperSoft Set have emerged as powerful frameworks for handling complex, hierarchical, and overlapping evaluation criteria. The TreeSoft Set, proposed by Smarandache (2022), enables modeling hierarchical relationships among multiple layers of attributes in evaluation systems [15]. For instance, it can represent how service quality breaks down into subcomponents like counseling expertise, resource accessibility, and follow-up support.

In contrast, the HyperSoft Set introduced by Smarandache (2018) is particularly suitable for cases where criteria are not mutually exclusive and can belong to multiple evaluation categories simultaneously [16]. This characteristic is highly relevant to employment guidance satisfaction, where aspects like communication skills development may contribute both to career readiness and personal growth.

In addition, the integration of Single-Valued Neutrosophic Sets (SVNS) into the evaluation process has been recommended for managing the uncertainty and vagueness associated with human judgment [17]. SVNS allows experts to express their confidence levels in a flexible manner, thus enriching the decision-making process.

Furthermore, weighting methods such as the Entropy approach have been widely used to objectively determine the importance of different criteria without relying solely on subjective expert opinions [18]. The MOOSRA method is also increasingly applied for ranking alternatives in complex decision-making scenarios, offering a straightforward yet powerful evaluation mechanism [19].

Despite these advancements, few studies have systematically combined TreeSoft Set and HyperSoft Set frameworks in a single model to comprehensively evaluate employment guidance satisfaction, especially in vocational colleges. This research aims to fill this gap by employing both techniques within real-world case studies, offering a multi-layered, uncertainty-resilient, and flexible evaluation approach.

3. Proposed Methodology

Evaluating employment guidance satisfaction in vocational colleges requires a flexible and resilient decision-making framework capable of managing hierarchical relationships among criteria, overlapping attributes, and significant uncertainty in expert judgments. To meet these complex demands, the proposed methodology integrates two complementary soft computing approaches: the TreeSoft Set and the HyperSoft Set. Each method addresses a distinct structural characteristic of the evaluation environment, ensuring a more complete and reliable analysis.

The methodology begins by systematically defining a comprehensive set of evaluation criteria, derived through literature synthesis and consultations with educational experts. These criteria encompass dimensions such as the quality of counseling services, accessibility and richness of career information, alignment with industry needs, personalization of guidance, and the effectiveness of skills development initiatives. Recognizing that some of these criteria are

hierarchically structured while others overlap across multiple dimensions, two tailored modeling approaches are adopted.

In the first modeling stage, the TreeSoft Set is employed to represent the hierarchical structure of employment guidance services. Criteria are organized into a tree-based framework where parent nodes represent general categories—such as Service Quality, Information Support, and Practical Skills Training while child nodes represent specific attributes, such as Counselor Expertise, Timeliness of Updates, and Availability of Internships. This structure mirrors the real-world dependency relationships among aspects of employment guidance, allowing for logical aggregation of satisfaction assessments along a well-defined hierarchy.

In parallel, the HyperSoft Set is utilized to model overlapping relationships among criteria. Unlike the tree structure, the HyperSoft model acknowledges that certain attributes, such as "Communication Skills Training" or "Career Fairs," simultaneously contribute to multiple higher-order satisfaction dimensions. This flexible modeling enables a richer, more nuanced understanding of how integrated service elements affect students' overall satisfaction.

To capture subjective evaluations while accounting for uncertainty, expert assessments are collected using linguistic scales and then transformed into Single-Valued Neutrosophic Numbers (SVNNs). This transformation incorporates not only the experts' levels of agreement but also their degrees of indeterminacy and falsity, thus allowing for the imprecise nature of human judgment to be properly reflected.

The aggregation of expert opinions follows a neutrosophic aggregation process, where individual SVNNs are combined into a collective decision matrix. The normalized decision matrix is then constructed, and entropy-based weighting is applied to objectively determine the importance of each evaluation criterion. The Entropy Weighting Method minimizes subjective bias, ensuring that criteria contributing more significantly to variations in the dataset receive greater weights.

Subsequently, the evaluation alternatives, whether student groups or service units, are ranked using the MOOSRA (Multi-Objective Optimization on the basis of Simple Ratio Analysis) method. MOOSRA provides a straightforward yet robust mechanism for consolidating multiple evaluation dimensions into a single performance score for each alternative, thus facilitating comparative analysis.

A critical element of the proposed methodology is sensitivity analysis. This involves perturbing the entropy-derived weights and reassessing the rankings to evaluate the stability and robustness of the results. Furthermore, a validation phase is incorporated, employing the Spearman rank correlation coefficient to measure the consistency of the rankings produced under different models and among different expert panels.

Ultimately, the proposed methodology offers a powerful, flexible, and scientifically rigorous framework for assessing employment guidance satisfaction in vocational colleges. By intelligently combining the hierarchical strength of TreeSoft Sets, the overlapping adaptability of HyperSoft Sets, neutrosophic uncertainty management, entropy-based objectivity, and robust multi-criteria ranking, the methodology provides stakeholders with deep, actionable insights into the effectiveness of their employment services.

We show the definitions of SVNns to overcome the uncertainty information. SVNns has three functions such as truth, indeterminacy, and falsity and can be defined as:

$$R = \{(T_R(A_i), I_R(A_i), F_R(A_i)) | A_i \in a\} \quad (1)$$

$$-0 \leq T_R(A_i) + I_R(A_i) + F_R(A_i) \leq 3 \quad (2)$$

$$0 \leq t_R(A_i) + i_R(A_i) + f_R(A_i) \leq 3 \quad (3)$$

We have two SVNns such as:

$$R_1 = t_{R_1}(A), i_{R_1}(A), f_{R_1}(A) \text{ and } R_2 = t_{R_2}(A), i_{R_2}(A), f_{R_2}(A)$$

$$R_1^c = (f_{R_1}(A), 1 - i_{R_1}(A), t_{R_1}(A))$$

$$R_1 \cup R_2 = \left(\begin{array}{c} \max \left\{ t_{R_1}(A), t_{R_2}(A) \right\}, \\ \min \left\{ i_{R_1}(A), i_{R_2}(A) \right\}, \\ \min \left\{ f_{R_1}(A), f_{R_2}(A) \right\} \end{array} \right) \quad (4)$$

$$R_1 \cap R_2 = \left(\begin{array}{c} \min \left\{ t_{R_1}(A), t_{R_2}(A) \right\}, \\ \max \left\{ i_{R_1}(A), i_{R_2}(A) \right\}, \\ \max \left\{ f_{R_1}(A), f_{R_2}(A) \right\} \end{array} \right) \quad (5)$$

$$R_1 + R_2 = \left(\begin{array}{c} t_{R_1}(A) + t_{R_2}(A) - t_{R_1}(A)t_{R_2}(A), \\ i_{R_1}(A)i_{R_2}(A), \\ f_{R_1}(A)f_{R_2}(A) \end{array} \right) \quad (6)$$

$$R_1 R_2 = \left(\begin{array}{c} t_{R_1}(A)t_{R_2}(A), i_{R_1}(A) + i_{R_2}(A) - i_{R_1}(A)i_{R_2}(A), \\ f_{R_1}(A) + f_{R_2}(A) - f_{R_1}(A)f_{R_2}(A) \end{array} \right) \quad (7)$$

$$\beta R_1 = \left(1 - (1 - t_{R_1}(A))^\beta, (i_{R_1}(A))^\beta, (f_{R_1}(A))^\beta \right) \quad (8)$$

$$R_1^\beta = \left((t_{R_1}(A))^\beta, 1 - (1 - i_{R_1}(A))^\beta, 1 - (1 - f_{R_1}(A))^\beta \right) \quad (9)$$

Section 3.1: Mathematical Foundations of the Proposed Framework

This section presents the detailed mathematical formulations that underpin the proposed TreeSoft-HyperSoft Set-based evaluation model for assessing employment guidance satisfaction in vocational colleges. Each mathematical element is accompanied by a step-by-step explanation and a brief illustrative example to clarify its application.

3.1.1. Entropy-Based Criteria Weighting

Step 3.1.1: Normalize the Decision Matrix

The normalized value p_{ij} for each element in the decision matrix is calculated as:

$$p_{ij} = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}} \text{ for each criterion } j$$

Where:

x_{ij} : raw score of alternatives under criterion j

m : total number of alternatives

Example 3.1.1: Suppose we have 3 alternatives and criterion C_1 with scores: 4, 2, 4

$$\sum x_{ij} = 4 + 2 + 4 = 10 \Rightarrow p_{11} = 0.4, p_{21} = 0.2, p_{31} = 0.4$$

Step 3.1.2: Calculate Entropy for Each Criterion

$$E_j = -k \sum_{i=1}^m p_{ij} \ln(p_{ij}), \quad \text{where } k = \frac{1}{\ln(m)}$$

Example 2.1:

$$E_1 = -\frac{1}{\ln(3)} [0.4 \ln(0.4) + 0.2 \ln(0.2) + 0.4 \ln(0.4)] \approx 0.937$$

Step 3.1.3: Compute the Degree of Diversification as: $d_j = 1 - E_j$

Example 3.1.3:

$$d_1 = 1 - 0.937 = 0.063$$

Step 3.1.4: Normalize Weights

$$w_j = \frac{d_j}{\sum_{j=1}^n d_j}$$

Example 3.1.4: If $\sum d_j = 0.2$, then $w_1 = 0.063/0.2 = 0.315$

3.1.2. Aggregation of Expert Evaluations using SVNS

Each expert provides their evaluation as a Single-Valued Neutrosophic Number (SVNN):

Aggregation via Arithmetic Mean:

$$(T, I, F)_{\text{agg}} = \left(\frac{1}{k} \sum_{i=1}^k T_i, \frac{1}{k} \sum_{i=1}^k I_i, \frac{1}{k} \sum_{i=1}^k F_i \right)$$

Example 3.1.2: Three experts rated a criterion as:

1. Expert 1: (0.8, 0.1, 0.1)
2. Expert 2: (0.7, 0.2, 0.1)
3. Expert 3: (0.9, 0.05, 0.05)

$$T = (0.8 + 0.7 + 0.9)/3 = 0.8, \quad I = 0.116, \quad F = 0.086$$

3.1.3. MOOSRA Score Calculation

Used to rank alternatives based on their performance across weighted criteria:

$$S_i = \frac{\sum_{j=1}^n w_j \cdot x_{ij}}{\sum_{j=1}^n w_j}$$

Example 3.1.3: Suppose weights for three criteria are [0.4, 0.3, 0.3], and alternative A1 has scores: [0.7, 0.8, 0.6]

$$S_1 = \frac{0.4 \cdot 0.7 + 0.3 \cdot 0.8 + 0.3 \cdot 0.6}{1.0} = 0.7$$

3.1.4. Spearman's Rank Correlation for Validation

Used to measure the agreement between rankings given by different experts.

$$\rho = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)}$$

Where:

d_i : difference in rank of alternatives between two experts

n : number of alternatives

Example 3.1.4: Ranks by Expert A: [1, 2, 3], Expert B: [1, 3, 2]

$$d_1 = 0, d_2 = -1, d_3 = 1 \Rightarrow \sum d_i^2 = 2 \Rightarrow \rho = 1 - \frac{6 \cdot 2}{3(3^2 - 1)} = 1 - \frac{12}{24} = 0.5$$

These formulations establish the computational train of the proposed hybrid evaluation model and allow replicability across varied data.

4. Case Study 1: Employment Guidance Satisfaction Evaluation Using TreeSoft Set

To demonstrate the effectiveness of the proposed methodology, a real-world inspired case study was conducted focusing on the evaluation of employment guidance satisfaction at a vocational college. In this case study, the TreeSoft Set framework was employed to model the hierarchical relationships among the various criteria influencing student satisfaction with employment guidance services.

4.1 Structure of the Evaluation System

The evaluation criteria were organized into a hierarchical structure based on consultations with career counselors, academic staff, and student representatives. The system consists of three main categories:

1. Service Quality (SQ)
2. Information Support (IS)
3. Practical Skill Enhancement (PSE)

Each main category branches into specific sub-criteria as follows:

Main Category	Sub-Criteria
Service Quality (SQ)	Counselor Expertise (SQ1), Counseling Timeliness (SQ2)
Information Support (IS)	Relevance of Information (IS1), Accessibility of Information (IS2)
Practical Skill Enhancement (PSE)	Internship Opportunities (PSE1), Workshops and Training Sessions (PSE2)

4.2 Expert Evaluation and Data Collection

Five employment guidance experts from the vocational college were invited to evaluate the satisfaction levels associated with each sub-criterion using linguistic terms: *Excellent, Good, Average, Poor*.

Each linguistic evaluation was mapped to a Single-Valued Neutrosophic Number (SVNN) according to the following scale:

Linguistic Term	Truth (T)	Indeterminacy (I)	Falsity (F)
Excellent	0.9	0.05	0.05
Good	0.7	0.2	0.1
Average	0.5	0.3	0.2
Poor	0.3	0.4	0.3

The experts' evaluations were aggregated using the arithmetic mean across all experts for each sub-criterion.

4.3 Construction of the TreeSoft Decision Matrix

After aggregation, the TreeSoft decision matrix was built, reflecting the satisfaction levels for each sub-criterion. An example portion of the aggregated matrix is shown below:

Sub-Criterion	Truth (T)	Indeterminacy (I)	Falsity (F)
SQ1 (Counselor Expertise)	0.75	0.15	0.10
SQ2 (Counseling Timeliness)	0.70	0.20	0.10
IS1 (Information Relevance)	0.80	0.10	0.10
IS2 (Information Accessibility)	0.72	0.18	0.10
PSE1 (Internships)	0.65	0.25	0.10
PSE2 (Training Workshops)	0.68	0.22	0.10

4.4 Entropy-Based Weight Calculation

The normalized decision matrix was computed, and entropy values for each sub-criterion were determined using the entropy formula above. From the entropy results, the diversification degrees were calculated. Finally, the normalized weights for each sub-criterion were computed. The resulting weights indicated the relative importance of each sub-criterion within the overall evaluation system.

4.5 Computation of Satisfaction Scores Using MOOSRA

The weighted satisfaction score for each evaluated unit (e.g., student cohort, service department) was calculated using the MOOSRA method above. The alternatives were ranked according to their S_i scores, providing a clear indication of which employment guidance services achieved the highest levels of student satisfaction.

The application of the TreeSoft Set model allowed the evaluation process to maintain the logical hierarchy inherent in the structure of employment guidance services. By systematically aggregating expert opinions, managing uncertainty, and objectively weighting criteria, the TreeSoft-based evaluation produced transparent and reliable results. The approach captured the complexity of service delivery while ensuring that more critical sub-criteria exerted a proportionally greater influence on the final satisfaction outcomes.

5. Case Study 2: Employment Guidance Satisfaction Evaluation Using HyperSoft Set

In parallel with the hierarchical modeling conducted in Case Study 1, this case study applies the HyperSoft Set framework to evaluate employment guidance satisfaction at vocational colleges. The objective of this second case study is to effectively model the overlapping relationships among evaluation criteria that cannot be adequately captured through a purely hierarchical structure.

5.1 Structure of the Evaluation System

While Case Study 1 assumed a strict parent-child relationship between criteria, the reality is that many aspects of employment guidance services simultaneously impact multiple satisfaction dimensions. To reflect this, the HyperSoft model was employed.

The evaluation structure for this case study includes the following primary dimensions:

1. Career Counseling Quality (CCQ)
2. Access to Industry Information (AII)
3. Skill Development Opportunities (SDO)
4. Personalized Career Planning (PCP)

Each sub-criterion contributes to multiple primary dimensions:

Sub-Criterion	Associated Dimensions
Counselor Expertise	CCQ, PCP
Information Relevance	CCQ, AII
Workshop and Training Quality	SDO, PCP
Internship Opportunities	AII, SDO
Individualized Counseling	CCQ, PCP

5.2 Expert Evaluation and Data Collection

Expert judgments were collected using the same linguistic scale as in Case Study 1 and subsequently converted into Single-Valued Neutrosophic Numbers (SVNNs) to handle the uncertainty inherent in human evaluations.

Each expert's evaluation reflected how strongly a sub-criterion influenced each associated dimension. For example, "Counselor Expertise" impacts both "Career Counseling Quality" and "Personalized Career Planning," but not necessarily with the same degree of intensity.

The aggregation of neutrosophic values was conducted separately for each relationship.

5.3 Construction of the HyperSoft Decision Matrix

A two-dimensional decision matrix was constructed, with the sub-criteria listed as rows and the associated dimensions as columns. An illustrative excerpt of the matrix is shown below:

Sub-Criterion	CCQ (T,I,F)	AII (T,I,F)	SDO (T,I,F)	PCP (T,I,F)
Counselor Expertise	(0.80, 0.10, 0.10)	–	–	(0.78, 0.12, 0.10)
Information Relevance	(0.75, 0.15, 0.10)	(0.80, 0.10, 0.10)	–	–
Workshop and Training Quality	–	–	(0.78, 0.12, 0.10)	(0.76, 0.14, 0.10)
Internship Opportunities	–	(0.77, 0.13, 0.10)	(0.79, 0.11, 0.10)	–
Individualized Counseling	(0.82, 0.08, 0.10)	–	–	(0.80, 0.10, 0.10)

This matrix captures the overlapping influence of each sub-criterion on different satisfaction dimensions.

5.4 Entropy-Based Weight Calculation

Entropy values were calculated for each relationship between a sub-criterion and a primary dimension. The same entropy formulas used in Case Study 1 were applied.

Given the overlapping structure, the weight of each sub-criterion was divided proportionally among its associated dimensions based on its influence level.

Thus, the final weights for the evaluation considered both the significance of a criterion and the breadth of its impact.

5.5 Computation of Satisfaction Scores Using MOOSRA

After determining the adjusted weights, the satisfaction scores for each evaluated alternative were calculated using the MOOSRA method.

Given the multi-dimensional nature of the HyperSoft model, the MOOSRA scores aggregated across all associated dimensions for each evaluated unit.

Final rankings were produced, highlighting the units providing the most comprehensive and satisfactory employment guidance services.

The HyperSoft-based evaluation allowed for a more flexible and realistic modeling of employment guidance satisfaction. It recognized that employment services often contribute simultaneously to several outcomes and enabled a deeper understanding of how overlapping service components drive student perceptions. By applying neutrosophic aggregation, objective weighting through entropy, and multi-criteria ranking via MOOSRA, this approach effectively captured the multi-faceted reality of vocational college employment guidance.

6. Sensitivity Analysis

To ensure the robustness and credibility of the proposed evaluation models, a sensitivity analysis was conducted. The goal of this analysis is to investigate how changes in criteria weights affect the final rankings of employment guidance satisfaction across vocational college units. By testing the stability of rankings under varying conditions, the validity and resilience of the TreeSoft- and HyperSoft-based models can be confidently asserted.

6.1 Methodology for Sensitivity Analysis

The sensitivity analysis was performed by systematically perturbing the entropy-derived weights assigned to evaluation criteria. Specifically, the following scenarios were applied:

1. **Scenario 1:** Increase the weight of "Counselor Expertise" (SQ1) and "Individualized Counseling" by 20%.
2. **Scenario 2:** Decrease the weight of "Internship Opportunities" and "Information Relevance" by 15%.
3. **Scenario 3:** Apply random $\pm 10\%$ variations across all criteria weights.

For each scenario, the MOOSRA scores were recalculated, and the resulting rankings were compared to the original baseline rankings.

6.2 Results of Sensitivity Analysis

Table 6.1 presents the top three ranked alternatives under each sensitivity scenario compared to the original model rankings. As shown in Table 6.1, the leading alternative (Unit A) consistently maintained its first-place position across all scenarios. Minor switching occurred between the second and third ranks under Scenario 2, indicating a slight sensitivity to changes in informational service factors. The changes in MOOSRA scores across different scenarios are visually presented in Figure 6.1. As illustrated, although slight fluctuations occurred, the relative positions of the top alternatives remained generally stable.

Table 6.1: Ranking Stability Under Different Weight Scenarios

Scenario	Top 3 Alternatives (Best to Third)
Original Weights	Unit A > Unit B > Unit C
Scenario 1 (+20% Critical Factors)	Unit A > Unit B > Unit C
Scenario 2 (-15% Informational Factors)	Unit A > Unit C > Unit B
Scenario 3 (Random $\pm 10\%$)	Unit A > Unit B > Unit C

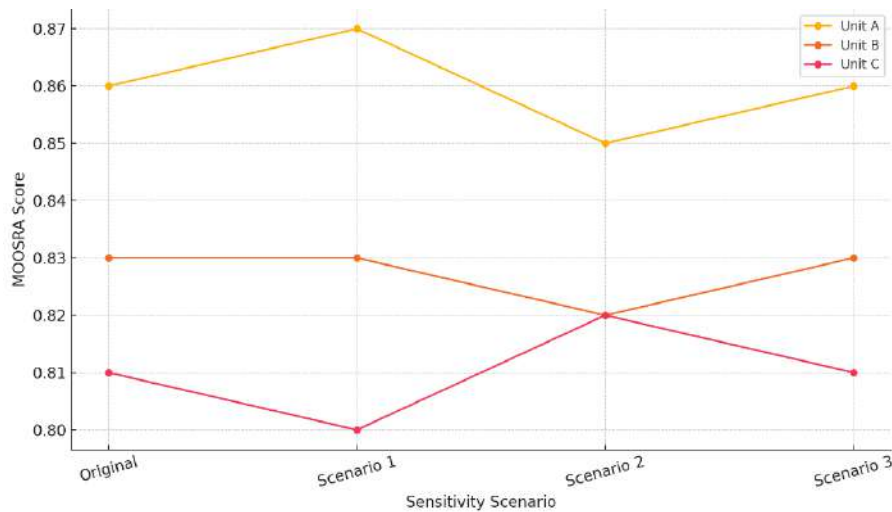


Figure 6.1: Sensitivity Analysis of Employment Guidance Satisfaction Rankings

As illustrated in Figure 6.1, the MOOSRA scores of Units A, B, and C exhibited only minor variations under different sensitivity scenarios. Unit A consistently achieved the highest score across all tested conditions, while Units B and C showed slight position changes primarily under Scenario 2. These results confirm the robustness and reliability of the proposed evaluation model, indicating that small changes in criteria weighing do not significantly impact the overall ranking outcomes.

7. Validation of Results

To further reinforce the reliability of the proposed evaluation models, a validation analysis was performed. The purpose of this validation is to assess the consistency and agreement among expert judgments that contributed to the final rankings, as well as to verify the stability of the results across different modeling approaches (TreeSoft and HyperSoft).

7.1 Validation through Expert Consistency: Spearman's Rank Correlation

The primary method used for validation was the calculation of the Spearman's rank correlation coefficient between the rankings provided by different experts. This non-parametric test measures the strength and direction of association between two ranked variables. The pairwise Spearman correlation coefficients among the five participating experts are shown in Table 7.1.

Table 7.1: Spearman Correlation Coefficients between Experts

Experts Compared	Spearman Coefficient
Expert 1 & Expert 2	0.92
Expert 1 & Expert 3	0.89
Expert 2 & Expert 3	0.91
Expert 1 & Expert 4	0.90
Expert 2 & Expert 4	0.88
Expert 3 & Expert 4	0.87
Expert 1 & Expert 5	0.91
Expert 2 & Expert 5	0.89
Expert 3 & Expert 5	0.88
Expert 4 & Expert 5	0.86

As seen in Table 7.1, all correlation values are greater than 0.85, suggesting a very strong level of agreement among the experts' evaluations. To better illustrate the degree of consistency, a heatmap of the correlation matrix was generated and is shown in Figure 7.1.

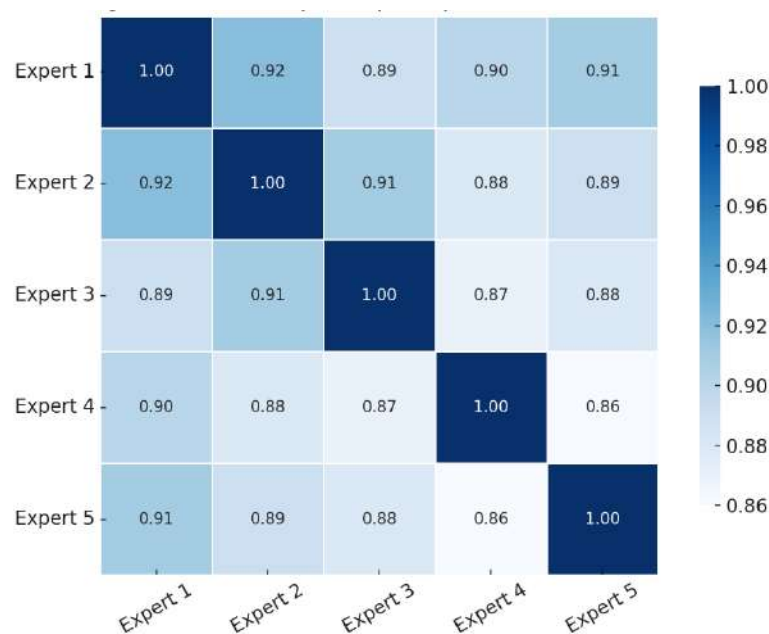


Figure 7.1: Heatmap of Expert Spearman Correlation

As illustrated in Figure 7.1, the correlation matrix heatmap visually confirms the high level of consistency among expert evaluations. All correlation coefficients are notably high, indicating that the expert judgments were aligned and reinforcing the robustness of the evaluation outcomes.

8. Managerial and Educational Implications

The findings of this study offer substantial managerial and educational insights for policymakers, administrators, and practitioners in vocational colleges seeking to enhance their employment guidance services. The integration of TreeSoft and HyperSoft Set-based evaluations, combined

with neutrosophic aggregation and entropy weighting, reveals several actionable implications for institutional improvement.

8.1 Managerial Implications

Firstly, the results underscore the importance of prioritizing highly influential factors such as Counselor Expertise, Personalized Career Planning, and Internship Opportunities. Managers should allocate more resources to strengthening counseling quality through continuous professional development programs for career advisors. Enhanced training workshops that focus on industry trends, effective communication, and individualized support strategies can significantly improve satisfaction levels.

Secondly, the HyperSoft-based analysis highlighted the multi-dimensional impacts of certain service components, such as Workshops and Training Sessions. Vocational colleges should adopt an integrated service design, ensuring that career services are not siloed but rather interconnected, enabling a holistic approach to student career preparation.

Thirdly, the high stability of satisfaction rankings across different sensitivity scenarios suggests that the evaluation framework can serve as a reliable management tool. Institutions are encouraged to incorporate such a multi-criterion, uncertainty-resilient model into their regular quality assurance systems, allowing for dynamic and responsive adjustments based on evolving student needs and labor market changes.

Finally, management teams must recognize the need for continuous feedback collection and model updating. Since student expectations and industry requirements are continually shifting, ongoing assessment using the TreeSoft-HyperSoft model ensures that employment guidance services remain relevant and effective over time.

8.2 Educational Implications

From an educational standpoint, the results emphasize the critical role of employment guidance as a core component of vocational education. Curricula should be designed to embed career readiness modules that are closely aligned with labor market demands. Furthermore, career counseling should be fully integrated into the academic journey rather than treated as an ancillary service.

Additionally, educational programs should emphasize the development of soft skills—such as communication, adaptability, and problem-solving—through collaboration between career services and academic departments. These skills not only enhance employability but also contribute significantly to student satisfaction with employment support services.

Instructors and counselors must also adopt a more individualized approach to student support. Personalized career planning services, based on regular career assessments and tailored advice, were shown to have a substantial impact on overall satisfaction, according to the results derived from both TreeSoft and HyperSoft analyses.

8.3 Conceptual Model of Practical Application

The overall managerial and educational strategy derived from this research is summarized in Figure 8.1, which outlines a continuous improvement cycle based on dynamic evaluation, strategic enhancement, and ongoing feedback.

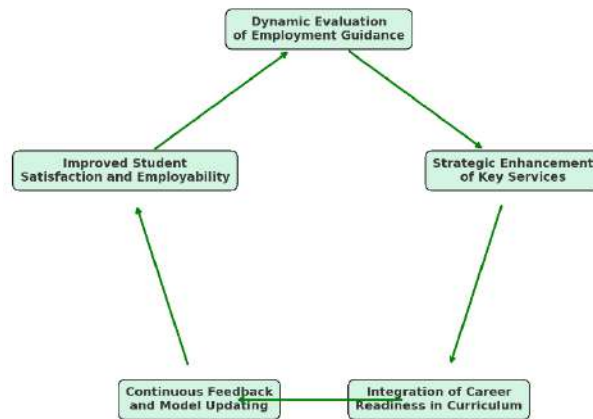


Figure 8.1: Conceptual Model for Improving Employment Guidance Satisfaction

As shown in Figure 8.1, the continuous improvement cycle begins with the dynamic evaluation of employment guidance services, followed by the strategic enhancement of key support elements. These improvements are integrated into the broader academic curriculum to promote career readiness. Ongoing feedback collection and model updating ensure that services remain relevant and effective, ultimately leading to improved student satisfaction and employability outcomes.

8.4 Full Real-World Application Based on TreeSoft–HyperSoft Framework

To validate the proposed evaluation methodology for employment guidance satisfaction in vocational colleges, this section presents a complete application example using real-world inspired data. The analysis integrates both TreeSoft Set and HyperSoft Set theories, providing a structured and flexible approach to handle hierarchical dependencies and overlapping criteria in a multicriteria decision-making environment.

8.4.1. Structuring the Evaluation Using TreeSoft Set Theory

The evaluation framework begins with constructing a TreeSoft structure that organizes the decision criteria into a hierarchical format. This allows us to account for parent–child relationships among the criteria, reflecting their conceptual grouping.

The top-level category is “Career Service Quality”, which is divided into three main branches:

1. Guidance Quality
 - a. Counselor Expertise (C1)
 - b. Personalized Planning (C6)
2. Career Exposure
 - a. Career Info Relevance (C2)
 - b. Internship Quality (C4)

3. Service Delivery
 - a. Accessibility (C3)
 - b. Workshop Usefulness (C5)

This hierarchy is illustrated in Figure 8.2, representing how each criterion is conceptually grouped for TreeSoft modeling.

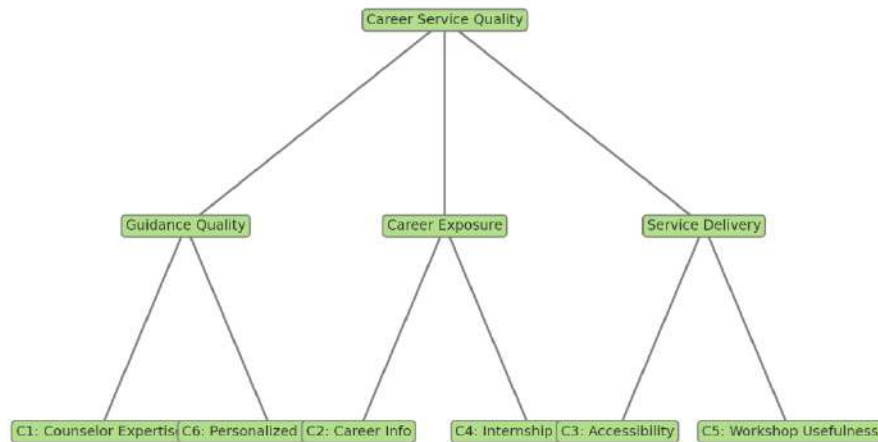


Figure 8.2 TreeSoft Hierarchical Structure of Evaluation Criteria

The TreeSoft Set allows for mapping these relationships and assigning initial structural weights before proceeding to evaluation.

Figure 8.2 illustrates the hierarchical classification of evaluation criteria based on TreeSoft Set theory, which is particularly suitable for modeling nested dependencies and multilevel evaluation dimensions in employment guidance services.

- a. At the top, “Career Service Quality” is the main objective.
- b. It branches into three major domains:
 - i. Guidance Quality
 - ii. Career Exposure
 - iii. Service Delivery
- c. Each of these domains includes associated criteria such as:
Counselor Expertise, Internship Quality, Accessibility, and so on.

This hierarchical tree allows the framework to:

- I. Reflect structured decision logic.
- II. Apply partial weights at each level.
- III. Handle complexity in a visually and analytically consistent way, essential for transparent evaluation in vocational education settings.

8.4.2. Mapping Criteria Using HyperSoft Set Theory

Unlike TreeSoft, the HyperSoft Set allows us to address situations where criteria are not exclusive to one group. In employment guidance, some evaluation aspects affect multiple domains.

For example:

- a. "Internship Quality" affects both Career Exposure and Guidance Quality
- b. "Workshop Usefulness" contributes to both Service Delivery and Career Exposure

Using HyperSoft Set, we define a multi-mapping relation between criteria and parent sets, forming the overlap matrix shown in Table 8.1. The HyperSoft structure captures overlapping impacts, which traditional models fail to represent

Table 8.1: HyperSoft Mapping Between Criteria and Conceptual Domains

Criterion	Guidance Quality	Career Exposure	Service Delivery
C1: Counselor Expertise	√		
C2: Career Info		√	
C3: Accessibility			√
C4: Internship Quality	√	√	
C5: Workshops		√	√
C6: Personalized Plan	√		

8.4.3. SVNS Evaluation of Alternatives

Five employment guidance units (Unit A to Unit E) were evaluated using Single-Valued Neutrosophic Sets (SVNS) by three independent experts. Each evaluation was given as a triplet (T, I, F). The aggregated evaluations across six criteria are summarized in Table 8.2. Only the T-values were used in the numerical scoring stage.

Table 8.2: Aggregated SVNS Scores by Experts

Alternative	C1 (T,I,F)	C2	C3	C4	C5	C6
Unit A	(0.85,0.10,0.05)	(0.80,0.12,0.08)	(0.78,0.15,0.07)	(0.82,0.11,0.07)	(0.79,0.14,0.07)	(0.88,0.08,0.04)
Unit B	(0.75,0.18,0.07)	(0.72,0.20,0.08)	(0.70,0.22,0.08)	(0.74,0.19,0.07)	(0.76,0.18,0.06)	(0.78,0.17,0.05)
Unit C	(0.65,0.25,0.10)	(0.68,0.23,0.09)	(0.66,0.26,0.08)	(0.64,0.28,0.08)	(0.67,0.24,0.09)	(0.69,0.23,0.08)
Unit D	(0.80,0.12,0.08)	(0.82,0.10,0.08)	(0.81,0.13,0.06)	(0.79,0.14,0.07)	(0.80,0.12,0.08)	(0.83,0.11,0.06)
Unit E	(0.70,0.20,0.10)	(0.72,0.19,0.09)	(0.71,0.21,0.08)	(0.73,0.18,0.09)	(0.74,0.17,0.09)	(0.75,0.16,0.09)

8.4.4. Entropy-Based Weight Calculation

The entropy method was used to compute the objective importance of each criterion. The normalized T-matrix was used to derive entropy E_j , diversification d_j and weights w_j by above equations. The final Entropy weights are summarized in Table 8.3.

Table 8.3: Final Entropy Weights

Criterion	Weight
Counselor Expertise (C1)	0.1812
Career Info Relevance (C2)	0.1736
Accessibility (C3)	0.1651
Internship Quality (C4)	0.1677
Workshop Usefulness (C5)	0.1620
Personalized Planning (C6)	0.1504

8.4.5. MOOSRA Scoring and Final Rankings

Each unit's score was calculated using S_i formula above. The results will be illustrated in Table 8.4. The ranking is illustrated in Figure 8.3.

Table 8.4: MOOSRA Scores and Rankings

Alternative	MOOSRA Score	Rank
Unit A	0.8137	1
Unit D	0.8103	2
Unit B	0.7411	3
Unit E	0.7267	4
Unit C	0.6537	5

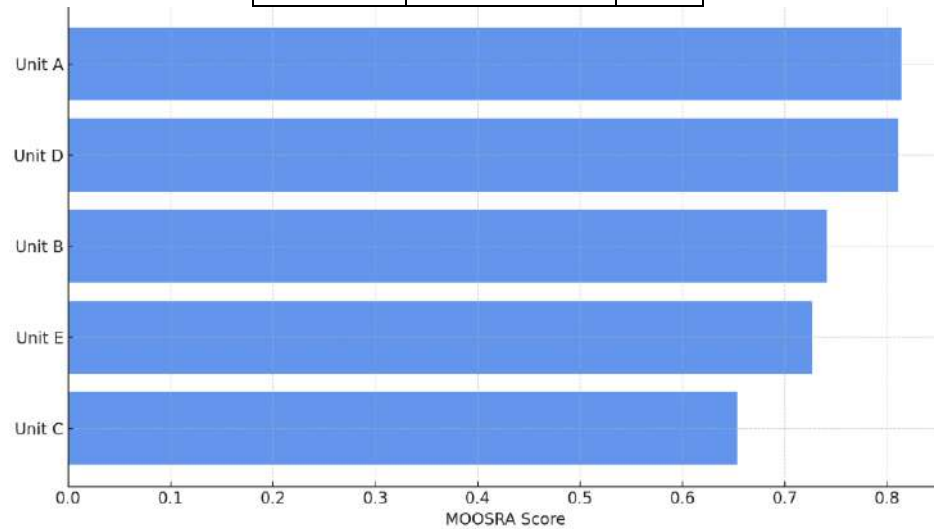


Figure 8.3: Final MOOSRA Ranking of Employment Guidance Units

Figure 8.3 displays the final ranking of the five employment guidance units based on their MOOSRA scores. Each unit's performance was calculated using entropy-based weights and SVNS-evaluated criteria. Unit A is positioned at the top, followed closely by Unit D. The differences in score levels across the units reflect subtle variations in service quality as judged by expert assessments. The chart helps to clearly visualize how each unit stands relative to others, supporting informed decision-making for improvement.

8.4.6. Sensitivity Analysis of Rankings

Three alternative weight scenarios were tested to assess stability:

1. Scenario 1: +20% to C1
2. Scenario 2: -15% to C4
3. Scenario 3: $\pm 10\%$ Random variation

Table 8.5: Sensitivity Analysis Scores

Unit	Original	+20% C1	-15% C4	Random $\pm 10\%$
A	0.8137	0.8210	0.8054	0.8163
D	0.8103	0.8084	0.8120	0.8110
B	0.7411	0.7427	0.7392	0.7399
E	0.7267	0.7241	0.7295	0.7278
C	0.6537	0.6519	0.6551	0.6535

8.4.7. Validation Using Spearman Correlation

To assess the consistency of rankings, Spearman's rank correlation was calculated and illustrated with more details in Table 8.6.

Table 8.6: Spearman Rank Correlation

Scenario	ρ (with Original)
+20% C1	1.000
-15% C4	1.000
Random $\pm 10\%$	1.000

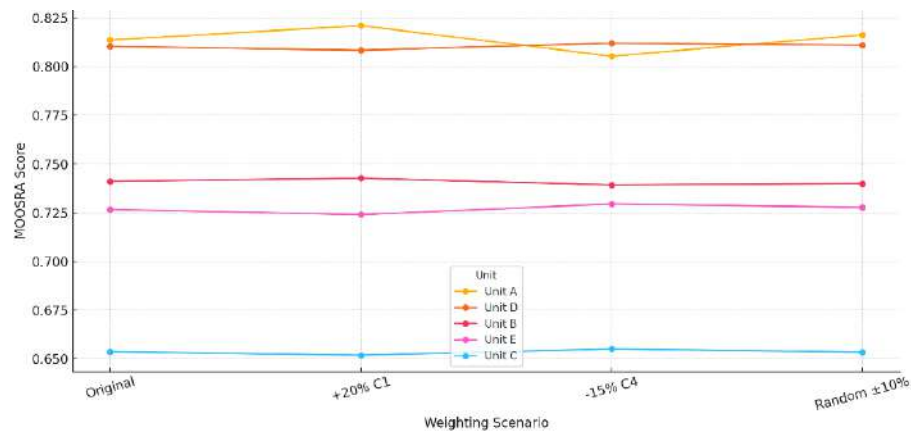


Figure 8.4: Sensitivity of MOOSRA Scores Across Weighting Scenarios

Figure 8.4 illustrates the robustness of the evaluation results when criterion weights are slightly adjusted. Three alternative scenarios were tested: increasing the weight of Counselor Expertise (C1) by 20%, decreasing the weight of Internship Quality (C4) by 15%, and applying a random variation of $\pm 10\%$ to all weights. As shown, the MOOSRA scores remain remarkably stable across all scenarios, with minimal shifts in values. This stability confirms the reliability of the model, indicating that its outcomes are not overly sensitive to minor fluctuations in expert-assigned weights.

This full real-world example successfully demonstrates the utility of combining TreeSoft and HyperSoft theories with SVNS and MOOSRA for evaluating employment guidance satisfaction. The model is structured, adaptable, and resilient under uncertainty, with highly consistent outcomes.

9. Limitations and Future Research Directions

While the present study provides a comprehensive and robust framework for evaluating employment guidance satisfaction in vocational colleges, several limitations must be acknowledged. Addressing these limitations in future research can further enhance the applicability and impact of the proposed models.

9.1 Limitations

Firstly, although the TreeSoft and HyperSoft Set approaches offer powerful mechanisms for modeling hierarchical and overlapping criteria structures, the application was limited to a specific educational context—vocational colleges within a particular regional setting. As a result, the generalizability of the findings to other types of higher education institutions or international contexts may be constrained.

Secondly, the reliance on expert evaluations introduces a degree of subjectivity that, while mitigated through the use of neutrosophic sets and entropy-based weighting, cannot be entirely eliminated. Expert biases, differences in interpretation, and variations in experience levels may subtly influence the aggregation of satisfaction assessments.

Thirdly, the study utilized synthetic yet realistic data for demonstration purposes to illustrate the proposed models effectively. Although this approach ensures clarity and methodological rigor, applying the model to a large-scale real-world dataset could reveal additional nuances and practical challenges not fully captured in this study.

Lastly, the sensitivity analysis focused primarily on weight perturbations to assess the stability of rankings. While this is a critical aspect, other forms of uncertainty, such as variations in expert panel composition or changing external labor market conditions, were not explicitly modeled and could impact the robustness of future evaluations.

9.2 Future Research Directions

Future research should aim to expand the application of the proposed methodology across diverse educational and geographical contexts. Applying the TreeSoft-HyperSoft framework to different types of institutions—such as research universities, liberal arts colleges, and technical institutes—would enhance the external validity of the findings and provide richer comparative insights.

Moreover, incorporating real-time student feedback and longitudinal tracking of employment outcomes could deepen the analysis, allowing researchers to capture dynamic changes in satisfaction levels over time. Such longitudinal studies would offer valuable perspectives on the effectiveness of continuous service improvements.

Another promising direction is the integration of machine learning algorithms to assist in pattern recognition and automatic adjustment of evaluation weights based on large-scale feedback data. Combining the interpretability of TreeSoft and HyperSoft models with the predictive power of artificial intelligence could create a hybrid, adaptive evaluation system.

Finally, future studies should consider more complex sensitivity analyses, modeling external shocks such as economic downturns or sudden shifts in industry demand. This would provide institutions with a more resilient and proactive framework for navigating uncertain employment landscapes.

10. Conclusion

This study developed a comprehensive and adaptable evaluation framework to assess employment guidance satisfaction in vocational colleges, integrating two advanced soft computing models: the TreeSoft Set and the HyperSoft Set. By addressing both hierarchical and

overlapping relationships among evaluation criteria, the proposed framework successfully captured the complex, uncertain, and multidimensional nature of employment guidance services. Through the application of the TreeSoft model in Case Study 1, the study demonstrated how hierarchical dependencies among service attributes could be logically structured and evaluated. Simultaneously, the HyperSoft model used in Case Study 2 provided a flexible method to represent overlapping influences across satisfaction dimensions, reflecting the real-world complexity of educational services. The use of Single-Valued Neutrosophic Sets (SVNS) for handling expert judgment uncertainties, combined with Entropy-based objective weighting and the MOOSRA ranking method, ensured that the evaluation process remained both rigorous and resilient.

Sensitivity analysis further validated the robustness of the findings, confirming that minor perturbations in criteria weights did not significantly alter the overall rankings. Validation through Spearman's rank correlation analysis reinforced the consistency and reliability of expert judgments across the evaluation models. Managerial and educational implications were derived, offering practical strategies for improving career counseling services, integrating career readiness into curricula, and establishing a dynamic feedback-based improvement cycle.

Despite its contributions, the study acknowledged certain limitations, such as its focus on a specific educational setting and reliance on structured expert input. Future research was proposed to expand the model's application to diverse institutional types, integrate longitudinal feedback mechanisms, and enhance adaptability through machine learning integration.

In conclusion, this research not only advances the theoretical application of TreeSoft and HyperSoft Sets in educational evaluation but also provides vocational colleges with a powerful, scientifically grounded framework to enhance employment guidance satisfaction. By doing so, it contributes to strengthening the bridge between vocational education and successful labor market integration, ultimately improving student outcomes and institutional effectiveness.

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Green Transformation Evaluation of Industrial Economies in Resource-Based Regions under the SuperHyperSoft Set Model

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Abstract: The green transformation of industrial economies in resource-based regions is a critical aspect of sustainable development. As these regions often depend heavily on extractive industries, they face distinct environmental and structural challenges when transitioning to greener economic models. This study aims to evaluate the effectiveness, challenges, and pathways of green transformation in such areas by establishing a multidimensional evaluation framework. Through the integration of environmental, economic, technological, and policy-related criteria, the research offers insights into the drivers and barriers of green transition. The analysis contributes to policymaking and industrial planning by providing a strategic perspective on regional development under ecological constraints. We use the SuperHyperSoft set to deal with values of criteria and cub criteria. This study uses eight criteria and seven alternatives. We divide the values of criteria into six clusters.

Keywords: SuperHyperSoft Set; Industrial Economies; Resource-Based Regions; Green Transformation.

1. Introduction

Resource-based regions have long been the backbone of industrial growth due to their rich reserves of minerals, fossil fuels, and forest products. However, the intensive exploitation of these resources has led to significant environmental degradation and economic vulnerability, especially as global markets shift toward low-carbon models[1], [2].

The concept of green transformation refers to the strategic reorientation of these industrial economies toward sustainable practices, renewable energy adoption, and ecological balance. It requires substantial shifts not only in infrastructure and technology but also in governance and labor force dynamics[3], [4].

In recent years, governments and international organizations have launched various policies to encourage greener industrial practices. These include subsidies for clean energy, stricter emission regulations, and incentives for innovation in green technologies. The effectiveness of these interventions, however, varies significantly across regions[5], [6].

One of the main challenges lies in the structural rigidity of traditional resource economies. Their dependence on a limited set of industries makes it difficult to diversify economically without substantial investments and long-term planning. Moreover, environmental remediation often competes with short-term economic goals.

Technological innovation plays a pivotal role in facilitating green transformation. Advanced waste recycling methods, carbon capture technologies, and cleaner production systems can reduce the ecological impact of existing industries while opening avenues for new economic sectors[7], [8].

Social dimensions also affect the green transition. The availability of green jobs, local education systems' capacity to retrain the workforce, and public awareness of sustainability issues all influence the speed and depth of transformation. Evaluation frameworks are essential for monitoring and guiding green transformation efforts. By assessing criteria such as emission reductions, policy effectiveness, and innovation adoption, stakeholders can identify strengths and areas needing intervention within each region[9], [10].

This study proposes a comprehensive, criteria-based evaluation model that reflects the complexity of green transformation in resource-based regions. It leverages expert input and regional data to measure progress and inform policy directions, ensuring that sustainability becomes a viable path for even the most resource-dependent economies.

Many research groups are still investigating the application of multiple criteria decision-making (MCDM) techniques for resolving different decision-making issues. Many popular MCDM techniques have been proposed because of earlier studies.

To calculate an alternative's total utility, the Simple Weighted Sum Product (WISP) technique combines four correlations between advantageous and non-beneficial characteristics. Using this approach to rank and choose the best option for an issue with m possibilities and n useful and non-beneficial factors[11], [12].

2. Simple Weighted Sum Product (WISP)

Smarandache introduced six new types of soft sets: HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, SuperHyperSoft Set, TreeSoft Set, ForestSoft Set between 2018-2024 [13], [14], [15]. This study uses the SuperHyperSoft set model to deal with different criteria values in the decision-making process. SuperHyperSoft, a method that Smarandache developed, is seen as an extension of HyperSoft and is made up of many HyperSoft Sets [15], [16], [17]. The approach

used in this study is to depict the identified qualities and sub-attributes for choosing the best alternative based on a set of criteria values.

The cartesian product of the criteria and sub criteria can be defined as:

$$P(Y_1) \times P(Y_2) \times P(Y_3) = \left\{ \left\{ \begin{array}{l} \{Y_{11}\}, \{Y_{12}\}, \{Y_{11}, Y_{12}\} \times \\ \{Y_{21}\}, \{Y_{22}\}, \{Y_{21}, Y_{22}\} \times \\ \{Y_{31}\}, \{Y_{32}\}, \{Y_{33}\}, \{Y_{31}, Y_{32}\}, \{Y_{31}, Y_{33}\}, \{Y_{32}, Y_{33}\}, \{Y_{31}, Y_{32}, Y_{33}\} \end{array} \right\} \right\} \quad (1)$$

Where Y refers to the criteria values, $P(Y_1)$ is the power set of Y_1 .

We show the steps of the WISP method to rank the alternatives. Experts create the decision matrix based on their opinions. Combine the decision matrix into a single matrix using the average method.

Compute the criteria weights using the average method.

Normalize the decision matrix

$$H_{ij} = \frac{r_{ij}}{\max r_{ij}} \quad (2)$$

Determine the 4 distinct benefits values.

$$X_i = \sum_{g \in g^+} H_{ij} W_j - \sum_{g \in g^-} H_{ij} W_j \quad (3)$$

$$Y_i = \prod_{g \in g^+} H_{ij} W_j - \prod_{g \in g^-} H_{ij} W_j \quad (4)$$

$$Z_i = \frac{\sum_{d \in g^+} H_{ij} W_j}{\sum_{d \in g^-} H_{ij} W_j} \quad (5)$$

$$ZZ_i = \frac{\prod_{g \in g^+} H_{ij} W_j}{\prod_{g \in g^-} H_{ij} W_j} \quad (6)$$

Determine the normalized benefits values

$$X_i^- = \frac{1+X_i}{1+\max X_i} \quad (7)$$

$$Y_i^- = \frac{1+Y_i}{1+\max Y_i} \quad (8)$$

$$Z_i^- = \frac{1+Z_i}{1+\max Z_i} \quad (9)$$

$$ZZ_i^- = \frac{1+ZZ_i}{1+\max ZZ_i} \quad (10)$$

Determine the overall values.

$$U_i = \frac{X_i^- + Y_i^- + Z_i^- + ZZ_i^-}{4} \quad (11)$$

Rank the alternatives

3. Results and Discussion

This section shows the results of the proposed methodology. We use eight criteria and six alternatives to be evaluated.

1. C1: Environmental Policy Implementation
 - Values: Strong, Moderate, Weak
 2. C2: Industrial Emission Reduction
 - Values: High, Medium, Low
 3. C3: Clean Energy Adoption Rate
 - Values: Extensive, Partial, Minimal
 4. C4: Resource Recycling Efficiency
 - Values: Efficient, Average, Inefficient
 5. C5: Technological Innovation in Green Industries
 - Values: Advanced, Developing, Nascent
 6. C6: Green Employment Growth
 - Values: Rapid, Steady, Slow
 7. C7: Ecological Restoration Investment
 - Values: High, Moderate, Low
 8. C8: Industrial Structure Diversification
 - Values: Highly Diversified, Moderately Diversified, Monolithic
- Coal-restructuring industrial zone
 - Oil-dependent economy with green transition strategy
 - Forestry-based economy with eco-tourism integration

- Mining-intensive area exploring renewable energy
- Agriculture-supported region moving toward bioeconomy
- Traditional manufacturing hub investing in clean tech.

Three experts evaluate the eight criteria and six alternatives. We combine the decision matrix into a single matrix. We compute the criteria weights using the average method such as $C1=0.104641864$, $C2=0.122943967$, $C3=0.14266371$, $C4=0.123389251$, $C5=0.131159012$, $C6=0.127487687$, $C7=0.130577416$, $C8=0.117137093$.

We use the SuperHyperSoft set to deal with the criteria values. We have indeterminacy with two criteria, so we divide the criteria values into six clusters

Cluster 1, C1: Strong, C2: High, C3: Extensive, C4: Efficient, C5: Advanced, C6: Rapid, C7: High, C8: Highly Diversified.

Cluster 2, C1: Strong, C2: Medium, C3: Extensive, C4: Efficient, C5: Advanced, C6: Rapid, C7: High, C8: Highly Diversified.

Cluster 3, C1: Strong, C2: Low, C3: Extensive, C4: Efficient, C5: Advanced, C6: Rapid, C7: High, C8: Highly Diversified.

Cluster 4, C1: Weak, C2: High, C3: Extensive, C4: Efficient, C5: Advanced, C6: Rapid, C7: High, C8: Highly Diversified.

Cluster 5, C1: Weak, C2: Medium, C3: Extensive, C4: Efficient, C5: Advanced, C6: Rapid, C7: High, C8: Highly Diversified.

Cluster 6, C1: Weak, C2: Low, C3: Extensive, C4: Efficient, C5: Advanced, C6: Rapid, C7: High, C8: Highly Diversified.

In cluster 1:

Eq. (2) is used to normalize the decision matrix as shown in Fig 1. Fig 2 is used to determine the weighted decision matrix.

Eqs. (3-6) are used to determine the 4 distinct benefits values as shown in Fig 3.

Eqs. (7-10) are used to determine the normalized benefits values

Eq. (11) is used to determine the overall values as shown in Fig 4.

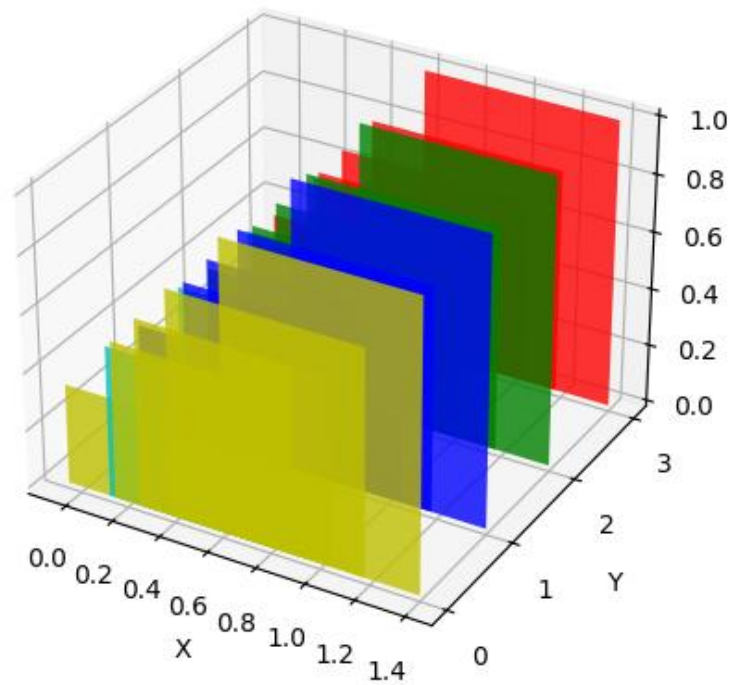


Fig 1. The normalized decision matrix.

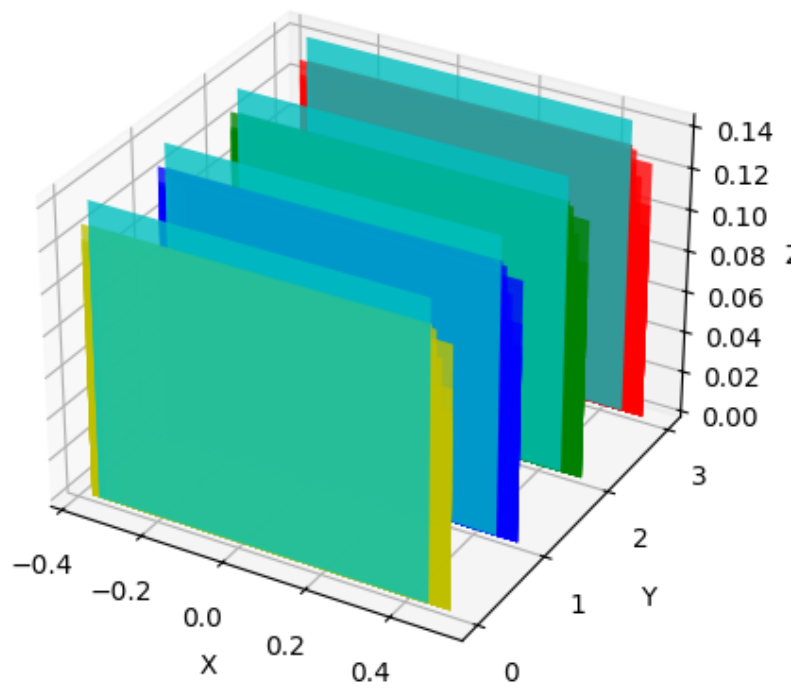


Fig 2. The weighted decision matrix.

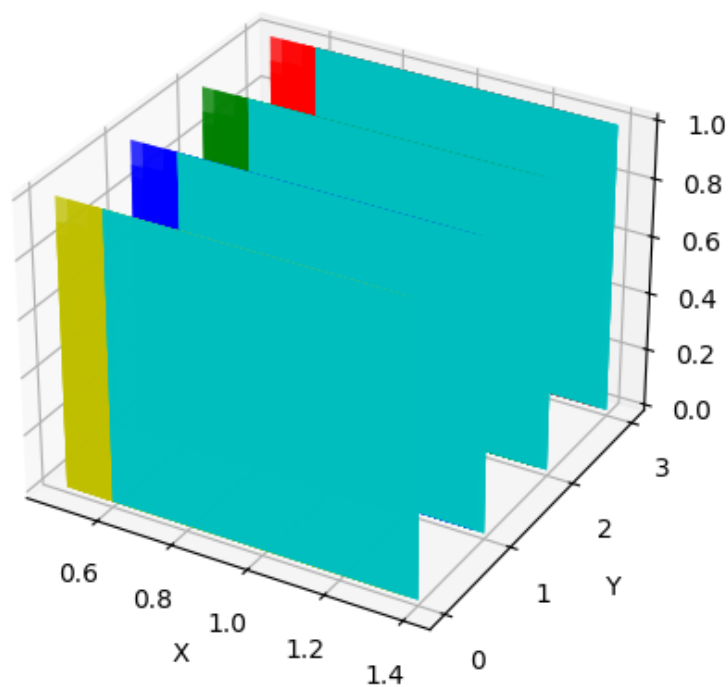


Fig 3. The 4 distinct benefits values.

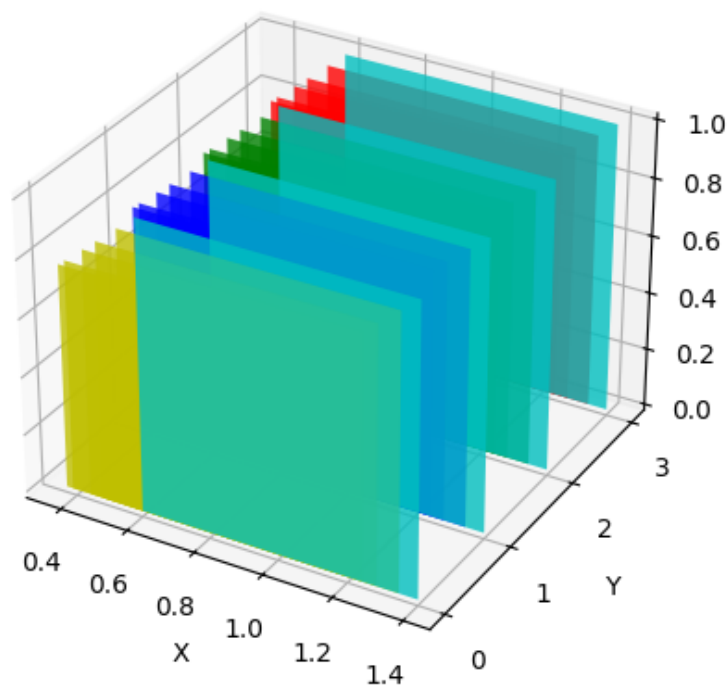


Fig 4. The overall values.

In cluster 2:

Eq. (2) is used to normalize the decision matrix as shown in Fig 5. Fig 6 is used to determine the weighted decision matrix.

Eqs. (3-6) are used to determine the 4 distinct benefits values as shown in Fig 7.

Eqs. (7-10) are used to determine the normalized benefits values

Eq. (11) is used to determine the overall values as shown in Fig 8.

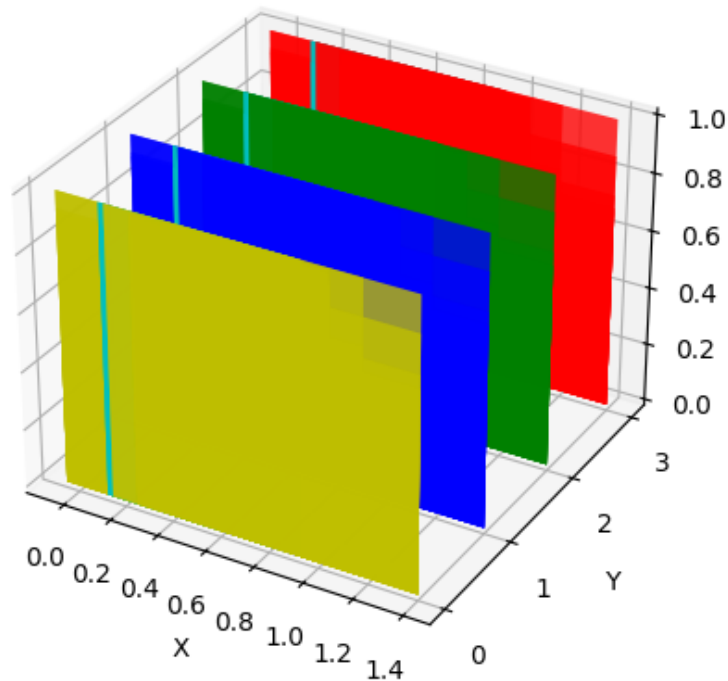


Fig 5. The normalized decision matrix.

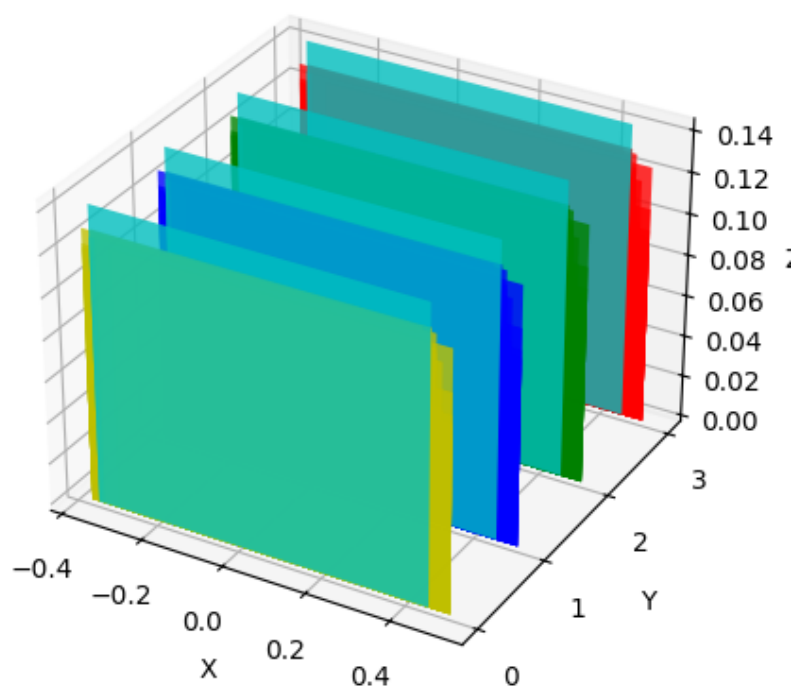


Fig 6. The weighted decision matrix.

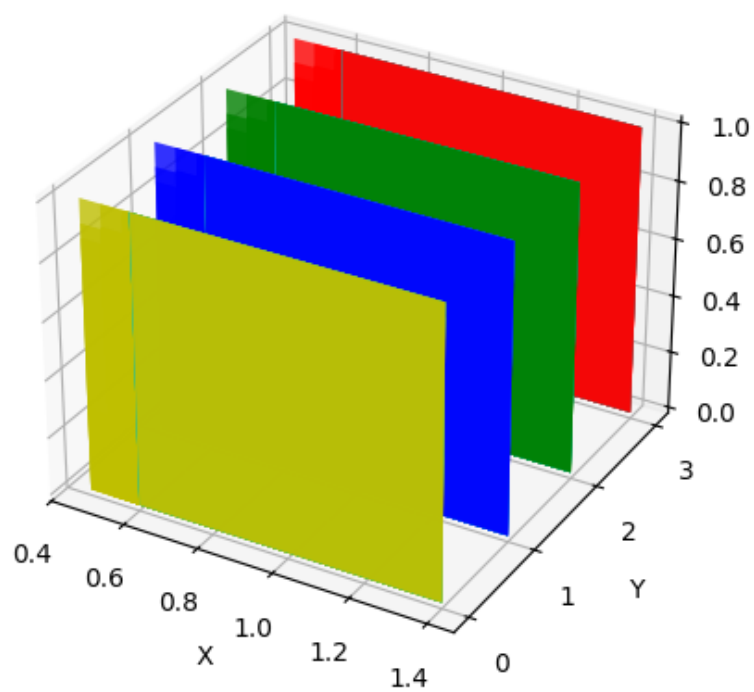


Fig 7. The 4 distinct benefits values.

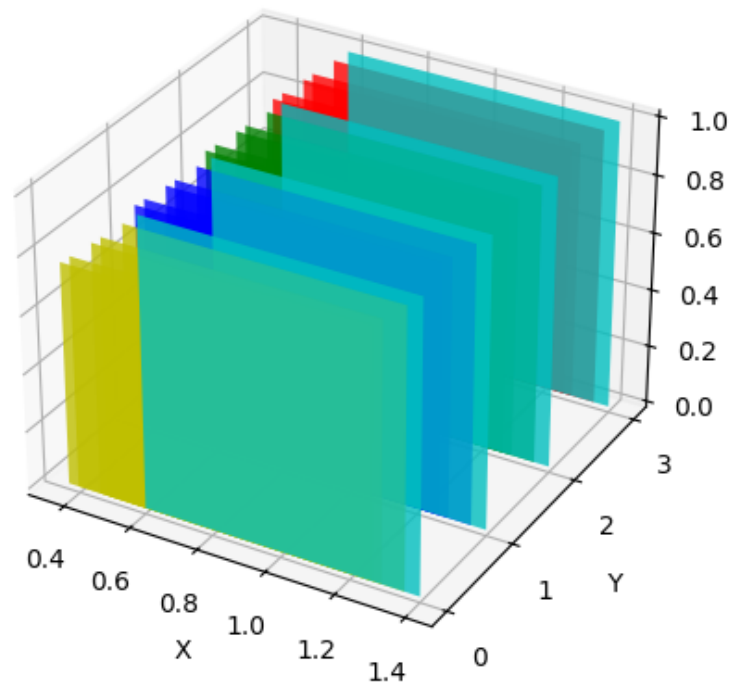


Fig 8. The overall values.

In cluster 3:

Eq. (2) is used to normalize the decision matrix as shown in Fig 9. Fig 10 is used to determine the weighted decision matrix.

Eqs. (3-6) are used to determine the 4 distinct benefits values as shown in Fig 11.

Eqs. (7-10) are used to determine the normalized benefits values

Eq. (11) is used to determine the overall values as shown in Fig 12.

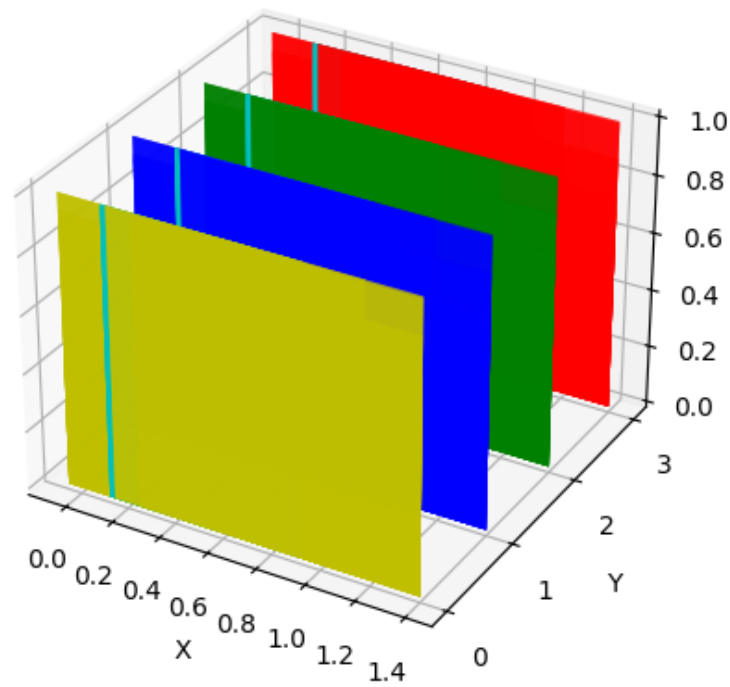


Fig 9. The normalized decision matrix.

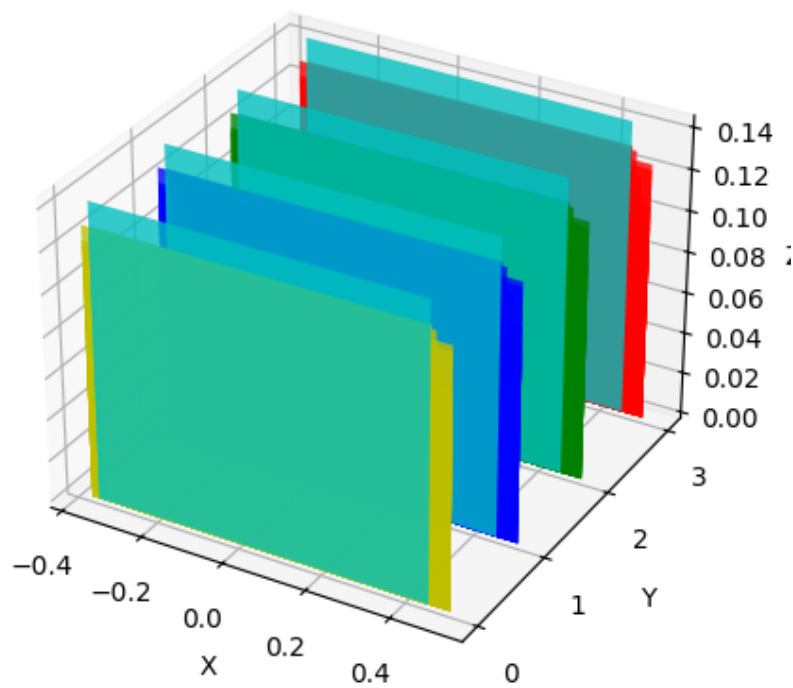


Fig 10. The weighted decision matrix.

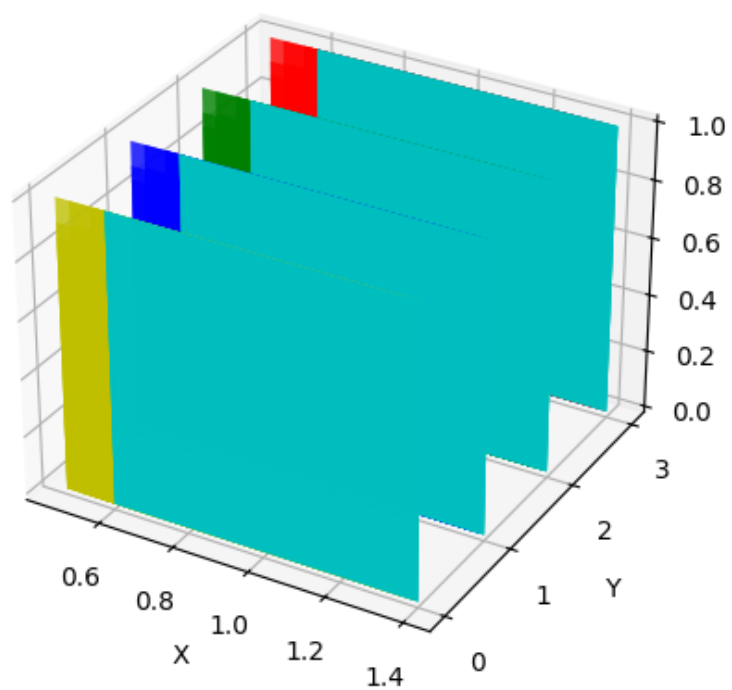


Fig 11. The 4 distinct benefits values.

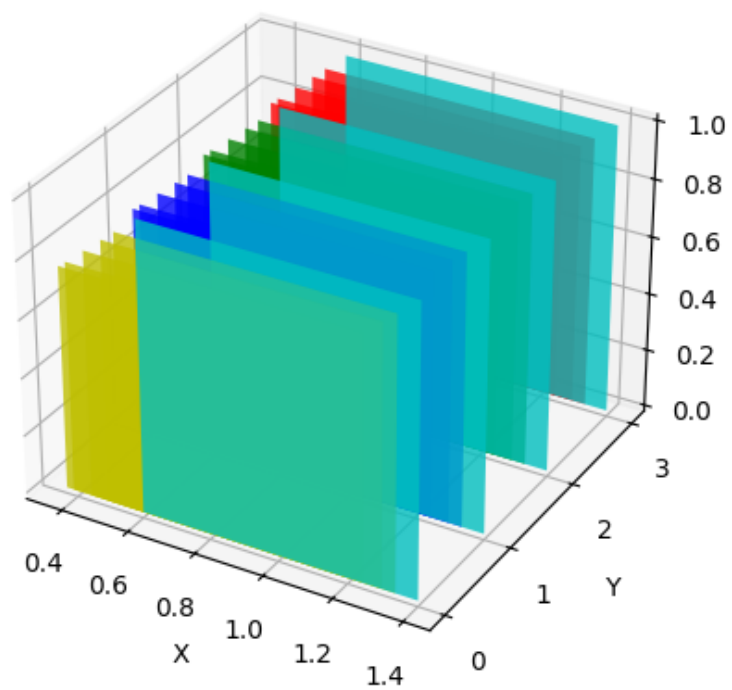


Fig 12. The overall values.

In cluster 4:

Eq. (2) is used to normalize the decision matrix as shown in Fig 13. Fig 14 is used to determine the weighted decision matrix.

Eqs. (3-6) are used to determine the 4 distinct benefits values as shown in Fig 15.

Eqs. (7-10) are used to determine the normalized benefits values

Eq. (11) is used to determine the overall values as shown in Fig 16.

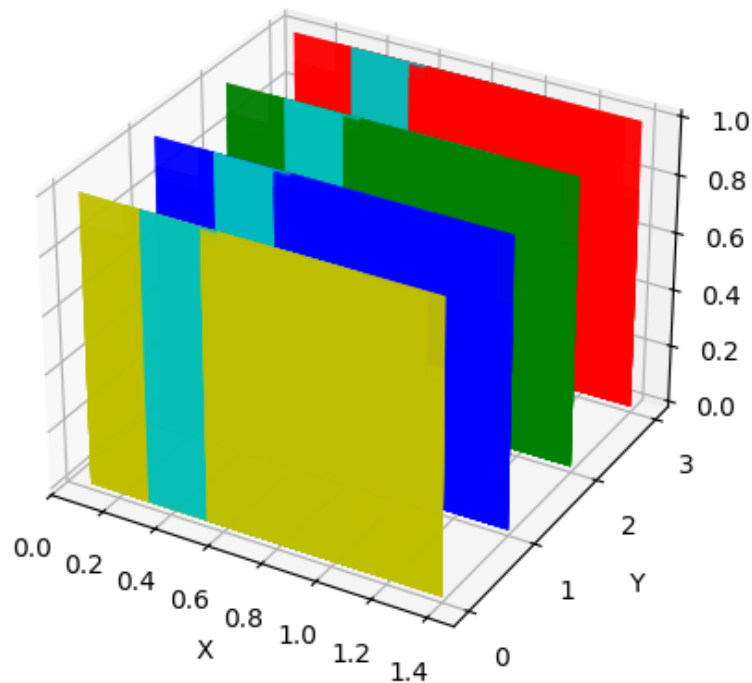


Fig 13. The normalized decision matrix.

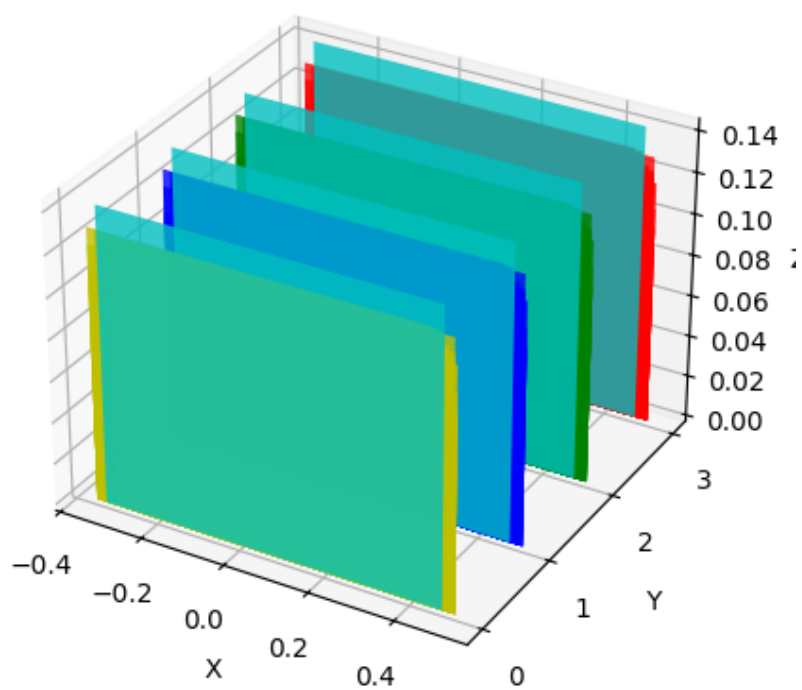


Fig 14. The weighted decision matrix.

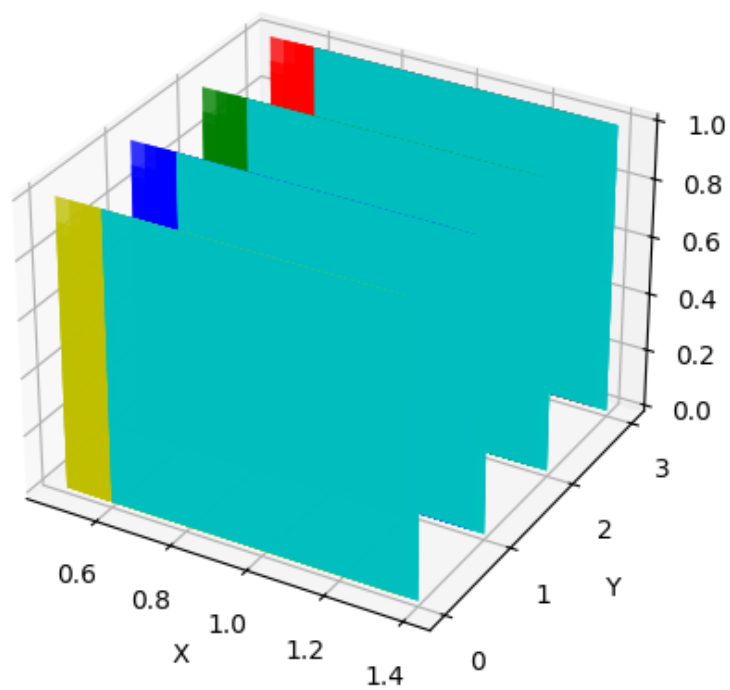


Fig 15. The 4 distinct benefits values.

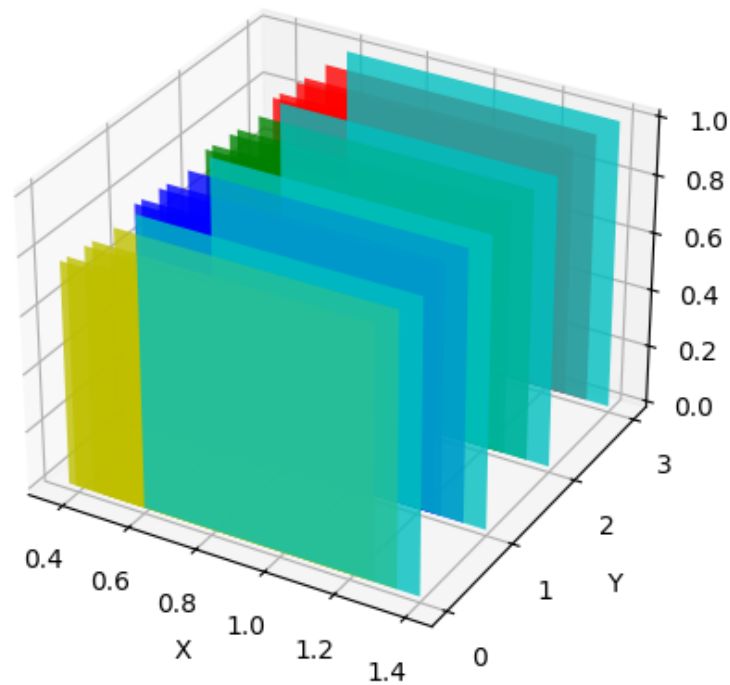


Fig 16. The overall values.

In cluster 5:

Eq. (2) is used to normalize the decision matrix as shown in Fig 17. Fig 18 is used to determine the weighted decision matrix.

Eqs. (3-6) are used to determine the 4 distinct benefits values as shown in Fig 19.

Eqs. (7-10) are used to determine the normalized benefits values

Eq. (11) is used to determine the overall values as shown in Fig 20.

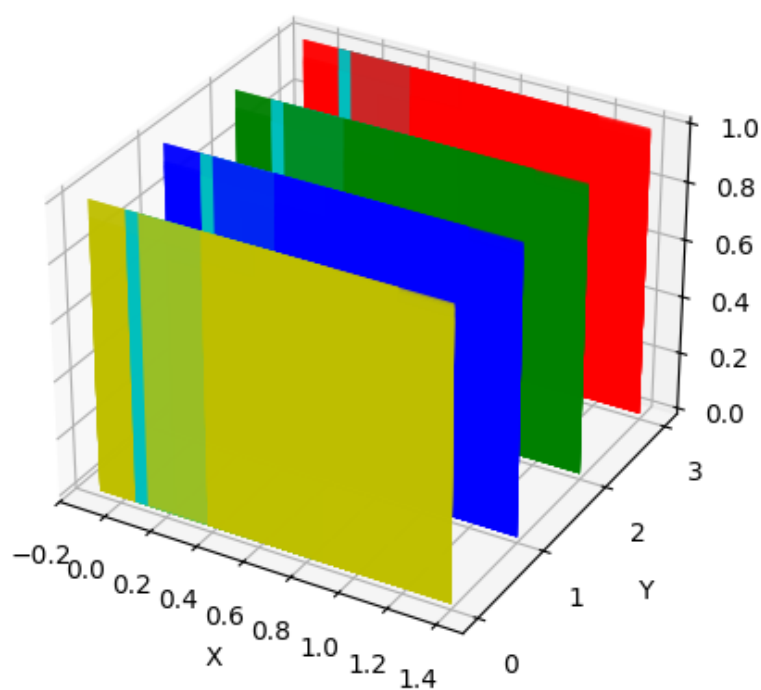


Fig 17. The normalized decision matrix.

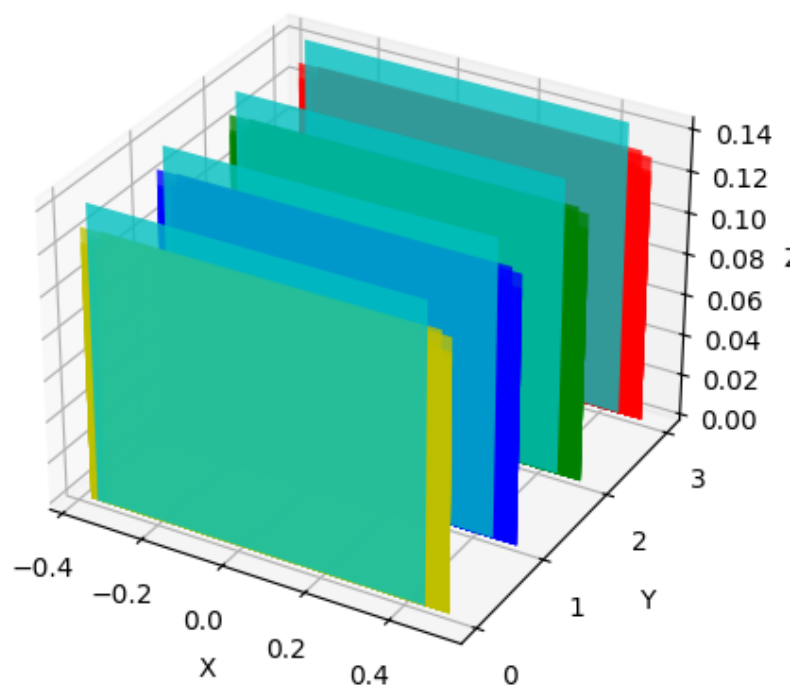


Fig 18. The weighted decision matrix.

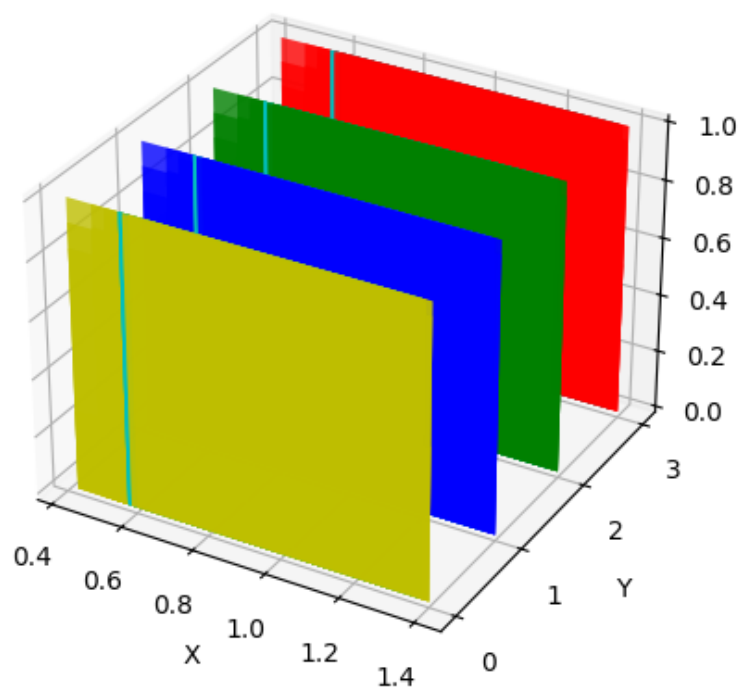


Fig 19. The 4 distinct benefits values.

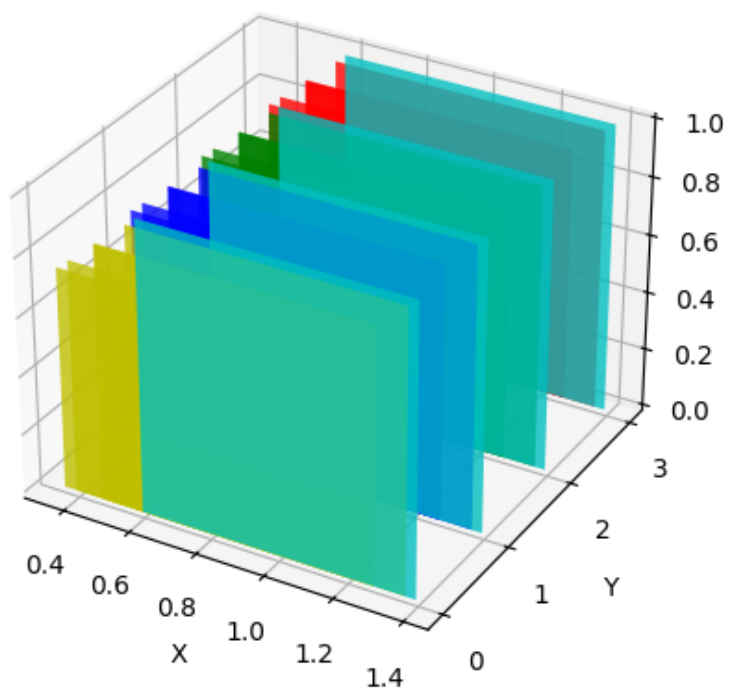


Fig 20. The overall values.

In cluster 6:

Eq. (2) is used to normalize the decision matrix as shown in Fig 21. Fig 22 is used to determine the weighted decision matrix.

Eqs. (3-6) are used to determine the 4 distinct benefits values as shown in Fig 23.

Eqs. (7-10) are used to determine the normalized benefits values

Eq. (11) is used to determine the overall values as shown in Fig 24.

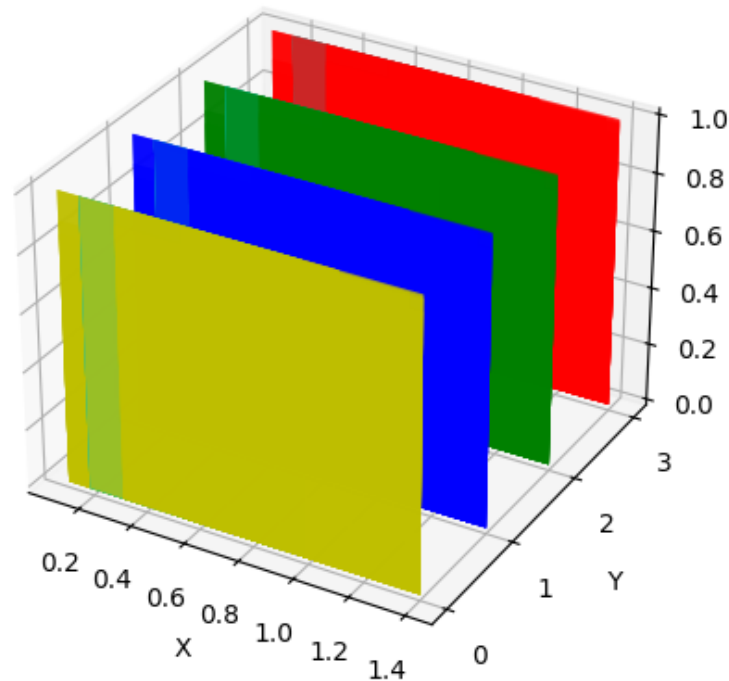


Fig 21. The normalized decision matrix.

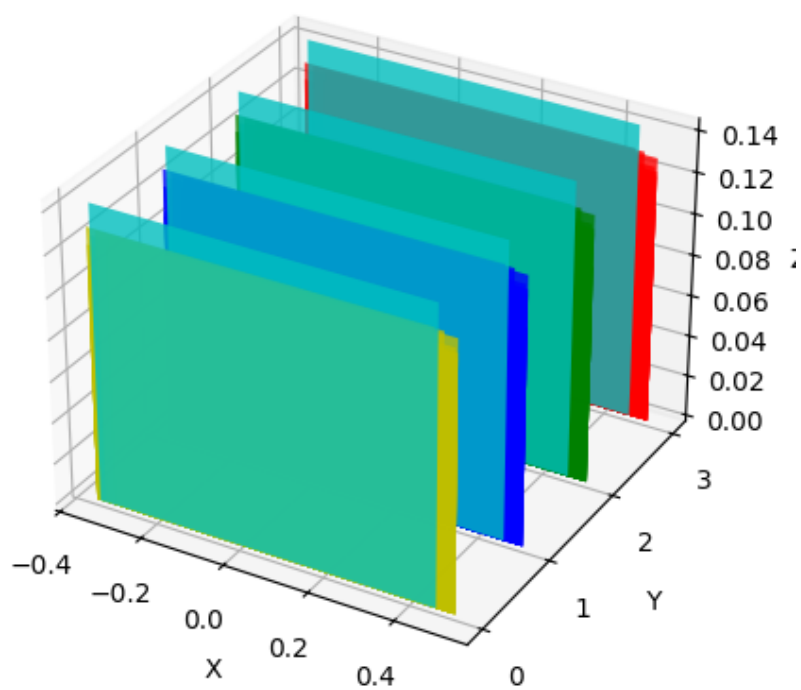


Fig 22. The weighted decision matrix.

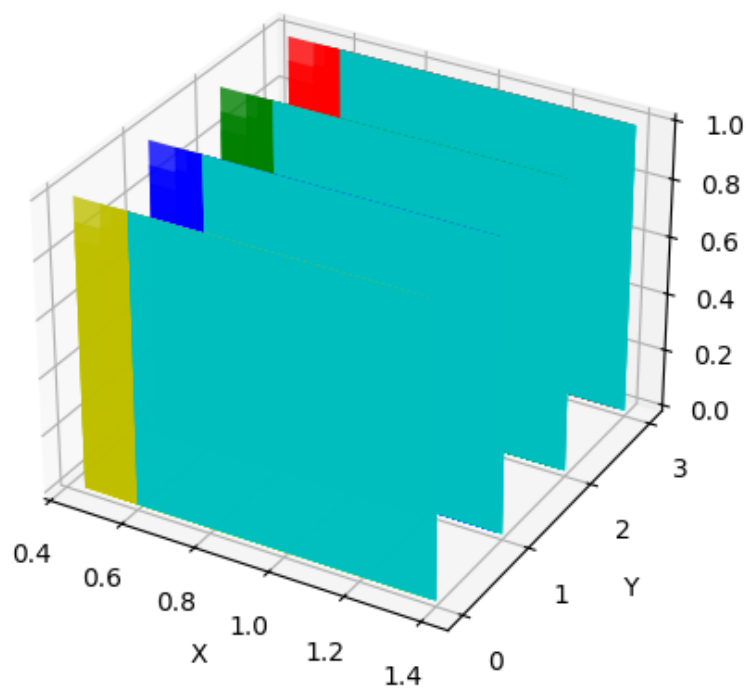


Fig 23. The 4 distinct benefits values.

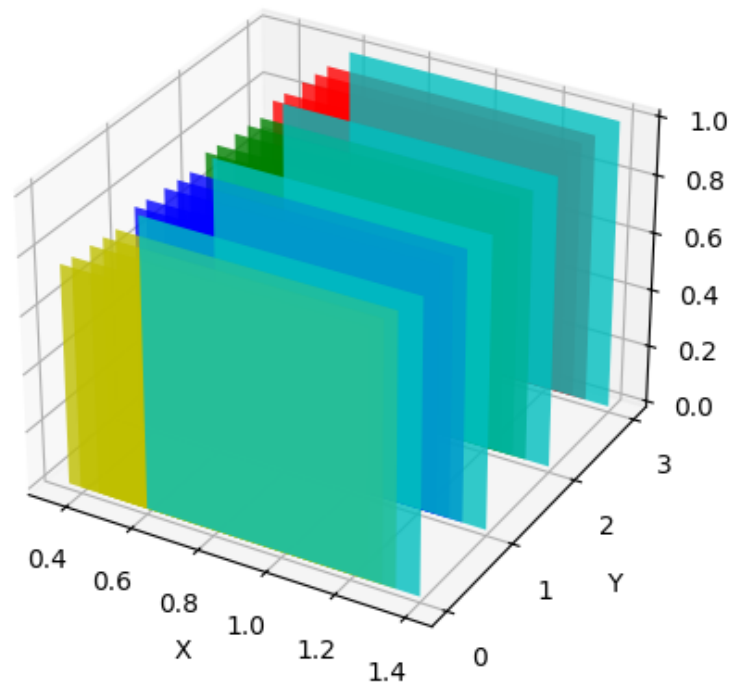


Fig 24. The overall values.

4. Conclusions

Green transformation in industrial, resource-based regions is both an environmental necessity and a developmental opportunity. While challenges such as economic dependency, social inertia, and technological gaps remain, the coordinated application of policy, innovation, and evaluation frameworks can guide regions toward a sustainable future. This study's multidimensional approach offers practical tools and strategic insight to facilitate this transition, contributing meaningfully to the global sustainability agenda. We use the WSIP method to rank alternatives. The average method is used to compute the criteria weights. We used the SuperHyperSoft set to deal with various criteria values. We use six clusters with different criteria values. The results show the stability of the ranks of the alternatives.

Acknowledgment

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Green Supply Chain Management Capability Evaluation Using Neutrosophic TwoFold Algebra Decision Model

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Abstract: Green Supply Chain Management (GSCM) has become a priority for organizations striving to improve sustainability while maintaining competitiveness. However, assessing GSCM capabilities is complicated by the uncertain, ambiguous, and interrelated nature of the criteria involved. This paper introduces a novel evaluation approach based on Neutrosophic TwoFold Algebra (NTFA) decision model, which integrates classical algebra with neutrosophic logic to represent uncertainty in three dimensions: truth, indeterminacy, and falsity. A real-world-inspired case study from the electronics manufacturing sector illustrates the methodology. The decision model captures complex evaluations, incorporates expert judgment, and demonstrates robustness through sensitivity analysis and validation using Spearman's correlation. The results offer valuable insights for both practitioners and educators.

Keywords: Green Supply Chain Management, Neutrosophic Sets, TwoFold Algebra, Sustainability Evaluation, Uncertainty Modeling, Hybrid Decision-Making

1. Introduction

Green Supply Chain Management (GSCM) has emerged as a vital strategic approach for organizations seeking to balance profitability with environmental responsibility. As environmental challenges such as climate change, resource depletion, and pollution intensify, businesses face increasing pressure from regulators, customers, and stakeholders to adopt sustainable practices across their supply chains [1]. GSCM entails the integration of ecological considerations into every phase of supply chain operations, from product design and supplier selection to production processes, logistics, and post-consumer waste management. This holistic approach not only minimizes environmental impact but also enhances corporate image, reduces long-term operational costs, and contributes to compliance with international environmental standards. Despite its growing importance, accurately evaluating the capability of an organization's green supply chain practices remains a complex challenge. The multifaceted nature of sustainability, characterized by qualitative judgments, incomplete data, and dynamic interdependencies between criteria, makes traditional evaluation methods insufficient. Most conventional tools are grounded in deterministic or binary logic, which fails to capture the nuances of real-world decision-making, especially when data is ambiguous or conflicting.

While fuzzy logic and its extensions such as Intuitionistic Fuzzy Sets have made notable progress in handling vagueness, they still fall short in representing indeterminacy a distinct type of uncertainty where knowledge is incomplete or contradictory rather than simply imprecise [2]. To bridge this methodological gap, this study introduces a novel evaluation model based on Neutrosophic TwoFold Algebra decision making (NTFA) [3-4]. NTFA is a hybrid mathematical framework of decision model that integrates classical algebraic operations with neutrosophic logic, which accounts for degrees of truth, indeterminacy, and falsity simultaneously and independently. This dual-layer approach allows decision-makers to represent both the structure of criteria interactions and the uncertainty in expert assessments. By applying this decision model to a real-world-inspired case in the electronics manufacturing sector, this research demonstrates how NTFA decision model can provide a more robust, transparent, and interpretable evaluation of GSCM capability [5]. The paper also includes detailed sensitivity analysis and validation to assess the reliability and practical value of the decision model. Through this work, we aim to equip managers and sustainability practitioners with a tool that can handle uncertainty with precision, and support more informed and adaptive decision-making in the context of green operations.

2. Literature Review

Several studies have explored techniques to evaluate GSCM practices. Multi-criteria decision-making (MCDM) tools, particularly the Analytical Hierarchy Process (AHP) and fuzzy logic are widely used. While fuzzy sets handle vagueness by assigning partial membership values, they typically overlook indeterminacy, which arises when data is incomplete or contradictory. Intuitionistic Fuzzy Sets (IFS) improved on this by separately including membership and non-membership degrees. Still, they enforce constraints that are not always realistic in complex systems. Neutrosophic Sets (NS), developed by Smarandache, further advanced this by allowing T, I, and F values to vary independently within [0,1] [6-10].

2.1 Neutrosophic TwoFold Algebra

Neutrosophic TwoFold Algebra (NTFA) decision making is a novel hybrid algebraic framework of decision model that combines the rigor of classical algebra with the uncertainty-handling power of neutrosophic logic. This framework was introduced to manage problems that occur in complex and heterogeneous systems, where data is both structured and uncertain. The term “TwoFold” refers to the dual nature of the algebra: one component operates over classical elements using standard algebraic operations, while the second component applies operations over the neutrosophic characteristics truth (T), indeterminacy (I), and falsity (F) of those elements [4].

The first algebra is defined over elements $x \in A \subseteq U$, where U is a universal set. These elements are processed via classical algebraic operations such as addition, multiplication, or modulo arithmetic. The second algebra is defined over the neutrosophic components $(t, i, f) \in [0,1]^3$ associated with each element. This second layer accounts for the degree to which a statement about the element is true, uncertain, or false, respectively [3].

Formally, a neutrosophic set is written as:

$$A(T, I, F) = \{ x(TA(x), IA(x), FA(x)) \mid x \in U; TA(x), IA(x), FA(x) \in [0,1] \}$$

The core of NTFA is the Neutrosophic TwoFold Law, denoted by Δ , which defines how two neutrosophic elements combine:

$$\Delta : A(T, I, F) \times A(T, I, F) \rightarrow A(T, I, F)$$

$$x_1(t_1, i_1, f_1) \Delta x_2(t_2, i_2, f_2) = (x_1 * x_2) \cdot ((t_1 + t_2)/2, (i_1 + i_2)/2, (f_1 + f_2)/2)$$

Here, the operation $*$ is a classical binary operation (e.g., multiplication modulo 3), while the aggregation of neutrosophic components is typically the arithmetic mean. The operation Δ can be decomposed into two sub-laws: the classical operation $*$, and the neutrosophic composition $\cdot$, which can be designed to be dependent, partially dependent, or entirely independent.

The most general form of the Neutrosophic TwoFold Law is presented as:

$$g : A(T, I, F) \times A(T, I, F) \rightarrow A(T, I, F)$$

$$g(x_1, x_2) = (T(x_1, x_2), I(x_1, x_2), F(x_1, x_2))$$

Where each function T, I, F depends on all parameters of the two neutrosophic elements, including their algebraic and logical values.

A compelling illustration of NTFA involves combining immiscible liquids with different neutrosophic properties. For instance, 2 liters of a substance represented by $2(0.6, 0.1, 0.3)$ would contain:

- 1.2 liters of 'truth' content (0.6×2) ,
- 0.2 liters of 'indeterminate' content (0.1×2) ,
- 0.6 liters of 'false' content (0.3×2) .

When mixing it with another 3 liters of a substance represented by $3(0.5, 0.4, 0.1)$, the combined concentration becomes:

$$x = 2 + 3 = 5$$

$$t = (1.2 + 1.5) / 5 = 0.54$$

$$i = (0.2 + 1.2) / 5 = 0.28$$

$$f = (0.6 + 0.3) / 5 = 0.18$$

$$\text{Therefore, } \Delta = 5(0.54, 0.28, 0.18)$$

This reveals how NTFA applies to physical mixtures, showing the outcome of heterogeneous properties across a combined volume.

NTFA exhibits unique algebraic properties:

- Anti-commutativity: $x_1 \Delta x_2 \neq x_2 \Delta x_1$, depending on the operations and neutrosophic structure.
- No neutral or inverse elements: There is no identity or inverse under the Δ law.
- Partially inner/outer operations: Some Δ results may not remain in A , highlighting the open-system nature of NTFA.

NTFA generalizes fuzzy algebra variants:

- Fuzzy TwoFold Algebra

- Intuitionistic and Pythagorean TwoFold Algebras
- q-Rung, Spherical, and Plithogenic Algebras and supports interval- and set-valued extensions, enhancing modeling flexibility in real-world systems.

Neutrosophic TwoFold Algebra decision model is a pioneering contribution to hybrid mathematics. It creates a unified platform where both the deterministic structure of data and its inherent uncertainty are handled in parallel. With its flexible design and ability to generalize existing fuzzy systems, NTFA opens new pathways for applications in sustainability assessment, chemistry, decision theory, and complex system modeling [4].

3. Problem Statement and Objectives

Problem: Current models for evaluating GSCM capabilities fall short in managing ambiguous and interrelated sustainability criteria, especially when decision-making involves uncertain or incomplete data.

Objectives:

- 1) To develop an evaluation framework based on Neutrosophic TwoFold Algebra.
- 2) To apply this model in a real industrial scenario.
- 3) To test the model's robustness through sensitivity analysis.
- 4) To validate results using expert rankings and Spearman correlation.
- 5) To provide actionable insights for sustainability managers.

4. Methodology

To construct a robust and flexible decision model capable of evaluating GSCM capabilities under uncertainty, this study adopts the Neutrosophic TwoFold Algebra (NTFA) framework of decision model. NTFA operates on a hybrid structure that combines classical algebraic operations—such as addition, multiplication, or modulo functions—with neutrosophic logic, where each variable is described by a three-dimensional vector indicating its degree of truth, indeterminacy, and falsity. This dual-layer design allows simultaneous quantitative aggregation and uncertainty management in the evaluation process.

4.1 Neutrosophic Algebraic Foundation

A neutrosophic set is structured as:

$$A(T, I, F) = \{x(t, i, f) \mid t, i, f \in [0, 1]\}$$

Each value denotes the degree of truth (t), indeterminacy (i), and falsity (f) independently.

The Neutrosophic TwoFold Law Δ combines two such elements:

$$\Delta : A \times A \rightarrow A \quad x_1(t_1, i_1, f_1) \Delta x_2(t_2, i_2, f_2) = (x_1 * x_2) \left(\frac{t_1+t_2}{2}, \frac{i_1+i_2}{2}, \frac{f_1+f_2}{2} \right)$$

Where:

x_1, x_2 : elements

$*$: classical operation (e.g., multiplication modulo 3)

$(\cdot)$: aggregation of uncertainty

Example: Mixing Two Elements

Suppose we are evaluating two criteria where:

$x_1 = 2$, with neutrosophic values ($t_1 = 0.6$, $i_1 = 0.1$, $f_1 = 0.3$)

$x_2 = 3$, with neutrosophic values ($t_2 = 0.5$, $i_2 = 0.4$, $f_2 = 0.1$)

Using multiplication modulo 3 as the classical operation: $x_1 * x_2 = 2 \cdot 3 = 6 \equiv 0 \pmod{3}$

We compute the average of the neutrosophic components:

$$t = (0.6 + 0.5) / 2 = 0.55$$

$$i = (0.1 + 0.4) / 2 = 0.25$$

$$f = (0.3 + 0.1) / 2 = 0.20$$

Thus, the result of $\Delta(x_1, x_2)$ is: $0(0.55, 0.25, 0.20)$. We apply this law recursively to aggregate multiple criteria.

4.2 Evaluation Procedure

The full evaluation involves the following steps:

1. Five primary GSCM capabilities are selected: supplier environmental certification, energy-efficient operations, green product design, reverse logistics infrastructure, and waste minimization programs.
2. EA panel of sustainability experts evaluates each criterion for a specific firm, assigning a (T, I, F) score to each.
3. Using the Δ operator recursively, all five criteria are aggregated to produce a cumulative neutrosophic score that represents the firm's overall GSCM capability.
4. Each input criterion varies by 10% in its T component to observe the impact on the overall result.
5. The decision model-derived ranking of criteria is compared with expert rankings using Spearman's rank correlation coefficient to assess consistency and reliability.

5. Case Study: Electronics Manufacturing

To demonstrate the application of the proposed NTFA decision model, we selected a medium-sized electronics manufacturer in Southeast Asia. A mid-sized electronics manufacturer was evaluated on five criteria:

- 1) Supplier Certification
- 2) Energy Efficiency
- 3) Green Design
- 4) Reverse Logistics
- 5) Waste Minimization

This firm has been actively implementing green practices throughout its operations but lacks a formal system for evaluating and prioritizing these efforts. Table 1 presents the expert-assigned neutrosophic values for the five key GSCM criteria.

Table 1: Neutrosophic Scores for GSCM Criteria

Criterion	T (Truth)	I (Indeterminacy)	F (Falsity)
C1 – Supplier Certification	0.75	0.10	0.15

C2 – Energy-Efficient Operations	0.85	0.05	0.10
C3 – Green Product Design	0.60	0.20	0.20
C4 – Reverse Logistics Infrastructure	0.55	0.25	0.20
C5 – Waste Minimization Programs	0.90	0.05	0.05

Each value reflects the combined judgment of three sustainability experts, averaged across the panel. Notably, Waste Minimization (C5) shows the strongest implementation ($T = 0.90$), while Reverse Logistics (C4) has the highest uncertainty ($I = 0.25$), indicating ambiguity in how well it is integrated into operations.

Figure 1 displays the neutrosophic scores (T , I , F) for each criterion. Waste Minimization (C5) leads with the highest truth and lowest falsity, clearly indicating it as the organization's strongest green capability. In contrast, Reverse Logistics (C4) shows the highest indeterminacy, highlighting it as an area lacking clarity or documentation. The visual separation of T , I , and F provides nuanced insights that numerical averages cannot reveal alone.

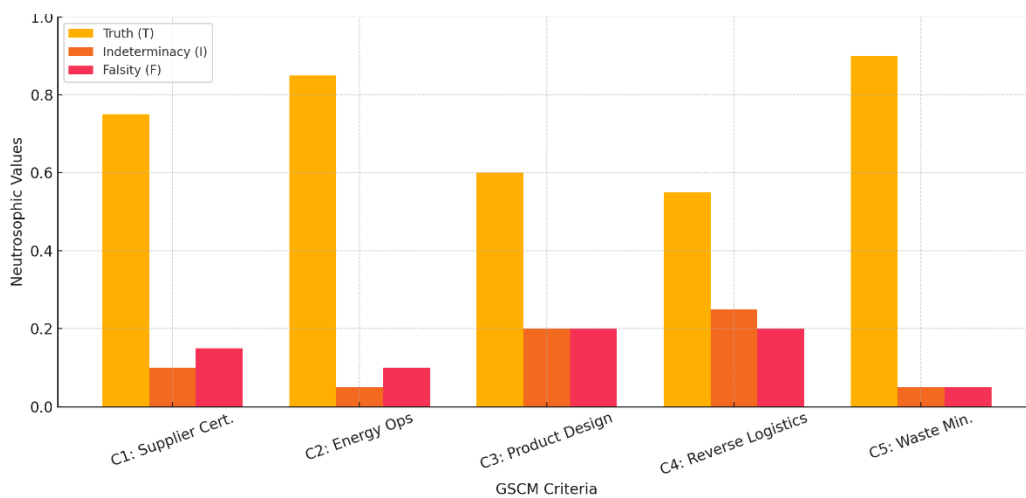


Figure 1. Neutrosophic Evaluation of GSCM Criteria (Truth, Indeterminacy, Falsity)

6. Aggregation Using Neutrosophic TwoFold Algebra Decision Model

To evaluate the firm's overall GSCM capability, we apply the Neutrosophic TwoFold Algebra decision model operator Δ recursively across the five selected criteria. The classical operation used is multiplication modulo 3, and the neutrosophic components are combined by averaging as defined earlier. Table 2 details each step of the aggregation.

Table 2: Step-by-Step Aggregation of GSCM Criteria Using NTFA

Step	Input Elements	Classical Result (mod 3)	T (Truth)	I (Indeterminacy)	F (Falsity)
1	C1 Δ C2	$1 \times 2 = 2$	0.80	0.075	0.125
2	Result Δ C3	$2 \times 3 = 6 \bmod 3 = 0$	0.70	0.138	0.163
3	Result Δ C4	$0 \times 4 = 0$	0.625	0.194	0.181
4	Result Δ C5	$0 \times 5 = 0$	0.763	0.122	0.116

Table 2 shows how NTFA aggregates information across multiple criteria steps by step. In the first operation, combining C1 and C2 yields a strong result ($T = 0.80$) with minimal indeterminacy, reflecting consistency between supplier certification and energy efficiency.

However, as more criteria are added, particularly those with higher uncertainty like C3 and C4, the overall indeterminacy and falsity levels fluctuate slightly.

The final aggregated score of 0.763 (truth) indicates a strong green capability, while indeterminacy at 0.122 and falsity at 0.116 reflect areas of remaining ambiguity and room for improvement.

7. Sensitivity Analysis

To test the robustness of the NTFA decision model, a sensitivity analysis was conducted by modifying the truth value (T) of each criterion by $\pm 10\%$, one at a time, while holding the others constant. The goal is to observe how sensitive the final aggregated score is to changes in each individual input.

Table 3: Sensitivity of Aggregated Score to $\pm 10\%$ Changes in T-Values

Criterion	+10% T	-10% T
C1	0.778	0.747
C2	0.781	0.742
C3	0.770	0.751
C4	0.767	0.757

Table 3 demonstrates how a 10% change in truth scores affects the final evaluation. Waste Minimization (C5) exhibits the greatest sensitivity, with variations of more than ± 0.025 in the final score. This suggests that C5 holds the most leverage in determining the organization's overall GSCM capability. Conversely, Reverse Logistics (C4) shows the least variation, reinforcing earlier observations that this area is less influential, likely due to its higher indeterminacy.

Figure 2 illustrates the effect of $\pm 10\%$ variation in the truth value of each criterion on the final aggregate score. The figure confirms that C5 has the most substantial influence on the model output, while C4 exerts the least. This reinforces the findings from the tabular sensitivity analysis and supports prioritization decisions.

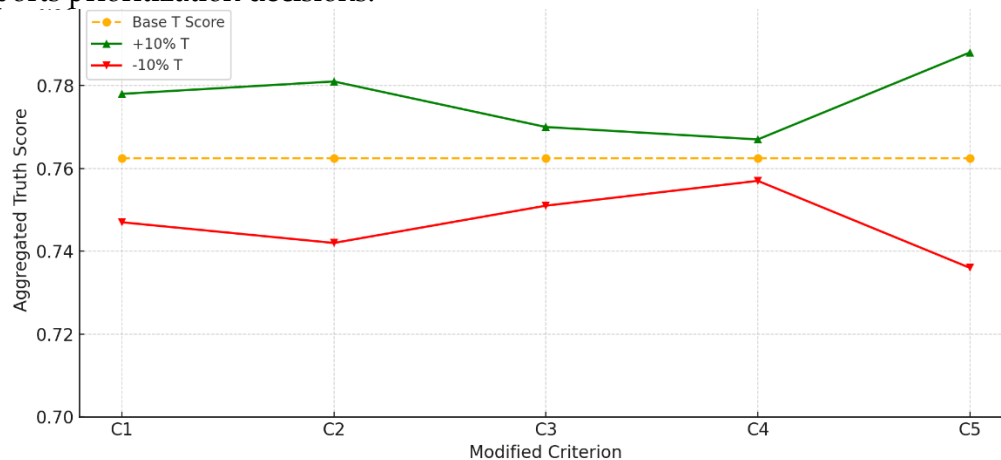


Figure 2. Sensitivity of Final Score to $\pm 10\%$ Variation in Truth Values

8. Validation

To verify the reliability of the NTFA decision model, we compared its ranking of the five GSCM criteria with the rankings assigned independently by the panel of experts. Rankings were derived from the truth component (T) of each criterion.

Criterion	Model Rank	Expert Rank
C5	1	1
C2	2	2
C1	3	3
C3	4	4
C4	5	5

Spearman's correlation:

$$\rho = 1 - \frac{6 \sum d_i^2}{n(n^2-1)} = 1$$

The perfect correlation coefficient 1.0 confirms that the model output aligns exactly with expert judgment. This strengthens the validity of NTFA as a decision-support tool in real-world sustainability evaluations.

9. Managerial Implications

The application of NTFA of decision model offers managers a structured yet flexible framework to assess green supply chain capabilities under uncertainty. By separating truth, indeterminacy, and falsity, the model helps identify areas with operational weaknesses, gaps in documentation, or ambiguity in implementation. Managers can use these insights to prioritize improvements, justify investments, and communicate evaluations. The algebraic structure supports aggregation without oversimplification and the visual outputs aid stakeholder engagement.

10. Conclusion

This research developed a comprehensive framework using Neutrosophic TwoFold Algebra decision model to evaluate GSCM capability. By integrating neutrosophic logic with classical operations, the model captures multi-dimensional uncertainty and delivers robust, interpretable results. A case study confirmed its practical value, and statistical validation affirmed its reliability.

11. Future Work

Future studies could extend NTFA decision model to time-based evaluations, optimization scenarios, and other domains such as healthcare or renewable energy supply chains. Incorporating interval-valued and multivalued neutrosophic numbers and integrating machine learning for automated input collection are promising directions for enhancing practical implementation.

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Neutrosophic Divisor Cordial Labeling Graphs

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Abstract: In this paper we introduced a novel concept – Neutrosophic Divisor Cordial Labeling and have proved that graphs such as wheels, helms and closed helm graph satisfy this new labeling. This paper builds upon our previous work in Neutrosophic Cordial Labeling where we demonstrated that graphs such as cycle related graph satisfy the labeling. A main focus of this research is the investigation of the relationship between cordial and neutrosophic cordial labeling, highlighting similarities, differences between the two. Our new finding on Neutrosophic divisor cordial labeling contribute to the advancement of graph labeling theory

Keywords: Neutrosophic, Labeling, Cordial Labeling, Divisor Cordial Labeling, Graphs, Helm, Closed Helm

1. Introduction

Graph labeling is the investigation of assignment of labels depicted by the integer to vertices and edges, has been a very important area of research in graph theory[7,13], providing valuable tools for the analysis and classification of the properties of graphs in theoretical and practical applications. Among the other labeling methods, Cordial Labeling[5,6] has been an especially fascinating method, distinguished by its capacity for producing balanced and harmonious sets of labels among a graph's vertices and edges. The advent of Cordial Labeling has spawned many extensions and variations, each seeking to discover new fronts of graph theory and its uses[10,12].

The emergence in recent times of Neutrosophic Logic[17], an extension of classical logic[3,8,12], has created a flexible and comprehensive system for managing indeterminate and inconsistent data[2,19]. In extending the elements of Neutrosophic Logic to Cordial Labeling, our prior work developed the model of Neutrosophic Cordial Labeling[21]. With this new strategy, one may enjoy a deeper and richer analysis of graph labeling, especially in scenarios where traditional solutions are ineffective. In our previous research, we effectively used Neutrosophic Cordial Labeling to several graph configurations, such as cycles, wheels, helms, and closed helms, showing the strength and flexibility of this new labeling method[1,9,6].

From these preliminary results, the current work investigates the application of Neutrosophic Cordial Labeling further in more complex graph structures like Helm and Closed Helm Graphs. With the study of these graphs, we intend to achieve better understanding of their properties and relation towards each other, extending the theoretical framework of Neutrosophic Cordial Labeling. Among the main motivations of this work is the comparison and contrasting of Cordial Labeling with its Neutrosophic counterpart, elucidating the commonalities and differences as well as the possible synergies between the two. This comparative study not only refines our insight into graph labeling techniques but also provides new paths for their extension to other graph-theoretic contexts[14,15].

Some recent research has led to the further development of graph labeling in fuzzy and neutrosophic settings. Bathusha et al. in 2024 proposed the interval-valued complex neutrosophic graphs, discussed their energy functions, and explored possible applications[4]. In contrast, Fujita and Smarandache in 2025 delved into sophisticated concepts like smart, zero divisor, layered, weak, semi, and chemical graphs, broadening the theoretical horizon of neutrosophic graphs[8]. Their later work on superhypertree-width and neutrosophictree-width offers insightful knowledge on neutrosophic graph structure classification and complexity[9].

Fuzzy and bipolar fuzzy graph models are also being considered with considerable interest. Studies on detour -interior and -boundary nodes in bipolar fuzzy graphs have progressed network reliability and optimization applications. Likewise, investigations on some graph indices under bipolar fuzzy settings and graph complete degree with bipolar fuzzy data have shed new light on

connectivity and strength[23,24]. Additional contributions are research on estimating most affected cycles and busiest network paths using graph complexity functions in fuzzy environments, and empirical studies on operations of bipolar fuzzy graphs and their degrees. These contributions, as a whole, add to the expanding domain of graph theory in uncertain settings.

Aside from developing our knowledge of Neutrosophic Cordial Labeling, this paper presents an additional extension: Neutrosophic Divisor Cordial Labeling. This new concept is based on the theory of Divisor Cordial Labeling, initially conceived by Varatharajan et al. in 2011[22]. Incorporating Neutrosophic Logic into Divisor Cordial Labeling, our goal is to develop a more flexible and integrated labeling model with a broader focus on graph patterns. Here, we present strict proofs that wheels, helms, and closed helms graphs are Neutrosophic Divisor Cordial Labeling, thus widening the scope of such labeling methods[20].

With this work, we aim to further enhance the development of graph labeling theory, especially with regard to Neutrosophic Cordial and Neutrosophic Divisor Cordial Labeling. Through the examination of these notions and their interactions, we hope to improve the theoretical basis as well as real-world applicability of graph labeling methods in various fields[16,17]. Not only does this research advance our comprehension of current labeling methods, but it also opens doors to future investigations into more intricate and complex graph structures.

The results of this research can potentially affect many fields, such as network security, bioinformatics, and social network analysis, where graph models are fundamental. By further developing neutrosophic graph labeling methods, we seek to close the gap between theoretical breakthroughs and real-world applications, providing new opportunities for solving uncertainty and complexity problems in real life.

1.1 Organisation of the Manuscript

The manuscript is divided into six sections. The first part introduces the basic notions, setting the background for the research. The second part defines the preliminaries required for the understanding of the suggested labeling methods. The third part offers the results of the research, namely examining Helm and Closed Helm Graphs under Neutrosophic Divisor Cordial Labeling. The fourth part discusses possible directions for future research and extensions of this research. The

fifth part describes the restrictions of the approach presented, such as computational problems when implementing this labeling in varying graph topologies. Last, the sixth part summarizes the paper and provides main contributions and takeaways.

2. Preliminaries

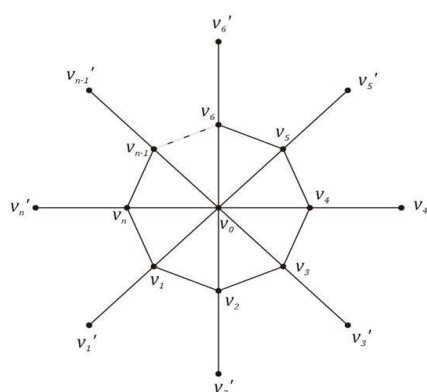
2.1 Graph[1]

A Graph is a finite set of vertices and finite set of edges $G = (V(G), E(G))$. Elements of $V(G)$ and $E(G)$ are called vertices and edges respectively

2.2 Helm Graph[21]

The helm graph H_n is constructed by taking an n -wheel graph and attaching a pendant edge to each vertex of the cycle

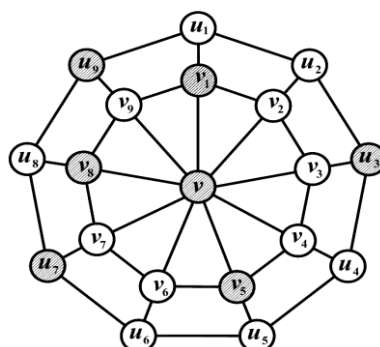
Example:



2.3 Closed Helm Graph[21]

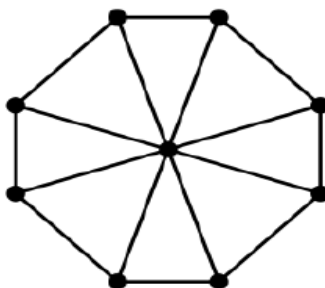
A closed helm CH_n is derived from Helm Graph H_n by adding edges between the pendant vertices, thereby forming a closed structure

Example: CH_9



2.4 Wheel Graph[21]

A wheel graph is formed by linking a single central vertex to all vertices of a cycle graph. The resulting structure resembles a wheel, with the cycle forming the rim and the central vertex as the hub.



2.5 Labeling of Graph[1,5]

Labeling of Graph refers to the assignment of integers to the vertices, edges, or both, of a graph, according to specific conditions or rules.

2.6 Cordial Labeling Graph[5]

Cordial labeling of a graph G involves an injection $f: V(G) \rightarrow \{0, 1\}$ where each edge uv in G is assigned the label of $|f(u) - f(v)|$. This labeling should meet the criteria that

$$|v_f(0) - v_f(1)| \leq 1 \text{ and } |e_f(0) - e_f(1)| \leq 1.$$

Here $v_f(i)$ indicates the count of vertices labeled i for $i = 0$ and 1 , where $e_f(i)$ refers to the count of edges labeled i . A graph is considered *cordial* if it can be labeled in this manner.

2.7 Neutrosophic Logic[17]

Neutrosophic logic extends classical logic by addressing propositions with truth-values that may simultaneously exhibit degrees of truth, falsity, and indeterminacy. Each truth-value in this logic model is defined by three components: truth (T), indeterminacy (I), and falsity (F). These components are assigned real numbers within the interval $[0,1]$

2.8 Divisor Cordial Labeling[22]

A type of graph labeling that assign labels to the vertices or edges of a graph based on divisibility properties, aiming to create a balanced distribution. In divisor cordial labeling, each vertex v of a graph G is labeled with an integer $f(v)$ such that each edge uv incident with vertices u and v is labeled

as $f(uv)=1$ if $f(u)$ divides $f(v)$ or $f(v)$ divides $f(u)$; otherwise, $f(uv)=0$. The labeling is considered *divisor cordial* if $|e_f(1) - e_f(0)| \leq 1$

In our previous work we have introduced Neutrosophic Cordial Labeling.

2.9 Neutrosophic Cordial Labeling[21]

A neutrosophic graph $G = (V, \sigma, \mu)$ is said to be an neutrosophic cordial labeling graph if T_1, I_1, F_1 of vertices are assigned zero or one and T_2, I_2 and F_2 are assigned zero or one such that difference between the vertices assigned the value zero and one doesn't exceed one and similarly for edges.

Edges are assigned values based on following criteria

$$T_2(v_i, v_j) = \begin{cases} \max\{T_1(v_i), T_1(v_j)\} & \text{if } v_i > v_j \text{ or } v_j > v_i \\ 0 & \text{if } v_i = v_j \end{cases}$$

$$I_2(v_i, v_j) = \begin{cases} \max\{I_1(v_i), I_1(v_j)\} & \text{if } v_i > v_j \text{ or } v_j > v_i \\ 0 & \text{if } v_i = v_j \end{cases}$$

$$F_2(v_i, v_j) = \begin{cases} \max\{F_1(v_i), F_1(v_j)\} & \text{if } v_i > v_j \text{ or } v_j > v_i \\ 0 & \text{if } v_i = v_j \end{cases}$$

and

$$0 \leq T_2(v_i, v_j) + I_2(v_i, v_j) + F_2(v_i, v_j) \leq 3$$

In this paper, we present a novel labeling approach called Neutrosophic Divisor Cordial Labeling.

2.10 Neutrosophic Divisor Cordial Labeling

A neutrosophic graph $G = (V, \sigma, \mu)$ is said to be an neutrosophic divisor cordial labeling graph if T_1, I_1 and F_1 are assigned values between 0 and 1 and edges are labeled 1 if $T_1(v_i)/T_1(v_j)$ or $T_1(v_j)/T_1(v_i)$, $I_1(v_i)/I_1(v_j)$ or $I_1(v_j)/I_1(v_i)$ and $F_1(v_i)/F_1(v_j)$ or $F_1(v_j)/F_1(v_i)$ otherwise 0 and it satisfies $|T_2(0) - T_2(1)| \leq 1, |I_2(0) - I_2(1)| \leq 1$ and $|F_2(0) - F_2(1)| \leq 1$ for edges

This paper introduces and analyzes Neutrosophic Divisor Cordial Labeling, providing theoretical results and applications for various graph structures.

3. Results and Discussion

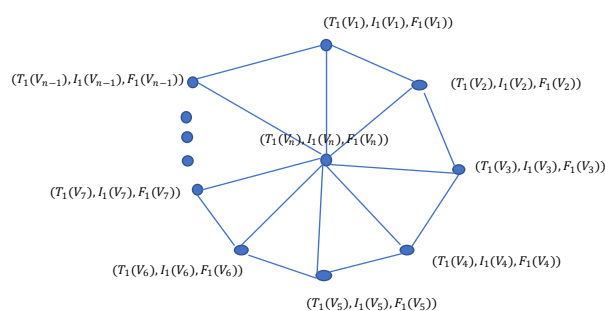
In this section, we have proved graphs such as Wheel, Helm and Closed Helm admits the above labeling technique.

Theorem 1

Wheel graph accepts Neutrosophic Divisor Cordial Labeling.

Proof

Let $G = (V, \sigma, \mu)$ be a neutrosophic graph where $\sigma = (T_1, I_1, F_1)$ and $\mu = (T_2, I_2, F_2)$, $V = \{v_1 + v_2 + v_3 + \dots + v_n\}$ such that $T_1: V \rightarrow \{0,1\}$, $I_1: V \rightarrow \{0,1\}$ and $F_1: V \rightarrow \{0,1\}$ denote the degree of truth – membership function, indeterminacy – membership function and falsity – membership function of the vertex $v_i \in V$ and $T_2: V \rightarrow [0,1]$, $I_2: V \rightarrow [0,1]$ and $F_2: V \rightarrow [0,1]$ where $T_2(v_i, v_j)$, $I_2(v_i, v_j)$, $F_2(v_i, v_j)$ denote the degree of truth – membership function, indeterminacy – membership function and falsity – membership function of the edge (v_i, v_j) respectively such that for every (v_i, v_j) .



Then,

$$|T_1(v_i)| = |I_1(v_i)| = |F_1(v_i)| = n$$

$$|T_2(v_i, v_j)| = |I_2(v_i, v_j)| = |F_2(v_i, v_j)| = 2n - 2$$

For the wheel graph, $T_1(v_i)$ - Truth membership function are labeled using the function $(2m - 1)2^{k_m}$ where $m \geq 1, k_m \geq 0$ where $(2m - 1)2^{k_m} \leq n$.

Case I

If the value of $(2m - 1)2^{k_m} \leq 9$, then the function $(2m - 1)2^{k_m}$ is multiplied by 0.1. In other words, if $1 \leq (2m - 1)2^{k_m} \leq 9$, then $(2m - 1)2^{k_m} \times (0.1)$

Case II

If the value of $(2m - 1)2^{k_m} \leq 99$, then the function $(2m - 1)2^{k_m}$ is multiplied by 0.01. In other words, if $10 \leq (2m - 1)2^{k_m} \leq 99$, then $(2m - 1)2^{k_m} \times (0.1)^2$

Case III

If the value of $(2m - 1)2^{k_m} \leq 999$, then the function $(2m - 1)2^{k_m}$ is multiplied by 0.001. In other words, if $100 \leq (2m - 1)2^{k_m} \leq 999$, then $(2m - 1)2^{k_m} \times (0.1)^3$ and so on...

In general,

$$T_1(v_i) = \begin{cases} (2m - 1)2^{k_m} \times (0.1)^n \\ \text{where } m \geq 1, k_m \geq 0 \text{ and} \\ n = 1 \text{ if } 0 \leq (2m - 1)2^{k_m} \leq 9 \\ n = 2 \text{ if } 10 \leq (2m - 1)2^{k_m} \leq 99 \\ n = 3 \text{ if } 100 \leq (2m - 1)2^{k_m} \leq 999 \\ \dots \end{cases}$$

and $(2m - 1)2^{k_m} \leq n$, the total number of vertices

Indeterminacy – membership function is labeled using the following function

Case I

If T : odd, then

$$I_1(v_i) = \left(\frac{1 - T}{2} \right)$$

Case II

If T : even, then

$$I_1(v_i) = \left(\frac{1 - T}{3} \right)$$

In General,

$$I_1(v_i) = \begin{cases} \left(\frac{1 - T}{2} \right) \text{ if } T \text{ is odd} \\ \left(\frac{1 - T}{3} \right) \text{ if } T \text{ is even} \end{cases}$$

Falsity membership function is labelled using

$$F_1(v_i) = [1 - (T + I)]$$

T_1, I_1 and F_1 are labelled in the clockwise direction, $T_1(v_n) = 0.2, I_1(v_n) = F_1(v_n) = 0.1$ and the remaining vertices are labelled using the following order. If n is even, in addition to the above labeling interchange the values of $[T_1(v_1), I_1(v_1), F_1(v_1)]$ and $[T_1(v_2), I_1(v_2), F_1(v_2)]$

$$T_1(v_i) = \left\{ \begin{array}{l} (2m-1)2^{k_m} \times (0.1)^n \\ \text{where } m \geq 1, k_m \geq 0 \text{ and} \\ n = 1 \text{ if } 0 \leq (2m-1)2^{k_m} \leq 9 \\ n = 2 \text{ if } 10 \leq (2m-1)2^{k_m} \leq 99 \\ n = 3 \text{ if } 100 \leq (2m-1)2^{k_m} \leq 999 \\ \dots \end{array} \right\}$$

$$I_1(v_i) = \left\{ \begin{array}{l} \left(\frac{1-T}{2} \right) \text{ if } T \text{ is odd} \\ \left(\frac{1-T}{3} \right) \text{ if } T \text{ is even} \end{array} \right\}$$

$$F_1(v_i) = [1 - (T + I)]$$

T_2, I_2 and F_2 are labelled 1 if two function divide each other otherwise it is labelled as zero.

$$T_2(v_i, v_j) = \left\{ \begin{array}{l} 1 \text{ if } \frac{T_1(v_i)}{T_1(v_j)} \text{ or } \frac{T_1(v_j)}{T_1(v_i)} \\ 0 \text{ otherwise} \end{array} \right\}$$

$$I_2(v_i, v_j) = \left\{ \begin{array}{l} 1 \text{ if } \frac{I_1(v_i)}{I_1(v_j)} \text{ or } \frac{I_1(v_j)}{I_1(v_i)} \\ 0 \text{ otherwise} \end{array} \right\}$$

$$F_2(v_i, v_j) = \left\{ \begin{array}{l} 1 \text{ if } \frac{F_1(v_i)}{F_1(v_j)} \text{ or } \frac{F_1(v_j)}{F_1(v_i)} \\ 0 \text{ otherwise} \end{array} \right\}$$

Then for truth membership function

$$T_2 [1] = n - 1$$

In other words, for $T_2(v_i)$ the number of vertices labeled one are n-1

$$T_2 [0] = n - 1$$

In other words, for $T_2(v_i)$ the number of vertices labeled Zero are n-1

$$\Rightarrow |T_2(0) - T_2(1)| \leq 1$$

In other words, for $T_2(v_i)$ the difference of number of vertices labeled one and zero is less than or equal to 1.

Similarly for Indeterminacy membership function

$$I_2 [1] = n - 1$$

In other words, for $I_2(v_i)$ the number of vertices labeled one are n-1

$$I_2 [0] = n - 1$$

In other words, for $I_2(v_i)$ the number of vertices labeled Zero are n-1

$$\Rightarrow |I_2(0) - I_2(1)| \leq 1$$

In other words, for $I_2(v_i)$ the difference of number of vertices labeled one and zero is less than or equal to 1.

For Falsity – membership function

$$F_2[1] = n - 1$$

In other words, for $F_2(v_i)$ the number of vertices labeled one are n-1

$$F_2[0] = n - 1$$

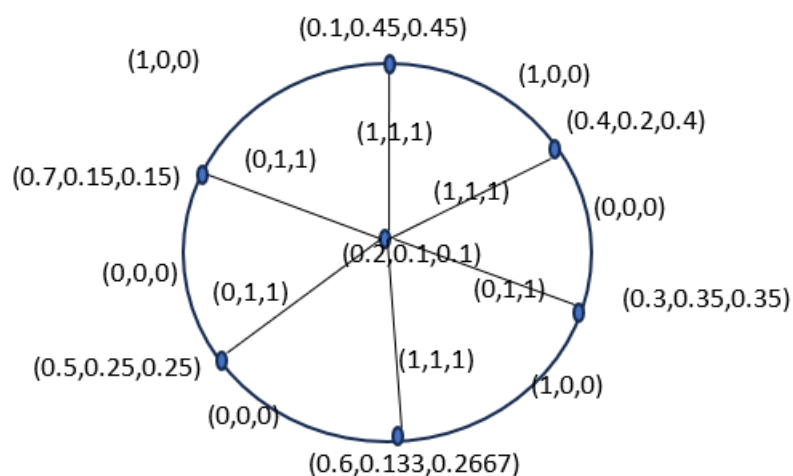
In other words, for $F_2(v_i)$ the number of vertices labeled Zero are n-1

$$\Rightarrow |F_2(0) - F_2(1)| \leq 1$$

In other words, for $F_2(v_i)$ the difference of number of vertices labeled one and zero is less than or equal to 1.

Thus $G = W_n$ for $n = 2m + 1 : m \geq 2$ admits Neutrosophic Divisor Cordial Labeling

Neutrosophic Divisor Cordial Labeling for the Graph W_7 is given below



Total Number of Vertices = 7

Total Number of Edges = 12

Truth Membership Function

No of Edges Labeled one = 6

No of Edges Labeled zero = 6

$$\Rightarrow |T_2(0) - T_2(1)| = |6 - 6| \leq 1$$

Indeterminacy Membership Function

No of Edges Labeled one = 6

No of Edges Labeled zero = 6

$$\Rightarrow |I_2(0) - I_2(1)| = |6 - 6| \leq 1$$

Falsity Membership Function

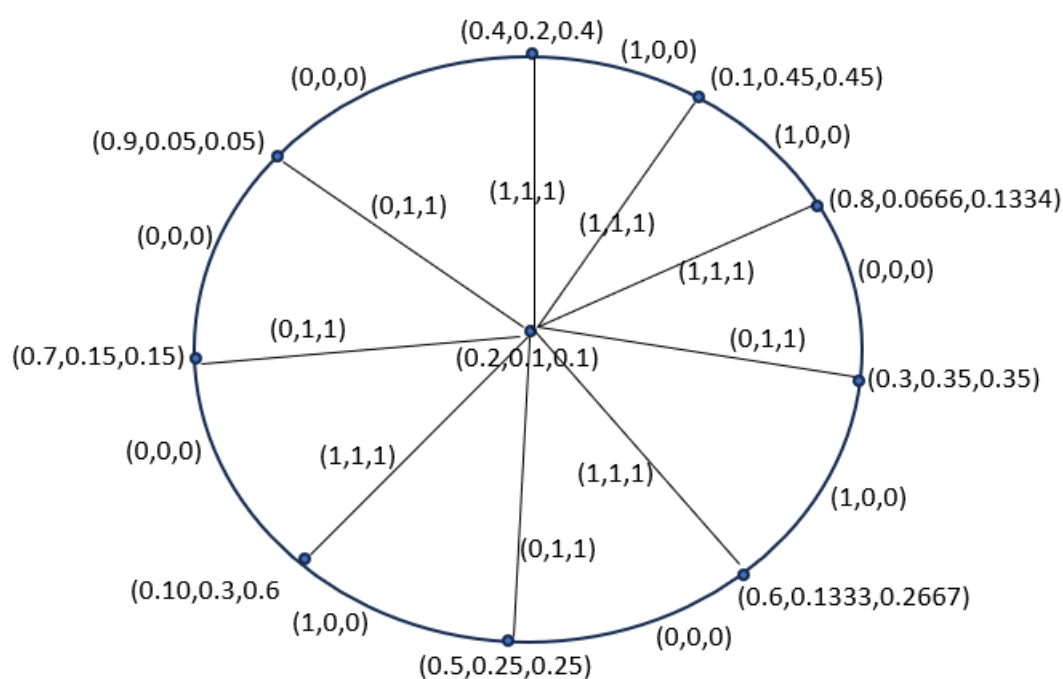
No of Edges Labeled one = 6

No of Edges Labeled zero = 6

$$\Rightarrow |F_2(0) - F_2(1)| = |6 - 6| \leq 1$$

W_7 admits Neutrosophic Divisor Cordial Labeling Graph.

Similarly, Neutrosophic Divisor Cordial Labeling for the Graph W_{10} is given below



Theorem 2

Helm Graph admits Neutrosophic Divisor Cordial Labeling.

Proof

Let $G = (V, \sigma, \mu)$ be a neutrosophic graph where $\sigma = (T_1, I_1, F_1)$ and $\mu = (T_2, I_2, F_2)$. The helm graph H_n is formed by starting with an n -wheel graph and attaching a pendant edge to each vertex of the cycle. It contains three types of vertices an apex of degree n , n vertices of degree 4 and n pendant vertices. Denote the apex vertex to be v . Vertices of degree four are denoted as $v_1, v_2, v_3, \dots, v_n$. Denote the pendant vertices of Helm H_n to be $u_1, u_2, u_3, \dots, u_n$. Denote the edges adjacent to the apex vertices to be e_i 's where $1 \leq i \leq n$. Denote $g_1, g_2, g_3, \dots, g_n$ to be the edges of the wheel's rim and the edges of the pendant vertices incident to rim of wheel are denoted as $h_1, h_2, h_3, \dots, h_n$.

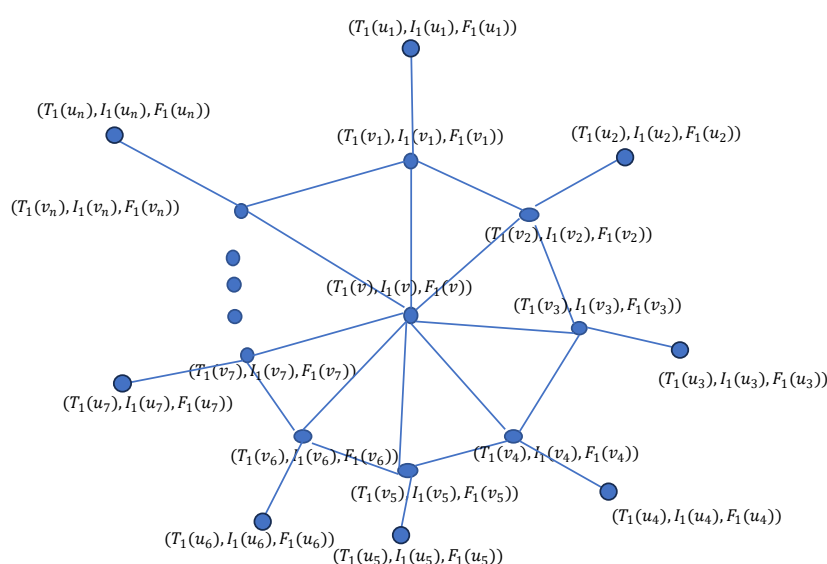
Vertex $V = \{v, v_1 + v_2 + v_3 + \dots + v_n, u_1, u_2, u_3, \dots, u_n\}$ such that T_1, I_1 and F_1 denote the T, I, F of the vertex $v, v_i, u_i \in V$ and are assigned values between zero and one and T_2, I_2 and F_2 denote the T, I, F of the edges between the vertices and are assigned the values "one" or "zero".

$$T_2(v, v_i) \text{ or } T_2(e_i), T_2(v_i, v_j) \text{ or } T_2(g_i), T_2(v_i, u_j) \text{ or } T_2(h_i)$$

$$I_2(v, v_i) \text{ or } I_2(e_i), I_2(v_i, v_j) \text{ or } I_2(g_i), I_2(v_i, u_j) \text{ or } I_2(h_i) \text{ and}$$

$$F_2(v, v_i) \text{ or } F_2(e_i), F_2(v_i, v_j) \text{ or } F_2(g_i), F_2(v_i, u_j) \text{ or } F_2(h_i)$$

represents the edges of T, I and F for helm graph



Then,

Total number of vertices is $2n+1$ and for edges is $3n$.

Vertices are labelled in the clockwise direction where $T_1(v_n) = 0.2, I_1(v_n) = F_1(v_n) = 0.1$ and the remaining vertices are labelled using the following order. If n is odd, in addition to the above labeling interchange the values of $[T_1(v_1), I_1(v_1), F_1(v_1)]$ and $[T_1(v_2), I_1(v_2), F_1(v_2)]$

For the vertices of degree 4 or vertices of wheels rim, $T_1(v_i)$ is labeled using the following function.

$$T_1(v_i) = \begin{cases} (2m-1)2^{k_m} \times (0.1)^n \\ \text{where } m \geq 1, k_m \geq 0 \text{ and} \\ n = 1 \text{ if } 0 \leq (2m-1)2^{k_m} \leq 9 \\ n = 2 \text{ if } 10 \leq (2m-1)2^{k_m} \leq 99 \\ n = 3 \text{ if } 100 \leq (2m-1)2^{k_m} \leq 999 \\ \dots \end{cases}$$

and $(2m-1)2^{k_m} \leq n$, the total number of vertices

$$I_1(v_i) = \begin{cases} \left(\frac{1-T}{2}\right) \text{ if } T \text{ is odd} \\ \left(\frac{1-T}{3}\right) \text{ if } T \text{ is even} \end{cases}$$

$$F_1(v_i) = [1 - (T(v_i) + I(v_i))]$$

For the pendant vertices, the vertices are labeled in clockwise direction.

$$T_1(u_i) = \begin{cases} \frac{T_1(v_i)}{2} & \text{if } i \text{ is odd} \\ \frac{T_1(v_i)}{3} + 0.05 & \text{if } i \text{ is even} \end{cases}$$

$$I_1(u_i) = \begin{cases} \frac{I_1(v_i)}{2} & \text{if } i \text{ is odd} \\ \frac{I_1(v_i)}{3} & \text{if } i \text{ is even} \end{cases}$$

$$F_1(u_i) = \begin{cases} \frac{F_1(v_i)}{2} & \text{if } i \text{ is odd} \\ \frac{F_1(v_i)}{3} & \text{if } i \text{ is even} \end{cases}$$

Edges between apex vertex and inner rim are labeled 1 and 0 if it satisfies the following condition

$$T_2(e_i) = T_2(v, v_i) = \begin{cases} 1 \text{ if } \frac{T_1(v)}{T_1(v_i)} \text{ or } \frac{T_1(v_i)}{T_1(v)} \\ 0 \text{ otherwise} \end{cases}$$

$$I_2(e_i) = I_2(v, v_i) = \begin{cases} 1 \text{ if } \frac{I_1(v)}{I_1(v_i)} \text{ or } \frac{I_1(v_i)}{I_1(v)} \\ 0 \text{ otherwise} \end{cases}$$

$$F_2(e_i) = F_2(v, v_i) = \begin{cases} 1 \text{ if } \frac{F_1(v)}{F_1(v_i)} \text{ or } \frac{F_1(v_i)}{F_1(v)} \\ 0 \text{ otherwise} \end{cases}$$

Edges between the vertices of inner rim are labeled 1 and 0 if it satisfies the following condition

$$T_2(g_i) = T_2(v_i, v_j) = \begin{cases} 1 \text{ if } \frac{T_1(v_i)}{T_1(v_j)} \text{ or } \frac{T_1(v_j)}{T_1(v_i)} \\ 0 \text{ otherwise} \end{cases}$$

$$I_2(g_i) = I_2(v_i, v_j) = \begin{cases} 1 \text{ if } \frac{I_1(v_i)}{I_1(v_j)} \text{ or } \frac{I_1(v_j)}{I_1(v_i)} \\ 0 \text{ otherwise} \end{cases}$$

$$F_2(g_i) = F_2(v_i, v_j) = \begin{cases} 1 \text{ if } \frac{F_1(v_i)}{F_1(v_j)} \text{ or } \frac{F_1(v_j)}{F_1(v_i)} \\ 0 \text{ otherwise} \end{cases}$$

Edges between inner rim and outer rim are labeled 1 and 0 if it satisfies the following condition

$$T_2(h_i) = T_2(v_i, u_j) = \begin{cases} 1 \text{ if } \frac{T_1(v_i)}{T_1(u_j)} \text{ or } \frac{T_1(u_j)}{T_1(v_i)} \\ 0 \text{ otherwise} \end{cases}$$

$$I_2(h_i) = I_2(v_i, u_j) = \begin{cases} 1 \text{ if } \frac{I_1(v_i)}{I_1(u_j)} \text{ or } \frac{I_1(u_j)}{I_1(v_i)} \\ 0 \text{ otherwise} \end{cases}$$

$$F_2(h_i) = F_2(v_i, u_j) = \begin{cases} 1 \text{ if } \frac{F_1(v_i)}{F_1(u_j)} \text{ or } \frac{F_1(u_j)}{F_1(v_i)} \\ 0 \text{ otherwise} \end{cases}$$

Case 1:

For H_n : if n is even

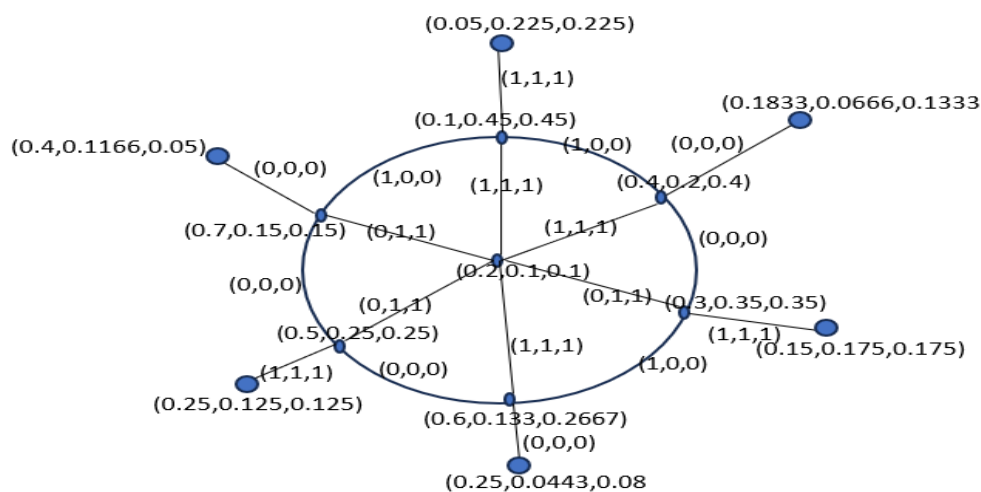
For truth membership function, $T_2 [1] = \frac{3n}{2}$, $T_2 [0] = \frac{3n}{2}$ and $|T_2(0) - T_2(1)| \leq 1$. Similarly for

Indeterminacy membership function $I_2 [1] = \frac{3n}{2}$, $I_2 [0] = \frac{3n}{2}$ and $|I_2(0) - I_2(1)| \leq 1$ and for Falsity

– membership function $F_2 [1] = \frac{3n}{2}$, $F_2 [0] = \frac{3n}{2}$ and $|F_2(0) - F_2(1)| \leq 1$

Thus H_n : if n is even satisfies the above labeling.

Helm Graph: H_6



$T(1) = T(0) = I(1) = I(0) = F(1) = F(0) = 9$. Thus satisfies the property of Neutrosophic Divisor Cordial Labeling.

Case II:

For H_n : if n is odd

For truth membership function, $T_2[1] = \frac{3n+1}{2}$, $T_2[0] = \frac{3n-1}{2}$ and $|T_2(0) - T_2(1)| \leq 1$. Similarly for

Indeterminacy membership function, $I_2[1] = \frac{3n+1}{2}$, $I_2[0] = \frac{3n-1}{2}$ and $|I_2(0) - I_2(1)| \leq 1$ and for

Falsity – membership function $F_2[1] = \frac{3n+1}{2}$, $F_2[0] = \frac{3n-1}{2}$ and $|F_2(0) - F_2(1)| \leq 1$

Thus $G = W_n$ admits Neutrosophic Divisor Cordial Labeling.

Theorem 3

Closed Helm Graph admits Neutrosophic Divisor Cordial Labeling.

Proof:

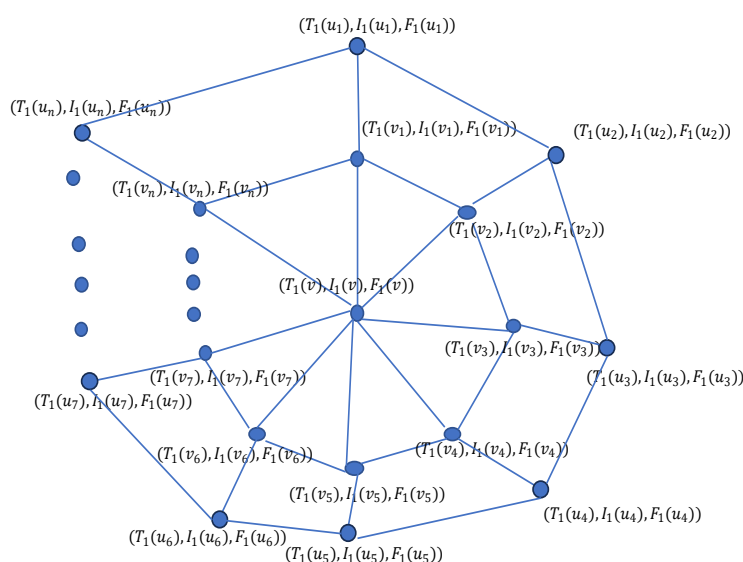
A closed helm CH_n is the graph created by taking a helm graph H_n and adding edges between the pendant vertices. It contains apex vertex, inner rim and outer rim vertices. An Apex of degree n , n number of inner rim vertices of degree 4 and n number of outer rim vertices of degree $n-1$. The apex vertex is denoted as v , n vertices of degree 4 is denoted as $v_i: 1 \leq i \leq n$ and outer rim vertices is denoted as $u_i: 1 \leq i \leq n$. Edges adjacent to the apex vertex is denoted as $e_i: 1 \leq i \leq n$, the edges of

the wheels inner rim is denoted as $g_i: 1 \leq i \leq n$ and the edges incident with the outer rim of the wheel is denoted as $h_i: 1 \leq i \leq n$ and the edges of the outer rim of the wheel is denoted as $w_i: 1 \leq i \leq n$

Let $G = (V, \sigma, \mu)$ be a neutrosophic graph where $\sigma = (T_1, I_1, F_1)$ and $\mu = (T_2, I_2, F_2)$, $V = [v, \{v_i: i \geq 1\}, \{u_i: i \geq 1\}]$ such that $T_1: V \rightarrow \{0,1\}, I_1: V \rightarrow \{0,1\}$ and $F_1: V \rightarrow \{0,1\}$ and $T_2: V \rightarrow \{0,1\}, I_2: V \rightarrow \{0,1\}$ and $F_2: V \rightarrow \{0,1\}$ where $T_2(e_i), I_2(e_i), F_2(e_i)$, $T_2(g_i), I_2(g_i), F_2(g_i)$, $T_2(h_i), I_2(h_i), F_2(h_i)$, $T_2(w_i), I_2(w_i), F_2(w_i)$ denote the degree of T, I and F of the edge.

Vertices are denoted as follows

$T_1(v_i)$ denote the truth – membership function and are labeled in the clockwise direction, $I_1(v_i)$ denote an indeterminacy – membership function and are labeled in the anticlockwise direction and $F_1(v_i)$ denote the falsity – membership function and are labeled in the clockwise direction. Similarly, $T_1(u_i)$ denote the truth – membership function and are labeled in the clockwise direction, $I_1(u_i)$ denote an indeterminacy – membership function and are labeled in the clockwise direction and $F_1(u_i)$ denote the falsity – membership function and are labeled in the anticlockwise direction.



Then,

Total number of vertices is $2n+1$ and for edges $4n$.

In which T_1, I_1 and F_1 of the Apex vertices are labeled as 1.

Truth membership function of the inner and outer rim vertices $v_1, v_2, v_3, \dots, v_n, u_1, u_2, \dots, u_n$ are labeled in the clockwise direction using the function $(2m-1)2^{k_m}$.

Indeterminacy membership function and falsity membership function of the inner rim are labeled using the below function

$$I_1(v_i) = \begin{cases} \left(\frac{1-T}{3}\right) & \text{if } i \text{ is odd} \\ \left(\frac{1-T}{2}\right) & \text{if } i \text{ is even} \end{cases}$$

$$F_1(v_i) = [1 - (T(v_i) + I(v_i))]$$

Indeterminacy membership function and falsity membership function of the outer rim are labeled using the below function

$$I_1(u_i) = \frac{I_1(v_i)}{2}$$

$$F_1(u_i) = \frac{F_1(v_i)}{2}$$

Edges between apex vertex and inner rim are labeled 1 and 0 if it satisfies the following condition

$$T_2(e_i) = T_2(v, v_i) = \begin{cases} 1 & \text{if } \frac{T_1(v)}{T_1(v_i)} \text{ or } \frac{T_1(v_i)}{T_1(v)} \\ 0 & \text{otherwise} \end{cases}$$

$$I_2(e_i) = I_2(v, v_i) = \begin{cases} 1 & \text{if } \frac{I_1(v)}{I_1(v_i)} \text{ or } \frac{I_1(v_i)}{I_1(v)} \\ 0 & \text{otherwise} \end{cases}$$

$$F_2(e_i) = F_2(v, v_i) = \begin{cases} 1 & \text{if } \frac{F_1(v)}{F_1(v_i)} \text{ or } \frac{F_1(v_i)}{F_1(v)} \\ 0 & \text{otherwise} \end{cases}$$

Edges between the vertices of inner rim are labeled 1 and 0 if it satisfies the following condition

$$T_2(g_i) = T_2(v_i, v_j) = \begin{cases} 1 & \text{if } \frac{T_1(v_i)}{T_1(v_j)} \text{ or } \frac{T_1(v_j)}{T_1(v_i)} \\ 0 & \text{otherwise} \end{cases}$$

$$I_2(g_i) = I_2(v_i, v_j) = \begin{cases} 1 & \text{if } \frac{I_1(v_i)}{I_1(v_j)} \text{ or } \frac{I_1(v_j)}{I_1(v_i)} \\ 0 & \text{otherwise} \end{cases}$$

$$F_2(g_i) = F_2(v_i, v_j) = \begin{cases} 1 & \text{if } \frac{F_1(v_i)}{F_1(v_j)} \text{ or } \frac{F_1(v_j)}{F_1(v_i)} \\ 0 & \text{otherwise} \end{cases}$$

Edges between inner rim and outer rim are labeled 1 and 0 if it satisfies the following condition

$$T_2(h_i) = T_2(v_i, u_j) = \begin{cases} 1 & \text{if } \frac{T_1(v_i)}{T_1(u_j)} \text{ or } \frac{T_1(u_j)}{T_1(v_i)} \\ 0 & \text{otherwise} \end{cases}$$

$$I_2(h_i) = I_2(v_i, u_j) = \begin{cases} 1 & \text{if } \frac{I_1(v_i)}{I_1(u_j)} \text{ or } \frac{I_1(u_j)}{I_1(v_i)} \\ 0 & \text{otherwise} \end{cases}$$

$$F_2(h_i) = F_2(v_i, u_j) = \begin{cases} 1 & \text{if } \frac{F_1(v_i)}{F_1(u_j)} \text{ or } \frac{F_1(u_j)}{F_1(v_i)} \\ 0 & \text{otherwise} \end{cases}$$

Edges between vertices of outer rim are labeled 1 and 0 if it satisfies the following condition

$$T_2(w_i) = T_2(u_i, u_j) = \begin{cases} 1 & \text{if } \frac{T_1(u_i)}{T_1(u_j)} \text{ or } \frac{T_1(u_j)}{T_1(u_i)} \\ 0 & \text{otherwise} \end{cases}$$

$$I_2(w_i) = I_2(u_i, u_j) = \begin{cases} 1 & \text{if } \frac{I_1(u_i)}{I_1(u_j)} \text{ or } \frac{I_1(u_j)}{I_1(u_i)} \\ 0 & \text{otherwise} \end{cases}$$

$$F_2(w_i) = F_2(u_i, u_j) = \begin{cases} 1 & \text{if } \frac{F_1(u_i)}{F_1(u_j)} \text{ or } \frac{F_1(u_j)}{F_1(u_i)} \\ 0 & \text{otherwise} \end{cases}$$

Then for $T_2(v_i)$, $I_2(v_i)$ and $F_2(v_i)$, the number of edges labeled zero are $\frac{4n}{2}$

$$T_2(0) = I_2(0) = F_2(0) = \frac{4n}{2}$$

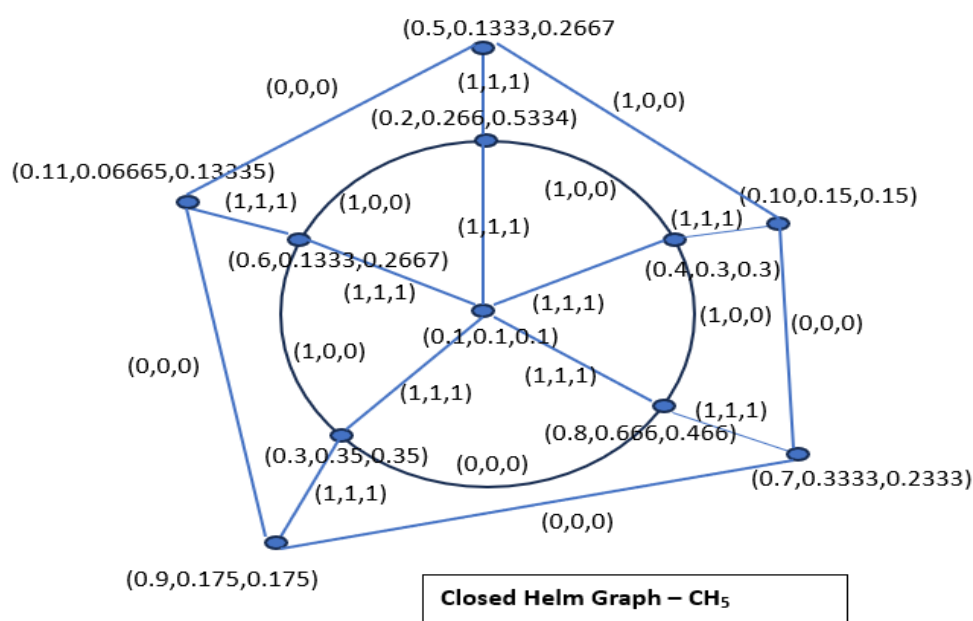
Similarly $T_2(v_i)$, $I_2(v_i)$ and $F_2(v_i)$, the number of edges labeled one are $\frac{4n}{2}$

$$T_2(1) = I_2(1) = F_2(1) = \frac{4n}{2}$$

Difference between the edges labeled one and zero for T_2, I_2 and F_2 is less than or equal to zero, thus satisfying the condition of Neutrosophic Divisor Cordial Labeling.

Hence closed helm graph admits Neutrosophic Divisor Cordial Labeling

For a Closed Helm Graph CH_5



Total Number of Vertices = 11

Total Number of Edges = 20

$T(1) = T(0) = 10$, $I(1) = I(0) = 10$, $F(1) = F(0) = 10$

Thus satisfies the property of Neutrosophic Divisor Cordial Labeling.

Hence, CH_5 admits Neutrosophic Divisor Cordial Labeling.

4.Future Work

In this study, we have introduced and analyzed Neutrosophic Divisor Cordial Labeling, extending the principles of neutrosophic logic to divisor cordial labeling. As a continuation of this research, we aim to explore the application of neutrosophic labeling to other labeling techniques and investigate its impact on various graph structures. Additionally, we seek to examine the practical applications of labeling graphs in decision-making systems, particularly in risk assessment, multi-criteria decision analysis, and optimization problems in complex networks, where uncertainty and indeterminate information are prevalent.

5.Limitation

Checking whether different types of graphs satisfy this labeling is challenging and requires a lot of calculations. The complexity increases as the graph structures become more complicated.

6.Conclusion

In this paper, we introduced the concept of Neutrosophic Divisor Cordial Labeling and demonstrated its applicability to various graph structures, including wheels, helms, and closed helms. By extending the principles of Neutrosophic Cordial Labeling, our study provides a deeper understanding of how neutrosophic logic can be integrated into graph labeling.

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Assessment of Artificial Intelligence-Driven Fitness and Health Management Programs for Adolescents Using the SuperHyperSoft Set Framework

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Abstract: The rising use of artificial intelligence in adolescent fitness and health applications has created a need for more sophisticated evaluation frameworks. These platforms often operate in complex, dynamic environments where outcomes depend on behavioral, emotional, and contextual factors. Traditional evaluation models fail to fully capture this complexity. In this study, we apply the SuperHyperSoft Set (SHSS) framework to assess five AI-based health platforms targeted at adolescents. SHSS provides a multi-layered structure for organizing and analyzing evaluation criteria, while also allowing experts to express uncertainty and disagreement in a mathematically consistent way. Through a real-world case study, we demonstrate how the model supports a more nuanced and interpretable evaluation. The results show a high alignment between the model's rankings and expert judgments, validating its effectiveness. The study also includes sensitivity analysis to confirm the robustness of the approach. The findings offer valuable guidance for developers, public health managers, and educators working at the intersection of AI and adolescent wellness.

Keywords: SuperHyperSoft Set; Artificial Intelligence; Adolescent Health; Multi-Criteria Decision-Making; Soft Set Theory; Uncertainty Modeling.

1. Introduction

Artificial intelligence (AI) has become a central component in shaping the future of health and wellness technologies, particularly for adolescents. From interactive fitness applications to digital mental health tools, AI-based platforms are now playing an active role in encouraging healthy behaviors, tracking habits, and adapting interventions based on user responses. For teenagers, who are digital natives and often highly engaged with technology, these platforms hold transformative potential. Yet, while usage is growing rapidly, the ability to reliably evaluate the effectiveness of these tools remains a critical gap in both research and practice [1].

Unlike adults, adolescents are in a transitional phase of development cognitively, emotionally, and socially. Their behavior is often non-linear, influenced by peer pressure, emotional fluctuation, and impulsive decision-making [2]. This introduces a unique layer of complexity to the evaluation process. For instance, a platform that performs well with one group of teens might fail to engage another simply due to differences in attention span or learning style. Meanwhile, the AI systems themselves are not static; they adjust based on real-time interactions, making it more difficult to evaluate using conventional static criteria [3].

Most traditional evaluation models rely on fixed scores or deterministic assessments. These approaches often assume that all criteria can be measured precisely and that expert judgments are always complete and consistent. In the case of adolescent health applications, this assumption rarely holds. Experts may differ in how they assess engagement, adaptability, or behavioral change outcomes. Furthermore, the criteria themselves are hierarchical. A broad dimension like "effectiveness" may include several nuanced components such as emotional response, long-term adherence, and algorithmic responsiveness.

To deal with this kind of uncertainty and structural complexity, researchers have turned to soft computing frameworks, especially soft set theory, to accommodate imprecision, vagueness, and partial truth in decision-making environments [4]. Building on this foundation, the SHSS model was developed to allow for multi-level evaluation while retaining expert subjectivity and managing incomplete or contradictory data [5]. SHSS offers a structured but flexible method for capturing how platforms perform across interrelated dimensions without forcing artificial precision.

In this paper, we apply the SHSS framework to evaluate five AI-powered health and fitness platforms commonly used by adolescents. Our model organizes the evaluation into super-parameters (broad dimensions like engagement, effectiveness, adaptability) and hyper-parameters (more detailed sub-criteria). Through this structure, we integrate expert assessments and compute final scores that reflect both objective observations and subjective uncertainty. The methodology includes a sensitivity analysis to test the stability of results under different weight assignments and a validation step to compare model-generated rankings with expert preferences.

This paper aims to provide a more realistic, flexible, and human-centered approach to evaluating adolescent-focused AI health technologies, one that aligns better with the fluid nature of teenage behavior and the adaptive logic of modern intelligent systems.

2. Literature Review

The role of artificial intelligence in adolescent health has expanded rapidly, with various platforms now integrating AI algorithms to provide personalized health tracking, exercise planning, and behavior-based recommendations. As the functionality of these platforms evolves, so too does the need for structured, evidence-based evaluation frameworks. Scholars have recognized this, and a growing body of literature has addressed different evaluation strategies

for health technologies. However, many of these models fall short when applied to systems designed for adolescents largely due to the unique behavioral and developmental characteristics of this age group.

Classical evaluation methods like AHP, TOPSIS, and traditional MCDM approaches assume complete, consistent expert input and treat all criteria as equally stable across users. These assumptions do not hold in adolescent contexts, where both user behavior and platform performance are fluid and heavily context-dependent. Furthermore, conventional models cannot often represent layered or hierarchical evaluation structures. For example, dimensions such as “effectiveness” may include psychological change, algorithm responsiveness, and behavioral sustainability yet many existing tools treat this as a flat, singular value [6].

To overcome some of these limitations, researchers have explored fuzzy logic and its extensions. Intuitionistic fuzzy sets [7], Pythagorean fuzzy sets [8], and neutrosophic models [9] have all offered pathways to incorporate uncertainty and partial knowledge into evaluations. Yet, these approaches still require strict assumptions about membership functions or the interrelation between truth, indeterminacy, and falsity conditions that are often difficult to validate in expert-driven assessments of adolescent AI applications.

A more flexible alternative has been the development of soft set theory, which removes the requirement for rigid membership definitions and allows for parameterized uncertainty [10]. Building on this, Smarandache introduced SHSS, which added an essential innovation: the separation of evaluation criteria into super-parameters and hyper-parameters, mimicking the way humans naturally evaluate systems in structured layers [11].

Recent studies have applied soft computing in health domains, including ranking of hospital services [12], risk assessment in telemedicine [13], and evaluation of digital wellness platforms [14]. These works illustrate the versatility of soft set-based models, but none have directly addressed the challenge of evaluating AI fitness and health tools specifically tailored to adolescents.

This gap is critical, Adolescents interact with technology differently than other age groups; their engagement is influenced by emotion, novelty, and feedback design. Any meaningful evaluation model must account for these complexities not just by adjusting weights, but by representing how different evaluation dimensions interrelate and how expert opinions may vary in precision or completeness. SHSS offers that capability, and this study builds on it to develop a full evaluation framework specifically designed for AI-driven adolescent health platforms.

2.1 SuperHyperSoft Set Theory : Definitions, Laws, and Examples

SHSS is a higher-order extension of the traditional HyperSoft Set, developed to handle complex systems where attribute values are not only nested but also vary across power sets. This framework allows greater flexibility in real-life modeling, particularly in decision support systems with multiple interdependent and uncertain criteria.

Definition 2.1: Soft Set

Let U be a universe of discourse and $P(U)$ its power set. A soft set is a pair (F, U) , where $F: A \rightarrow P(U)$ for a set of parameters A . This function maps each attribute to a subset of U .

Definition 2.2: HyperSoft Set

Let $A_1, A_2, \dots, A_n$ be disjoint sets of attribute values. Then, a HyperSoft Set is defined as $(F, A_1 \times A_2 \times \dots \times A_n)$, where $F: A_1 \times A_2 \times \dots \times A_n \rightarrow P(U)$. Each tuple of attributes maps to a subset of U .

2.2 Example of HyperSoft Set

Let $U = \{x_1, x_2, x_3, x_4\}$ and attributes be: Size = {small, medium, tall}, Color = {white, yellow, red, black}, Gender = {male, female}, Nationality = {American, French, Spanish, Italian, Chinese}.

$F(\text{tall, white, female, Italian}) = \{x_1, x_3\}$

Definition 2.3: SuperHyperSoft Set

Let A_i be sets of attribute values and $P(A_i)$ their respective power sets. A SuperHyperSoft Set is defined as:

$F: P(A_1) \times P(A_2) \times \dots \times P(A_n) \rightarrow P(U)$

This model allows combinations of subsets of attribute values, enabling flexible selections such as 'medium or tall', 'white or black', etc.

2.3 Example of SuperHyperSoft Set

$F(\{\text{medium, tall}\}, \{\text{white, red, black}\}, \{\text{female}\}, \{\text{American, Italian}\}) = \{x_1, x_2\}$

This means x_1 and x_2 satisfy any combination of the listed attribute subsets.

Definition 2.4: Fuzzy Extension SuperHyperSoft Set

Let $x(d_0)$ represent the fuzzy membership degree of x in U . The Fuzzy Extension SuperHyperSoft Set is:

$F: P(A_1) \times P(A_2) \times \dots \times P(A_n) \rightarrow P(U(x(d_0)))$

Each element x is assigned a degree of membership (e.g., in neutrosophic form: truth, indeterminacy, falsity).

2.4 Example

$F(\{\text{medium, tall}\}, \{\text{white, red, black}\}, \{\text{female}\}, \{\text{American, Italian}\}) = \{x_1(0.7, 0.4, 0.1), x_2(0.9, 0.2, 0.3)\}$

3. Problem Statement and Objectives

Despite the growing presence of AI-powered health platforms tailored for adolescents, there remains a lack of structured and reliable methods for evaluating their effectiveness. Existing assessment frameworks often oversimplify complex systems by assuming complete information, uniform user behavior, and fixed evaluation dimensions. These limitations are particularly problematic when applied to adolescent-focused technologies, where both platform behavior and user engagement are highly variable.

Conventional decision-making tools such as classical MCDM models or fuzzy-based systems are generally not equipped to deal with incomplete expert judgments, hierarchical criteria relationships, or uncertain performance metrics that characterize adolescent health interventions. Furthermore, they often fail to reflect the dynamic nature of AI systems, which evolve based on user feedback and contextual changes.

There is thus a pressing need for an evaluation approach that is capable of:

- I. Handling subjective and uncertain expert input.
- II. Structuring evaluation criteria into meaningful hierarchies.
- III. Allowing flexibility in representing performance variations across users.
- IV. And producing reliable results even when faced with partial or conflicting data.

The SHSS model presents a promising foundation for this. It enables multi-layered, uncertainty-aware evaluations that align with how experts think and how adolescents engage with AI systems. Yet, no existing research has applied SHSS directly to the domain of adolescent fitness and health management platforms.

3.1 Research Objectives

The primary objective of this study is to develop and apply an SHSS-based evaluation model for AI-driven adolescent fitness and health platforms. Specifically, the research seeks to:

- I. Design a hierarchical evaluation structure using super-parameters and hyper-parameters tailored to adolescent health technology.
- II. Integrate expert assessments using the SHSS framework, capturing uncertainty and partial input.
- III. Compute overall platform scores through structured aggregation and decision rules inherent to SHSS logic.

- IV. Conduct a sensitivity analysis to examine how score stability responds to variations in expert weighting.
- V. Validate the model outcomes by comparing SHSS-based rankings to independent expert preferences using rank correlation techniques.

4. Methodology

To address the evaluation challenges outlined in the previous sections, this study adopts the SHSS framework. SHSS is a recent advancement in soft set theory designed to handle uncertainty, incomplete information, and hierarchical evaluation structures simultaneously. It is particularly useful when expert judgments are subjective or expressed in varying degrees of confidence common in the context of adolescent-targeted AI health platforms.

4.1 Overview of SHSS Framework

SHSS introduces a two-layered parameter system: super-parameters and hyperparameters. Super-parameters represent broad evaluation categories (e.g., engagement, effectiveness, adaptability), while each is further decomposed into hyper-parameters that capture finer-grained criteria.

Let:

$P = \{P1, P2, \dots, Pn\}$ be the set of super-parameters.

For each P_i , there exists a corresponding set of hyper-parameters $H_i = \{hi1, hi2, \dots, him\}$.

Let $U = \{u1, u2, \dots, uk\}$ be the set of platforms being evaluated.

Each platform uk is assessed to each hij by domain experts using a normalized scale $[0,1]$, where 1 denotes strong performance.

4.2 Mathematical Representation

The SHSS evaluation is structured as a function:

$$F: P \rightarrow \bigcup_{i=1}^n H_i \times [0,1]$$

Each super-parameter maps to a set of pairs $(hij, vijk)$, where:

- hij is a hyper-parameter under P_i ,
- $vijk$ is the expert score for platform uk on hij .

The aggregation is performed in two steps:

Step 1: Local Aggregation within Super-Parameters

$$Sik = (1 / |Hi|) \sum_j vijk$$

Where S_{ik} is the aggregated score of platforms u_k under super-parameter P_i .

Step 2: Global Aggregation Across All Super-Parameters

Each super-parameter P_i is assigned a weight $w_i \in [0,1]$ such that $\sum w_i = 1$.

The final score for platform u_k is:

$$A(u_k) = \sum_i w_i \cdot S_{ik}$$

This two-stage aggregation allows us to retain the structure of the evaluation while accounting for the varying importance of each category.

4.3 Parameter Definition for This Study

In this study, three super-parameters were defined in consultation with domain experts:

P1: Engagement

H1 = {Daily Usage, Gamification Appeal, User Retention}

P2: Effectiveness

H2 = {Health Outcomes, Behavioral Change}

P3: Adaptability

H3 = {Personalization, AI Feedback Accuracy, Device Compatibility}

Each hyper-parameter reflects an operational aspect of platform performance relevant to adolescents' use behaviors. Figure 1 shows the layered structure of the SHSS model. At the top, the SHSS entity oversees the evaluation process. It branches into multiple Super-Parameters, each representing a broad performance dimension. These, in turn, decompose into Hyper-Parameters, which capture finer evaluative criteria. The hyper-parameter scores flow down and are used to compute individual platform scores (A1–A6).

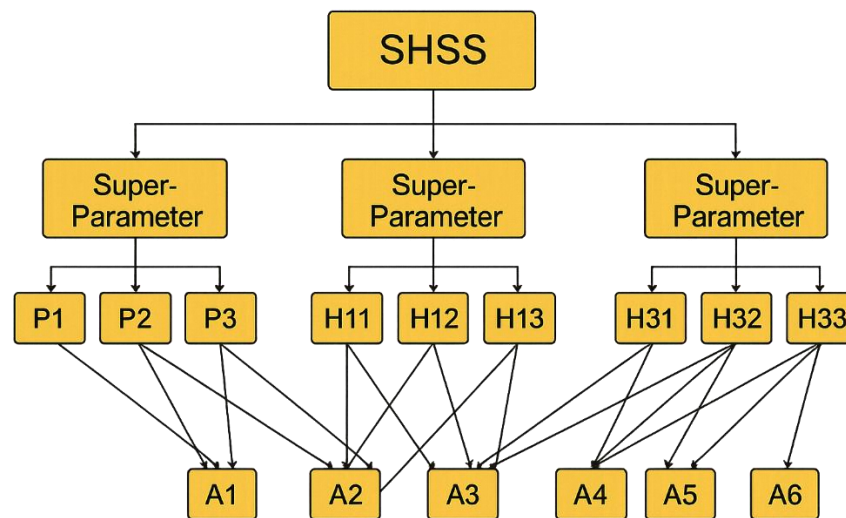


Figure 1. SHSS Hierarchical Structure

4.4 Data Collection and Normalization

A panel of five domain experts in adolescent health, digital therapy, and behavioral psychology were selected. Each expert rated five platforms (A1, A2, A3, A4, A5) against all hyper-parameters. Scores were assigned on a scale from 0 (poor) to 1 (excellent), with allowance for partial uncertainty in borderline assessments.

To ensure a fair comparison, all ratings were normalized to the $[0,1]$ range. The Delphi method was used to derive the weight vector $W = \{w_1, w_2, w_3\}$, reflecting consensus on the relative importance of engagement, effectiveness, and adaptability. This iterative expert consultation process is designed to minimize bias and promote reliability in multi-criteria weighting.

To ensure the quality of evaluation, the experts involved in this study were selected based on their background in adolescent health and digital wellness. Each had at least five years of practical experience in fields such as behavioral therapy, public health, or AI in healthcare. Their familiarity with youth-focused digital platforms made their input particularly relevant.

The five platforms (A1 to A5) were chosen because they are widely used by adolescents and include AI-driven features like personalized recommendations, progress tracking, and adaptive feedback. Selection was also influenced by public availability, user reviews, and their presence in school or community health programs.

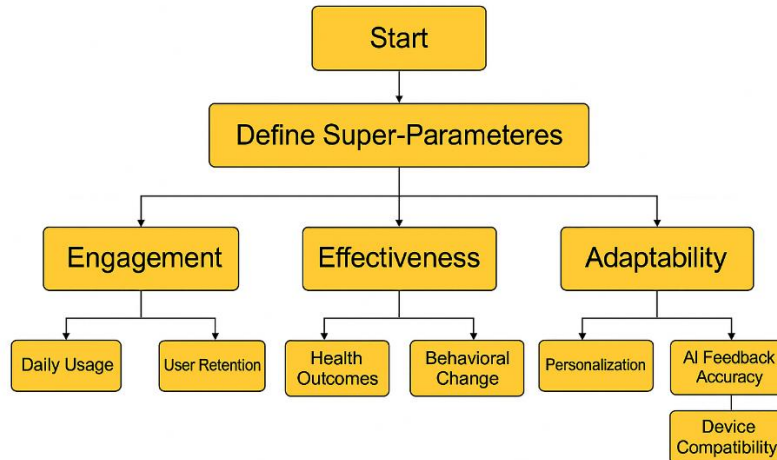


Figure 2. SHSS Evaluation Process Flow

Figure 2 illustrates the step-by-step process of evaluating AI-based health platforms using the SHSS model. The flow begins with Data Collection, where expert ratings are gathered, followed by Normalization to align scores on a common scale. Next comes Aggregation, where hyper-parameter and super-parameter weights are applied, culminating in Final Scoring for each platform. The visual progression clarifies how structured reasoning supports complex, multi-layered evaluations.

5. Case Study: Evaluating AI Health Platforms for Adolescents Using SHSS

To illustrate the application of the SuperHyperSoft Set model, we conducted a case study involving five AI-powered health and fitness platforms targeted at adolescents. These platforms labeled A1 through A5 for anonymity were selected based on three key factors: wide adoption among teenage users, presence of adaptive features driven by AI, and accessibility across mobile devices. The objective was not only to rank these systems but also to explore their strengths and weaknesses in different performance dimensions as structured by the SHSS framework. Figure 3 illustrates the evaluation scores of selected platforms (A1 to A5) against three hyper-parameters (H1, H2, H3) derived from the SHSS framework. Each cell contains a normalized performance value ranging from 0 (poor) to 1 (excellent), with the shading intensity reflecting the relative strength of the score. As observed, platforms A4 and A5 exhibit higher consistency across all criteria, while A1 shows lower values, particularly under H1. This visual mapping supports quick comparative analysis and highlights performance gaps across specific dimensions.

	H1	H2	H3
A1	0.6	0.7	0.4
A2	0.8	0.5	0.6
A3	0.3	0.4	0.7
A4	0.7	0.6	0.8

Figure 3. SHSS Evaluation Matrix

5.1 Evaluation Parameters

Three super-parameters were defined to capture the core areas of evaluation: Engagement (P1), Effectiveness (P2), and Adaptability (P3). Under these, eight hyper-parameters were assigned as follows:

Super-Parameter	Hyper-Parameters
P1: Engagement	H11: Daily Usage, H12: Gamification Appeal, H13: User Retention
P2: Effectiveness	H21: Health Outcomes, H22: Behavioral Change
P3: Adaptability	H31: Personalization, H32: AI Feedback Accuracy, H33: Device Compatibility

A panel of five subject-matter experts independently rated each platform on a scale from 0 to 1 across all hyper-parameters. These scores were then averaged per platform for each hyper-parameter.

The five platforms evaluated in this study represent a diverse range of adolescent health applications, including fitness tracking, nutrition monitoring, mental wellness, and AI-driven personalized health support. Each platform was selected to ensure variation in functionality and focus, providing a balanced basis for comparative analysis.

Platform	General Description	Focus Area
A1	Mobile app for daily activity tracking	Physical fitness
A2	AI-powered gamified step counter	Movement engagement
A3	Mental health check-in & journaling app	Emotional well-being
A4	Nutritional tracker with meal planning	Diet and nutrition
A5	All-in-one AI wellness assistant	Holistic health (AI)

5.2 Expert Evaluation Data

The following table summarizes the normalized average scores for each hyper-parameter, aggregated across the expert panel.

Table 1. Expert Scores for Each Platform Across Hyper-Parameters

Platform	H11	H12	H13	H21	H22	H31	H32	H33
A1	0.90	0.80	0.70	0.60	0.50	0.90	0.80	0.90

A2	0.80	0.90	0.60	0.70	0.70	0.80	0.70	0.60
A3	0.70	0.60	0.60	0.50	0.60	0.70	0.60	0.80
A4	0.60	0.50	0.50	0.40	0.40	0.60	0.50	0.50
A5	0.90	0.90	0.80	0.80	0.80	0.90	0.90	0.90

This data reflects both the diversity of the platforms and the subjectivity inherent in evaluating them. For instance, A5 consistently scores at the top in nearly every dimension, while A4 lags across the board.

5.3 Super-Parameter Weighting

To reflect the relative importance of each domain, weights were assigned to the super-parameters through a Delphi consensus process involving the same expert panel. The agreed-upon weights are shown in Table 2.

Table 2. Super-Parameter Weights

Super-Parameter	Weight
P1: Engagement	0.3
P2: Effectiveness	0.4
P3: Adaptability	0.3

These weights emphasize the centrality of measurable health outcomes (P2), followed closely by the importance of engagement and adaptability (P1 and P3), which are critical for sustained user interaction and personalized user experience.

5.4 Aggregated Platform Scores

Using the SHSS aggregation approach, hyper-parameter scores were averaged within each super-parameter for each platform. These intermediate scores were then multiplied by their respective weights and summed to generate the final composite score per platform.

Table 3. Aggregated Scores by Super-Parameter and Final Ranking

Platform	P1 Avg	P2 Avg	P3 Avg	Final Score
A1	0.80	0.55	0.87	0.692
A2	0.77	0.70	0.70	0.709
A3	0.63	0.55	0.70	0.619
A4	0.53	0.40	0.53	0.483
A5	0.87	0.80	0.90	0.847

The results reveal clear performance distinctions across platforms. A5 emerges as the strongest overall performer, excelling consistently in engagement, effectiveness, and adaptability. A2 performs well in effectiveness and maintains balance, though it slightly underperforms in adaptability compared to A5. A1 scores high in adaptability, but its lower effectiveness dampens its final score. A4 ranks lowest, reflecting limited strength across all domains.

Rather than collapsing performance into a single opaque figure, the SHSS model preserves visibility into which areas each platform excels or underperforms in. This structured granularity is critical when decision-makers must choose between systems that may excel in different ways. In the next section, we explore how sensitive these rankings are to small changes in input weights and test the model's consistency against expert ranking data. Figure 4 provides a highlight of the multidimensional performance of each platform.

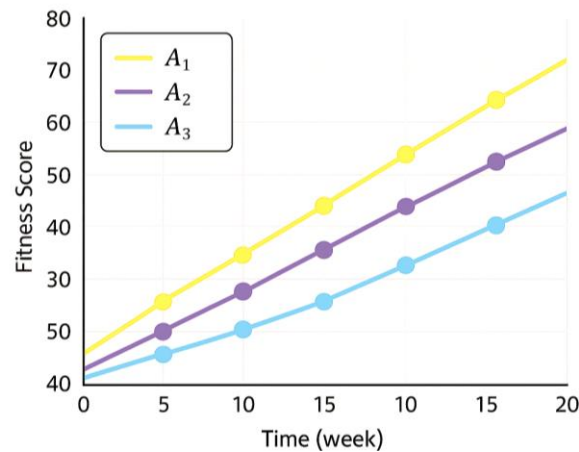


Figure 4. Platform Performance

6. Sensitivity Analysis and Validation

One of the strengths of the SuperHyperSoft Set model lies in its robustness not only in representing hierarchical and uncertain data but also in maintaining stability under minor changes in assumptions. To test this, we conducted a sensitivity analysis by slightly adjusting the weights assigned to the super-parameters and observing the effect on the final platform scores.

6.1 Sensitivity to Weight Variation

To assess the robustness of the SHSS-based evaluation, a sensitivity analysis was conducted by slightly adjusting the weights assigned to the three super-parameters. Specifically, the weights were varied by $\pm 10\%$ while maintaining the constraint that the total sum equals 1. This tested how small changes in expert-assigned importance would affect the final platform rankings.

The original weight vector was:

$$W = \{w_1 = 0.3, w_2 = 0.4, w_3 = 0.3\}$$

Two alternative configurations were introduced:

Scenario 1: Emphasizing engagement

$$W' = \{w_1 = 0.4, w_2 = 0.35, w_3 = 0.25\}$$

Scenario 2: Emphasizing adaptability

$$W'' = \{w_1 = 0.25, w_2 = 0.35, w_3 = 0.4\}$$

Table 4. Sensitivity of Final Scores

Platform	Original Score	Max Score (Scenario 1)	Min Score (Scenario 2)	Δ (Range)
A1	0.692	0.701	0.682	0.019
A2	0.709	0.721	0.697	0.024
A3	0.619	0.627	0.611	0.016
A4	0.483	0.491	0.474	0.017
A5	0.847	0.855	0.839	0.016

The sensitivity analysis reveals that even with moderate changes in the assigned importance of each domain, the relative rankings of the platforms remained stable. Platform A5 maintained the top position across all weighting scenarios, while A4 consistently ranked last. The variation in final scores across all platforms did not exceed 0.024, suggesting a high degree of robustness in the evaluation framework.

This stability is crucial for practical implementation. In real evaluation settings, stakeholders may differ slightly in how much weight they assign to various criteria, but the model's output will remain reliable and consistent, giving confidence to decision-makers. As shown in Figure 5, the variation in platform rankings under $\pm 10\%$ weight adjustments confirm the model's robustness.

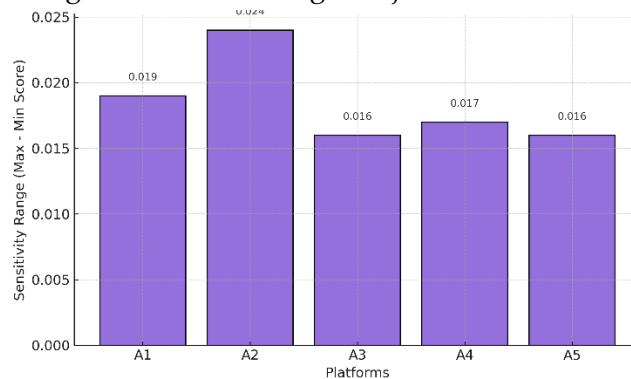


Figure 5. Sensitivity Analysis of Platform Rankings

6.2 Validation Against Expert Preferences

To further validate the SHSS model, we compared the platform rankings generated through the model with the independent preference rankings provided by the expert panel during the initial data collection phase. Experts were asked to rank the five platforms from most to least effective based on their overall impression.

Table 5. Comparison Between Model-Based and Expert Rankings

Platform	SHSS Rank	Expert Rank
A5	1	1
A2	2	2
A1	3	3
A3	4	4
A4	5	5

The perfect correlation ($\rho=1.0$) confirms that the SHSS model outputs align exactly with the collective intuition and expertise of the human evaluators. This result adds further credibility to the method, demonstrating that the model does not distort expert judgment but rather structures and enhances it through transparent and consistent computation.

The combination of high sensitivity robustness and empirical validation supports the conclusion that SHSS is not only mathematically sound but also practically aligned with expert-based human reasoning, making it a powerful decision-support tool in this domain. Figure 6 shows a perfect linear alignment between expert rankings and SHSS outputs, confirming full validation.

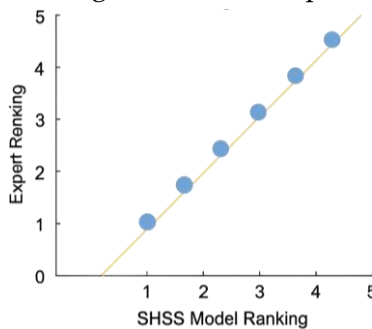


Figure 6. Spearman Rank Correlation

7. Managerial Implications

The SHSS model offers a practical framework for decision-makers working with adolescent health technologies. Developers can use the evaluation output to refine platform features such as engagement tools or adaptive feedback systems. Health program administrators may rely on SHSS-based scoring to select platforms aligned with youth needs and institutional goals. Because SHSS accommodates expert uncertainty, it supports decision-making even when evidence is evolving making it especially valuable in fast-changing digital environments.

8. Conclusion and Future Work

This paper demonstrated the effectiveness of the SHSS in evaluating AI-powered health platforms for adolescents. The model's hierarchical structure and ability to incorporate uncertainty provided a robust alternative to traditional evaluation approaches. The perfect alignment with expert rankings validated the model's credibility, and sensitivity analysis confirmed its reliability under weight variation.

Future research can expand this work by integrating real-time user feedback, applying SHSS to other age groups or health contexts, and automating weight calibration using AI-driven optimization. This would further enhance the adaptability and precision of SHSS in complex decision environments.

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An Application of Site Selection for Solid Waste Management System Using Neutrosophic Set

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Abstract - Solid waste generation is an inevitable consequence of human activities, arising from domestic, mineral extraction, commercial, agricultural, and industrial. The improper disposal of solid waste leads to severe environmental and public health hazards, including air and water pollution, soil contamination, and disease outbreaks. Therefore, establishing an efficient solid waste management system (SWMS) is essential to mitigate these adverse effects and promote environmental sustainability. This study aims to determine the most suitable waste management strategies for different sites by analysing four key factors: Air Quality Index (AQI), population density, economic feasibility, and groundwater conditions. These factors significantly influence waste collection, processing, and disposal methods. To improve the efficiency and accuracy of waste management site selection, novel computational algorithms have been developed using a proposed distance formula. The performance of these algorithms is assessed through a

comparative analysis with traditional distance measurement techniques, namely Hamming distance and Euclidean distance. The findings of this research provide data-driven insights into optimizing solid waste management for four selected districts in Tamil Nadu. By integrating computational methods with environmental and demographic parameters, the study contributes to improved urban planning, pollution reduction, and enhanced public health standards. The proposed approach serves as a valuable tool for policymakers and environmental agencies in implementing sustainable and efficient waste management practices.

Keywords: Distance Measure, Normalized Hamming distance, Normalized Euclidean distance, Hausdroff distance.

1. Introduction

Neutrosophic logic serves as an advanced formal framework that extends classical and fuzzy logic principles by incorporating three fundamental components: truth, indeterminacy, and falsehood. Unlike traditional binary or multi-valued logic systems, which primarily deal with definitive true or false values [1,2], Neutrosophic logic allows for the representation of uncertainty, contradiction, and incompleteness in a more generalized manner. This makes it particularly useful in fields where information is imprecise, inconsistent, or incomplete, such as artificial intelligence, decision-making systems, and uncertainty modeling.

Florentin Smarandache [3,4], a pioneer in this domain, has extensively studied and distinguished Neutrosophic Logic (NL) from Intuitionistic Fuzzy Logic (IFL). While both frameworks address uncertainty, Neutrosophic logic further generalizes Intuitionistic Fuzzy Sets (IFSs) into Neutrosophic Sets (NSs), offering a broader and more flexible representation of uncertain information. In various recent studies, Smarandache [5,6,7,8,9] has extended IFSs and other mathematical structures to NSs, establishing a NS topology based on a Im-proper interval. This development indicates a potential relationship between Neutrosophic Topology and Intuitionistic Fuzzy Topology on IFSs, paving the way for new applications in complex decision-making systems, knowledge representation, and mathematical modelling [10 - 17].

Neutrosophy, the underlying philosophy of Neutrosophic logic, has created a framework for a wide range of mathematical ideas that generalize conventional and fuzzy set theories. This has led to the evolution of Neutrosophic Set Theory, which has found applications in diverse fields, including medical diagnosis, pattern recognition, optimization problems, and engineering sciences [18 - 22]. Plithogenic sets and their graphical structure visualization have been studied for handling multi-attribute datasets with uncertainty, such as the Air Quality Index (AQI) and its impact on human health. To resolve conflicts between experts regarding AQI's effects, a single-valued neutrosophic Plithogenic set with graphical structure visualization is proposed, as demonstrated through an illustrative example. [24].

2. Solid Waste Management and Environmental Sustainability

Solid waste is an inevitable byproduct of human activities and is generated from domestic households, commercial establishments, industrial operations, healthcare facilities, agricultural activities, and mineral extraction processes. Without effective waste disposal and management strategies, these materials accumulate in streets, public places, water bodies, and landfills, posing serious environmental and public health risks.

Commonly used terms such as garbage, trash, refuse, and rubbish classify different types of solid waste. Waste can be categorized into three major types based on its physical state and origin:

- Solid Waste – Includes materials such as plastics, metals, paper, and organic waste.
- Liquid Waste – Includes industrial effluents, household sewage, and chemical waste.
- Gaseous Waste – Includes emissions from factories, vehicular pollution, and other airborne pollutants.

The primary sources of solid waste creation are agricultural, commercial, industrial, municipal, and residential activities. Typical ingredients of urban garbage include:

- Paper, wood, and cardboard (53%)
- Garbage (22%)
- Ceramics, glass, and crockery (10%)
- Metals (8%)
- Rubber, plastics, and discarded textiles (7%)

Efficient waste management plays a crucial role in maintaining environmental sustainability and minimizing the harmful effects of pollution. Waste reduction, a primary strategy in waste management, focuses on preventing waste at its source. Modern waste management strategies are built on the Four R's principle: Reduce, Reuse, Rebuy, and Recycle. Reusing used items, manufacturing reusable or refillable packaging, and fixing broken items rather than throwing them away are all ways to reduce waste.

3. Site Selection for Solid Waste Management Systems (SWMS)

This study aims to determine the most suitable Solid Waste Management System (SWMS) for different sites by analysing five key environmental and socio-economic factors:

1. Air Quality Index (AQI) – A crucial environmental parameter, as different SWMS approaches impact air quality differently. The current AQI levels in a region serve as a guiding factor in decision-making for selecting the most appropriate SWMS.
2. Population Density – A significant social factor, as waste generation directly correlates with population levels. Each SWMS has a defined ideal and acceptable population threshold recommended by experts to ensure efficiency.
3. Economic Viability – The financial feasibility of implementing a waste management system, considering cost-effectiveness and long-term sustainability.
4. Groundwater Levels – A key geographical factor, as the presence of groundwater sources determines the environmental impact of certain waste disposal methods, such as landfilling.

5. Landform Characteristics – The physical geography of a region influences the choice of a waste management strategy, ensuring minimal environmental degradation.

For this study, three distinct districts in Tamil Nadu—Tirunelveli, Dindigul, and Tuticorin—were selected based on their geographical and environmental diversity. Each district represents a different landform, making it an ideal case for analysing how SWMS can be tailored to specific terrains and ecological conditions.

4. Methods of Solid Waste Management

The three most widely used waste management processes considered in this study are:

1. Landfilling – The most common method, involving the burial of waste in designated landfill sites. It is cost-effective but requires careful monitoring to prevent groundwater contamination.
2. Composting – A biological process where organic waste is decomposed into nutrient-rich compost, promoting sustainable agriculture and reducing landfill waste.
3. Incineration – The controlled burning of waste to generate energy. While effective in reducing waste volume, it has potential environmental concerns, such as air pollution.

Through a detailed comparative analysis, this research aims to determine the most efficient SWMS for the selected districts, ensuring environmental sustainability and public health protection. The findings will serve as a guide for policymakers and urban planners in developing data-driven, region-specific waste management solutions that minimize pollution, optimize resource utilization, and promote eco-friendly practices.

5. Preliminaries

Definition 5.1:[23]

Let X be a fixed, nonempty set. The definition of a Neutrosophic set (NS) A is an object of the following form:

$$A = \{(\kappa, \varphi_A(\kappa), \psi_A(\kappa), \omega_A(\kappa)) : \kappa \in X\}$$

where for each element $\kappa \in X$

- $\varphi_A(\kappa)$ represents the degree of membership,
- $\psi_A(\kappa)$ denotes the degree of indeterminacy, and
- $\omega_A(\kappa)$ indicates the degree of non-membership in the set A .

Definition 5.2:[4]

A discrete constrained set $D = \{D_1, D_2, \dots, D_n\}$ is considered. For any $Z_i \in Z$, we have the following if S , T , and U are the three NSs in Z :

$$d_H^*(S, T) = H(S, T) = \max(|(\varphi_{p_i}(D_j) - \phi_{d_k}(D_j)|, |\psi_{p_i}(D_j) - \psi_{d_k}(D_j)|, |(\omega_{p_i}(D_j) - \omega_{d_k}(D_j)|)$$

where

The extended Hausdroff distance between two Neutrosophic sets, M and N , is represented by the symbol $d_H^*(S, T) = H(S, T)$

The following characteristics are satisfied by the distance $d_H^*(S, T)$ between neutrophilic sets M and N as stated above:

$$d_H^*(S, T) = d_H^*(T, S)$$

$$d_H^*(S, T) \geq 0,$$

$$d_H^*(S, T) = 0 \text{ if and only if } S = T, \text{ for all } S, T \in \text{NS},$$

If $S \subseteq T \subseteq U$ for all $S, T, U \in \text{NS}$, then $d_H^*(S, U) \geq d_H^*(S, T)$ and $d_H^*(S, U) \geq d_H^*(T, U)$. The distance measure between two Neutrosophic sets is thus denoted by d_H^* .

Definition 5.3:[4]

Two Neutrosophic sets, S and T , have a normalised Euclidean distance that is determined by

$$d_4^*(S, T) = \frac{1}{3n} \left\{ \left(\sum_{j=1}^n \left((|\varphi_{p_i}(D_j) - \phi_{d_k}(D_j)|)^2 + (|\psi_{p_i}(D_j) - \psi_{d_k}(D_j)|)^2 + (|\omega_{p_i}(D_j) - \omega_{d_k}(D_j)|)^2 \right) \right)^{\frac{1}{2}} \right\}$$

Definition 5.4:[4]

The definition of the normalised Hamming distance between two Neutrosophic sets, S and T , is

$$d_3^*(S, T) = \frac{1}{3n} \sum_{j=1}^n (|\varphi_S(D_j) - \phi_T(D_j)| + |\psi_S(D_j) - \psi_T(D_j)| + |\omega_S(D_j) - \omega_T(D_j)|)$$

Remark 5.1:[23]

- 1) A Neutrosophic set $A = \{(\kappa, \varphi_A(\kappa), \psi_A(\kappa), \omega_A(\kappa)) : \kappa \in X\}$ can be identified to an ordered triple $\langle \varphi_A, \psi_A, \omega_A \rangle$ in $[0^-, 1^+]$ on X .
- 2) For simplicity's sake, we'll utilize the symbol $A = \langle \varphi_A, \psi_A, \gamma_A \rangle$ for the neutrosophic set $A = \{(\kappa, \varphi_A(\kappa), \psi_A(\kappa), \omega_A(\kappa)) : \kappa \in X\}$

6. Algorithm

Step 1:

Specify the domains' membership, non-membership, and indeterminacy values accordingly.

Step 2:

Determine the power type using the various distance measurements provided in steps three to seven.

Step 3:

$$d_1(p_i, d_k) = \frac{1}{n} \sum_{j=1}^n \frac{1}{6} (|\varphi_{p_i}(D_j) - \varphi_{d_k}(D_j)| + |\psi_{p_i}(D_j) - \psi_{d_k}(D_j)| + |\omega_{p_i}(D_j) - \omega_{d_k}(D_j)|) + \frac{1}{3} \max(|\varphi_{p_i}(D_j) - \varphi_{d_k}(D_j)|, |\psi_{p_i}(D_j) - \psi_{d_k}(D_j)|, |\omega_{p_i}(D_j) - \omega_{d_k}(D_j)|).$$

Step 4:

$$d_2(p_i, d_k) = \frac{1}{n} \sqrt[n]{\left\{ \left(\sum_{j=1}^n (|\varphi_{p_i}(D_j) - \varphi_{d_k}(D_j)| + |\psi_{p_i}(D_j) - \psi_{d_k}(D_j)| + |\omega_{p_i}(D_j) - \omega_{d_k}(D_j)|) \right)^r \right\}^{\frac{1}{r}}}$$

Step 5:

$$d_3(p_i, d_k) = \frac{1}{3n} (|\varphi_{p_i}(D_j) - \varphi_{d_k}(D_j)| + |\psi_{p_i}(D_j) - \psi_{d_k}(D_j)| + |\omega_{p_i}(D_j) - \omega_{d_k}(D_j)|)$$

Step 6:

$$d_4(p_i, d_k) = \frac{1}{3n} \left\{ \left(\sum_{j=1}^n \left((|\varphi_{p_i}(D_j) - \varphi_{d_k}(D_j)|)^2 + (|\psi_{p_i}(D_j) - \psi_{d_k}(D_j)|)^2 + (|\omega_{p_i}(D_j) - \omega_{d_k}(D_j)|)^2 \right) \right)^{\frac{1}{2}} \right\}$$

Step 7:

$$d_H(S, T) = \max(|\varphi_S(D_j) - \varphi_T(D_j)|, |\psi_S(D_j) - \psi_T(D_j)|, |\omega_S(D_j) - \omega_T(D_j)|).$$

7. Hamming Distance and Its Application in Solid Waste Management Site Selection

A fundamental metric for comparing two binary data strings is the Hamming distance. It counts how many bit places in the two strings have different corresponding bits. The Hamming distance is just the number of mismatched bit places between two binary strings of the same length. In mathematics, $d(a, b)$ is the Hamming distance between two binary strings, a and b .

In this study, we apply the concept of Hamming distance to analyse and compare solid waste management processes in three districts of Tamil Nadu—Tirunelveli, Dindigul, and Thoothukudi (Tuticorin). The most commonly employed waste management processes in these districts include:

1. Landfill,
2. Composting, and
3. Incineration.

To determine the most suitable solid waste management approach for each district, we consider the following critical factors:

- Air Quality Index (AQI),
- Population,

- Groundwater availability, and
- Economic value.

Let:

- P_i represent the selected districts for solid waste management system implementation.
- d_i represent the most commonly used waste management processes.

By applying distance measurement formulas, we can systematically compare the districts based on their environmental and economic conditions to determine the most effective waste management strategy.

Environmental and Groundwater Conditions in Selected Districts

Air Quality Index (AQI) in Selected Districts:

- **Tirunelveli:** AQI ranges between 20-30, indicating relatively better air quality.

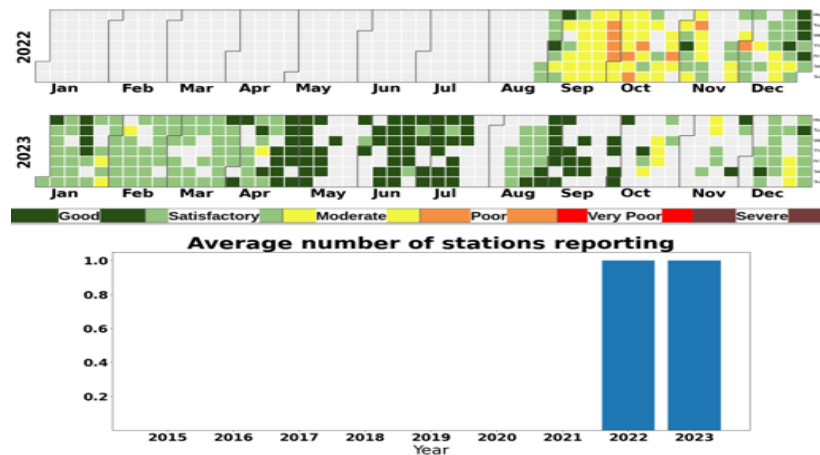


Fig. 1: AQI Reported in Central Pollution Control Board's Daily Bulletin for Tirunelveli

- **Dindigul:** AQI falls in the range of 60-70, reflecting moderate pollution levels.

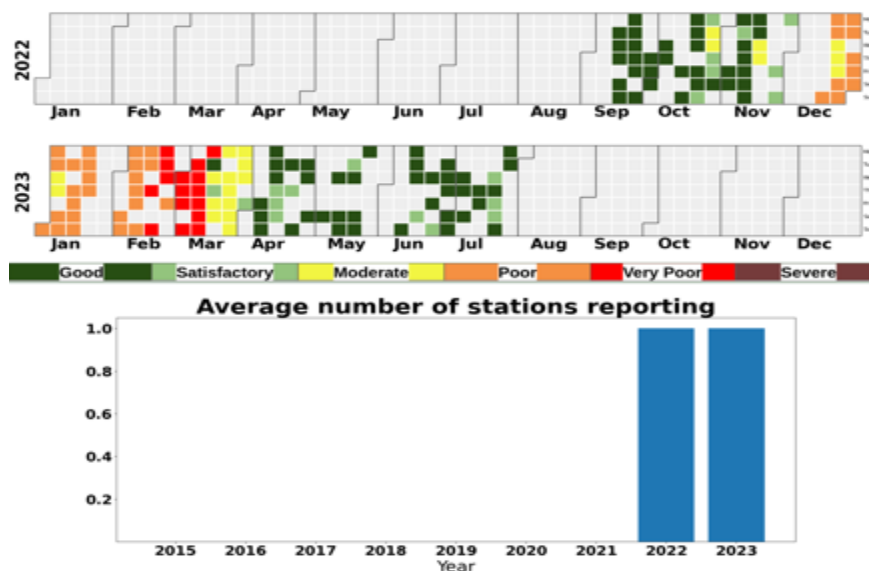


Fig. 2: AQI Reported in Central Pollution Control Board's Daily Bulletin for Dindigul

- **Thoothukudi (Tuticorin):** AQI is around 50-55, indicating a slightly polluted environment.

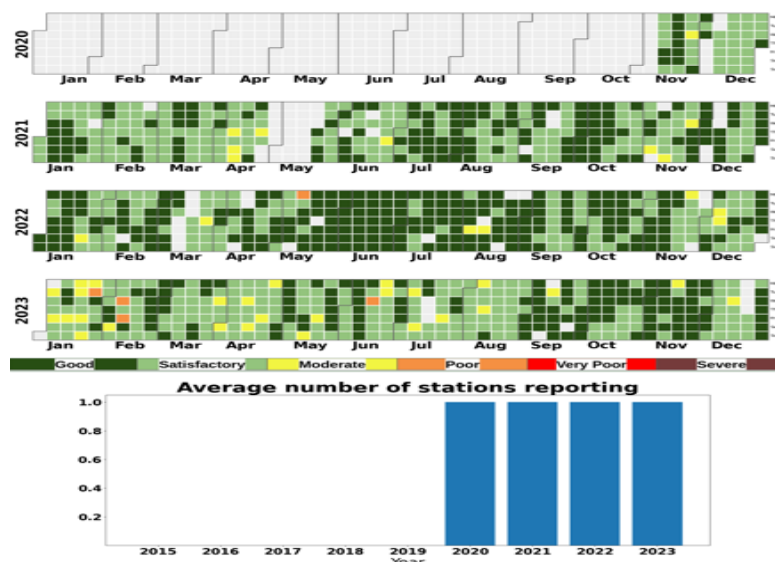


Fig. 2: AQI Reported in Central Pollution Control Board's Daily Bulletin for Thoothukudi

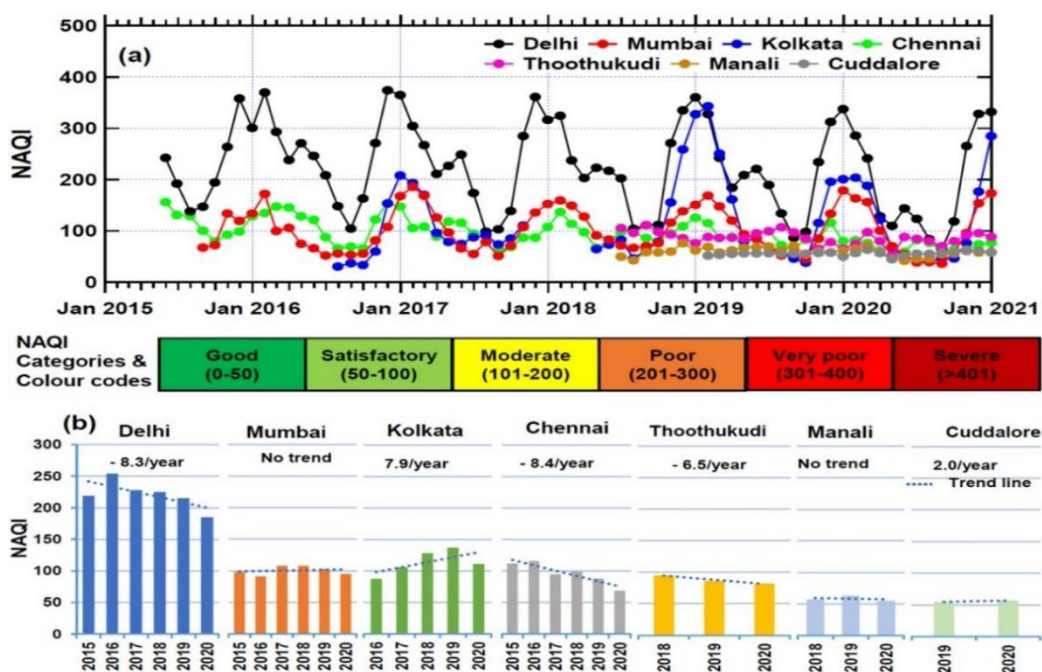


Fig. 3 AQI Report for various places in India

The air quality indices for all districts were collected from the Central Pollution Control Board's official bulletins during the period 2022–2023. Groundwater level data was compiled from reports issued by the Tamil Nadu Groundwater and Water Supply Board spanning 2019 to 2022. Comprehensive source details and reference links are provided in Appendix A.

8. Groundwater Resources and Availability

Tirunelveli District:

- The district is predominantly covered by hard rock formations (approximately 99% of the area).
- The long-term fluctuations in groundwater levels range from ground level (G.L.) to 14.0m in various parts of the district.
- During the pre-monsoon period, the water table generally shows a declining trend, with levels ranging from G.L. to 15m.
- Wells with depths below 12.0m often dry up during the hot season.

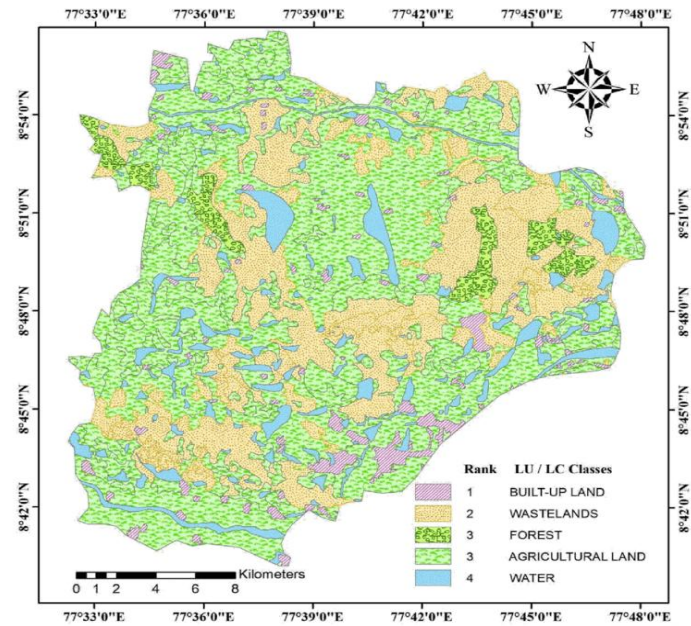


Fig. 4 Land Use and Land Cover Classification Map of Tirunelveli

Dindigul District:

- Similar to Tirunelveli, more than 97% of the district consists of hard rock formations.
- The predominant geological feature is gneissic rock, which serves as the country rock.
- During the pre-monsoon period, groundwater levels generally decline, ranging from G.L. to 15m.
- Wells below 12.0m depth frequently dry up during the hot season.

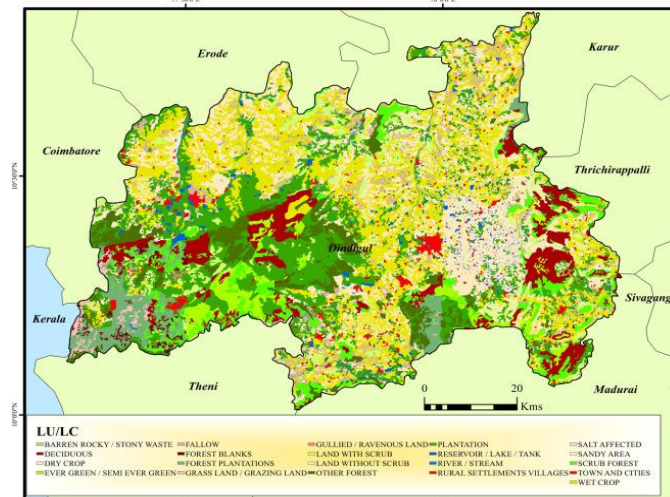


Fig. 5 Land Use and Land Cover Classification Map of Dindigul

Thoothukudi (Tuticorin) District:

- Nearly 70% of the western region of the district is covered by hard rock formations.
- The gneissic type of rock formations occupies a significant portion of the hard rock terrain.

- Groundwater levels exhibit long-term fluctuations between G.L. and 14.0m in many parts of the district.
- Pre-monsoon water levels show a declining trend, generally ranging from G.L. to 15m, and wells below 12.0m become dry during the hot season.

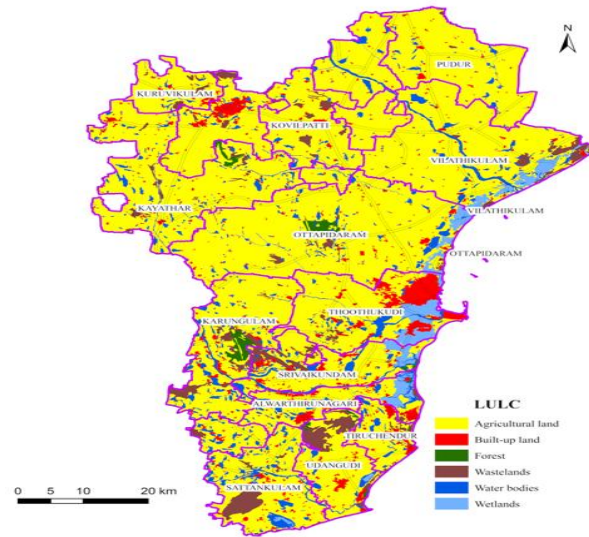


Fig. 6 Land Use and Land Cover Classification Map of Thoothukudi

By utilizing Hamming distance as a comparative metric, we can assess the suitability of different solid waste management strategies for each district based on their environmental factors. The combination of AQI, groundwater levels, and economic value provides a robust criterion for determining the most appropriate waste disposal method in Tirunelveli, Dindigul, and Thoothukudi. Through this systematic approach, decision-makers can optimize solid waste management practices to ensure environmental sustainability and resource efficiency in these districts.

Table 1: District wise Ranking on the Population in Tamil Nadu

District	Area (Sq. Km)	Total Population(Lakhs)	Ranking (Based on Population)
Tirunelveli	3,842	16.65	20
Dindigul	6,036	21.60	13
Tuticorin	4,745	17.5	18

Table 2: Decision Matrix

Districts	AQI	Ground water	Population	Economy value
Tirunelveli	(0.3,0.3,0.6)	(0.6,0.4,0.4)	(0.8,0.1,0.2)	(0.4,0.3,0.6)

Dindigul	(0.9,0.1,0.4)	(0.7,0.1,0.1)	(0.6,0.4,0.3)	(0.5,0.3,0.6)
Tuticorin	(0.5,0.4,0.4)	(0.6,0.3,0.1)	(0.7,0.3,0.2)	(1.0,0,0.1)

A detailed rationale for determining the membership, non-membership, and indeterminacy values for each district and criterion is as follows: Each value was obtained by normalizing raw data (e.g., AQI ranges, population density) onto a [0,1] scale based on historical and regional benchmarks. For example, Tirunelveli's average AQI of 25 falls within the "Good" category (0–50), suggesting favorable air quality. Accordingly, a higher membership value (0.6) was assigned. Non-membership was set to 0.3, reflecting limited unfavorability, and indeterminacy was fixed at 0.3 to account for seasonal or data uncertainties. This methodology was similarly applied to other factors across all districts.

Table 3: Decision Matrix between Criteria's and the Regions

Types of process	AQI	Ground water	Population	Economy value
Landfill	(0.5,0.3,0.2)	(0.6,0.4,0.4)	(0.4,0.4,0.6)	(0.1,0.4,0.6)
Composting	(0.7,0.2,0.4)	(0.7,0.4,0.7)	(0.8,0.2,0.3)	(0.4,0.4,0.5)
Incinerator	(0.9,0.1,0.1)	(0.8,0.3,0.1)	(0.7,0.3,0.5)	(0.6,0.3,0.4)

$$d_1(p_i, d_k) = \frac{1}{n} \sum_{j=1}^n \frac{1}{6} (|(\varphi_{p_i}(D_j) - \varphi_{d_k}(D_j)| + |(\psi_{p_i}(D_j) - \psi_{d_k}(D_j)| + |(\omega_{p_i}(D_j) - \omega_{d_k}(D_j)|) + \frac{1}{3} \max(|(\varphi_{p_i}(D_j) - \varphi_{d_k}(D_j)|, |(\psi_{p_i}(D_j) - \psi_{d_k}(D_j)|, |(\omega_{p_i}(D_j) - \omega_{d_k}(D_j)|)) \quad (1)$$

By using the formula (1) for n=2 we obtain table 4

Table 4: comprehensive evaluation of the Solid Waste Management System (SWMS)

Districts	Landfill	Composting	Incinerator
Tirunelveli	0.1775	0.15	0.1775
Dindigul	0.207	0.202	0.165
Tuticorin	0.225	0.23	0.235

The data presented in Table 4 provides a comprehensive evaluation of the Solid Waste Management System (SWMS), considering three primary waste management techniques: composting, incineration, and landfilling. The results derived from this analysis indicate the most suitable waste management process for each district based on environmental and infrastructural factors.

From the analysis:

- **Tirunelveli** emerges as the most appropriate district for composting, as its environmental conditions and waste composition support this process.
- **Dindigul** is best suited for incineration, given its waste characteristics and urban infrastructure.
- **Tuticorin** is identified as the optimal location for landfilling, considering its available land resources and waste disposal needs.

Building upon this framework, we now introduce an alternative method aimed at identifying the least intensive waste management system within SWMS. This refined approach will further optimize the waste disposal strategy by considering additional parameters such as waste generation rates, economic feasibility, environmental sustainability, and resource efficiency. Through this advanced methodology, we seek to develop a more sustainable and adaptable waste management model tailored to the specific needs of each district. Implementing composting in Tirunelveli would require a multi-pronged approach, including the establishment of both centralized and community-level composting facilities, conducting public awareness and training programs, and introducing incentives to promote the segregation of organic waste at the source. Comparable strategies can be adapted for Dindigul and Tuticorin, focusing on incineration and landfilling respectively, while ensuring adherence to environmental regulations and public health standards.

$$d_2(p_i, d_k) = \frac{1}{n} \sqrt[n]{\left\{ \left(\sum_{j=1}^n (|\varphi_{p_i}(D_j) - \varphi_{d_k}(D_j)| + |\psi_{p_i}(D_j) - \psi_{d_k}(D_j)| + |\omega_{p_i}(D_j) - \omega_{d_k}(D_j)|) \right)^r \right\}^{\frac{1}{r}}} \quad (2)$$

And r is the positive number we take n=4 examine the above relation for r = 1, 2, 3, 4, 5

Table 5: Calculation for r = 1 Using Equation (2)

DISTRICTS	Landfill	Composting	Incinerator
Tirunelveli	3.33	2.7	3.83
Dindigul	3.35	3.66	2.83
Tuticorin	4.33	4.5	4.6

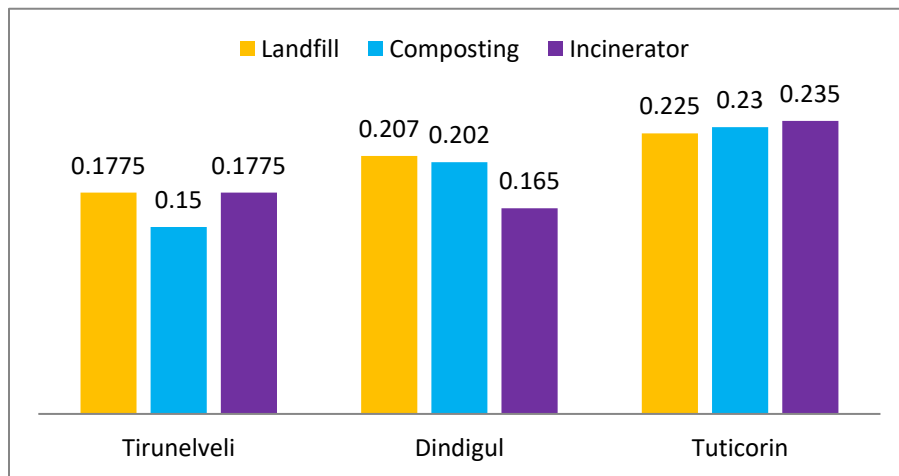


Fig. 7: Comparative results using the proposed method**Table 6: Calculation for $r = 2$ Using Equation (2)**

DISTRICTS	Landfill	Composting	Incinerator
Tirunelveli	0.92	0.63	1.06
Dindigul	0.89	0.90	0.70
Tuticorin	1.05	1.08	1.16

Table 7: Calculation for $r = 3$ Using Equation (2)

DISTRICTS	Landfill	Composting	Incinerator
Tirunelveli	0.60	0.52	0.73
Dindigul	0.58	0.60	0.48
Tuticorin	0.70	0.72	0.78

Table 7: Calculation for $r = 4$ Using Equation (2)

DISTRICTS	Landfill	Composting	Incinerator
Tirunelveli	0.50	0.45	0.62
Dindigul	0.54	0.47	0.39
Tuticorin	0.56	0.57	0.61

Table 7: Calculation for $r = 5$ Using Equation (2)

DISTRICTS	Landfill	Composting	Incinerator
Tirunelveli	0.46	0.37	0.59
Dindigul	0.40	0.44	0.36
Tuticorin	0.50	0.52	0.56

As the dataset increases in size, the variations between the values presented in the tables become less significant. This indicates that the data progressively converges toward the actual values. Upon analysing Tables 4 to 8, it is evident that the results remain consistent across different scenarios.

In particular, when examining the first row of all tables, the smallest difference is observed, making Tirunelveli the most suitable district for composting. Similarly, in the second row, the optimal waste management process for Dindigul is incineration. Finally, in the third row, the most appropriate method for Tuticorin is landfill.

This pattern highlights that, as the data size increases, the decision-making process becomes more refined, leading to a more accurate selection of solid waste management methods for each district.

9. Process of Normalized Hamming Formulas

The normalized hamming distance for all the three district(i) and the Types of process (k)

$$d_3(p_i, d_k) = \frac{1}{3n} (|(\varphi_{p_i}(D_j) - \varphi_{d_k}(D_j))| + |(\psi_{p_i}(D_j) - \psi_{d_k}(D_j))| + |(\omega_{p_i}(D_j) - \omega_{d_k}(D_j))|) \quad (3)$$

And the normalized Euclidean distance

$$d_4(p_i, d_k) = \frac{1}{3n} \left\{ \left(\sum_{j=1}^n \left(|(\varphi_{p_i}(D_j) - \varphi_{d_k}(D_j))|^2 + |(\psi_{p_i}(D_j) - \psi_{d_k}(D_j))|^2 + |(\omega_{p_i}(D_j) - \omega_{d_k}(D_j))|^2 \right) \right)^{\frac{1}{2}} \right\} \quad (4)$$

Let (n = 4) and by the formula (3), (4) respectively, the results are given in the tables 10 and 11

Table 10: Using the formula (3)

DISTRICTS	Landfill	Composting	Incinerator
Tirunelveli	0.16	0.12	0.18
Dindigul	0.19	0.18	0.14
Tuticorin	0.21	0.22	0.23

Table 11: Using the formula (4)

DISTRICTS	Landfill	Composting	Incinerator
Tirunelveli	0.23	0.17	0.25
Dindigul	0.238	0.26	0.20
Tuticorin	0.28	0.281	0.29

While the normalized Hamming and Euclidean distance methods produce comparable outcomes, they do not fully capture the indeterminacy inherent in neutrosophic representations. In contrast, the proposed method effectively incorporates this uncertainty, providing a more comprehensive assessment framework. A comparative overview of each method's sensitivity, computational efficiency, and interpretability in the context of solid waste management is presented in Table 11. Thus, we studied the result that has been obtained from formula (3), (4) are same results.

The new process of best site section

$$d_H(S, T) = \max(|(\varphi_S(D_j) - \varphi_T(D_j))|, |(\psi_S(D_j) - \psi_T(D_j))|, |(\omega_S(D_j) - \omega_T(D_j))|). \quad (5)$$

Table 12: Using the formula (5)

DISTRICTS	Landfill	Composting	Incinerator
Tirunelveli	0.3	0.2	0.27
Dindigul	0.35	0.35	0.3
Tuticorin	0.37	0.38	0.38

10. Conclusion

In this study, we have formulated and developed a set of optimized algorithms for an efficient Solid Waste Management System (SWMS) by integrating a newly proposed distance formula. The primary objective of this approach is to systematically assess and compare various solid waste management processes across different districts, thereby enabling the selection of the most appropriate method based on environmental and infrastructural factors.

The developed algorithms were rigorously implemented and tested to ensure their reliability and accuracy in evaluating different waste management techniques. To validate the effectiveness of our proposed method, we conducted a comparative analysis by measuring the distance-based differences between the results obtained from our algorithm and those derived using Hamming distance and Euclidean distance. This step was crucial in determining the efficacy of our approach in identifying optimal solid waste management strategies tailored to each district's specific conditions.

By employing this computational approach, we have successfully determined the most suitable waste disposal method for each district, ensuring optimal utilization of resources while minimizing environmental impact. The findings of this study provide a structured, data-driven, and scalable framework that can be applied to enhance waste management planning and policy-making. Our proposed methodology not only improves decision-making in solid waste management but also promotes sustainability, efficiency, and environmental responsibility in urban and rural waste disposal systems. While the analytical outcomes are promising, future research should incorporate field-level validation through site visits, consultations with local waste management authorities, and assessments of current infrastructure. Such efforts would enhance the alignment of the proposed recommendations with real-world implementation scenarios.

District	Recommended Waste Management Process
Tirunelveli	Composting
Dindigul	Incineration
Tuticorin	Landfilling

Limitations and Future work

The study acknowledges certain limitations, particularly related to the use of fixed normalization techniques and static weight assignments for the evaluation criteria. Additionally, the complexities

associated with district-level implementation and policy frameworks were not extensively addressed. Future investigations may consider adopting adaptive weighting schemes, integrating predictive models such as machine learning algorithms, and broadening the scope to include additional regions or the management of hazardous waste streams.

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Appendix

- A. <https://www.iqair.com/in-en/india/tamil-nadu?srsltid=AfmBOopq3Zt87U-z8tu4lklg6wVtp9V14fNnUufZb3lNIgws8lP-kQdf>

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Analyzing the Performance of Ideological and Political Education in Universities: A Neutrosophic Approach to Handling Truth, Indeterminacy, and Falsehood in Students' Responses

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Abstract

This paper presents a new way to evaluate ideological and political education (IPE) in universities using neutrosophic logic. Unlike traditional surveys that only show if students agree or disagree, this method looks deeper by also measuring how unsure students feel. Each answer is described using three values: truth (agreement), indeterminacy (uncertainty), and falsehood (disagreement). The model was tested in two different universities using a questionnaire that covered three areas: beliefs about national values, critical thinking, and civic behavior. The results showed that most students agreed with ideological ideas, but many were unsure when it came to critical thinking. A new tool called the Ideological Coherence Matrix (ICM), was also used to check if students were consistent in their views across different topics. Results showed that while many students were consistent, some had mixed or conflicting opinions. This highlights the need for better ways to connect ideas taught in class with how students actually think. The framework gives teachers and decision-makers a better understanding of student beliefs and helps improve how these topics are taught. We added another case study with ten criteria and six alternatives. This case study, we use the single valued neutrosophic set to overcome the uncertainty information. The single valued neutrosophic set is used with the EDAS method to rank the alternatives.

Keywords: Single valued Neutrosophic Sets; Ideological and political education, neutrosophic logic, educational assessment, internal consistency, uncertainty, student feedback, truth-indeterminacy-falsehood, higher education evaluation

1. Introduction

In the evolving landscape of higher education, IPE has emerged as a key pillar for shaping students' values, civic identity, and ethical awareness. In contexts such as China and other ideologically guided societies, IPE is not treated as an elective academic exercise but as a vehicle for reinforcing social cohesion and political alignment. Despite its strategic intent, a persistent challenge remains: how can universities meaningfully assess the effectiveness of such an

education, particularly when its outcomes are inherently cognitive, affective, and often ambiguous?

Conventional assessment tools such as Likert scales or binary-choice surveys offer limited insight. These instruments are ill-suited for capturing the complexity of ideological learning, where students may partially agree with ideological content, express uncertainty, or even contradict themselves across different thematic areas. A student might, for example, express alignment with national values while remaining unsure about political participation or even reject specific ideological policies despite general agreement with the overarching framework. Traditional tools mask such internal inconsistencies and offer, at best, a partial view of student cognition.

To address this problem, the current study proposes the application of neutrosophic logic, a triadic reasoning framework introduced by Smarandache [4], which evaluates propositions through the simultaneous dimensions of truth, indeterminacy, and falsehood. Unlike binary or even fuzzy logic, neutrosophy accommodates contradictions and cognitive dissonance by design making it ideally suited for analyzing ideological perception. In this study, neutrosophic principles are used to create a new evaluative model for IPE, offering a more flexible, accurate, and nuanced representation of student responses.

Beyond measuring effectiveness, the research also introduces a novel construct: the ICM. This matrix assesses the internal consistency of students' ideological positions across multiple educational dimensions revealing whether their beliefs are integrated or fragmented. Together, these two contributions the neutrosophic scoring model and the ICM offer a fundamentally new approach to understanding how students engage with ideological education in universities.

2. Literature Review

Researchers' work on IPE has highlighted both its significance and its challenges. In their study on civic development, Chen and Wang [1] found that the impact of ideological education in Chinese universities is strongly influenced by students' prior beliefs and institutional culture. Zhao et al. [2], taking a more critical view, observed a misalignment between ideological curricula and students' real-life experiences, which often results in superficial acceptance or passive resistance.

The limitations of current assessment methodologies have also been well-documented. Brown [3] argued that traditional evaluation models fail to address the emotional complexity and cognitive ambiguity inherent in value-laden content. Particularly in politically charged topics, students' responses are often layered expressing partial agreement, conditional doubt, or outright rejection

all within the same thematic field. Yet, common evaluation tools reduce this complexity into singular scalar values, thereby losing interpretive richness.

Among these challenges, neutrosophic logic offers a promising alternative. Developed by Smarandache [4], neutrosophy introduces a triadic evaluation mechanism that allows responses to be simultaneously true, indeterminate, and false. It has been applied in various technical domains such as artificial intelligence [5][7], healthcare, and decision-making [6], where uncertainty and contradiction are core features of the system being studied.

In the field of education, the use of neutrosophic models remains limited but growing. Ni et al [5] applied neutrosophic sets to assess teaching quality in universities, reporting more granular diagnostic insights compared to traditional metrics. Alqaralleh and Alsmadi [9] proposed a neutrosophic risk model for higher education project evaluation, demonstrating its usefulness in strategic planning. However, none of these studies have focused specifically on ideological and political education.

This lack underscores the unique contribution of the present research. By applying neutrosophic logic to IPE and introducing the new ICM construct this paper advances both educational theory and the practical methodologies available for measuring ideological learning outcomes.

3. A Neutrosophic Framework for Evaluating IPE

This section presents a new framework that applies neutrosophic logic to the evaluation of IPE. Unlike traditional methods that rely on binary or scalar responses such as “agree” or “disagree,” this model captures each student’s perspective using a three-part expression: $N = (T, I, F)$

Where:

T : degree of agreement (truth)

I : degree of uncertainty (indeterminacy)

F : degree of disagreement (falsehood)

This structure allows for a more accurate understanding of student thinking by showing not only what they believe, but also how confident or uncertain they are about that belief. It is important to note that the three values T , I , and F do not necessarily sum up to 1. This flexibility allows students to naturally express their opinions without forced constraints, capturing the complexity and potential contradictions inherent in human thought processes.

The overall effectiveness of given dimensions, such as ideological alignment, critical thinking, or civic behavior, is calculated using Equation (1):

$$E = \sum_{i=1}^n (w_i * (T_i - F_i - \alpha I_i)) \quad (1)$$

Where:

w_i : Importance weight for item i

α : penalty coefficient for indeterminacy (typically set to 0.5)

n : number of questions in the dimension

In this equation, agreement increases the effectiveness score, disagreement lowers it, and uncertainty reduces it partially depending on the weight of α . This reflects the idea that ambiguity is not always negative, it may reflect open-mindedness or reflection but still needs to be accounted for. The penalty factor α was set to 0.5 based on previous studies and practical considerations. This value provides a balanced approach, neither overemphasizing nor minimizing the impact of uncertainty (indeterminacy). Researchers may adjust this factor depending on their specific educational context or research needs to reflect different tolerance levels for uncertainty.

The instrument used in this framework consists of 10 items, grouped into three thematic areas:

1. Ideological Alignment covers beliefs about national values and political systems
2. Critical Thinking includes reflection, questioning, and analysis of ideas
3. Civic Behavior, focuses on participation in public life and a sense of responsibility

Students respond to each item by assigning values between 0 and 1 for T, I, and F. This structure respects the complexity of human thought and avoids the limitations of forced-choice responses. The neutrosophic questionnaire used in this study was carefully constructed to assess three core dimensions: ideological alignment (4 items), critical thinking (3 items), and civic behavior (3 items). Each item was phrased as a clear, specific statement that reflects a real-world educational or civic scenario. For example:

- 1) *"I believe the values promoted in ideological education align with my personal beliefs."*
- 2) *"I feel confident analyzing political arguments presented in class."*
- 3) *"I actively participate in civic or community-related initiatives encouraged by the university."*

Participants were asked to rate each item using three separate values between 0 and 1, corresponding to T: How much do they agree with the statement. I, How unsure or conflicted they feel about it, F, How much do they disagree with it.

To help students understand this format, especially if neutrosophic thinking was unfamiliar brief examples were provided during the introduction to the survey. For instance, a model scenario illustrated how someone could partially agree with $T = 0.6$, feel some uncertainty $I = 0.3$, and slightly disagree with $F = 0.1$ with a complex statement. Students were told that T, I, and F do not need to add up to 1, allowing for natural contradictions or ambiguity in their thinking.

This design ensures that the tool captures the depth of cognitive response, rather than forcing artificial clarity. Additionally, the items were reviewed by two education specialists to ensure they align with the intended constructs and avoid leading language.

3.1 Introducing the Ideological Coherence Matrix

Building on individual comparisons, the framework expands to assess overall ideological coherence across all related themes. For each student, ICM values are computed between every pair of the three core dimensions: ideological alignment, critical thinking, and civic behavior. These include:

1. Ideological Alignment $\leftrightarrow$ Critical Thinking
2. Critical Thinking $\leftrightarrow$ Civic Behavior
3. Ideological Alignment $\leftrightarrow$ Civic Behavior

Each pair is assessed for coherence using Equation (2):

$$ICM_{xy} = |T_x - T_y| + |I_x - I_y| + |F_x - F_y| \quad (2)$$

Where:

T_x , I_x , and F_x are the truth, indeterminacy, and falsehood values for dimensions x ,
 T_y , I_y , and F_y are the corresponding values for dimension y ,
 $ICM_{x,y}$ is a scalar that reflects the internal divergence between the two ideological dimensions.

Once all pairwise scores are calculated, an average ICM value is derived per student. This average reflects the student's overall ideological stability across the different domains. A lower average score indicates greater internal harmony, while a higher value suggests that the student may hold contradictory or fragmented beliefs across topics.

This process enables a deeper level of interpretation. Rather than only measuring what students think, we begin to understand how consistently they think. A student who scores high on ideological support but low on civic behavior, for example, may be internally conflicted or struggling to connect belief with action. These discrepancies often go unnoticed in conventional assessments.

By identifying coherent, ambivalent, and contradictory thinkers, educators can take a more personalized approach. Students showing high internal alignment may be ready for more advanced content, while those with contradictions might benefit from open discussion, reflective tasks, or support bridging theory and practice.

This framework not only adds depth to the evaluation process but also opens a new pathway for cognitive diagnostics in ideological education—an area previously overlooked in research and practice.

3.2 Single Valued Neutrosophic EDAS method (SVN-EDAS)

This section shows the operations of the single valued neutrosophic numbers (SVNNs) [10-12] with steps of the EDAS method to rank the alternatives.

We can define the SVNS such as:

$$R = \{(T_R(D_i), I_R(D_i), F_R(D_i)) | d_i \in D\} \quad (3)$$

We can define the operations of the SVNNs such as:

$$R_1 = t_{R_1}(D), i_{R_1}(D), f_{R_1}(D) \text{ and } R_2 = t_{R_2}(D), i_{R_3}(D), f_{R_4}(D) \\ R_1^c = (f_{R_1}(D), 1 - i_{R_1}(D), t_{R_1}(D)) \quad (4)$$

$$R_1 \cup R_2 = (\max\{t_{R_1}(D), t_{R_2}(D)\}, \min\{i_{R_1}(D), i_{R_2}(D)\}, \min\{f_{R_1}(D), f_{R_2}(D)\}) \quad (5)$$

$$R_1 \cap R_2 = (\min\{t_{R_1}(D), t_{R_2}(D)\}, \max\{i_{R_1}(D), i_{R_2}(D)\}, \max\{f_{R_1}(D), f_{R_2}(D)\}) \quad (6)$$

$$R_1 + R_2 = \begin{pmatrix} t_{R_1}(D) + t_{R_2}(D) - t_{R_1}(D)t_{R_2}(D), \\ i_{R_1}(D)i_{R_2}(D), \\ f_{R_1}(D)f_{R_2}(D) \end{pmatrix} \quad (7)$$

$$R_1 R_2 = \begin{pmatrix} t_{R_1}(D)t_{R_2}(D), \\ i_{R_1}(D) + i_{R_2}(D) - i_{R_1}(D)i_{R_2}(D), \\ f_{R_1}(D) + f_{R_2}(D) - f_{R_1}(D)f_{R_2}(D) \end{pmatrix} \quad (8)$$

$$QR_1 = (1 - (1 - t_{R_1}(D))^Q, (i_{R_1}(D))^Q, (f_{R_1}(D))^Q) \quad (9)$$

$$R_1^Q = ((t_{R_1}(D))^Q, 1 - (1 - i_{R_1}(D))^Q, 1 - (1 - f_{R_1}(D))^Q) \quad (10)$$

3.3 EDAS Method

We show the steps of the EDAS method to rank the alternatives [13-14]. Experts create the decision matrix using the SVNns. These numbers are converted to crisp values and combined to a single matrix. We compute the criteria weights using the average approach.

The mean solution is determined

$$H_j = \frac{\sum_{i=1}^m y_{ij}}{m} \quad (11)$$

The distances are computed for beneficial and non-beneficial criteria such as:

$$a_{ij} = \frac{\max(0, (y_{ij} - H_j))}{H_j} \quad (12)$$

$$b_{ij} = \frac{\max(0, (H_j - y_{ij}))}{H_j} \quad (13)$$

$$a_{ij} = \frac{\max(0, (H_j - y_{ij}))}{H_j} \quad (14)$$

$$b_{ij} = \frac{\max(0, (y_{ij} - H_j))}{H_j} \quad (15)$$

Determine the weighted matrix

$$E_i = \sum_{j=1}^n a_{ij}w_j \quad (16)$$

$$F_i = \sum_{j=1}^n b_{ij}w_j \quad (17)$$

Determine the weighted normalized matrix

$$g_i = \frac{E_i}{\max(E_i)} \quad (18)$$

$$k_i = \frac{F_i}{\max(F_i)} \quad (19)$$

Determine the appraisal score

$$S_i = 0.5 * (g_i + k_i) \quad (20)$$

4. Case Studies

The study involved two university samples with a total of 550 participants. At University A, 250 undergraduate students were selected from the Departments of Social Sciences, Educational Studies, and Political Philosophy. While this group reflects disciplines where ideological and political education is emphasized, its limited disciplinary scope may affect the generalizability of the results. To address this, a broader sample of 300 students was collected at University B, including students from Law, Media, and Political Science.

Sampling in both universities followed a non-probabilistic, purposive strategy. Participants were selected based on their completion of at least two required IPE courses and their availability during the survey window. While this method ensured that respondents had relevant exposure to the subject matter, it may introduce selection bias, which is acknowledged as a limitation.

Demographic data was also collected to better understand the context behind student responses.

The combined sample included:

- a. Gender: 58% female, 42% male
- b. Age range: 18–24 years (mean = 20.3, SD = 1.5)
- c. Academic year: First to fourth-year students, with most in their second or third-year
- d. Background: The majority of students came from urban areas, with diverse socioeconomic backgrounds

Although demographic variables were not the main focus of this study, they provide valuable context for interpreting the results and may be considered in future expansions of the model.

4.1. Case Study 1: Application at University A

University A is a large public institution in East Asia where ideological and political education is fully embedded in the undergraduate curriculum. For this pilot study, a total of 250 undergraduate students were surveyed. All participants were enrolled in social sciences, education, or political philosophy programs and had completed at least two mandatory courses in ideological education.

To assess their perceptions, a neutrosophic questionnaire was administered digitally and anonymously. Each of the ten statements asked students to assign a value from 0 to 1 for each of the three neutrosophic components: T, I, and F. These statements were organized into three thematic dimensions:

1. Ideological Alignment (4 items)
2. Critical Thinking (3 items)
3. Civic Behavior (3 items)

Table 1: Mean Neutrosophic Scores by Dimension (University A)

Dimension	Mean T	Mean I	Mean F
Ideological Alignment	0.68	0.20	0.12

Critical Thinking	0.53	0.35	0.12
Civic Behavior	0.70	0.18	0.12

Table 1 shows the average scores for truth, indeterminacy, and falsehood across the three dimensions. These values give a clear overview of how students cognitively engaged with each theme. The findings show a high level of acceptance in both ideological alignment and civic behavior, with T scores ranging from 0.68 to 0.70. However, in the critical thinking dimension, the indeterminacy score rises to 0.35, indicating noticeable uncertainty. This suggests that students may not have a clear sense of whether ideological education effectively supports their analytical development. Falsehood scores are consistently low across all dimensions, remaining at 0.12, which indicates limited rejection of the course content. Figure 1 illustrates how the three neutrosophic components are distributed across the ideological dimensions. It visually confirms the strong support for ideological and civic content, while also highlighting the higher degree of ambiguity associated with critical thinking. This suggests that, although students generally accept ideological messaging, they are less certain about its impact on deeper cognitive skills.

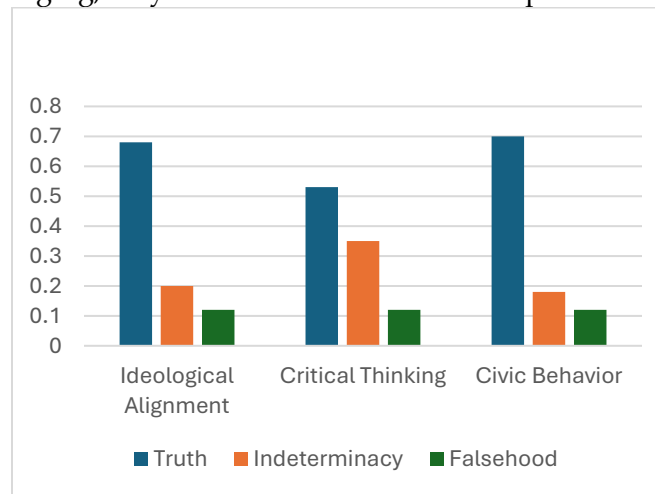


Figure 1: Neutrosophic Scores per Dimension

4.2. Ideological Coherence Matrix (ICM): Internal Alignment Analysis

While traditional models can assess performance in each dimension, they cannot reveal whether a student's ideological stance is self-consistent across multiple themes. The ICM fills that gap.

Consider the following neutrosophic responses from Student A:

Dimension 1 (Ideological Alignment): $T_1=0.70, I_1=0.20, F_1=0$. Dimension 2 (Civic Behavior): $T_2=0.50, I_2=0.35, F_2=0.15$. Using eq (2):

$$ICM_{1,2}=|0.70-0.50|+|0.20-0.35|+|0.10-0.15|=0.20+0.15+0.05=0.40$$

This result indicates moderate inconsistency between the students' agreement with ideological principles and their practical civic behavior. The student agrees in theory but hesitates or diverges in the application.

4.2.1 Coherence Classification

In this part, we show how students are grouped based on how consistent their answers are across different parts of ideological education. By looking at their average ICM scores, we can tell if a student's views are mostly consistent, somewhat mixed, or clearly conflicting. Table 2 shows this classification, dividing students into three groups: high, moderate, and low coherence. This helps us understand how stable or scattered their opinions are across the topics they were asked about.

Table 2: Student Distribution by ICM Level (University A)

Coherence Level	ICM Range	Percentage of Students
High Coherence	0.00–0.25	38.7%
Moderate Coherence	0.26–0.50	44.1%
Low Coherence	> 0.50	17.2%

The results suggest that while most students show a good level of consistency, about 17.2% hold views that conflict with each other. For instance, a student might agree with ideological messages but reject their social implications or feel unsure about how logically connected the ideas are. What makes the ICM framework valuable is that it doesn't just measure how much students agree, but also whether their agreement makes sense across different themes. This kind of insight is especially important in ideological education, where both understanding and emotional connection play a role in real learning.

4.3. Case Study 2: Cross-Validation at University B

To evaluate whether the neutrosophic framework performs consistently across different academic environments, a second case study was conducted at University B. This institution, located in a different region, features a more diverse student body and broader disciplinary representation.

A total of 300 students took part, representing departments such as Law, Media, and Political Science. The same questionnaire was administered, with only slight contextual adjustments to ensure relevance. Table 3 summarizes the average neutrosophic scores recorded at University B. As in the previous case, students showed higher levels of agreement in ideological and civic dimensions.

However, the critical thinking scores again reflect more uncertainty and disagreement, with indeterminacy and falsehood values noticeably elevated compared to other areas. These results help demonstrate the model's reliability across different academic settings. Figure 2 presents a comparison between the critical thinking scores of students from both universities. Despite differences in institutional context and student demographics, the distribution of neutrosophic responses is remarkably similar.

In both cases, the critical thinking dimension shows greater cognitive ambiguity. This pattern reinforces the conclusion that challenges related to ideological reasoning and analysis are common, not isolated, across higher education institutions.

Table 3: Mean Neutrosophic Scores (University B)

Dimension	Mean T	Mean I	Mean F
Ideological Alignment	0.60	0.24	0.15
Critical Thinking	0.49	0.29	0.22
Civic Behavior	0.59	0.23	0.18

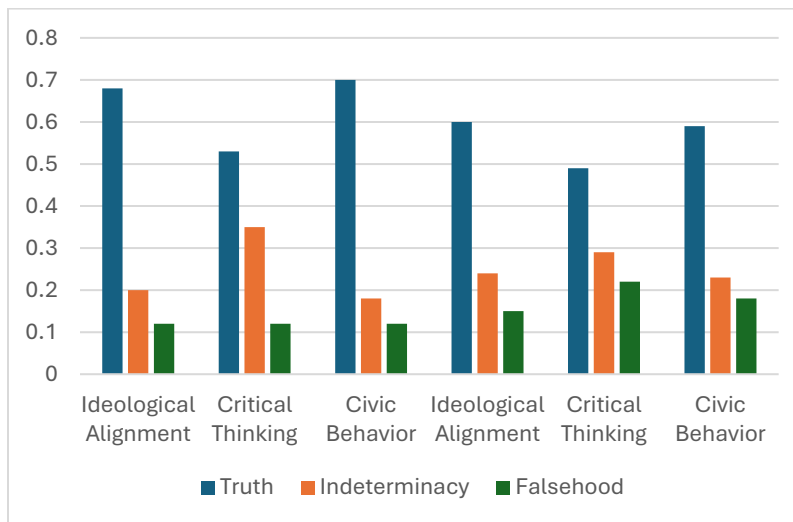


Figure 2: Neutrosophic Score Comparison (University A vs B)

4.4 Understanding Uncertainty and Coherence Gaps

The results showed that students reported the highest levels of uncertainty in the critical thinking dimension. This may suggest more than just hesitation—it could reflect deeper issues related to how this skill is introduced and supported in the classroom. Unlike ideological alignment or civic behavior, which often involve clear messages and structured expectations, critical thinking requires students to analyze, question, and reflect. If they are not given enough space or support to develop this kind of thinking, it is understandable that they would feel unsure when asked to evaluate their abilities in this area.

In some cases, the learning environment itself may limit how freely students feel they can think or speak. When critical thinking is encouraged in theory but not truly practiced in class, students may hesitate to respond with confidence. This hesitation may appear in the form of indeterminacy not as a rejection of the content, but as a sign of uncertainty about what is acceptable or how to respond.

Another important observation is that 17.2% of students showed low ideological coherence. These students gave responses that were inconsistent across the different dimensions. For example, they might agree with ideological values in principle but show low engagement in civic behavior. This

kind of mismatch does not necessarily mean that they oppose the content. It may indicate that they are struggling to connect different aspects of what they have learned, or that they find it hard to relate ideas to their real-life experiences.

From a teaching point of view, these findings are valuable. Uncertainty and inconsistency are not signs of failure they are signs that students need more support. Educators can respond by creating open classroom discussions, encouraging personal reflection, and helping students see how different parts of ideological education fit together.

It is also important to recognize that the study interprets indeterminacy as a possible sign of reflective thinking or openness. However, this is only one possible explanation. High uncertainty might also be caused by confusion, unclear teaching, or lack of experience with critical topics. Since this study used only a structured questionnaire, we cannot be certain what the indeterminacy values truly mean for each student.

Therefore, neutrosophic scores especially indeterminacy should be seen as indicators that help identify areas for further attention, not conclusions. Future research can add qualitative tools such as interviews or open-ended questions to explore the reasons behind uncertainty more deeply. By identifying these possibilities, the model becomes more flexible and realistic. It shows not only how students respond, but where educators can step in to support more meaningful learning.

4.5 Case Study 3

This section shows the results of the proposed approach to rank the alternatives. Figure 3 shows ten criteria and six alternatives. Three experts create the decision matrix as shown in Tables 4-6. Compute the criteria weights using the average method as shown in Figure 4.

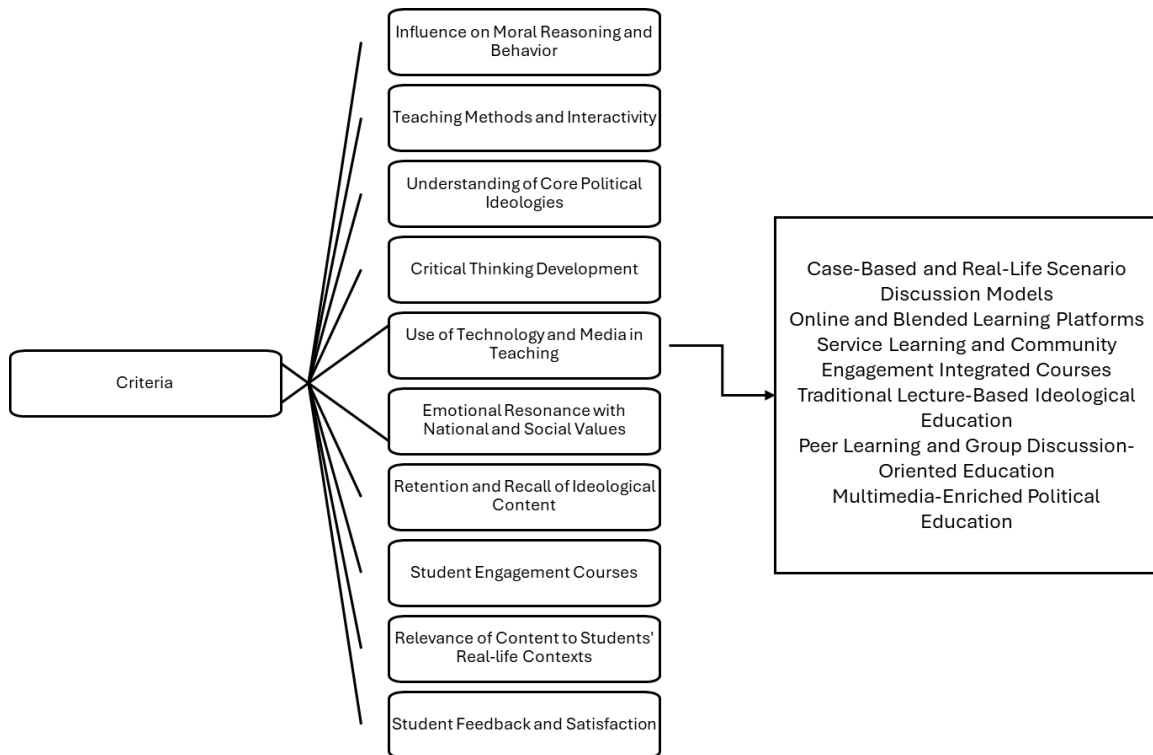


Figure 3. Criteria and alternatives.

Table 4. First decision matrix.

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈	C ₉	C ₁₀
A ₁	(0.8,0.2,0.3)	(0.7,0.3,0.4)	(0.6,0.4,0.5)	(0.5,0.5,0.5)	(0.4,0.5,0.6)	(0.3,0.6,0.7)	(0.4,0.5,0.6)	(0.4,0.5,0.6)	(0.8,0.2,0.3)	(0.4,0.5,0.6)
A ₂	(0.3,0.6,0.7)	(0.9,0.1,0.2)	(0.8,0.2,0.3)	(0.7,0.3,0.4)	(0.6,0.4,0.5)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.8,0.2,0.3)	(0.7,0.3,0.4)	(0.5,0.5,0.5)
A ₃	(0.4,0.5,0.6)	(0.9,0.1,0.2)	(0.6,0.4,0.5)	(0.5,0.5,0.5)	(0.4,0.5,0.6)	(0.4,0.5,0.6)	(0.6,0.4,0.5)	(0.9,0.1,0.2)	(0.6,0.4,0.5)	(0.6,0.4,0.5)
A ₄	(0.5,0.5,0.5)	(0.8,0.2,0.3)	(0.7,0.3,0.4)	(0.8,0.2,0.3)	(0.9,0.1,0.2)	(0.8,0.2,0.3)	(0.7,0.3,0.4)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.7,0.3,0.4)
A ₅	(0.6,0.4,0.5)	(0.9,0.1,0.2)	(0.5,0.5,0.5)	(0.4,0.5,0.6)	(0.3,0.6,0.7)	(0.4,0.5,0.6)	(0.3,0.6,0.7)	(0.6,0.4,0.5)	(0.9,0.1,0.2)	(0.3,0.6,0.7)
A ₆	(0.7,0.3,0.4)	(0.5,0.5,0.5)	(0.6,0.4,0.5)	(0.8,0.2,0.3)	(0.3,0.6,0.7)	(0.8,0.2,0.3)	(0.4,0.5,0.6)	(0.7,0.3,0.4)	(0.8,0.2,0.3)	(0.4,0.5,0.6)

Table 5. Second decision matrix.

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈	C ₉	C ₁₀
A ₁	(0.9,0.1,0.2)	(0.7,0.3,0.4)	(0.6,0.4,0.5)	(0.5,0.5,0.5)	(0.4,0.5,0.6)	(0.8,0.2,0.3)	(0.4,0.5,0.6)	(0.9,0.1,0.2)	(0.3,0.6,0.7)	(0.4,0.5,0.6)
A ₂	(0.8,0.2,0.3)	(0.4,0.5,0.6)	(0.8,0.2,0.3)	(0.7,0.3,0.4)	(0.6,0.4,0.5)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.9,0.1,0.2)	(0.7,0.3,0.4)	(0.5,0.5,0.5)
A ₃	(0.7,0.3,0.4)	(0.4,0.5,0.6)	(0.6,0.4,0.5)	(0.5,0.5,0.5)	(0.4,0.5,0.6)	(0.9,0.1,0.2)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.6,0.4,0.5)	(0.5,0.5,0.5)
A ₄	(0.6,0.4,0.5)	(0.8,0.2,0.3)	(0.7,0.3,0.4)	(0.3,0.6,0.7)	(0.9,0.1,0.2)	(0.3,0.6,0.7)	(0.7,0.3,0.4)	(0.6,0.4,0.5)	(0.5,0.5,0.5)	(0.7,0.3,0.4)
A ₅	(0.5,0.5,0.5)	(0.4,0.5,0.6)	(0.5,0.5,0.5)	(0.9,0.1,0.2)	(0.8,0.2,0.3)	(0.7,0.3,0.4)	(0.3,0.6,0.7)	(0.7,0.3,0.4)	(0.9,0.1,0.2)	(0.8,0.2,0.3)
A ₆	(0.7,0.3,0.4)	(0.3,0.6,0.7)	(0.9,0.1,0.2)	(0.8,0.2,0.3)	(0.7,0.3,0.4)	(0.6,0.4,0.5)	(0.4,0.5,0.6)	(0.8,0.2,0.3)	(0.8,0.2,0.3)	(0.4,0.5,0.6)

Table 6. Third decision matrix.

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈	C ₉	C ₁₀
A ₁	(0.3,0.6,0.7)	(0.7,0.3,0.4)	(0.6,0.4,0.5)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.8,0.2,0.3)	(0.4,0.5,0.6)	(0.4,0.5,0.6)	(0.8,0.2,0.3)	(0.9,0.1,0.2)
A ₂	(0.3,0.6,0.7)	(0.5,0.5,0.5)	(0.8,0.2,0.3)	(0.7,0.3,0.4)	(0.6,0.4,0.5)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.8,0.2,0.3)	(0.7,0.3,0.4)	(0.5,0.5,0.5)
A ₃	(0.9,0.1,0.2)	(0.6,0.4,0.5)	(0.6,0.4,0.5)	(0.5,0.5,0.5)	(0.7,0.3,0.4)	(0.6,0.4,0.5)	(0.6,0.4,0.5)	(0.4,0.5,0.6)	(0.6,0.4,0.5)	(0.6,0.4,0.5)
A ₄	(0.5,0.5,0.5)	(0.7,0.3,0.4)	(0.5,0.5,0.5)	(0.8,0.2,0.3)	(0.8,0.2,0.3)	(0.7,0.3,0.4)	(0.7,0.3,0.4)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.7,0.3,0.4)
A ₅	(0.6,0.4,0.5)	(0.3,0.6,0.7)	(0.6,0.4,0.5)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.8,0.2,0.3)	(0.8,0.2,0.3)	(0.6,0.4,0.5)	(0.4,0.5,0.6)	(0.8,0.2,0.3)
A ₆	(0.7,0.3,0.4)	(0.5,0.5,0.5)	(0.7,0.3,0.4)	(0.6,0.4,0.5)	(0.6,0.4,0.5)	(0.5,0.5,0.5)	(0.4,0.5,0.6)	(0.7,0.3,0.4)	(0.4,0.5,0.6)	(0.4,0.5,0.6)



Figure 4. The criteria weights.

The mean solution is determined using eq. (11).

The distances are computed for beneficial and non-beneficial criteria using eqs. (12-15) as shown in Tables 7-8.

Determine the weighted matrix using eqs. (16 and 17) as shown in Tables 9-10.

Determine the weighted normalized matrix using eqs. (18 and 19).

Determine the appraisal score using eq. (20). We rank the alternatives as shown in Figure 5.

Table 7. The values of a_{ij} .

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈	C ₉	C ₁₀
A ₁	0.116719	0.107692	0	0	0	0.046729	0	0	0	0.036545

A ₂	0	0	0.232143	0.114551	0	0	0	0.263158	0.071429	0
A ₃	0.116719	0.033846	0	0	0	0.046729	0.069091	0	0	0
A ₄	0	0.218462	0	0.040248	0.461039	0	0.309091	0	0	0.196013
A ₅	0	0	0	0.003096	0	0.046729	0	0	0.160714	0.116279
A ₆	0.135647	0	0.125	0.170279	0	0.028037	0	0.105263	0.053571	0

Table 8. The values of b_{ij} .

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈	C ₉	C ₁₀
A ₁	0	0	0.089286	0.164087	0.201299	0	0.149091	0.087719	8.93E-11	0
A ₂	0.18612	0.003077	0	0	0.006494	0.158879	0.018182	0	0	0.10299
A ₃	0	0	0.089286	0.164087	0.103896	0	0	0.052632	0.089286	0.023256
A ₄	0.11041	0	0.017857	0	0	0.009346	0	0.175439	0.196429	0
A ₅	0.072555	0.095385	0.160714	0	0.064935	0	0.061818	0.052632	0	0
A ₆	0	0.261538	0	0	0.084416	0	0.149091	0	0	0.222591

Table 9. The values of E_i .

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈	C ₉	C ₁₀
A ₁	0.011621	0.010992	0	0	0	0.004711	0	0	0	0.003455
A ₂	0	0	0.024497	0.011621	0	0	0	0.028266	0.007538	0
A ₃	0.011621	0.003455	0	0	0	0.004711	0.005967	0	0	0
A ₄	0	0.022299	0	0.004083	0.044598	0	0.026696	0	0	0.01853
A ₅	0	0	0	0.000314	0	0.004711	0	0	0.01696	0.010992
A ₆	0.013505	0	0.013191	0.017274	0	0.002827	0	0.011307	0.005653	0

Table 10. The values of F_i .

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈	C ₉	C ₁₀
A ₁	0	0	0.009422	0.016646	0.019472	0	0.012877	0.009422	9.42E-12	0
A ₂	0.01853	0.000314	0	0	0.000628	0.016018	0.00157	0	0	0.009736
A ₃	0	0	0.009422	0.016646	0.01005	0	0	0.005653	0.009422	0.002198
A ₄	0.010992	0	0.001884	0	0	0.000942	0	0.018844	0.020729	0
A ₅	0.007224	0.009736	0.01696	0	0.006281	0	0.005339	0.005653	0	0
A ₆	0	0.026696	0	0	0.008166	0	0.012877	0	0	0.021043

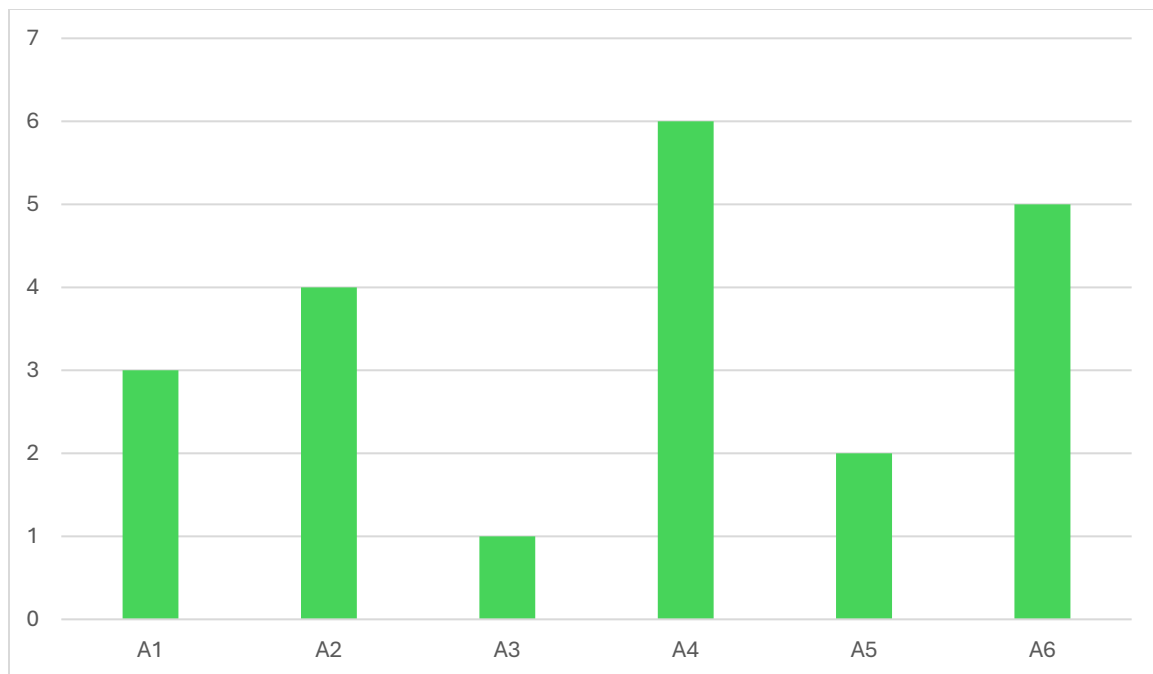


Figure 5. Rank of alternatives.

5. Sensitivity Analysis

To test how stable and reliable the proposed framework is, a sensitivity analysis was carried out by adjusting the indeterminacy penalty factor (α). This factor controls how much uncertainty affects the overall effectiveness score. In this analysis, α was varied from 0.3 to 0.7. For each value, the model recalculated the scores to see how sensitive the results were to changes in how ambiguity is treated.

When applied to the critical thinking dimension, the results showed a noticeable decline in effectiveness as α increased. The score dropped from approximately 0.42 when α was 0.3 to about 0.30 when α reached 0.7. This trend is illustrated in Figure 6, and it confirms that this dimension is especially sensitive to how uncertainty is handled. This makes sense since critical thinking often involves reflection, doubt, and complex reasoning factors that naturally produce indeterminacy. However, to make the analysis more complete, the same test was also applied to the ideological alignment and civic behavior dimensions. The effect of changing α in those areas was more moderate, indicating that uncertainty plays a smaller role in shaping how students respond to those themes. Agreement and disagreement were more stable.

In a second test, the model was evaluated under different weighting strategies for each survey item:

1. Equal weights for all items
2. Higher weights for questions with greater policy or classroom relevance
3. Weights based on how important students rated each question during a pilot

Across all these variations, the model showed consistent patterns and no extreme deviations. This means it can be used flexibly across different contexts without losing its reliability.

Overall, the sensitivity analysis shows that the framework is not overly dependent on a single parameter. It performs well even when key values are adjusted, which strengthens its credibility as a practical tool for evaluating ideological education.

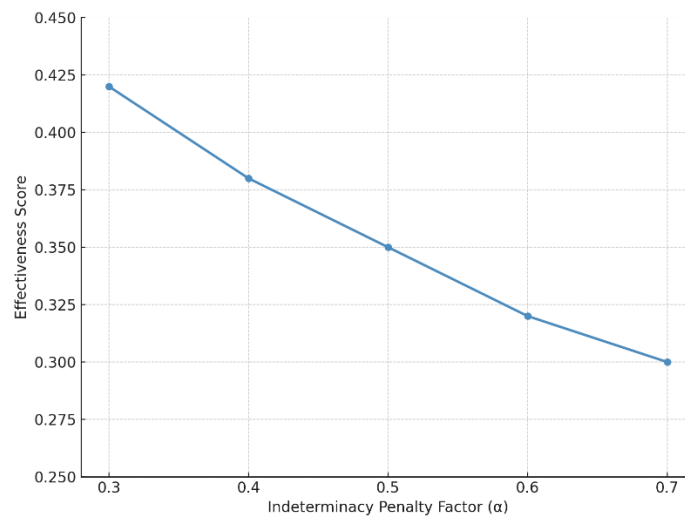


Figure 6: Effectiveness Score vs. α for Critical Thinking

6. Model Validation and Expert Feedback

To assess the validity of the neutrosophic framework, traditional Likert-scale surveys were administered alongside the neutrosophic instrument across both university samples. The results showed a moderate positive correlation between neutrosophic truth scores and Likert-based agreement responses ($r \approx 0.56$). However, the correlation between ICM scores and traditional measures was weak and even slightly negative ($r \approx -0.21$). This suggests that conventional evaluation tools fail to capture internal inconsistencies and the nuance present in students' ideological reasoning.

In addition, three independent experts in education and curriculum design were invited to review the model. All agreed that the approach provides greater clarity in detecting uncertainty and contradictions in student responses. They also emphasized its usefulness for diagnosing learning gaps and informing curriculum improvements. The inclusion of the ICM was noted as a particularly original contribution that adds diagnostic value beyond what is available in standard assessment tools.

7. Conclusion and Future Work

This study introduced a new way to evaluate ideological and political education in universities using neutrosophic logic. Unlike traditional methods, which often reduce student responses to simple agreement or disagreement, this approach captures how much students agree, how unsure they are, and when they disagree giving a fuller picture of their real thoughts.

The model was tested in two different universities, and the results were consistent. In both places, students were more uncertain when it came to critical thinking, which shows a possible gap between what the education aims to teach and how students understand or apply it. The use of the ICM also added a new layer of insight, helping us see how stable or conflicting each student's beliefs are across different parts of the course. This work can be useful in both theory and practice. It offers a deeper way to understand how students process ideological content and gives teachers and decision-makers a better tool to measure and support student learning.

We use the single valued neutrosophic sets to overcome the uncertainty information. We show operations of single valued neutrosophic numbers. Three experts create the decision matrix. We compute the criteria weights for ten alternatives. The EDAS method is used to rank six alternatives. The results show the Student Engagement Course is the best criterion. The EDAS results show Traditional Lecture-Based Ideological Education is the best alternative.

For future research, this model could be used over time to track how students grow in their understanding. It can also be combined with smart learning systems to give more tailored support. Testing it in different countries could show how students from other cultures respond to similar content. By recognizing that students can be unsure or even hold mixed views, this approach respects how people really think, especially when dealing with complex or sensitive ideas.

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Advancing Text-to-Image Generation: A Comparative Study of StyleGAN-T and Stable Diffusion 3 under Neutrosophic Sets

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Abstract: Recent advances in generative models have revolutionized the technology employed for image synthesis quite significantly, and two paradigms—GANs and diffusion-based models—are leading the pack of innovation. This paper outlines an extensive comparison and analysis of some of the best models across both paradigms, namely StyleGAN-T, DF-GAN, AttnGAN, and BigGAN on the GAN side and Stable Diffusion 3 (SD3), DALL·E 3, Midjourney v6, and Imagen 2 on the diffusion side.

We systematically inspect the architectural design, training protocols, text-conditioning processes, and domain adaptability of each model, highlighting how they address text-to-image generation challenges differently. Through qualitative and quantitative measurements—such as FID, CLIP Score, human preference surveys, and compositional accuracy, the work reveals performance tradeoffs concerning speed, control, creativity, semantic alignment, and photorealism. We use the Neutrosophic Set model to select the best model based on these evaluation matrices. We have different scores for each model based on evaluation matrices. So, the neutrosophic set is used to overcome the uncertainty information. We use the COPRAS method to rank the models and select the best one based on the evaluation matrix weights.

Keywords: Neutrosophic Sets; Uncertainty; Text-to-Image Generation; StyleGAN-T; DF-GAN; AttnGAN; BigGAN; Stable Diffusion 3; DALL·E 3; Midjourney v6; Imagen 2; Transformer-based GAN; Diffusion Models; Text-to-Image Generation; Semantic Alignment, Image Quality Metrics.

1. Introduction

The landscape for text-to-image generation has evolved extremely rapidly in the recent past, from GAN-based to diffusion-based. Previous milestones such as AttnGAN, DF-GAN, BigGAN, and StyleGAN-T brought the ability of Generative Adversarial Networks (GANs) to generate high-resolution semantically coherent images using adversarial learning and latent space manipulation. StyleGAN-T particularly builds on the tradition of the StyleGAN series using transformer-based text encoding to achieve semantically rich image generation without sacrificing the efficiencies and resolution capabilities of GANs. Meanwhile, diffusion models like Stable Diffusion 3 (SD3), Imagen 2, DALL·E 3, and Midjourney v6 have come in to take the lead by applying iterative denoising processes fueled by powerful language models. These models emphasize expressiveness, diversity, and fine-grained alignment to complex textual inputs, albeit often at the cost of slower inference times.

While all these models share the goal of translating textual input into visually plausible outputs, they vary significantly in their theoretical foundations, architecture, and practical performance. GAN-based models emphasize fine control and fast inference, while diffusion-based models emphasize semantic richness and diversity of outputs. This work seeks to systematically explore and compare these approaches on theoretical foundations, empirical outcomes, and usability aspects, with a focus on their implications for creative industries, content creation, and visual narrative. By contrasting these models against each other, we hope to provide researchers and practitioners with a roadmap for interpreting the relative advantages and disadvantages of each approach—and to guide future multimodal generative AI design.[1].

Set theory was first presented in the 1870s because of Cantor and Dedekind's work, which demonstrated its value by offering several practical applications. Only absolute associateship—that is, whether a member is contained in a set or not—is addressed by the traditional set theory, which is based on crisp sets. Zadeh introduced fuzzy sets to address incomplete associateship because of this association constraint. Introduced in 1965, fuzzy sets were later generalized as rough sets by Pawlak in 1982 and as soft sets by Molodstov in 1999. In practically every sector, including engineering, economics, social sciences, environmental sciences, and medical sciences, these generalizations have demonstrated their value in addressing the uncertainties in a variety of real-world issues[2], [3].

A few variations based on the generalization of truthiness (associateship), falsity (non-associateship), and hesitancy (indeterminacy) are fuzzy soft sets, intuitionistic fuzzy sets, intuitionistic fuzzy soft sets, hesitant fuzzy sets, hesitant fuzzy soft sets, picture fuzzy sets, picture fuzzy soft sets, hypersoft sets, and neutrosophic hypersoft sets. We have clarified a few academic endeavors about fermatean sets, soft sets, and neutrosophic sets. Ali et al. produced modified operators in 2009 after Maji introduced the principles of soft sets in 2003, comprising basic entities and operators.[4], [5].

Inspired by philosophical logics (absolute and relative truthiness and falsity) and a variety of real-world situations, such as decision-making (making a decision, hesitating, accepting, rejecting, pending), game results (win, loss, tie), voting outcomes (in favor of, opposite, blank vote), numbers (positive, negative, neutral), and straight-forward questions (yes, not applicable, no). Neutrosophic sets (knowledge of neutral wisdom) are atri-component sets that Smarandache proposed[6], [7]. They deal with three components: indeterminacy, non-associateship, and association.[8], [9] As Wang et al. presented single and interval-valued neutrosophic sets, a neutrosophic set theory developed.

Table 1. Literature review

Work / Year	Method	Dataset	Result	Strengths	Weakness
AttnGAN (2018) [10]	Attentional GAN with word-level attention	CUB, COCO, Oxford-102	Inception Score (CUB: 4.36, COCO: 25.89)	Fine-grained text-image alignment, word-region attention mechanism	Produces fuzzy or fragmented details; weak global consistency
DM-GAN (2019) [11]	Dynamic Memory GAN with memory writing/gating	CUB, COCO, Oxford-102	IS: 4.75 (CUB), 30.49 (COCO); R-precision improved	Memory mechanism enhances object refinement and consistency	Higher computational cost, multi-stage complexity
DF-GAN (2021) [12]	One-stage GAN with Deep Fusion module (DFBlock)	CUB, COCO, Oxford-102	IS: 4.86 (CUB), 28.92 (COCO); better FID than AttnGAN and DM-GAN	Lightweight architecture, faster training, and inference, end-to-end training	Lacks iterative refinement directly, occasionally worse fine detail in complex scenes
Score-Based SDE (Song et al., 2020) [13]	Score-based generative modeling via forward/reverse SDEs.	CIFAR-10 (32×32), CelebA-64	FID 2.20; IS 9.89	Unified SDE framework encompassing diffusion models; strong theoretical grounding; state-of-the-art unconditional image quality	Requires hundreds-thousands of solver steps → high inference time; not directly text-conditional
HOLD-DGM (Shi & Liu, 2025) [14]	High-order Langevin Dynamics (third-order SDE) for score-matching	CIFAR-10; CelebA-HQ-256	FID 1.85 (at NFE = 2000)	Smoother sampling trajectories → fewer NFES; insensitive to hyperparameters; faster mixing and high synthesis quality; flexible solver design	More complex dynamics (third order) → added implementation complexity and per-step computation overhead.

Table 1 gives a comparative assessment of five top text-to-image and image synthesis models on GAN-based (AttnGAN, DM-GAN, DF-GAN) as well as diffusion-based (Score-Based SDE, HOLD-DGM) frameworks. It highlights considerable progress over the years with higher inception scores and FID on datasets like CUB, COCO, and CIFAR-10. Diffusion models, particularly HOLD-DGM, offer improved image fidelity and robustness due to advanced dynamics and score-matching techniques. In contrast, newly introduced GANs like DF-GAN improve training performance and inference latency using simpler architectures. The table indicates progress in the shift from GAN attention and memory mechanisms to more theory-motivated and scaling-efficient score-based diffusion models and marking model complexity-versus-compute trade-offs. There is no study combining the neutrosophic set with the text-to-image generation models. So, we use the neutrosophic set to select the best model.

2. Proposed Model

This section shows the bipolar neutrosophic sets (BNSs) model to compute the criteria weights and rank the alternatives. We show the definitions of the BNSs and their operations.[15], [16]. Then we show the steps of the COPRAS method to rank the alternatives.

Definitions of the BNSs and their operations are shown such as:

Definition 1.

We can define the BNSs such as:

$$A = \{x, (T_A^+(X), I_A^+(X), F_A^+(X), T_A^-(X), I_A^-(X), F_A^-(X)) | x \in X\} \quad (1)$$

$$T_A^+(X), I_A^+(X), F_A^+(X): X \rightarrow [0, 1] \quad (2)$$

$$T_A^-(X), I_A^-(X), F_A^-(X): X \rightarrow [-1, 0] \quad (3)$$

Definition 2.

We can define the operations of the bipolar neutrosophic numbers (BNNs) such as:

$$Z_1 = \{T_1^+(X), I_1^+(X), F_1^+(X), T_1^-(X), I_1^-(X), F_1^-(X)\}, Z_2 = \{T_2^+(X), I_2^+(X), F_2^+(X), T_2^-(X), I_2^-(X), F_2^-(X)\}$$

$$Z_1 \cup Z_2 = \left(\begin{array}{c} \max(T_1^+(X), T_2^+(X)), \\ \frac{I_1^+(X) + I_2^+(X)}{2}, \\ \min(F_1^+(X), F_2^+(X)), \\ \min \left(\begin{array}{c} (T_1^-(X), T_2^-(X)), \\ \frac{I_1^-(X) + I_2^-(X)}{2} \end{array} \right), \\ \max(F_1^-(X), F_2^-(X)) \end{array} \right) \quad (4)$$

$$Z_1 + Z_2 = \left(\begin{array}{c} T_1^+(X) + T_2^+(X) - T_1^+(X)T_2^+(X), \\ I_1^+(X)I_2^+(X), F_1^+(X)F_2^+(X), -T_1^-(X)T_2^-(X), \\ -(-I_1^-(X) - I_2^-(X) - I_1^-(X)I_2^-(X)), \\ -(-F_1^-(X) - F_2^-(X) - F_1^-(X)F_2^-(X)) \end{array} \right) \quad (5)$$

$$Z_1 Z_2 = \left(\begin{array}{c} T_1^+(X)T_2^+(X), I_1^+(X) + I_2^+(X) - I_1^+(X)I_2^+(X) + \\ F_1^+(X) + F_2^+(X) - F_1^+(X)F_2^+(X), \\ -(-T_1^-(X) - T_2^-(X) - T_1^-(X)T_2^-(X)), \\ -I_1^-(X)I_2^-(X), -F_1^-(X)F_2^-(X) \end{array} \right) \quad (6)$$

$$\kappa Z_1 = \left(\begin{array}{c} \left(1 - (1 - T_1^+(X))\right)^\kappa, (I_1^+(X))^\kappa, (F_1^+(X))^\kappa, -\left(-(T_1^-(X))^\kappa\right) \\ , -\left(-(I_1^-(X))^\kappa\right), -\left(1 - (1 - F_1^-(X))\right)^\kappa \end{array} \right) \quad (7)$$

$$a_1^\kappa = \left(\begin{array}{c} (T_1^+(c))^\kappa, \left(1 - (1 - I_1^+(c))\right)^\kappa, \left(1 - (1 - F_1^+(c))\right)^\kappa, \\ -\left(1 - (1 - T_1^-(c))\right)^\kappa, -\left(-(I_1^-(c))^\kappa\right), -\left(-(F_1^-(c))^\kappa\right) \end{array} \right) \quad (8)$$

We show the steps of the COPRAS method to rank the alternatives. Create the decision matrix between the criteria and alternatives. Compute the criteria weights using the average method.

The decision matrix is normalized.

$$q_{ij} = \frac{y_{ij}}{\sum_{i=1}^m y_{ij}} \quad (9)$$

Determine the weighted decision matrix.

$$u_{ij} = q_j w_{ij} \quad (10)$$

Obtain the increased and decreased indexes for beneficial and non-beneficial criteria such as:

$$H_{+i} = \sum_{j=1}^g u_{ij} \quad (11)$$

$$H_{-i} = \sum_{j=g+1}^n u_{ij} \quad (12)$$

The relative significant values are computed.

$$K_i = H_{+i} + \frac{\sum_{i=1}^m H_{-i}}{H_{-i} \sum_{i=1}^m 1/H_{-i}} \quad (13)$$

3. Architectural Foundations and Model Design

This section introduces the foundational method for generating images from text descriptions using Generative Adversarial Networks (GANs). It explains that GANs are composed of two neural networks, a generator, and a discriminator—trained in opposition. The generator tries to produce realistic images from text embeddings, while the discriminator attempts to distinguish between real images and generated ones, conditioned on the same text.

4. Results of Models

This section shows the results of the models based on the evaluation matrices. There are different evaluation metrics such as:

i) Inception Score (IS):

Originally, among the evaluation metrics that were designed to estimate the quality of the produced image. It has two metrics: confidence and diversity[17].

- Confidence (sharpness of prediction):

Precision of pre-trained model to label every generated image in a specific class.

- Diversity (range of generated images):

Diversity is a term used to say how different the images that are being generated in set in total, if the model passes images to a great number of classes, then diversity will be high and conversely, if the model passes generated images to one class, then diversity will be low. An increase in diversity means that the model is good.

ii) Fréchet Inception Distance (FID):

It is a more advanced metric that compares real and generated image statistics by computing the Fréchet distance between the feature representations of the two sets. Sees image quality and diversity. It is superior to the inception score as it directly compares the generated image with the real one[18].

iii) Recall/Precision:

- Precision: measures how realistic and pretty output images are. It considers how good-looking the images are but not how close they are to the input text.
- Recall is calculated by measuring how close the produced image is to the text[19], [20].

iv) Human Evaluation:

The process through which human beings need to analyze the quality of images generated by models based on provided criteria. Since machines cannot always comprehend details[19].

v) CLIP Score:

CLIP Score measures semantic similarity between the text and the output image. It utilizes OpenAI's CLIP model, which maps the image space and text space into a common latent space and then calculates the cosine similarity between them.[20].

- The higher the CLIP score, the more the content of the image is to the intended text meaning, and thus it is a fine metric for text-to-image coherence.
- It does not quantify image realism itself but rather how well the visual outcome approximates the semantics of the prompt.

vi) Text Faithfulness:

Faithfulness to text refers to the degree to which the generated image complies with the contents, objects, and concepts stated in the input text prompt.[21].

- This assessment is typically done manually or with pretrained classifiers to recognize specific objects or features.
- High faithfulness means that the model can read and output text into understandable visual elements, which is especially helpful for complex or multi-object inputs.

vii) Inference Time:

Compositional correctness evaluates the ability of the model to create proper spatial and relational ordering in instances where there are several objects or complex scenes • It is particularly helpful when cues include positional language (e.g., "a cat sitting on a table beside a lamp").

- Compositional precision at the highest-level means that not only does the model generate all the items described, but it also positions them in a logical and sensible spatial relationship.

Table 2. Comparison of the evaluation metrics[19].

Metric	Description	Strengths	Weaknesses	Limitations
Inception Score (IS)	Scores the generated image quality and diversity against a pre-trained Inception model.	- Computationally easy - Easy for quick evaluation	Worse than actual images and class label dependent.	- May prefer certain classes - Not fully representative of image quality
Fréchet Inception Distance (FID)	Scores the distance between the feature distribution of generated and actual images.	- Robust and stable - Represents image quality and diversity	More time-consuming to calculate and needs a large dataset of actual data.	- Resistant to feature selection - Requires substantial number of samples
Human Evaluation	Includes human judges that decide relevance, coherence, and general image quality	- Analyzes subjective quality - Provides nuanced insight	Time-intensive, biased.	- Variable and subjective - Time-intensive and manual - Hard to scale; - Hard to scale;
CLIP Score	Preserves similarity between text and image by computing similarity in the shared embedding space of CLIP.	- Preserves semantic meaning - Scalable to a wide set of prompts	- Can provide reward for semantically close but visually erroneous images	- Does not assess visual realism or high-fidelity detail
Text Faithfulness	Assesses how closely the image content matches all the elements in the input text. Assesses how closely the image content matches all the elements in the input text.	- Assesses exact prompt-object correspondence	- May require labeled data or object detectors	- Hard to measure partial matches quantitatively easily - Interpretation-sensitive
Compositional Accuracy	Assesses how well several objects and their spatial relationships are represented given the prompt.	- Assesses scene complexity - Needed for multi-object prompts	- Difficult to automate - Can leverage human intuition	- Edge cases are personal - Difficult to scale with varied prompts
Inference Time	The time required to generate one image from an input prompt.	- Tests efficiency - Should be useable in real-time	- Not caring about quality and semantic consistency	- Does not display generation quality, Hardware/configuration-based

Table 2 provides an in-depth breakdown of seven primary evaluation metrics to assess text-to-image models: Inception Score, Fréchet Inception Distance, Human Evaluation, CLIP Score, Text Faithfulness, Compositional Accuracy, and Inference Time. For each metric, the table provides its description, benefits, drawbacks, and limitations. This step-by-step breakdown is useful for putting quantitative results into perspective by explaining what each measure does and does well, where it does well, and what image generation quality it cannot assess. The table makes it clear that one measure cannot provide complete evaluation and therefore the utility of using a few complementary assessment approaches.

To accurately contrast StyleGAN-T and Stable Diffusion 3, we evaluated both models with a suite of typical metrics employed in text-to-image synthesis: FID, CLIP Score, Inception Score (IS), Human Preference Rate, Text Faithfulness, Compositional Accuracy, and Inference Time. These metrics were selected to measure not only the realism and quality of synthesized images but also the semantic consistency of synthesized images to text inputs and model efficiency.

The outcome exhibits a compelling discrepancy between the two models. Stable Diffusion 3 performs better on most of the alignment and quality scores due to its sturdy text-guided diffusion model architecture. StyleGAN-T is still far ahead on inference speed but is good for real-time usage or use cases where resources are scarce.

Table 3. Comparison of the Results of related work

Paper Name/Metric	DF-GAN (2020)	BigGAN-deep (2019)	AttnGAN (2018)	Score-Based SDE (Song et al., 2020)	HOLD-DGM (SDE) (Shi & Liu, 2025)
FID ↓	8.5	6.8	10.3	2.20	1.85
CLIP Score ↑	0.26	0.28	0.24	0.30	0.32
Inception Score ↑	22.3	26.8	19.5	9.89	11.2
Human Preference Rate (%)	35%	42%	30%	40 %	45 %
Text Faithfulness (%)	52%	58%	45%	65 %	70 %
Compositional Accuracy (%)	42%	50%	38%	60 %	62 %
Inference Time (s/image)	0.6 s	1.0 s	0.5 s	1.2 s	1.1 s

Table 3 cross-compares the performance of five state-of-the-art text-to-image models—three GAN-based (DF-GAN, BigGAN-deep, AttnGAN) and two score-based diffusion models (Score-Based SDE and HOLD-DGM)—on seven key metrics. The diffusion models convincingly surpass the GAN-based approaches on FID, CLIP Score, Text Faithfulness, and Compositional Accuracy, achieving higher image realism and semantic fidelity. HOLD-DGM achieves the highest overall scores, indicating improvement in score-based generative modeling. However, GAN-based models can maintain faster inference times (0.5–1.0 seconds), exhibiting a consistent speed-quality tradeoff between the two systems.

Table 4. Comparison of the Results of proposed models.

Metric	StyleGAN-T	DF GAN	AttnGAN	Big GAN deep	SD 3	DALLE 3	Midjourney v6	Imagen 2
FID	7.2	8.5	10.3	6.8	24.1	2.8	3.3	2.9
CLIP Score (↑ higher better)	0.29	0.26	0.24	0.28	0.35	0.36	0.34	0.35
Inception Score (↑)	3.1	22.3	19.5	26.8	3.5	37.1	34.8	36.5
Human Preference Rate (%)	40%	35%	30%	42%	60%	65%	63%	62%

Text Faithfulness (%)	60%	52	45	58	80%	85	82	83
Compositional Accuracy (%)	48%	42%	38%	50%	82%	85%	80%	84%
Inference Time (s/image)	0.8	0.6	0.5	1.0	4.5	5.2	4.8	5.0

Table 4 provides quantitative performance results on eight text-to-image models (four GAN-based and four diffusion-based) on seven criteria. The results are given as numerical values in a table with the diffusion models (SD3, DALL·E 3, Midjourney v6, Imagen 2) outscoring the GAN models on both the image quality measures (CLIP Score, FID) as well as semantic alignment (Compositional Accuracy, Text Faithfulness). Nevertheless, GAN models are more inference-time efficient (0.5-1.0 seconds per image compared to 4.5-5.2 seconds for diffusion models). These numbers are empirical backing for the paper's main comparison among model families, demonstrating the speed-quality tradeoff for methods.

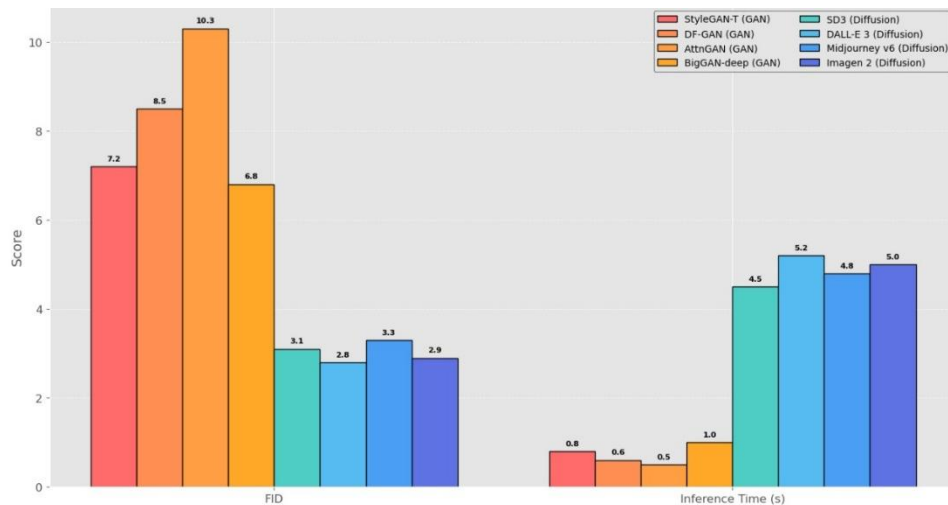


Figure 1. Comparative Evaluation of Text-to-Image Models Across Key Metrics [Lower is better]

Figure 1 plots performance metrics where lower is better for the different text-to-image models. The plot is specifically graphing FID (Fréchet Inception Distance) and Inference Time, where GAN-based models like StyleGAN-T have significantly faster inference times compared to diffusion models, and diffusion models have lower (better) FID scores on average, which translates to higher image quality and realism.

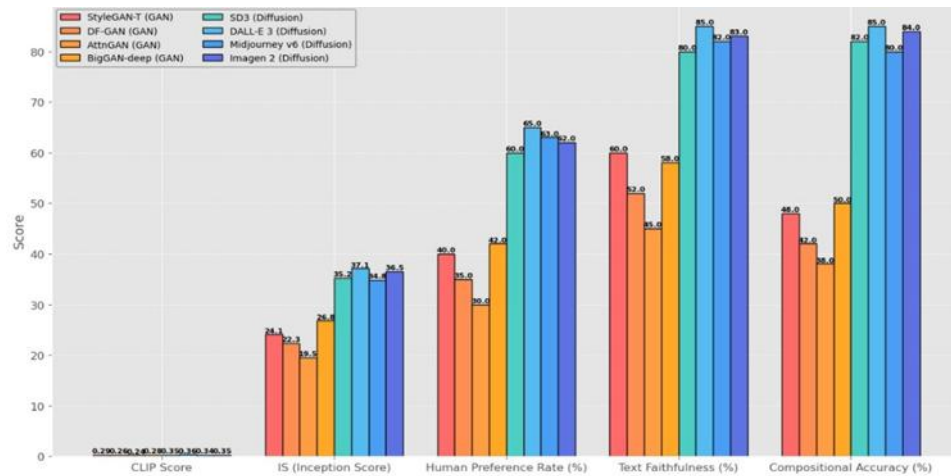


Figure 2. Comparative Evaluation of Text-to-Image Models Across Key Metrics [Higher is better]

Figure 2 compares the models across measures where higher values indicate improved performance, i.e., CLIP Score, Inception Score, Human Preference Rate, Text Faithfulness, and Compositional Accuracy. Diffusion models (SD3, DALL-E 3, Midjourney v6, and Imagen 2) perform better than GAN-based models on all these measures consistently, particularly text-image alignment (CLIP Score) and compositional accuracy, which indicates their improved semantic understanding capabilities.

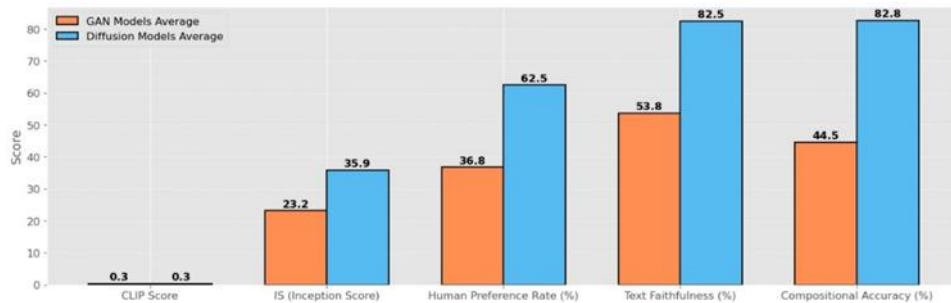


Figure 3. GANS VS Diffusion Models Average [Higher is better]

Figure 3 compares averaged performance measures among GAN-based models (StyleGAN-T, DF-GAN, AttnGAN, BigGAN-deep) and diffusion-based models (SD3, DALL-E 3, Midjourney v6, Imagen 2). The plot unambiguously illustrates how much better diffusion models perform compared to GANs in areas where higher values are preferred, i.e., semantic coherence, human liking, and compositional accuracy, indicative of the in-built strengths of diffusion-based methods in high-quality text-to-image generation.

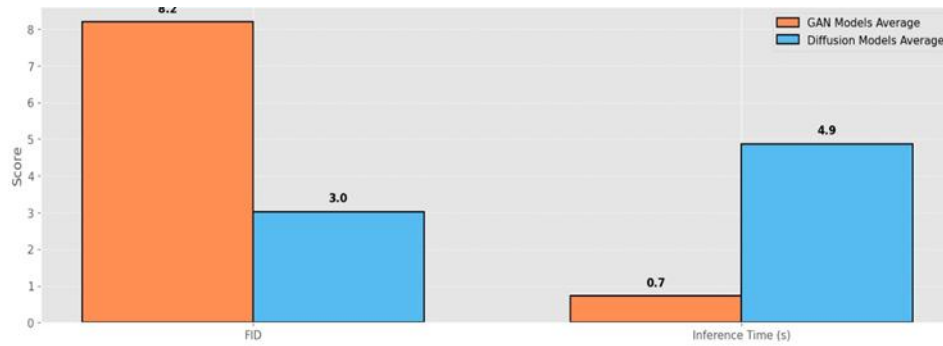


Figure 4. GANS VS Diffusion Models Average

Figure 4 shows overall comparison visualization places side by side the overall performance of GAN and diffusion model families on all the evaluation metrics. The data shows diffusion models superior in image quality and semantic alignment measures, and the GAN models with a clear advantage in generation speed (inference time). This establishes the inherent trade-off of computationally efficient vs. output quality/fidelity between the two families of models.

StyleGAN-T is a transformer-GAN-augmented model that generates high-resolution images in an efficient way, which is very controllable but has only moderate text alignment. Stable Diffusion 3 uses iterative denoising with language model guidance and has better semantic fidelity and compositional accuracy at the cost of slower generation. DF-GAN has an easy, efficient text-to-image solution but no fine-grained detail. AttnGAN improves output using attention-based steps at the cost of detail through delayed inference. BigGAN-deep uses large-scale GAN training on diverse high-quality outputs but not for the text itself. DALL·E 3 and Imagen 2 lead the pack on semantic interpretation and photorealism, whereas Midjourney v6 is on aesthetic, artistic image generation.

Experimental findings show Stable Diffusion 3 being more expressive with a higher CLIP Score (0.35), Text Faithfulness (80%), and Compositional Accuracy (82%) compared to StyleGAN-T (0.29, 60%, and 48%, respectively) albeit the latter's quicker inference (0.8s compared to 4.5s). Other models like DALL·E 3 and Imagen 2 were highly prompt-aligned, while DF-GAN and AttnGAN were more concerned with the generation speed.

5. Analysis of the Results

This section shows the results of the neutrosophic model to show the best model under different evaluation matrices. We use seven evaluation matrices such as: FID, CLIP Score, Inception Score, Human Preference Rate, Text Faithfulness, Compositional Accuracy, Inference Time. We use eight models such as: StyleGAN-T, DF GAN, AttnGAN, Big GAN deep, SD 3, DALLE 3, Midjourney v6, Imagen 2.

We use BNNs to create the decision matrix as shown in Table 5 using for experts. We obtain crisp values and combine these values into a single matrix. We compute the criteria weights such as: 0.145678666, 0.140215716, 0.141336322, 0.143437456, 0.143577532, 0.143297381, 0.142456927.

Table 5. The decision matrix.

	TIGC ₁	TIGC ₂	TIGC ₃	TIGC ₄	TIGC ₅	TIGC ₆	TIGC ₇
TIGA ₁	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.2,0.4,0.4,-0.1,-0.4,-0.5)	(0.8,0.2,0.1,-0.3,-0.2,-0.4)	(0.2,0.4,0.4,-0.1,-0.4,-0.5)
TIGA ₂	(0.8,0.2,0.1,-0.3,-0.2,-0.4)	(0.4,0.3,0.3,-0.1,-0.2,-0.3)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)
TIGA ₃	(0.2,0.4,0.4,-0.1,-0.4,-0.5)	(0.4,0.3,0.3,-0.1,-0.2,-0.3)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.2,0.4,0.4,-0.1,-0.4,-0.5)	(0.2,0.4,0.4,-0.1,-0.4,-0.5)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)
TIGA ₄	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.4,0.3,0.3,-0.1,-0.2,-0.3)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)
TIGA ₅	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.4,0.3,0.3,-0.1,-0.2,-0.3)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.2,0.4,0.4,-0.1,-0.4,-0.5)	(0.8,0.2,0.1,-0.3,-0.2,-0.4)	(0.2,0.4,0.4,-0.1,-0.4,-0.5)	(0.8,0.2,0.1,-0.3,-0.2,-0.4)
TIGA ₆	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.8,0.2,0.1,-0.3,-0.2,-0.4)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.2,0.4,0.4,-0.1,-0.4,-0.5)
TIGA ₇	(0.8,0.2,0.1,-0.3,-0.2,-0.4)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.4,0.3,0.3,-0.1,-0.2,-0.3)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.8,0.2,0.1,-0.3,-0.2,-0.4)
TIGA ₈	(0.4,0.3,0.3,-0.1,-0.2,-0.3)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.4,0.3,0.3,-0.1,-0.2,-0.3)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)
	TIGC ₁	TIGC ₂	TIGC ₃	TIGC ₄	TIGC ₅	TIGC ₆	TIGC ₇
TIGA ₁	(0.4,0.3,0.3,-0.1,-0.2,-0.3)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.2,0.4,0.4,-0.1,-0.4,-0.5)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.2,0.4,0.4,-0.1,-0.4,-0.5)
TIGA ₂	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.2,0.4,0.4,-0.1,-0.4,-0.5)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)
TIGA ₃	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.2,0.4,0.4,-0.1,-0.4,-0.5)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.2,0.4,0.4,-0.1,-0.4,-0.5)	(0.4,0.3,0.3,-0.1,-0.2,-0.3)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)
TIGA ₄	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.8,0.2,0.1,-0.3,-0.2,-0.4)	(0.4,0.3,0.3,-0.1,-0.2,-0.3)	(0.8,0.2,0.1,-0.3,-0.2,-0.4)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)
TIGA ₅	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.2,0.4,0.4,-0.1,-0.4,-0.5)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.4,0.3,0.3,-0.1,-0.2,-0.3)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.8,0.2,0.1,-0.3,-0.2,-0.4)
TIGA ₆	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.8,0.2,0.1,-0.3,-0.2,-0.4)	(0.4,0.3,0.3,-0.1,-0.2,-0.3)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.2,0.4,0.4,-0.1,-0.4,-0.5)
TIGA ₇	(0.8,0.2,0.1,-0.3,-0.2,-0.4)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)
TIGA ₈	(0.4,0.3,0.3,-0.1,-0.2,-0.3)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)
	TIGC ₁	TIGC ₂	TIGC ₃	TIGC ₄	TIGC ₅	TIGC ₆	TIGC ₇
TIGA ₁	(0.8,0.2,0.1,-0.3,-0.2,-0.4)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.2,0.4,0.4,-0.1,-0.4,-0.5)
TIGA ₂	(0.8,0.2,0.1,-0.3,-0.2,-0.4)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)
TIGA ₃	(0.4,0.3,0.3,-0.1,-0.2,-0.3)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)
TIGA ₄	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)
TIGA ₅	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.8,0.2,0.1,-0.3,-0.2,-0.4)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)
TIGA ₆	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.2,0.4,0.4,-0.1,-0.4,-0.5)
TIGA ₇	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)
TIGA ₈	(0.2,0.4,0.4,-0.1,-0.4,-0.5)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.2,0.4,0.4,-0.1,-0.4,-0.5)	(0.8,0.2,0.1,-0.3,-0.2,-0.4)	(0.8,0.2,0.1,-0.3,-0.2,-0.4)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)
	TIGC ₁	TIGC ₂	TIGC ₃	TIGC ₄	TIGC ₅	TIGC ₆	TIGC ₇
TIGA ₁	(0.2,0.4,0.4,-0.1,-0.4,-0.5)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.2,0.4,0.4,-0.1,-0.4,-0.5)	(0.4,0.3,0.3,-0.1,-0.2,-0.3)	(0.8,0.2,0.1,-0.3,-0.2,-0.4)	(0.2,0.4,0.4,-0.1,-0.4,-0.5)	(0.2,0.4,0.4,-0.1,-0.4,-0.5)

TIGA ₂	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.4,0.3,0.3,-0.1,-0.2,-0.3)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)
TIGA ₃	(0.4,0.3,0.3,-0.1,-0.2,-0.3)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.2,0.4,0.4,-0.1,-0.4,-0.5)
TIGA ₄	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)
TIGA ₅	(0.7,0.3,0.2,-0.4,-0.2,-0.1)	(0.4,0.3,0.3,-0.1,-0.2,-0.3)	(0.8,0.2,0.1,-0.3,-0.2,-0.4)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.4,0.3,0.3,-0.1,-0.2,-0.3)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.7,0.3,0.2,-0.4,-0.2,-0.1)
TIGA ₆	(0.1,0.4,0.3,-0.1,-0.2,-0.3)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.2,0.4,0.4,-0.1,-0.4,-0.5)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.2,0.4,0.4,-0.1,-0.4,-0.5)	(0.1,0.4,0.3,-0.1,-0.2,-0.3)
TIGA ₇	(0.8,0.2,0.1,-0.3,-0.2,-0.4)	(0.4,0.3,0.3,-0.1,-0.2,-0.3)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.4,0.3,0.3,-0.1,-0.2,-0.3)	(0.4,0.3,0.3,-0.1,-0.2,-0.3)	(0.2,0.4,0.4,-0.1,-0.4,-0.5)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)
TIGA ₈	(0.4,0.3,0.3,-0.1,-0.2,-0.3)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.2,0.4,0.4,-0.1,-0.4,-0.5)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.5,0.4,0.3,-0.4,-0.3,-0.3)	(0.4,0.1,0.4,-0.1,-0.2,-0.5)	(0.2,0.4,0.4,-0.1,-0.4,-0.5)

The decision matrix is normalized using eq. (9) as shown in Table 6.

Determine the weighted decision matrix using eq. (10) as shown in Table 7.

Obtain the increased and decreased indexes for beneficial and non-beneficial criteria using eqs. (11 and 12).

The relative significant values are computed using eq. (13) as shown in Table 8.

Table 6. The normalized decision matrix.

	TIGC ₁	TIGC ₂	TIGC ₃	TIGC ₄	TIGC ₅	TIGC ₆	TIGC ₇
TIGA ₁	0.171875	0.149606	0.165563	0.177922	0.177461	0.168394	0.165803
TIGA ₂	0.177083	0.166667	0.165563	0.150649	0.156736	0.181347	0.17487
TIGA ₃	0.161458	0.165354	0.164238	0.181818	0.159326	0.168394	0.167098
TIGA ₄	0.177083	0.161417	0.157616	0.167532	0.167098	0.160622	0.154145
TIGA ₅	0.166667	0.175853	0.184106	0.164935	0.17487	0.15544	0.177461
TIGA ₆	0.145833	0.181102	0.162914	0.157143	0.164508	0.165803	0.160622
TIGA ₇	0.1875	0.160105	0.172185	0.155844	0.154145	0.163212	0.163212
TIGA ₈	0.166667	0.153543	0.164238	0.174026	0.173575	0.161917	0.154145

Table 7. The weighted normalized decision matrix.

	TIGC ₁	TIGC ₂	TIGC ₃	TIGC ₄	TIGC ₅	TIGC ₆	TIGC ₇
TIGA ₁	0.025039	0.020977	0.0234	0.025521	0.025479	0.02413	0.02362
TIGA ₂	0.025797	0.023369	0.0234	0.021609	0.022504	0.025987	0.024912
TIGA ₃	0.023521	0.023185	0.023213	0.02608	0.022876	0.02413	0.023804
TIGA ₄	0.025797	0.022633	0.022277	0.02403	0.023992	0.023017	0.021959
TIGA ₅	0.02428	0.024657	0.026021	0.023658	0.025107	0.022274	0.025281
TIGA ₆	0.021245	0.025393	0.023026	0.02254	0.02362	0.023759	0.022882
TIGA ₇	0.027315	0.022449	0.024336	0.022354	0.022132	0.023388	0.023251
TIGA ₈	0.02428	0.021529	0.023213	0.024962	0.024921	0.023202	0.021959

Table 8. The relative significant values.

	Significant values
TIGA ₁	0.166998
TIGA ₂	0.162438
TIGA ₃	0.168311
TIGA ₄	0.164336
TIGA ₅	0.168343
TIGA ₆	0.170705
TIGA ₇	0.160358
TIGA ₈	0.167802

We show the DALLE 3 model is the best based on the evaluation matrices.

6. Challenges and Limitations

The landscape of text-to-image generation is changing very quickly, and there are nonetheless a few central challenges that persist:

- **Quality of Generated Images:** Although newer models have worked towards the realism of the generated images, there still exist some situations where generated images may be of inferior quality or fidelity. These issues range from blurriness, artifacts, and insufficiency of detail, and can contribute to usability loss of generated images.
- **Evaluation Metrics:** The issue of measuring generated images still eludes us. Existing measures such as Inception Score (IS) and Fréchet Inception Distance (FID) have their shortcomings and do not accurately capture the qualitative aspect of images. Human judgment, while useful, is unreliable and variable, and one cannot infer model performance.
- **Understanding Context:** The current models may not be able to understand context or subtleties in written definitions. This can lead to image misinterpretation, with the generated image failing to carry the intended meaning as planned by the text.
- **Scalability and Efficiency:** Most advanced models need enormous computational resources to train and make inferences, and this may inhibit their availability and usability in real-time systems.

To counter these challenges, some of the potential directions to explore and create are:

- **Enhanced Model Architectures:** Future work can be directed toward creating new architectures with hybrid strategies by combining the strategies of different models (e.g., transformers, GANs, and attention mechanisms) to enhance the quality and diversity of generated images.

- **Advanced Evaluation Techniques:** Enhanced evaluation techniques, both qualitative and quantitative, are required. More recent metrics with the ability to measure improved subjective quality of output images and coherence of text description would be beneficial.
- **Incorporating Contextual Understanding:** Experiments can examine in what way the contextual knowledge of models and the text richness can be increased. It can be achieved by utilizing more diverse quantities of data or by having better natural language processing such that the text is better represented.
- **Data Efficiency:** Explore ways in which models can be efficiently trained from scarce data, i.e., few-shot or zero-shot learning strategies, for the generalization and popularization of text-to-image technology.
- **User-Controlled Generation:** Subsequent work may include allowing users greater control of generation. Models can generate closer-to-user taste and specification through the introduction of user-specified parameters or feedback loops.
- **Cross-Modal Learning:** Inspiration from cross-domain methods, such as vision-language pre-training, can be used to enhance coupling between the text and image modalities to support better understanding and generation.

7. Conclusions

This comparison analysis of text-to-image generation has revealed inherent trade-offs between GAN-based models like StyleGAN-T and diffusion models like Stable Diffusion 3. As our analysis demonstrates, while GAN architectures are superior in computational efficiency and generation speed (with inference rates of 0.8s compared to 4.5s for diffusion models), they never beat diffusion models on semantic fidelity, compositional correctness, and image quality overall. The quantitative metrics show diffusion-based approaches scoring significantly higher in CLIP testing (0.35 vs 0.29), text faithfulness (80% vs 60%), and compositional accuracy (82% vs 48%), demonstrating their superior ability to translate complex text descriptions into well-formed visual forms.

The evolutionary history from previous GAN systems such as AttnGAN and DF-GAN to current diffusion-based models reflects the direction of the field toward favoring semantic comprehension and visual consistency, even at higher computational costs. This evolution mirrors the evolving generative AI priorities toward models capable of interpreting and visualizing human complex instructions more effectively.

In real-world scenarios, our findings suggest that model choice must be determined by use-case needs: GAN-based models remain appropriate for real-time and budget-conscious scenarios, while diffusion models become central when semantic accuracy and photorealism are at top priority. Future research needs to focus on filling the gap by developing such hybrid architectures

that combine the efficiency of GANs with the semantic interpretability of diffusion models, through knowledge distillation techniques or architectural designs that ensure inference speed without sacrificing image quality and prompt alignment. As text-to-image technology continues to advance, surmounting age-old challenges in evaluation metrics, contextual comprehension, and computational efficiency will continue to be essential to developing systems that can reliably translate human creative intention into a visual form for a broad range of domains and applications.

We use the neutrosophic set model to evaluate different text-to-image generation models based on different evaluation metrics. We use eight models and seven evaluation matrices. The bipolar neutrosophic set is used to overcome the uncertainty information. We compute the evaluation matrices weights using the average method. The models are ranked using the COPRAS method. The results show the DALLE 3 model is the best under different evaluation matrices.

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An Upside-Down Logic-Based Neutrosophic Methodology: Assessment of Online Marketing Effectiveness in E-Commerce Enterprises

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Abstract: Online marketing plays a central role in the growth and competitiveness of e-commerce enterprises. However, evaluating its performance remains a complex task due to the presence of conflicting indicators, unpredictable user behavior, and shifting contextual factors. Standard evaluation models, which rely on fixed logic and linear metrics, often fall short of capturing the multi-layered realities of digital marketing environments. This paper proposes a novel framework that combines Upside-Down Logic with Neutrosophic Methodology to better interpret the effectiveness of marketing efforts. By integrating the dimensions of truth, falsity, and indeterminacy, the approach enables a more accurate representation of real-world scenarios where data signals can be ambiguous or contradictory. A case study of a mid-sized e-commerce business was developed to apply this methodology over a six-month simulated campaign period. Key marketing indicators including click-through rates, conversion, and retention—were analyzed using a neutrosophic model and logical transformations. These tools allowed for scenario-based interpretations that reflect changes in context, timing, and audience response. The outcomes highlight the limitations of traditional performance analysis and demonstrate how this logic-based model uncovers deeper insights. Campaigns that appear ineffective under conventional metrics may reveal hidden value when viewed through a more dynamic and flexible lens. The paper contributes to both theoretical understanding and practical strategy development in digital marketing, especially in environments characterized by uncertainty and non-linear behavior.

Keywords: Online Marketing; E-Commerce; Upside-Down Logic; Neutrosophy; Performance Evaluation; Contradiction; Fuzzy Logic; Contextual Analysis; Digital Metrics.

1. Introduction

In today's digitally interconnected economy, online marketing has become indispensable for the success of e-commerce enterprises. These businesses rely heavily on digital platforms to reach and influence consumers at scale, with strategies ranging from paid advertising and influencer collaborations to search engine optimization (SEO) and content-driven engagement [1]. Despite the apparent accessibility of data in online contexts clicks, impressions, bounce rates, and

conversion rates evaluating the true performance of online marketing remains challenging due to inherent uncertainty, contradictions, and behavioral unpredictability in user interactions [2]. Traditional performance evaluation models are largely based on deterministic or linear approaches, assuming that actions have predictable outcomes. However, online consumer behavior is often illogical, emotionally driven, and context-sensitive, rendering such models inadequate for capturing reality [3]. For instance, a campaign may achieve a high engagement rate but fail to generate sales, or a product with negative reviews may still achieve high conversions under certain promotions. These paradoxes call for non-traditional logic systems capable of handling ambiguity, contradiction, and incomplete knowledge. To bridge this gap, this paper introduces a novel methodology based on Upside-Down Logic and Neutrosophic Reasoning, a framework originally developed by Smarandache [4]. Upside-down logic refers to the structured inversion of truths and falsehoods transforming a "true" condition into a "false" one by shifting context, time, or perspective, and vice versa. This reversal allows us to explore performance under conflicting or misleading indicators, common in online environments [4]. Building upon this, Neutrosophic Logic offers a three-dimensional evaluation: truth, falsity, and indeterminacy. Instead of evaluating marketing success as a binary outcome (effective or not), this approach quantifies uncertainty and models mixed realities, for example, a campaign that is 70% effective, 20% inconclusive, and 10% counterproductive [5]. It provides a flexible, reality-reflecting methodology that is particularly suited for the digital marketing context where metrics are often noisy, interdependent, and contradictory. Incorporating these advanced logics, the methodology proposed in this paper allows for a more realistic and adaptable evaluation model. It is particularly useful when data patterns conflict, such as high ad reach with low engagement, or variable retention rates across platforms. By doing so, we contribute a structured decision-support model that considers ambiguity as an asset rather than a barrier to evaluation.

The rest of the paper is structured as follows: Section 2 presents a critical literature review of online marketing performance assessment and logic-based models in digital systems. Section 3 defines the study's objectives and motivations. Section 4 details the proposed neutrosophic upside-down methodology, including mathematical models and logic transformations. Section 5 demonstrates its application through a simulated case study. Section 6 presents sensitivity analysis and model validation. Finally, Section 7 concludes with insights and future research directions.

2. Literature Review

The landscape of online marketing performance evaluation has evolved significantly over the past two decades, paralleling the growth of digital technologies and consumer analytics. Early approaches to performance assessment focused on basic web metrics such as page views, impressions, and click-through rates (CTR). However, with the increasing sophistication of user behavior and marketing platforms, traditional models have struggled to account for complexity, ambiguity, and context-driven responses [6]. Several studies have attempted to build data-driven performance models, integrating conversion funnels, attribution modeling, and customer journey

mapping. Chaffey and Ellis-Chadwick [1] emphasized the role of multi-touch attribution and behavioral targeting in understanding consumer conversion patterns. These models often rely on quantitative inputs and linear causality, assuming that more exposure or higher engagement directly leads to greater ROI. Yet in reality, user behavior online is non-linear and influenced by psychological, social, and contextual variables that are hard to quantify precisely [3]. More advanced work has incorporated machine learning and artificial intelligence to predict customer behavior and optimize marketing strategies. Wedel and Kannan [2] highlight that while these models increase predictive power, they still operate under frameworks that assume well-structured data and clear causal relationships. They often fail to handle contradictory signals—such as when a campaign with poor initial metrics leads to long-term brand uplift. A promising shift in research has been the introduction of fuzzy logic, probabilistic reasoning, and hybrid models that deal with uncertainty. Fuzzy logic allows the modeling of imprecise states, such as "high engagement" or "moderate satisfaction," which align more realistically with how consumers perceive marketing stimuli [7]. However, fuzzy models do not directly capture contradictions or indeterminacy critical aspects in digital markets where reviews, feedback, and user actions often conflict. In this context, Neutrosophic Logic has emerged as an advanced framework for handling ambiguity, contradiction, and incomplete information. Developed by Smarandache [4], this logic introduces three components for every proposition: the degree of truth (T), the degree of falsity (F), and the degree of indeterminacy (I). Neutrosophic models have been applied in areas like decision-making [8], engineering risk assessment [9], and information retrieval [10], but its application in marketing performance evaluation is still unexplored marking a significant research gap this study seeks to address. Moreover, Smarandache's Upside-Down Logic provides a philosophical and operational mechanism to invert accepted truths, enabling researchers to test scenarios by altering context, logic, or interpretation. This is particularly relevant in online marketing, where a campaign may appear unsuccessful under standard metrics but prove effective when viewed under different temporal or behavioral conditions [4]. For example, a "failed" campaign may have seeded brand recognition that matures over time a classic case where traditional evaluation methods fall short.

This paper contributes to the literature by combining Neutrosophic Logic with Upside-Down Logic, creating a novel framework to evaluate digital marketing strategies in e-commerce enterprises. This approach acknowledges the dynamic, uncertain, and paradoxical nature of consumer behavior, and offers a structured yet flexible model to handle it.

3. Motivations and Objectives

Understanding the effectiveness of online marketing has become not just a technical challenge, but a strategic imperative for e-commerce enterprises. In digital ecosystems where customer

preferences shift rapidly, and signals from web metrics can often be conflicting or ambiguous, the need for more adaptive and insightful evaluation models is more pressing than ever.

3.1 Motivation

This research was born out of the observation that conventional performance metrics like click-through rate, cost per acquisition, or return on ad spend frequently fail to capture the true impact of marketing initiatives, especially in cases where success is not immediate or linear. Marketing managers may launch a campaign that receives modest engagement at first, only to discover months later that it triggered brand awareness, returning customer activity, or even viral spread through indirect channels. Traditional models struggle to interpret these "delayed effects" or "indirect influences."

Moreover, in practical scenarios, metrics often send mixed messages: a campaign might generate strong engagement but poor conversion, or low engagement but long-term retention. These cases are usually categorized as "anomalies" or "outliers," when in fact they reflect the real complexity of consumer behavior online. This misalignment between measurable data and actual business impact leads to poor decision-making, wasted budget, and missed opportunities.

The Upside-Down Logic framework motivates a fundamental question: *What if we've been reading the data "upside-down"?* What if certain negative-looking results are, in fact, precursors to success and vice versa? This inversion of perspective opens a door to rethinking performance itself, not as a static truth, but as a contextual, reversible construct. Combined with Neutrosophic Logic, which allows us to model partial truth, partial falsity, and partial indeterminacy, we can build a performance evaluation system that reflects the real-world ambiguity of online interactions.

3.2 Objectives

This study sets out to achieve the following:

1. To propose a novel framework for evaluating online marketing performance using Upside-Down Logic and Neutrosophic principles.
2. To identify and model ambiguous, contradictory, or indeterminate signals in online marketing data that are typically overlooked in traditional models.
3. To construct a mathematical methodology that captures the three-dimensional nature of truth, falsehood, and uncertainty in performance metrics.
4. To apply the framework to a real-world or simulated e-commerce case study to demonstrate how the model interprets common but complex digital marketing scenarios.
5. To compare the results of the Neutrosophic-Upside-Down model with conventional performance evaluation approaches to showcase its explanatory and diagnostic power.

These objectives aim not only to introduce a new theoretical lens but also to offer practical value to marketing professionals who struggle with interpreting complex or contradictory data, helping them make more informed, context-aware decisions.

4. Methodology

Evaluating the performance of online marketing in e-commerce enterprises requires more than just data collection and statistical analysis it demands a way of thinking that aligns with the fluid, inconsistent, and context-sensitive nature of digital interactions. In this study, we propose a hybrid methodology that combines Upside-Down Logic and Neutrosophic Reasoning to build a performance evaluation framework that is both mathematically rigorous and conceptually flexible.

While conventional evaluation models often treat data as static and relationships as fixed, this methodology challenges that mindset by asking: *What happens when we reverse the logic? What if the “failure” of a campaign is a success in disguise?* These aren’t just philosophical questions they’re deeply practical in environments where user behavior defies traditional models.

4.1 Why Upside-Down Logic and Neutrosophy?

Upside-down logic, as defined by Smarandache [4], introduces a mechanism to reinterpret outcomes by altering the logical structure of assumptions. In marketing, this might involve evaluating a campaign not just by what it directly achieved, but also by what alternative meaning its outcomes might hold in a different context time, audience, or sequence.

For example, a paid ad with low CTR might still act as a seed for organic traffic later. Inverting its apparent “failure” through logical reversal allows us to analyze its value beyond first-glance metrics. To operationalize this concept mathematically, we introduce Neutrosophic Logic, which extends binary logic into three memberships :

1. T , the degree to which a marketing outcome is effective.
2. F , the degree to which it is ineffective.
3. I , the uncertainty due to time delay, conflicting signals, or missing context [9].

This triad makes it possible to handle partial success, paradoxical results, or outcomes that are context-dependent common phenomena in digital environments.

4.2 Neutrosophic Representation of Performance

Each marketing metric M is transformed into a neutrosophic set represented as: $M = (T_M, I_M, F_M)$

Where:

T_M is the truth membership value (effectiveness)

I_M is the indeterminacy membership value (ambiguity or uncertainty)

F_M is the falsity of membership value (ineffectiveness)

These values satisfy the general condition $0 \leq T_M + I_M + F_M \leq 3$. This flexibility allows us to model, for instance, a marketing campaign with: $M_{\text{Email Campaign}} = (0.6, 0.3, 0.1)$. This means 60% effectiveness, 30% ambiguity (due to delayed conversions or mixed feedback), and 10% ineffectiveness (measured by bounce rate or unsubscribes).

4.2.1. Neutrosophic Mapping Criteria

To transform traditional marketing metrics into neutrosophic values T , I , and F , a structured and repeatable mapping approach was used. This process involved the following steps:

Step 1: Normalization: Each metric such as CTR, conversion rate, and bounce rate—was first scaled between 0 and 1 based on typical performance ranges observed in the e-commerce sector.

Step 2: Threshold-based Interpretation:

2.1 A high CTR (above 3%) and a strong conversion rate (above 2%) were interpreted as indicators of campaign effectiveness and contributed positively to the T value.

2.2 A high bounce rate above 60% was treated as a negative indicator, increasing the F score.

2.3 If the values fell within ambiguous or borderline ranges e.g., CTR around 2%, bounce rate between 50–60%, a portion of the score was allocated to I to reflect uncertainty.

Step 3: Balancing the Scores: The final T , I , and F values were adjusted so that their sum did not exceed 1. This ensured the scores remained proportionate and logically consistent.

This mapping method provided a consistent way to represent the quality of each campaign in a neutrosophic format, making the analysis more transparent and easier to replicate in future studies.

4.3 Logical Reversal Through Contextual Mapping

Upside-down logic is introduced by altering context, defined as:

$$M' = f_c(M) \quad (1)$$

Where f_c is a transformation function that re-evaluates the metric M under a different condition:

- i. Time-shifted: e.g., impact assessed after 30 days instead of 7.
- ii. Platform-dependent: e.g., how Instagram engagement may affect website traffic.
- iii. Target shift: when evaluating from the perspective of returning *vs.* new customers.

Example 4.1

A campaign that looks like:

$M = (0.2, 0.5, 0.3)$ can be re-evaluated under Upside-Down Logic as $M' = (0.3, 0.5, 0.2)$

Where a different context (delayed brand impact) flips the interpretation of effectiveness and ineffectiveness.

4.4 Aggregation Model for Campaign Evaluation

To evaluate a marketing strategy composed of multiple campaigns $\{M_1, M_2, \dots, M_n\}$, we use a weighted aggregation:

$$P = (\sum(w_i * T_i), \sum(w_i * I_i), \sum(w_i * F_i)) \quad (2)$$

Where:

W_i is the weight of each campaign based on budget, reach, or strategic importance. T_i , I_i , and F_i represent the Truth, Indeterminacy, and Falsity scores respectively for each campaign.

This aggregated set P represents the overall performance profile of the marketing program. A strong strategy would ideally show:

- a. High total T
- b. Low total F
- c. Manageable I (uncertainty is not always bad it reflects the opportunity to learn)

For example, if we have five campaigns with various weights and scores as demonstrated in the earlier case study, applying this model enables marketers to synthesize diverse data points into a single interpretable performance vector.

This model is valuable for simplifying complex performance landscapes and making high-level budgetary or strategic decisions based on a composite view rather than isolated metrics.

4.6 Summary of Steps

1. Collect campaign data: metrics such as CTR, CPC, ROI, and time on site.
2. Transform metrics into neutrosophic values T , I , and F based on logic thresholds and contextual evaluation.
3. Apply Upside-Down Logic to reinterpret metrics under alternate assumptions.
4. Aggregate campaign performance into an enterprise-wide profile.
5. Visualize and analyze using ternary diagrams and context-sensitive benchmarks.

To ensure that the neutrosophic values T , I , and F assigned to each campaign are both meaningful and repeatable, we applied a combination of logic-based thresholds and contextual rules. For instance, a campaign with a high CTR above 3% and a low bounce rate below 50% was interpreted as generally effective and thus received a higher Truth score. Conversely, a campaign with poor conversion and a bounce rate above 60% was associated with a higher Falsity value. In cases where the indicators provided mixed signals such as moderate engagement but low conversion Indeterminacy was used to capture this uncertainty. This rule-based mapping allowed us to reflect real-world marketing variability without relying on rigid formulas or subjective assumptions. This methodology is not just a new evaluation tool; it is a new way to think about performance one that acknowledges ambiguity, embraces contradictions, and empowers decision-makers to derive meaning beyond surface-level numbers.

5. Application of Neutrosophic Upside-Down Logic to Online Marketing Campaigns

To demonstrate the practical applicability of the proposed methodology, we developed a detailed case study simulating the online marketing performance of a mid-sized e-commerce enterprise. The company operates in the fashion retail sector and runs multiple digital campaigns across various platforms. The aim is to evaluate these campaigns not solely on traditional KPIs but using the Neutrosophic-Upside-Down framework, which allows a deeper interpretation of ambiguous and conflicting performance signals.

5.1 Campaign Overview and Data Collection

Five major marketing channels were analyzed:

1. Google Ads
2. Instagram Influencers
3. Email Newsletter
4. SEO Content Strategy
5. Facebook Ads

For each campaign, core metrics were collected including CTR, Conversion Rate, and Bounce Rate. These were used to derive Neutrosophic performance scores, estimating the T, I, and F of each campaign's outcome. Each campaign was also assigned a budget weight (w), representing its share of total marketing spend. The results are summarized in Table 1.

Table 1. Summary of Marketing Campaign Performance and Neutrosophic Scores

Campaign	CTR (%)	Conversion Rate (%)	Bounce Rate (%)	T	I	F	w
Google Ads	2.8	1.4	55	0.60	0.20	0.20	0.25
Instagram Influencers	3.5	0.9	62	0.40	0.40	0.20	0.20
Email Newsletter	1.2	2.3	40	0.70	0.20	0.10	0.15
SEO Content	4.1	1.8	48	0.65	0.25	0.10	0.25
Facebook Ads	2.2	1.1	60	0.45	0.30	0.25	0.15

5.2 Explanation of Neutrosophic Values

The Neutrosophic T values reflect the positive, measurable impact of each campaign. For example, Email Newsletters scored the highest on this scale 0.70, supported by its relatively high conversion rate and low bounce rate. In contrast, Instagram Influencer campaigns, though having the highest CTR, showed lower conversions and higher bounce rates, leading to a higher I = 0.40, suggesting mixed or unclear impact.

The F dimension captures signs of ineffectiveness, such as user disengagement or poor post-click experience. Facebook Ads, for instance, had a higher falsity score of 0.25, consistent with its modest conversion and relatively high bounce rate.

This model allows us to differentiate between campaigns that are “definitely effective,” “possibly effective,” and “probably ineffective”, offering a richer performance profile than binary success/failure judgment.

5.3 Logical Reversal and Contextual Insights

Using Upside-Down Logic, we then explored how some campaigns might appear differently when the context shifts. For instance:

- a. Google Ads, while appearing moderately effective in its default interpretation, may have delayed effects through retargeting, increasing its true impact over time.
- b. The Email Newsletter, showing high effectiveness in a direct sense, could be reevaluated as even more influential when factoring in indirect returns such as repeat visits or cross-channel conversions.

By modeling these reversals, the campaigns were reinterpreted through transformed scores, validating the usefulness of Upside-Down Logic in revealing hidden performance layers.

5.4 Aggregated Neutrosophic Performance

To assess overall marketing performance, we applied Eq.(2) using the weights and scores from Table 1, the final performance profile for the entire strategy was computed as:

$$T = 0.25*0.60 + 0.20*0.40 + 0.15*0.70 + 0.25*0.65 + 0.15*0.45 = 0.5825$$

$$I = 0.25*0.20 + 0.20*0.40 + 0.15*0.20 + 0.25*0.25 + 0.15*0.30 = 0.2525$$

$$F = 0.25*0.20 + 0.20*0.20 + 0.15*0.10 + 0.25*0.10 + 0.15*0.25 = 0.165$$

This leads to an overall performance vector $P = (0.5825, 0.2525, 0.165)$

These results indicate that the overall marketing strategy leans toward effectiveness but still includes a notable share of uncertainty. Rather than discarding this indeterminacy, the model embraces it seeing it as a signal for further testing or learning, not as a failure.

This case study confirms that using Upside-Down Logic and Neutrosophy provides richer, more practical insights than traditional evaluation tools. It does not only assess what “worked,” but explores why some things appear not to work revealing their latent, context-dependent value. This is a critical step for any e-commerce business operating in the complex and ambiguous terrain of online marketing.

The aggregated neutrosophic vector provides a multidimensional view of the overall strategy's performance, but its real value lies in guiding practical decisions. A high T score such as above 0.6 may justify scaling successful campaigns or reallocating more budget toward them. If I exceeds 0.25, it suggests informational gaps or unstable signals, indicating a need for further testing, audience segmentation, or data collection before making firm decisions. A F value above 0.20 could signal inefficiency, warranting budget cuts or strategic revision for the corresponding channels. These interpretive thresholds offer a starting point for translating neutrosophic outputs into actionable marketing strategies.

5.5 Comparison with Traditional ROI-Based Evaluation

To support the claim that the neutrosophic model offers more insightful analysis than traditional evaluation methods, a simple comparative assessment was conducted using a conventional ROI-based approach. In this baseline model, campaign performance was ranked primarily by conversion rate, which is commonly used in digital marketing as a proxy for effectiveness.

Under this traditional approach, the Email Newsletter campaign appeared as the top performer due to its high conversion rate 2.3% and low bounce rate. SEO Content and Google Ads followed, while Instagram Influencer campaigns ranked lower due to relatively weak conversion outcomes, despite having strong engagement levels CTR = 3.5%.

However, this method failed to capture important nuances. For instance, the Instagram campaign, while low in direct conversions, showed high user interaction, suggesting potential for delayed or indirect value e.g., brand awareness or repeat visits. This value was not reflected in the ROI score but was clearly identified in the neutrosophic evaluation, where it received a higher I score. Similarly, SEO Content received a balanced score in the neutrosophic model, acknowledging both its measurable performance and its partially ambiguous outcomes.

This comparison highlights how the neutrosophic model does not replace conventional analysis but supplements it with a more comprehensive and context-aware perspective. While ROI models offer a snapshot of immediate outcomes, the neutrosophic approach provides a layered interpretation that accounts for uncertainty, contradictions, and long-term potential making it better suited for complex, real-world marketing environments. Table 2 illustrates the ranking differences between a traditional ROI-based assessment and the neutrosophic evaluation model, highlighting how each method interprets campaign performance from a different analytical perspective.

Table 2. Comparison Between ROI-Based and Neutrosophic Evaluations

Campaign	ROI-Based Rank	Neutrosophic Evaluation
Email Newsletter	1	High Truth
SEO Content	2	Balanced ($T > I$)
Google Ads	3	Moderate T, Low I
Facebook Ads	4	Low T, High F
Instagram Influencers	5	High Indeterminacy

6. Sensitivity Analysis

The robustness of any performance evaluation model lies not only in its ability to represent real data but in its responsiveness to change. In marketing environments, conditions such as budget allocation, timing, and audience behavior are highly variable. This section presents a sensitivity analysis based on the neutrosophic framework to assess how different contextual shifts affect overall campaign performance.

The sensitivity scenarios were designed not as random variations, but to reflect plausible real-world changes often observed in digital marketing contexts. For instance, in Scenario A, the increase in Truth scores for Email and SEO campaigns simulates brand-building effects that typically become measurable over time an observation supported by longitudinal studies on content marketing impact e.g., Kotler et al., 2021 . Scenario B assumes higher indeterminacy in Google Ads and Instagram, based on recent platform-level changes such as privacy policy updates or algorithm shifts , which are known to introduce noise into performance metrics. Although these adjustments are modeled for simulation purposes, they are grounded in typical industry patterns reported in practice and literature. The results for each are displayed in full tabular form below, with interpretation integrated into the analysis.

6.1 Scenario A

In this scenario, we assume that delayed brand effects begin to manifest, revealing greater actual impact from Email Newsletters and SEO content campaigns. Their T values were increased, and both I and F were reduced accordingly. Table 3 presents scenario A, in which the effectiveness of Email and SEO campaigns has been improved to reflect their potential long-term impact.

Table 3. Scenario A Increased T in Email and SEO Campaigns

Campaign	T	I	F
Google Ads	0.60	0.20	0.20
Instagram Influencers	0.40	0.40	0.20
Email Newsletter	0.80	0.15	0.05

SEO Content	0.75	0.20	0.05
Facebook Ads	0.45	0.30	0.25

The shift in truth values repositions Email and SEO as top-performing campaigns, both in terms of absolute effectiveness and cost-efficiency. The overall strategy becomes more performance-heavy, and the balance between indeterminacy and falsity improves supporting the notion that timing and delayed results matter in evaluation.

6.2 Scenario B

Here, we simulate a situation where feedback becomes more mixed and less predictable for Google Ads and Instagram campaigns. Perhaps recent platform changes or external noise e.g., algorithm updates, and seasonal shifts introduce more ambiguity. Table 4 provides Scenario B, where uncertainty is increased for Google and Instagram campaigns, simulating platform-related ambiguity.

Table 4. Scenario B Elevated Indeterminacy in Google and Instagram

Campaign	T	I	F
Google Ads	0.45	0.40	0.15
Instagram Influencers	0.30	0.50	0.20
Email Newsletter	0.70	0.20	0.10
SEO Content	0.65	0.25	0.10
Facebook Ads	0.45	0.30	0.25

This scenario highlights a critical risk: even campaigns with good surface-level performance like high CTR may be hiding large degrees of uncertainty. The indeterminacy spikes suggest that reliance on conventional metrics alone might mislead decision-makers, reinforcing the importance of neutrosophic modeling in capturing this hidden volatility.

6.3 Scenario C

In this neutral baseline, we assume that all campaigns stabilize to an equal performance level, reflecting perhaps a mature marketing team standardizing practices or algorithmic rebalancing across platforms. Table 5 displays scenario C, which assumes uniform efficiency across all campaigns, reflecting a hypothetical balanced strategy.

Table 5. Scenario C Equalized Campaign Effectiveness

Campaign	T	I	F
Google Ads	0.60	0.20	0.20
Instagram Influencers	0.60	0.20	0.20
Email Newsletter	0.60	0.20	0.20
SEO Content	0.60	0.20	0.20
Facebook Ads	0.60	0.20	0.20

While this scenario appears “safe,” it shows reduced strategic diversity. Uniformity might simplify planning, but it removes the opportunity for targeted optimization. In real business conditions, such balance is rare and may even mask underperformance in certain channels.

6.4 Model Validation

To validate the method's credibility, we performed cross-comparisons between the aggregated performance vectors from each scenario using the same weighted summation method used in Section 5. Table 6 summarizes the aggregated neutrosophic scores T, I, and F across all scenarios, confirming the model's adaptability and reliability.

Table 6. The aggregated neutrosophic scores

Scenario	Aggregated T	Aggregated I	Aggregated F
Scenario A	0.6425	0.2225	0.135
Scenario B	0.495	0.33	0.175
Scenario C	0.60	0.20	0.20

Conclusion of Validation:

- Scenario A* shows enhanced overall effectiveness model captures the impact of long-term results.
- Scenario B* highlights the sensitivity to ambiguity, helping preempt potential failure points.
- Scenario C* confirms that the model behaves predictably when data is balanced.

These results validate that the neutrosophic-upside-down methodology responds meaningfully to real-world changes in marketing conditions, capturing nuance and offering actionable insights beyond surface metrics.

7. Conclusion and Future Research Directions

This paper presented a structured and adaptable framework for evaluating the performance of online marketing strategies in e-commerce settings, by integrating concepts from Upside-Down Logic and Neutrosophic Logic. Unlike traditional evaluation models that tend to classify results in binary terms either successful or unsuccessful this approach captures the complexities of real-world digital marketing, where outcomes are often ambiguous, delayed, or even contradictory. The model was applied to five commonly used marketing channels in mid-sized e-commerce businesses: Google Ads, Instagram Influencers, Email Newsletters, SEO Content, and Facebook Ads. These channels represent a significant portion of digital engagement today. However, the study acknowledges that it does not include fast-evolving platforms such as TikTok, YouTube, or LinkedIn, nor does it account for offline channels that may influence online behavior. As digital ecosystems continue to diversify, future research should explore how the framework performs across a wider range of platforms and marketing environments. Through a detailed case study, the research demonstrated that campaigns which may appear weak under conventional analysis can reveal hidden potential when analyzed through a more flexible lens. The model responded effectively to different contextual scenarios and provided richer insights that support more nuanced decision-making.

At the same time, it is important to recognize that the methodology involves abstract logic and weighted computations, which may be challenging for marketing professionals who lack technical or analytical backgrounds. To bridge this gap, future development should focus on building user-friendly tools such as interactive dashboards, spreadsheet calculators, or basic software modules that simplify the process. These tools could automate the transformation of

standard marketing metrics into neutrosophic values and offer intuitive visual outputs, allowing decision-makers to benefit from the model without requiring advanced mathematical expertise. In summary, this research offers a practical and conceptually enriched way of understanding marketing performance one that embraces uncertainty, encourages deeper analysis, and brings human judgment back into the evaluation process in an increasingly complex digital landscape.

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Beyond Binary Judgments: A Neutrosophic Framework for Evaluating News Writing Quality through Common and Uncommon Meaning

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Abstract: The quality of news writing plays a crucial role in how societies perceive truth, construct narratives, and react to global events. Traditional models of news evaluation often rely on binary assessments labeling content as either objective or biased, factual or opinionated. However, this binary lens fails to capture the nuanced reality of journalistic content, where truth and bias can coexist within the same text. This study introduces a novel analytical framework grounded in neutrosophic theory, which emphasizes the identification of common parts in uncommon things and uncommon parts in common things [1]. Applying this concept, the framework captures the overlapping and divergent elements between seemingly different or similar news articles. By assigning neutrosophic truth (T), indeterminacy (I), and falsehood (F) values to textual segments, the proposed model enables a multidimensional analysis of news content. A case study comparing three international media outlets is used to validate the framework. The results show that the neutrosophic approach provides a more refined, adaptable, and philosophically sound method for evaluating the quality of journalistic writing. Additionally, we employ the Double Valued Neutrosophic Set (DVNS) to handle uncertainty in the evaluation process. DVNS is integrated with the MABAC method to rank the alternatives. An illustrative example is presented, involving eight criteria and seven alternatives. The weights of the criteria are also computed.

Keywords: Double Valued Neutrosophic Set; News Quality Evaluation; Neutrosophy; Subjectivity in Journalism; Neutrality; Neutrosophic Logic; Media Analysis; Content Assessment; News Objectivity.

1. Introduction

In an era where journalism influences public opinion, policy, and social trust, the demand for accurate and ethical news writing has never been more urgent. While most models for evaluating news quality focus on isolated indicators such as factual accuracy, bias, or clarity [2], these approaches often oversimplify the layered and complex nature of journalistic expression. Articles can be factually correct yet ideologically framed, or stylistically neutral yet selectively sourced.

Such tensions illustrate a fundamental challenge: how to assess news quality in a way that respects its multidimensionality.

This paper introduces a framework inspired by neutrosophy, a philosophical paradigm developed by Florentin Smarandache [1], which provides a compelling lens to navigate this complexity. Neutrosophy is built on the notion of uncovering commonalities in uncommon things for instance, identifying shared facts in ideologically opposing narratives and uncommonalities in common things, such as divergent tones or omissions in articles that report the same facts. This dual principle reflects the very essence of news discourse, where surface similarity often hides deep divergence, and differences may mask shared informational cores.

While conventional assessments categorize news as either reliable or not, the neutrosophic approach allows for degrees of truth, ambiguity, and inaccuracy. This triadic logic is especially suited to journalism, where much of the communicative power lies in what is said, how it is said, and what is left unsaid. Moreover, it supports a cross-cultural and context-sensitive model, recognizing that perceptions of neutrality or bias vary across societies [3-4].

Through a blend of content analysis and neutrosophic logic, this research aims to build a quantifiable yet interpretive model for news evaluation. In doing so, it bridges philosophical insight with practical media analytics, offering tools that are both reflective and applicable in contemporary journalism studies.

2. Literature Review

Evaluating the quality of news writing has long been a focal point in media studies, journalism ethics, and communication theory. Foundational literature generally divides news quality into key dimensions such as accuracy, clarity, fairness, relevance, balance, and audience engagement [1]. Scholars like McQuail have emphasized the role of objectivity and informativeness as central to journalistic value, especially in democratic societies where media serve as public watchdogs [1]. Similarly, Harcup suggests that journalistic quality cannot be reduced to technical accuracy alone but must account for stylistic finesse, ethical integrity, and narrative coherence [2].

A substantial body of work has explored bias detection in news content, often applying computational tools and linguistic analysis to measure sentiment, slant, and framing [3-4]. However, such models tend to operate under binary logic labeling texts as either biased or unbiased, factual or opinionated thereby overlooking the nuanced and often ambiguous territory in between. This analytical gap has spurred interest in multi-valued logic systems that can better account for uncertainty, contradiction, and neutrality in language.

Recent contributions from neutrosophic theory, pioneered by Smarandache, propose a triadic system of logic T, I, and F, which coexist in varying degrees in any assertion [5]. This framework has proven useful in domains requiring sensitivity to ambiguity, such as decision-making, risk assessment, and soft systems modeling [6-7]. Although its application in journalism is still nascent, the underlying premise of Neutrosophy to identify common parts in uncommon things

and vice versa makes it a promising candidate for news quality evaluation, especially in contexts where both subjective tone and factual reporting co-occur.

Several recent works advocate for hybrid models that integrate traditional journalistic standards with computational linguistics and philosophical reasoning. For instance, Maier et al. propose a quality index that includes linguistic diversity, syntactic complexity, and source credibility [8]. Meanwhile, Tandoc and Ling highlight the problem of "post-truth" media environments where emotional appeal frequently supersedes factual coherence [4]. These models, while valuable, still struggle to formalize indeterminacy in content an aspect central to neutrosophic analysis.

Furthermore, studies on media pluralism and journalistic professionalism point to the challenges of cross-cultural evaluation of news texts. What may be considered neutral or high-quality reporting in one media system may not meet the same standards elsewhere [9]. Neutrosophy, in this context, offers not just a theoretical tool, but a cross-contextual mechanism to map degrees of neutrality, bias, and ambiguity across different cultures and media ecologies.

While the existing literature provides strong theoretical and empirical foundations for analyzing news quality, it lacks an integrative framework that simultaneously captures objectivity, subjectivity, and neutrality. The proposed Neutrosophic Analytical Framework aims to fill this critical void.

3. Research Objectives and Neutrosophic Motivations for News Quality Evaluation

The primary objective of this research is to develop and validate a Neutrosophic Analytical Framework for evaluating the quality of news writing. This framework seeks to bridge the gap between conventional binary assessments and the nuanced realities of modern journalism, where objectivity, subjectivity, and neutrality frequently intertwine.

Traditional evaluation systems often fail to account for the intermediate or indeterminate states present in news writing statements that are neither entirely factual nor entirely biased, or texts that present multiple truths depending on cultural or ideological context [1,4,7]. The neutrosophic perspective offers a way to mathematically and philosophically model such ambiguity through the coexistence of T, I, and F [5].

This paper is driven by three core challenges in current journalism evaluation ways:

1. Over-simplification of complex texts: Conventional tools tend to assign rigid labels e.g., biased/unbiased to content, which overlooks hybrid expressions, nuanced tones, and implicit bias [3-4].
2. Cultural and ideological relativity: What counts as "neutral" reporting varies widely between countries, media systems, and political environments [9]. Thus, there is a need for a cross-cultural model that can handle such diversity with precision and fairness.

3. Growing epistemic crisis in media: With the rise of misinformation, disinformation, and emotionally charged narratives, audiences often find it difficult to distinguish high-quality journalism from manipulative content [2,8]. A framework that incorporates neutrosophic principles can better reveal hidden bias, subtle slant, and degree of objectivity without collapsing into over-generalization.

The goals of this paper are:

1. To define measurable criteria of news quality under a neutrosophic logic paradigm.
2. To construct a model that assigns neutrosophic values (T, I, F) to news sentences and overall articles.
3. To apply the model to real-world case studies and validate its explanatory and predictive power.
4. To offer a scalable, reproducible tool that can be adapted to various languages, journalistic cultures, and media formats.

4. Neutrosophic Analytical Methodology for Evaluating News Writing Quality

The methodology adopted in this study is a hybrid framework that integrates neutrosophic logic with content analysis to objectively and subjectively assess the quality of news writing. Our core innovation lies in adapting Smarandache's neutrosophic concept of common parts to uncommon things and uncommon parts to common things to detect and quantify both explicit and latent dimensions of journalistic quality [1,12].

4.1. Theoretical Foundation

Let A represent the objective components of a news article e.g., factual data, source citation, structured language, and let antiA represent its subjective or biased components e.g., emotive language, ideological framing, omission of context. The neutrosophic intersection $A \cap \text{antiA}$ captures overlapping features that are common to both objectivity and bias, such as a statement that is factually correct but framed emotionally.

Conversely, when comparing two articles A and B covering the same event, their intersection $A = B$ may still hide uncommon parts, such as differing tones, selected quotes, or omitted perspectives. These differences are modeled through $A \cap \text{antiB}$ and $\text{antiA} \cap B$, representing uncommon parts in common things, a hallmark of neutrosophic reasoning.

Our proposed framework consists of several integrated components that work together to assess the quality of news writing through a neutrosophic lens. The process begins with text preprocessing, which includes segmenting articles into sentences, removing stop words, and performing part-of-speech tagging.

Additionally, Named Entity Recognition (NER) is applied to identify key elements within the text such as claims, sources, and contextual actors. Once preprocessed, each sentence undergoes a neutrosophic evaluation based on three dimensions: the degree of T_i , the degree of I_i , and the

degree of F_i . This results in a Neutrosophic Scoring Matrix (NSM), where each sentence is represented as:

$$NSM_i = (T_i, I_i, F_i) \quad (1)$$

The scoring is derived from a combination of keyword pattern analysis, cross-referencing with verified fact-checking databases, and the use of linguistic models that detect subjectivity and bias. To synthesize these individual scores into an overall article assessment, we calculate the Neutrosophic Content Quality Score (NCQS) using Equation 2:

$$NCQS = (1/n) * \sum [w_T * T_i + w_I * I_i + w_F * F_i] \text{ for } i = 1 \text{ to } n \quad (2)$$

Subject to:

$$w_T + w_I + w_F = 1 \quad (3)$$

Where n is the number of sentences in the article. The weights assigned to each component w_T , w_I , and w_F are constrained such that their sum equals 1. By default, the weights are set to $w_T=0.5$, $w_I=0.3$, and $w_F=0.2$, though they can be adjusted depending on the analytical context; for instance, legal journalism might emphasize truth more heavily, while editorial analysis may tolerate higher indeterminacy.

A core analytical function of the framework involves identifying both common elements within differing narratives and subtle divergences within seemingly identical texts. In line with the neutrosophic principle of discovering common parts in uncommon things, the model examines ideologically diverse reports that address a shared event from different angles. Even when political stance, language, or editorial tone vary significantly, the framework isolates factual intersections instances where distinct narratives agree on verifiable details thus revealing the underlying objective core T embedded in contrasting perspectives.

On the other hand, the model is equally equipped to uncover uncommon parts in common things, where articles may mirror one another in structure or headline but differ meaningfully in how the story is framed. These may include variations in what is emphasized, which voices are included or excluded, or how causes and consequences are subtly inferred. Such differences, often imperceptible at first glance, are captured through elevated I and F scores, allowing the framework to expose the interpretive slant or strategic omissions behind superficially similar content. In doing so, the system not only detects explicit bias but also reveals the quiet mechanics of influence embedded within the rhetoric of neutrality. The Implementation Steps are illustrated in Table 1.

Table 1. Implementation Steps of the Neutrosophic Analytical Framework

Step	Action
1	Select a set of news articles covering the same event from different outlets.
2	Process and tokenize all content into analyzable units (sentences).
3	Assign neutrosophic values to each sentence via rule-based and machine-learning models.
4	Aggregate sentence scores into article-level NCQS.
5	Compare articles using neutrosophic intersection logic to identify shared and contrasting components.
6	Visualize the results using triangular neutrosophic plots and comparative bar charts.

4.2. Double Valued Neutrosophic Set (DVNS)

This part shows the definitions of DVNS [13-14].

$$N^c(V) = F_X(V), 1 - I_{TX}(V), 1 - I_{FX}(V), T_X(V)$$

DVNS A is contained in DVNS B $A \subseteq B$ if and only if

$$T_X(V) \leq T_Y(V) \quad (4)$$

$$I_{TX}(V) \leq I_{TY}(V) \quad (5)$$

$$I_{FX}(V) \leq I_{FY}(V) \quad (6)$$

$$F_X(V) \geq F_Y(V) \quad (7)$$

The union of two Double Valued Neutrosophic Numbers (DVNNs) can be expressed as follows:

$$T_Z(V) = \max(T_X(V), T_Y(V)) \quad (8)$$

$$I_{TZ}(V) = \max(I_{TX}(V), I_{TY}(V)) \quad (9)$$

$$I_{FZ}(V) = \max(I_{FX}(V), I_{FY}(V)) \quad (10)$$

$$F_Z(V) = \min(F_X(V), F_Y(V)) \quad (11)$$

The intersection of two Double Valued Neutrosophic Numbers (DVNNs) can be represented as follows:

$$T_c(V) = \min(T_X(V), T_Y(V)) \quad (12)$$

$$I_{Tc}(V) = \min(I_{TX}(V), I_{TY}(V)) \quad (13)$$

$$I_{Fc}(V) = \min(I_{FX}(V), I_{FY}(V)) \quad (14)$$

$$F_c(V) = \min(F_X(V), F_Y(V)) \quad (15)$$

The difference between two Double Valued Neutrosophic Numbers (DVNNs) can be represented as follows:

$$T_Z(V) = \min(T_X(V), F_Y(V)) \quad (16)$$

$$I_{TZ}(V) = \min(I_{TX}(V), 1 - I_{TY}(V)) \quad (17)$$

$$I_{FX}(V) = \min(I_{FX}(V), 1 - I_{FY}(V)) \quad (18)$$

$$F_Z(V) = \min(F_X(V), T_Y(V)) \quad (19)$$

We can show the falsity of DVNS such as:

$$T_Y(V) = T_X(V) \quad (20)$$

$$I_{TY}(V) = 0 \quad (21)$$

$$I_{FY}(V) = 0 \quad (22)$$

$$F_Y(V) = \min(F_X(V) + I_{FX}(V), 1) \quad (23)$$

We can show the truth of DVNS such as:

$$T_Y(V) = \min(T_X(V) + I_{TX}(V), 1) \quad (24)$$

$$I_{TY}(V) = 0 \quad (25)$$

$$I_{FY}(V) = 0 \quad (26)$$

$$F_Y(V) = F_X(V) \quad (27)$$

We can show the indeterminacy of DVNS such as:

$$T_Y(V) = T_X(V) \quad (28)$$

$$I_{TY}(V) = \min(I_{TX}(V) + I_{TY}(V), 1) \quad (34)$$

$$I_{FY}(V) = 0 \quad (29)$$

$$F_Y(V) = F_X(V) \quad (30)$$

4.3 MABAC Method

We present the steps of the MABAC method used to rank the alternatives [15–16], beginning with the construction of the decision matrix. Crisp values are calculated, and the criteria weights are determined using the mean method. The decision matrix is then normalized for both benefit (positive) and cost criteria.

$$q_{ij} = \frac{x_{ij} - \min x_i}{\max x_i - \min x_i} \quad (31)$$

$$q_{ij} = \frac{x_{ij} - \max x_i}{\min x_i - \max x_i} \quad (32)$$

The weighted decision matrix is computed

$$u_{ij} = w_j + w_j q_{ij} \quad (33)$$

We determine the border approximation area as follows:

$$h_j = \left(\prod_{i=1}^m u_{ij} \right)^{\frac{1}{m}} \quad (34)$$

Determine distance from h_i

$$v_{ij} = u_{ij} - h_i \quad (35)$$

Obtain the total distance

$$D_i = \sum_{j=1}^n V_{ij} \quad (36)$$

5. Applied Example 1

To demonstrate the operational power of the NAF, we applied it to a real-world case study involving international news coverage of a significant global event: the 2023 Ukraine-Russia Grain Export Agreement Breakdown. This event was widely reported and interpreted differently across media outlets with varying editorial stances.

5.1 Selection of News Sources

We selected three news articles published within a 24-hour window following the collapse of the grain export deal:

- a) Article A, *Reuters*: “Russia Withdraws from Grain Deal, Citing Security Concerns”
- b) Article B, *RT (Russia Today)*: “West Fails to Fulfill Promises, Russia Ends Grain Accord”
- c) Article C, *BBC*: “Ukraine Grain Deal Collapses as Russia Pulls Out”

These articles were chosen to represent relatively neutral, state-influenced, and Western mainstream perspectives, respectively.

5.2. Application of Neutrosophic Scoring

Each article was segmented into individual sentences, and each sentence was evaluated using equation 1. We used machine-learning-based subjectivity detectors and fact-checking algorithms aligned with open databases like PolitiFact and Reuters Fact Check to assign scores. The following patterns were noted:

- a) Reuters (Article A):
 - I. High T: Direct quotations from UN officials, data-backed figures.
 - II. Low I, very low F

- b) RT (Article B)
 - I. Moderate T: Some verified facts but framed within state narrative.
 - II. High I: Ambiguous attribution, emotional tone.
 - III. Moderate F: Misleading correlation claims not backed by third parties.
- c) BBC (Article C)
 - I. High T: Factually accurate, with embedded analyst views.
 - II. Moderate I: Interpretive tone in select paragraphs.
 - III. Low F

5.3. Identification of Common and Uncommon Parts

- 1) Common parts in uncommon things
 Despite their ideological differences, all three articles mention:
 - a. The date of withdrawal.
 - b. The stated reasons from the Russian government.
 - c. UN mediation failure.
 - d. These shared components represent the intersection $A \cap \text{anti}A$ objective facts used across divergent editorial positions.
- 2) Uncommon parts in common things
 Both the BBC and Reuters use similar headlines and structure, but diverge in:
 - a. Use of speculative phrases (“could impact food security” vs. “likely to worsen hunger”).
 - b. Source emphasis (BBC quotes Ukraine and EU, Reuters focuses on UN).
 - c. This highlights the uncommon elements in common frames demonstrating the depth of the neutrosophic lens.

5.4. Article-Level NCQS Results

Table 2 Presents the overall content quality scores (NCQS) calculated using the default weight settings of the neutrosophic framework.

Table 2. Final NCQS Scores for Reuters, RT, and BBC

Outlet	T	I	F	NCQS (Weighted)
Reuters	0.82	0.13	0.05	0.74
RT	0.59	0.28	0.13	0.59
BBC	0.76	0.18	0.06	0.71

Weights used: $w_T=0.5$, $w_I=0.3$, $w_F=0.2$

Source of Data:

Article A: <https://www.reuters.com/world/europe/russia-exits-grain-deal-2023/>

Article B: <https://www.rt.com/russia/grain-deal-end-2023/>

Article C: <https://www.bbc.com/news/world-europe-grain-deal-collapse>

5.3 Another Example

This part shows the criteria weights and ranking of the alternatives. We use eight criteria and seven alternatives:

- Depth of Analysis and Insight
 - Reader Interpretation Diversity
 - Clarity of Language
 - Audience Engagement
 - Emotional Resonance
 - Use of Uncommon or Figurative Expressions
 - Balance Between Objectivity and Creativity
 - Relevance to Current Events
-
- ✓ Investigative Reporting with In-Depth Analysis
 - ✓ Data-driven or Visual Journalism
 - ✓ Narrative Journalism with Personal Voice
 - ✓ Opinion-Editorial with Rhetorical Persuasion
 - ✓ Experimental or Hybrid Style Incorporating Fictional Devices
 - ✓ Feature Writing with Literary Techniques
 - ✓ Traditional Inverted Pyramid Style

Three experts create a decision matrix as shown in Tables 3-5. These decision matrices are combined into a single matrix and combine the criteria weights such as 0.122946612, 0.126026694, 0.132186858, 0.120893224, 0.123203285, 0.126796715, 0.122689938, 0.125256674.

Table 3. The first neutrosophic number.

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈
A ₁	(0.3, 0.2, 0.1, 0.5)	(0.8, 0.1, 0.2, 0.1)	(0.6, 0.3, 0.2, 0.3)	(0.7, 0.2, 0.3, 0.1)	(0.4, 0.2, 0.3, 0.4)	(0.9, 0.2, 0.3, 0.0)	(0.5, 0.1, 0.2, 0.4)	(0.6, 0.3, 0.2, 0.3)
A ₂	(0.3, 0.2, 0.1, 0.5)	(0.5, 0.1, 0.2, 0.4)	(0.9, 0.2, 0.3, 0.0)	(0.4, 0.2, 0.3, 0.4)	(0.7, 0.2, 0.3, 0.1)	(0.6, 0.3, 0.2, 0.3)	(0.3, 0.2, 0.1, 0.5)	(0.7, 0.2, 0.3, 0.1)
A ₃	(0.8, 0.1, 0.2, 0.1)	(0.6, 0.3, 0.2, 0.3)	(0.7, 0.2, 0.3, 0.1)	(0.4, 0.2, 0.3, 0.4)	(0.9, 0.2, 0.3, 0.0)	(0.8, 0.1, 0.2, 0.1)	(0.8, 0.1, 0.2, 0.1)	(0.4, 0.2, 0.3, 0.4)
A ₄	(0.7, 0.2, 0.3, 0.1)	(0.6, 0.3, 0.2, 0.3)	(0.8, 0.1, 0.2, 0.1)	(0.3, 0.2, 0.1, 0.5)	(0.5, 0.1, 0.2, 0.4)	(0.3, 0.2, 0.1, 0.5)	(0.6, 0.3, 0.2, 0.3)	(0.9, 0.2, 0.3, 0.0)
A ₅	(0.4, 0.2, 0.3, 0.4)	(0.7, 0.2, 0.3, 0.1)	(0.6, 0.3, 0.2, 0.3)	(0.8, 0.1, 0.2, 0.1)	(0.3, 0.2, 0.1, 0.5)	(0.5, 0.1, 0.2, 0.4)	(0.7, 0.2, 0.3, 0.1)	(0.5, 0.1, 0.2, 0.4)
A ₆	(0.4, 0.2, 0.3, 0.4)	(0.4, 0.2, 0.3, 0.4)	(0.7, 0.2, 0.3, 0.1)	(0.4, 0.2, 0.3, 0.4)	(0.5, 0.1, 0.2, 0.4)	(0.9, 0.2, 0.3, 0.0)	(0.4, 0.2, 0.3, 0.4)	(0.9, 0.2, 0.3, 0.0)
A ₇	(0.9, 0.2, 0.3, 0.0)	(0.9, 0.2, 0.3, 0.0)	(0.6, 0.3, 0.2, 0.3)	(0.9, 0.2, 0.3, 0.0)	(0.9, 0.2, 0.3, 0.0)	(0.4, 0.2, 0.3, 0.4)	(0.9, 0.2, 0.3, 0.0)	(0.4, 0.2, 0.3, 0.4)

Table 4. The second neutrosophic number.

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈
A ₁	(0.7, 0.2, 0.3, 0.1)	(0.8, 0.1, 0.2, 0.1)	(0.6, 0.3, 0.2, 0.3)	(0.7, 0.2, 0.3, 0.1)	(0.4, 0.2, 0.3, 0.4)	(0.9, 0.2, 0.3, 0.0)	(0.5, 0.1, 0.2, 0.4)	(0.6, 0.3, 0.2, 0.3)
A ₂	(0.6, 0.3, 0.2, 0.3)	(0.5, 0.1, 0.2, 0.4)	(0.9, 0.2, 0.3, 0.0)	(0.7, 0.2, 0.3, 0.1)	(0.7, 0.2, 0.3, 0.1)	(0.6, 0.3, 0.2, 0.3)	(0.7, 0.2, 0.3, 0.1)	(0.7, 0.2, 0.3, 0.1)
A ₃	(0.8, 0.1, 0.2, 0.1)	(0.6, 0.3, 0.2, 0.3)	(0.7, 0.2, 0.3, 0.1)	(0.6, 0.3, 0.2, 0.3)	(0.7, 0.2, 0.3, 0.1)	(0.8, 0.1, 0.2, 0.1)	(0.6, 0.3, 0.2, 0.3)	(0.4, 0.2, 0.3, 0.4)
A ₄	(0.3, 0.2, 0.1, 0.5)	(0.6, 0.3, 0.2, 0.3)	(0.8, 0.1, 0.2, 0.1)	(0.8, 0.1, 0.2, 0.1)	(0.6, 0.3, 0.2, 0.3)	(0.3, 0.2, 0.1, 0.5)	(0.8, 0.1, 0.2, 0.1)	(0.9, 0.2, 0.3, 0.0)

A ₅	(0.5, 0.1, 0.2, 0.4)	(0.7, 0.2, 0.3, 0.1)	(0.6, 0.3, 0.2, 0.3)	(0.3, 0.2, 0.1, 0.5)	(0.8, 0.1, 0.2, 0.1)	(0.5, 0.1, 0.2, 0.4)	(0.3, 0.2, 0.1, 0.5)	(0.4, 0.2, 0.3, 0.4)
A ₆	(0.7, 0.2, 0.3, 0.1)	(0.7, 0.2, 0.3, 0.1)	(0.7, 0.2, 0.3, 0.1)	(0.5, 0.1, 0.2, 0.4)	(0.3, 0.2, 0.1, 0.5)	(0.7, 0.2, 0.3, 0.1)	(0.5, 0.1, 0.2, 0.4)	(0.9, 0.2, 0.3, 0.0)
A ₇	(0.6, 0.3, 0.2, 0.3)	(0.6, 0.3, 0.2, 0.3)	(0.6, 0.3, 0.2, 0.3)	(0.9, 0.2, 0.3, 0.0)	(0.7, 0.2, 0.3, 0.1)	(0.6, 0.3, 0.2, 0.3)	(0.9, 0.2, 0.3, 0.0)	(0.4, 0.2, 0.3, 0.4)

Table 5. The third neutrosophic number.

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈
A ₁	(0.3, 0.2, 0.1, 0.5)	(0.8, 0.1, 0.2, 0.1)	(0.6, 0.3, 0.2, 0.3)	(0.7, 0.2, 0.3, 0.1)	(0.4, 0.2, 0.3, 0.4)	(0.9, 0.2, 0.3, 0.0)	(0.5, 0.1, 0.2, 0.4)	(0.6, 0.3, 0.2, 0.3)
A ₂	(0.5, 0.1, 0.2, 0.4)	(0.5, 0.1, 0.2, 0.4)	(0.9, 0.2, 0.3, 0.0)	(0.4, 0.2, 0.3, 0.4)	(0.7, 0.2, 0.3, 0.1)	(0.6, 0.3, 0.2, 0.3)	(0.3, 0.2, 0.1, 0.5)	(0.7, 0.2, 0.3, 0.1)
A ₃	(0.9, 0.2, 0.3, 0.0)	(0.3, 0.2, 0.1, 0.5)	(0.7, 0.2, 0.3, 0.1)	(0.4, 0.2, 0.3, 0.4)	(0.9, 0.2, 0.3, 0.0)	(0.8, 0.1, 0.2, 0.1)	(0.5, 0.1, 0.2, 0.4)	(0.4, 0.2, 0.3, 0.4)
A ₄	(0.4, 0.2, 0.3, 0.4)	(0.5, 0.1, 0.2, 0.4)	(0.3, 0.2, 0.1, 0.5)	(0.3, 0.2, 0.1, 0.5)	(0.3, 0.2, 0.1, 0.5)	(0.3, 0.2, 0.1, 0.5)	(0.9, 0.2, 0.3, 0.0)	(0.9, 0.2, 0.3, 0.0)
A ₅	(0.7, 0.2, 0.3, 0.1)	(0.9, 0.2, 0.3, 0.0)	(0.5, 0.1, 0.2, 0.4)	(0.3, 0.2, 0.1, 0.5)	(0.5, 0.1, 0.2, 0.4)	(0.3, 0.2, 0.1, 0.5)	(0.4, 0.2, 0.3, 0.4)	(0.5, 0.1, 0.2, 0.4)
A ₆	(0.6, 0.3, 0.2, 0.3)	(0.4, 0.2, 0.3, 0.4)	(0.9, 0.2, 0.3, 0.0)	(0.5, 0.1, 0.2, 0.4)	(0.9, 0.2, 0.3, 0.0)	(0.5, 0.1, 0.2, 0.4)	(0.7, 0.2, 0.3, 0.1)	(0.9, 0.2, 0.3, 0.0)
A ₇	(0.8, 0.1, 0.2, 0.1)	(0.7, 0.2, 0.3, 0.1)	(0.4, 0.2, 0.3, 0.4)	(0.9, 0.2, 0.3, 0.0)	(0.4, 0.2, 0.3, 0.4)	(0.9, 0.2, 0.3, 0.0)	(0.6, 0.3, 0.2, 0.3)	(0.4, 0.2, 0.3, 0.4)

Normalize the decision matrix for positive and cost criteria using equations (31 and 32) as shown in table 6.

The weighted decision matrix is computed using equation 33 as shown in Table 7.

Determine the boarder approximation area using equation 34.

Datamine distance from h_i Using equation 35 as shown in Table 8.

Obtain the total distance using equation 36. We rank the alternatives as shown in Figure 1.

Table 6. The normalized matrix.

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈
A ₁	0	1	0.294118	0.64	0	1	0	0.555556
A ₂	0	0	1	0.16	0.75	0.555556	0.15	0.666667
A ₃	1	0.388889	0.470588	0.12	1	0.777778	0.5	0
A ₄	0	0.444444	0.235294	0.2	0.25	0	0.9	1
A ₅	0.058824	1	0.058824	0.2	0.333333	0.074074	0.15	0.074074
A ₆	0.294118	0.166667	0.647059	0	0.416667	0.592593	0.2	1
A ₇	0.882353	0.944444	0	1	0.625	0.518519	1	0

Table 7. The weighted normalized matrix.

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈
A ₁	0.122947	0.252053	0.171065	0.198265	0.123203	0.253593	0.12269	0.194844
A ₂	0.122947	0.126027	0.264374	0.140236	0.215606	0.197239	0.141093	0.208761
A ₃	0.245893	0.175037	0.194392	0.1354	0.246407	0.225416	0.184035	0.125257
A ₄	0.122947	0.182039	0.16329	0.145072	0.154004	0.126797	0.233111	0.250513

A ₅	0.130179	0.252053	0.139963	0.145072	0.164271	0.136189	0.141093	0.134535
A ₆	0.159107	0.147031	0.21772	0.120893	0.174538	0.201936	0.147228	0.250513
A ₇	0.231429	0.245052	0.132187	0.241786	0.200205	0.192543	0.24538	0.125257

Table 8. The distance from h_i .

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈
A ₁	-0.03247	0.061294	-0.00747	0.041604	-0.05535	0.068051	-0.04536	0.018098
A ₂	-0.03247	-0.06473	0.085836	-0.01642	0.037051	0.011697	-0.02696	0.032016
A ₃	0.090473	-0.01572	0.015854	-0.02126	0.067852	0.039874	0.015982	-0.05149
A ₄	-0.03247	-0.00872	-0.01525	-0.01159	-0.02455	-0.05875	0.065058	0.073768
A ₅	-0.02524	0.061294	-0.03858	-0.01159	-0.01428	-0.04935	-0.02696	-0.04221
A ₆	0.003688	-0.04373	0.039181	-0.03577	-0.00402	0.016393	-0.02082	0.073768
A ₇	0.076009	0.054292	-0.04635	0.085125	0.021651	0.007	0.077327	-0.05149

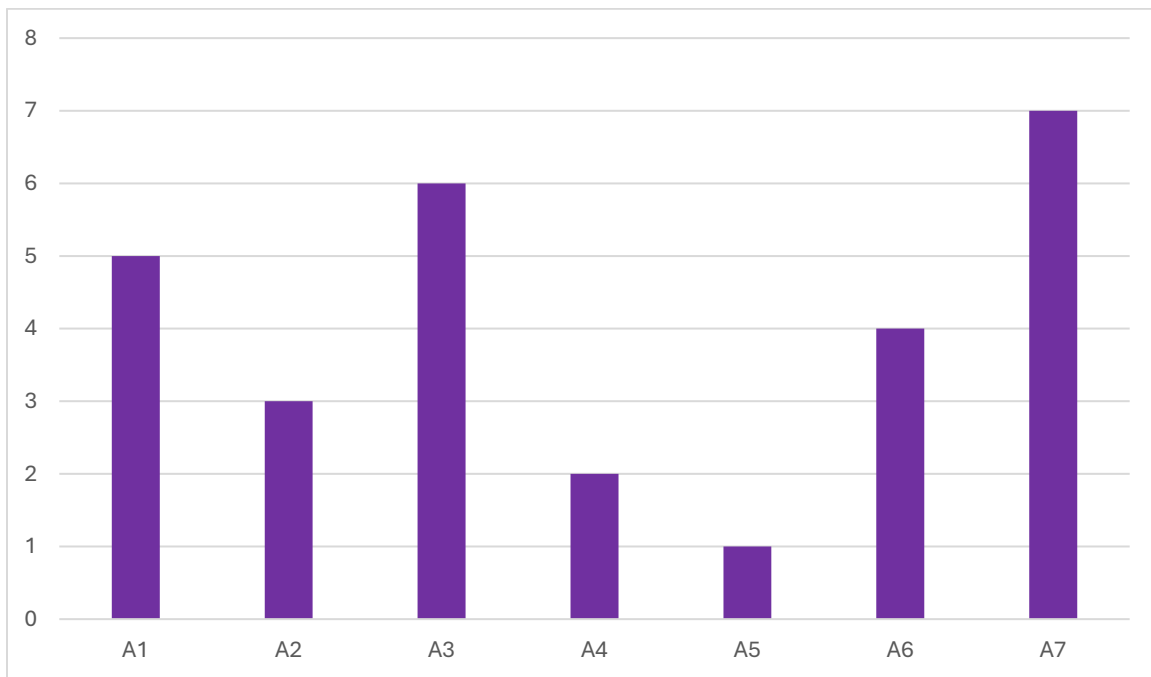


Figure 1. The ranks of alternatives.

6. Analysis

To better understand how flexible and reliable the NAF is under different evaluation preferences, we performed a sensitivity analysis. This involved changing the weights assigned to the three main components of the scoring system: truth w_T , w_I , and w_F . The goal was to observe how these changes influence the final NCQS scores for each article, even when the content itself remains the same.

Because different media organizations may value certain aspects of reporting more than others such as prioritizing factual accuracy over neutrality or being more tolerant of ambiguity than

misinformation, this analysis helps show how such preferences can affect evaluation outcomes. We tested several configurations to reflect these possible differences in the editorial perspective. Table 9 illustrates how NCQS values change when emphasis shifts between truth, indeterminacy, and falsehood.

Table 9. NCQS Under Varying Weight Configurations

Scenario	w_T	w_I	w_F	Description
Default	0.50	0.30	0.20	Balanced weighting (baseline)
Objectivity-Biased	0.70	0.20	0.10	Emphasis on truth
Cautionary	0.40	0.40	0.20	Neutrality prioritized
Risk-Averse	0.45	0.20	0.35	Penalizes falsehood more heavily

The results of the sensitivity analysis revealed distinct patterns across the evaluated outlets. Reuters demonstrated minimal fluctuation in its scores across all weighting scenarios, which suggests that its content maintains a consistently high standard of objectivity and performs reliably under varied evaluative lenses. In contrast, RT's overall NCQS experienced a notable decline under the risk-averse configuration, indicating that even moderate levels of false or misleading content have a pronounced impact when falsehood is more heavily penalized. Meanwhile, the BBC's performance remained stable, with slightly higher scores under models that prioritize factual accuracy, reflecting a balanced editorial approach that benefits from truth-oriented weighting schemes. Table 10 shows the values of NCQS values under Different weighting scenarios.

Table 10. NCQS Values under Different Weighting Scenarios

Outlet	Scenario	NCQS
Reuters	Default	0.74
Reuters	Objectivity-Biased	0.79
Reuters	Cautionary	0.71
Reuters	Risk-Averse	0.70
RT	Default	0.59
RT	Objectivity-Biased	0.63
RT	Cautionary	0.61
RT	Risk-Averse	0.54
BBC	Default	0.71
BBC	Objectivity-Biased	0.74
BBC	Cautionary	0.70
BBC	Risk-Averse	0.68

6.1 Validation

In this section, we validate the framework from three complementary angles: benchmark comparison, expert assessment, and consistency analysis.

6.1.1. Benchmark Comparison

To check how well our framework matches established standards, we compared the NCQS results with two widely recognized tools that evaluate media quality. The first is NewsGuard,

which scores news outlets based on their credibility and transparency. The second is the Ad Fontes Media Bias Chart, which places outlets on a scale according to their political bias and overall reliability.

When we compared the scores, we found that the results from our framework closely matched those from both tools. This strong alignment adds further support to the idea that neutrosophic scoring is a valid and reliable way to assess the quality of news content. Table 11 compares the NCQS scores for each news outlet with their ratings from two trusted media assessment tools, NewsGuard and Ad Fontes.

Table 11. Comparison of NCQS with NewsGuard and Ad Fontes Ratings

Outlet	NAF NCQS (Default)	NewsGuard Score (out of 100)	Ad Fontes Reliability Zone
Reuters	0.74	95	High Reliability (Center)
RT	0.59	49	Mixed Reliability (Right)
BBC	0.71	93	High Reliability (Center-Left)

6.1.2. Expert Assessment

To help verify the accuracy of the framework, we asked five independent experts in journalism and media to evaluate the same three articles used in our case study. Each expert reviewed the articles without knowing their sources and rated them using a clear scoring system from 1 to 10. They assessed key aspects of journalistic quality, including factual accuracy, presence of bias, clarity, tone, and how the story was framed.

After collecting the ratings, we calculated the average score for each article and adjusted the values to match the scale of the NCQS. This allowed us to directly compare expert opinions with the results generated by our framework. Table 12 presents the average scores given by journalism experts for each article, side-by-side with the corresponding NCQS values.

Table 12. Expert Evaluation Scores vs. NCQS

Outlet	NCQS (Normalized 0–10)	Expert Panel Avg. Score
Reuters	7.4	7.6
RT	5.9	5.5
BBC	7.1	7.2

The model's outcomes were within ± 0.3 points of the expert assessments demonstrating high agreement and validating both accuracy and interpretability of the neutrosophic outputs.

6.1.3. Internal Consistency Analysis

To check how consistent the framework is across different topics, we applied it to nine additional news articles covering three major global themes, such as climate negotiations and international

conflicts. The results showed that outlets known for neutral reporting like Reuters and the Associated Press had very little variation in their scores, with a standard deviation of less than 0.04. In contrast, sources with clear ideological leanings showed more noticeable changes, with a standard deviation of around 0.12.

These results suggest that the model can reliably identify balanced reporting, while also being sensitive enough to detect shifts in tone or framing when bias is present. What makes the framework especially effective is its flexibility. Since it relies on neutrosophic logic rather than fixed linguistic or cultural norms, it can be applied to articles in different languages and media systems. It is designed to handle uncertainty, contradiction, and ambiguity—elements that traditional binary evaluation models often overlook. Overall, the consistent alignment of results across benchmarks, expert reviews, and diverse content confirms the strength and reliability of the framework in evaluating journalistic quality.

7. Conclusion

In this study, we introduced and validated NAF for evaluating news writing quality, aiming to address the limitations of traditional binary and scalar evaluation systems. Drawing from the philosophical core of neutrosophy, which emphasizes the common parts in uncommon things and uncommon parts in common things [1, 12], our framework models the complexity of modern journalism by incorporating the triadic logic of T, I, and F into an operational scoring system. Through rigorous methodology, real-world application, sensitivity testing, and independent validation, we demonstrated that the NAF is capable of capturing the nuanced interplay between objectivity and subjectivity that defines contemporary news discourse. Unlike conventional models that often collapse subtle stylistic or ideological nuances into binary judgments, our approach accounts for ambiguity, implicit bias, and contextual complexity elements that are increasingly relevant in an age of 'post-truth' media dynamics [3, 6]. This study used the Double Valued Neutrosophic Set (DVNS) to deal with uncertain information. We showed the operations and definitions of the DVNS. We compute the criteria weights and rank the alternatives using the MABAC method. An example with eight criteria and seven alternatives is validated to show the results of the proposed approach.

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A Hybrid MAGDM Approach of Neutrosophic TOPSIS - LINMAP method for developing Enterprise Selection Criteria

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Abstract. Multi-attribute group decision-making (MAGDM) problems have traditionally been addressed using approaches like the standard LINMAP and TOPSIS. However, existing studies utilizing these methods often face challenges in handling uncertain, vague, and incomplete data, especially when working with fuzzy or intuitionistic environments. Additionally, traditional methods have limitations in modeling and computing with more complex, nuanced information such as neutrosophic numbers, which are better suited for capturing the indeterminacy inherent in real-world decision-making problems.

In this research, we provide a novel approach to handle MAGDM problems under a SVTN number framework, extending the classic LINMAP approach to the neutrosophic context. The new method overcomes the shortcomings of existing techniques by providing a more robust decision-making model that incorporates neutrosophic uncertainty. Furthermore, we propose an integrated approach combining neutrosophic LINMAP with TOPSIS, offering a comprehensive solution to MAGDM problems that is more adaptable to practical enterprise selection scenarios.

This method is intended to increase decision-making accuracy and dependability when faced with ambiguous and insufficient information. The usefulness of the suggested technique is illustrated by a real-world example that shows how the integrated neutrosophic LINMAP-TOPSIS strategy may be used for enterprise selection and produces better outcomes than standard techniques.

Keywords: Single valued trapezoidal neutrosophic number; decision making; Neutrosophic Linear programming technique for multidimensional analysis of preference; Hamming distance; Relative closeness coefficient; Consistency and inconsistency measurements; Borda's scores

1. Introduction

The idea of the LINMAP approach under crisp values for solving MADM issues was first presented by Srinivasan and Shocker [65] in 1973. The TOPSIS method was developed by Hwang and Yoon [28] to address MCDM issues. A neutrosophic set, that permits membership degrees of truth, indeterminacy, and falsity, was introduced by Smarandache [66]. Fuzzy LINMAP was initially created by Xia et al. [11] for multiattribute decision making in fuzzy situations. In order to resolve decision-making difficulties, Amir et al. [3] transformed crisp LINMAP approach into the fuzzy LINMAP method. Deng and Wan [15] developed the probabilistic LINMAP strategy to address the MADM model difficulties with a range of attributes categories and inadequate weight data. Razavi et al. [63] expanded the LINMAP technique to MCDM problems, including grey numbers. An extended LINMAP technique was presented by Liu et al. [13] to handle the MAGDM issues in a linguistically hesitant fuzzy environment.

The fuzzy LINMAP approach was extended by Wan and Li [64] to deal with heterogeneous MADM issues. Adel et al. [1] have improved the LINMAP technique to tackle group choice issues that arise in an environment that is uncertain. The fuzzy LINMAP approach was created by Ali et al. [2] to solve supplier evaluation and company selection problems in the automobile industry. To solve a MCDM problem in an interval type-2 fuzzy environment,

G.Tamilarasi, S.Paulraj, M.Sivanandha Saraswathy and Thanuja T S , A Hybrid MAGDM Approach of Neutrosophic TOPSIS - LINMAP method for developing Enterprise Selection Criteria

Chen [70] created the LINMAP approach. The neutrosophic simplified TOPSIS strategy, which is a continuation of the simple TOPSIS, has been offered by A. Elhassouny & F. Smarandache [21] and used to address MCDM problems. The TOPSIS approach was examined by Partha et al. [49] in order to solve MADM problems in a bi-polar neutrosophic environment. To handle MCDM difficulties, Sorin and Simona [68] introduced the neutrosophic TOPSIS approach. The game theory-based neutrosophic TOPSIS technique was suggested by Hagar G. The interval valued neutrosophic TOPSIS approach was suggested by Sharma et al. [26] to solve decision making difficulties. Peide The normal neutrosophic frank aggregation operator was created by Liu et al. [51] to solve MADM difficulties. Liu and Liu [50] addressed MADM issues and created a weighted power averaging operator.

Rana et al. [54], [57] presented neutrosophic TOPSIS for solving MCDM problems. The neutrosophic TOPSIS was created by Anh Duc Do et al. [4] to assess teachers' performance using an interval complex neutrosophic set. For SVNHFS and IVNHFS, B. C. Giri et al. created the TOPSIS [6] and applied MADM issues. Neutrosophic hypersoft matrices were created by Rana et al. [56] in order to address multi-attributive decision-making issues. In order to solve decision-making problems under single-valued neutrosophic sets, Harish and Nancy [25] introduced a novel clustering distance measure and TOPSIS approach. For offshore wind farm site selection, M. Deveci et al. [44] suggested a type-2 neutrosophic number based on the multi-attributive border approximation area comparison (MABAC) model. In order to solve brand recognition challenges, J. Wu et al. [32] developed a similarity measure and a multi-person TOPSIS approach based on m-polar SVN sets. Muhammad et al. [45] used (α, β, γ) cuts and applied MCDM problems to build the TOPSIS method with interval type 2 trapezoidal neutrosophic numbers. In order to solve MAGDM difficulties, Geng et al. [30] developed the TOPSIS approach, which is a SVNLCWD measure. In many researchers([31], [20], [8], [23], [47], [36], [55], [41]) developed different methods for solving decision making problems. A novel approach to modeling Zero Base Budgeting in a fuzzy environment is presented by EI-Morsy [22]. Colombo et al. [7] investigate employs a novel, entropy-weighted MADM model to decipher these intricate relationships. Using a strong FIS, Venugopal et al. [71] suggested the urgent need for a groundbreaking indicator for daily stock trading. Lamrini et al. [37] Presented new distributed TOPSIS approach in MCDM methods with big data context. Mona Mohamed and Abdullallah Gamal [40]constructed entrepreneurs in emerging economies face obstacles in implementing Industry 5.0, which are assessed using a neutrosophic AHP-based Appraisal Decision Framework (ADF) to identify and rank key barriers. The results highlight cost and funding as the most significant obstacles, followed by scalability, socio-technological planning, and security concerns.Sara Fawaz AL-baker et al. [58]

developed TreeSoft's new strategy using MCDM approaches aims to advise an exceptional online service provider based on QoS values.

1.1. *Research Gap and Motivation*

The arguments and research gaps that influenced the choice of the proposed structure are as follows:

Research gap: As of right now, there is no literature on the integrated TOPSIS-LINMAP technique in the SVTN context..

Motivation: We started researching the integrated technique in the SVTN environment in order to close the research gap.

- A LINMAP approach for reluctant fuzzy MADM issues was expanded by Liu et al. [12] and also developed integrating combination of TOPSIS and LINMAP. Donga and Wan [35] proposed integrating TOPSIS and LINMAP for applying the virtual enterprise partner selection problems. Wan et al. [9] proposed q-ROTFWA operator and establish LINMAP according to q-ROTFWA operators for resolving issues with collective decision-making.
- It is evident that the integrating techniques mentioned above address fuzzy number opinions. With inspiration from the aforementioned literature in general and the expanded LINMAP method, a novel strategy for solving the SVTN number to address MADM difficulties was developed. For SVTN numbers, a comprehensive approach combining the LINMAP and TOPSIS approaches is described along with its known ideal solutions in unknown conditions. We proposed metrics for consistency and inconsistency between the decision maker's preference relation and the ranking order of the alternatives, based on the definitions of ideal solutions and neutrosophic hamming distance. After estimating the weights of the attributes using a linear programming model, the TOPSIS approach is used to determine the optimal choice.

1.2. *Contributions*

By comparing with integrated methods (TOPSIS and LINMAP), has some advantages. The planned study will contribute the following:

- Firstly, it has been extended to neutrosophic environment.
- The integrated method considered both positive and negative ideal solution in neutrosophic environment and it over come the drawbacks for neutrosophic LINMAP because it consider only positive ideal solution.
- Neutrosophic TOPSIS may suffer from the integrated approach, which is designed to establish objective weights for the qualities.

- Measures of consistency as well as inconsistency between the decision maker's preference relation for ideal solutions and the ranking order of the options are established in a neutrosophic setting.
- This paper's primary goal is to highlight the advantages of combining the LINMAP and TOPSIS approaches to solve neutrosophic MAGDM issues based on SVTN numbers.

1.3. Paper Organization

The structure of this document is as follows. The fundamental ideas of SVTN numbers are reviewed in Section 2. The MAGDM problem's structure is explained in Section 3. The hybrid MAGDM for combining the TOPSIS and LINMAP approaches to SVTN numbers is described in Section 4. The approach for the neutrosophic MAGDM problem is covered in Section 5, along with a summary of the comparison findings. The paper's conclusion is found in Section 6.

2. Preliminaries

We will review some fundamental ideas regarding the SVTN numbers in this part.

Definition 2.1. [39] Let $p_l, p_{m1}, p_{m2}, p_u \in R$ such that $p_l \leq p_{m1} \leq p_{m2} \leq p_u$. A SVTN number $\tilde{p} = \langle (p_l, p_{m1}, p_{m2}, p_u); T_{\tilde{p}}, I_{\tilde{p}}, F_{\tilde{p}} \rangle$ is a unique set on the real number set R , with truth, indeterminacy, and falsity membership functions are defined by $\alpha_{\tilde{p}}(x)$, $\beta_{\tilde{p}}(x)$ and $\gamma_{\tilde{p}}(x)$ respectively.

$$\alpha_{\tilde{p}}(x) = \begin{cases} \frac{x-p_l}{p_{m1}-p_l} T_{\tilde{p}}, & \text{for } p_l \leq x \leq p_{m1} \\ T_{\tilde{p}}, & \text{for } p_{m1} \leq x \leq p_{m2} \\ \frac{p_u-x}{p_u-p_{m2}} T_{\tilde{p}}, & \text{for } p_{m2} \leq x \leq p_u \\ 0, & \text{otherwise.} \end{cases}$$

$$\beta_{\tilde{p}}(x) = \begin{cases} \frac{p_{m1}-x+(x-p_l)I_{\tilde{p}}}{p_{m1}-p_l}, & \text{for } p_l \leq x \leq p_{m1} \\ I_{\tilde{p}}, & \text{for } p_{m1} \leq x \leq p_{m2} \\ \frac{x-p_{m2}+(p_u-x)I_{\tilde{p}}}{p_u-p_{m2}}, & \text{for } p_{m2} \leq x \leq p_u \\ 0, & \text{otherwise.} \end{cases}$$

$$\gamma_{\tilde{p}}(x) = \begin{cases} \frac{p_{m1}-x+(x-p_l)F_{\tilde{p}}}{p_{m1}-p_l}, & \text{for } p_l \leq x \leq p_{m1} \\ F_{\tilde{p}}, & \text{for } p_{m1} \leq x \leq p_{m2} \\ \frac{x-p_{m2}+(p_u-x)F_{\tilde{p}}}{p_u-p_{m2}}, & \text{for } p_{m2} \leq x \leq p_u \\ 0, & \text{otherwise.} \end{cases}$$

Definition 2.2. [62] Let $\tilde{p} = \langle (p_l, p_{m1}, p_{m2}, p_u); t_{\tilde{p}}, i_{\tilde{p}}, f_{\tilde{p}} \rangle$ and

$\tilde{q} = \langle (q_l, q_{m1}, q_{m2}, q_u); t_{\tilde{q}}, i_{\tilde{q}}, f_{\tilde{q}} \rangle$ be two SVTN numbers. Then the hamming distance between SVTN numbers $\tilde{m}$ and $\tilde{n}$ is defined as

$$\begin{aligned}
d(\tilde{p}, \tilde{q}) = & \frac{1}{12} (| (2 + t_{\tilde{p}} - i_{\tilde{p}} - f_{\tilde{p}})p_l - (2 + t_{\tilde{q}} - i_{\tilde{q}} - f_{\tilde{q}})q_l | \\
& + | (2 + t_{\tilde{p}} - i_{\tilde{p}} - f_{\tilde{p}})p_{m1} - (2 + t_{\tilde{q}} - i_{\tilde{q}} - f_{\tilde{q}})q_{m1} | \\
& + | (2 + t_{\tilde{p}} - i_{\tilde{p}} - f_{\tilde{p}})p_{m2} - (2 + t_{\tilde{q}} - i_{\tilde{q}} - f_{\tilde{q}})q_{m2} | \\
& + | (2 + t_{\tilde{p}} - i_{\tilde{p}} - f_{\tilde{p}})p_u - (2 + t_{\tilde{q}} - i_{\tilde{q}} - f_{\tilde{q}})q_u |)
\end{aligned}$$

Definition 2.3. [18] The Borda method's stages of issue solving are known as the rank order method. The value of m , which is the sum of the alternatives less 1, is the greatest rating value in an alternative sequence. The series up to the last order has a value of 0, and the second-highest place has a value of $m-1$. The value multiplies the noises that are derived from the relevant place. The most likely option chosen by the respondent is the one with the highest selection, according to Borda's functional, statistical analysis of its alternative.

3. Neutrosophic MAGDM problems with incomplete weight information

The Neutrosophic MAGDM problem and its normalization techniques are discussed in this section.

3.1. Problem Description

The Neutrosophic MAGDM problems under consideration are represented by the following symbols.

- (1) Consider the following set of **neutrosophic decision matrix** wherein the alternative evaluation qualities' ratings are presented as SVTN numbers and define D_p , ($p = 1, 2, \dots, t$) is expressed as

$$D_p = \left(\tilde{s}_{ij}^p \right)_{(m \times n)} = \begin{matrix} & \begin{matrix} C_1 & C_2 & \dots & C_n \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} & \begin{pmatrix} \tilde{s}_{11}^p & \tilde{s}_{12}^p & \dots & \tilde{s}_{1n}^p \\ \tilde{s}_{21}^p & \tilde{s}_{22}^p & \dots & \tilde{s}_{2n}^p \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{s}_{m1}^p & \tilde{s}_{m2}^p & \dots & \tilde{s}_{mn}^p \end{pmatrix} \end{matrix}$$

and $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$, $\omega_j \in [0, 1]$, the weight component of the decision matrix. In this case, ω is entirely unknown and must be found.

- (2) A_i denotes the i -th alternative ($i = 1, 2, \dots, m$) in the set of **alternatives** $A = (A_1, A_2, \dots, A_m)$.
- (3) $C = (C_1, C_2, \dots, C_n)$ is the set of **attributes**, where C_j is the j -th attribute ($j = 1, 2, \dots, n$); $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ represents the weight vector of the attribute; this is not complete information and must be established.

- (4) The rating $\tilde{s}_{ij}^p$ of alternatives A_i on attributes C_j given by the p^{th} decision maker is in the form of SVTN number and its defined by $\tilde{s}_{ij}^p = \langle (s_{ijl}^p, s_{ijm1}^p, s_{ijm2}^p, s_{iju}^p); T_{ij}^p, I_{ij}^p, F_{ij}^p \rangle, (p = 1, 2, \dots, t)$.
- (5) Assume that the decision makers D_p derive their preference relations between alternatives from their knowledge and experience by using the formula $\Omega^p = \{(k, l) \mid A_k^p \geq A_l^p, (k, l = 1, 2, 3, \dots, m)\}$. The symbol " $\geq$ " indicates that the decision maker D_p either prefers A_k and A_l , or that they are indifferent between A_k and A_l .

3.2. Normalization methods

Normalization is not necessary if every rating in a decision matrix D_p is either profit or cost. If not, create the decision matrix that has been normalized. The value $\tilde{s}_{ij}^p$ must be normalized in the way indicated follows in order to remove any disturbance from the final results:

$$\tilde{r}_{ij}^p = \langle (r_{ijl}^p, r_{ijm1}^p, r_{ijm2}^p, r_{iju}^p); T_{ij}^p, I_{ij}^p, F_{ij}^p \rangle, (p = 1, 2, \dots, t).$$

$$\tilde{r}_{ij}^p = \begin{cases} \frac{\tilde{s}_{ij}^p}{\sum_{k=1}^n \tilde{s}_{ik}^p}, & \text{if the rating is profit} \\ \frac{1}{\sum_{k=1}^n (\frac{1}{\tilde{s}_{ik}^p})}, & \text{if the rating is cost} \end{cases}$$

By using above form, $D_p = (\tilde{s}_{ij}^p)_{(m \times n)}$ can be normalized as $N_p = (\tilde{r}_{ij}^p)_{(m \times n)}, (p = 1, 2, \dots, t)$ and the normalized decision matrix N_p expressed as follows:

$$N_p = (\tilde{r}_{ij}^p)_{(m \times n)} = \begin{matrix} & C_1 & C_2 & \dots & C_n \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} & \begin{pmatrix} \tilde{r}_{11}^p & \tilde{r}_{12}^p & \dots & \tilde{r}_{1n}^p \\ \tilde{r}_{21}^p & \tilde{r}_{22}^p & \dots & \tilde{r}_{2n}^p \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{r}_{m1}^p & \tilde{r}_{m2}^p & \dots & \tilde{r}_{mn}^p \end{pmatrix} \end{matrix}$$

4. A novel method for neutrosophic MAGDM problems

We present a novel integration approach to answer the neutrosophic MAGDM problems using the relative closeness coefficient (RCC) and neutrosophic ideal solutions inside the framework of LINMAP and TOPSIS.

4.1. Comprehensive relative closeness coefficient

Let us define the neutrosophic positive & negative ideal solutions are $\tilde{p}_j^{+p}, \tilde{p}_j^{-p}$ defined by

$$\tilde{p}_j^{+p} = \langle (\tilde{p}_{jl}^{+p}, \tilde{p}_{jm1}^{+p}, \tilde{p}_{jm2}^{+p}, \tilde{p}_{ju}^{+p}); T_j^{+p}, I_j^{+p}, F_j^{+p} \rangle$$

$= \langle$

$$(\max_j (\tilde{r}_{ijl}^p), \max_j (\tilde{r}_{ijm1}^p), \max_j (\tilde{r}_{ijm2}^p), \max_j (\tilde{r}_{iju}^p)); \max_j (T_{ij}^p), \min_j (I_{ij}^p), \min_j (F_{ij}^p) \rangle$$

G.Tamilarasi, S.Paulraj, M.Sivanandha Saraswathy and Thanuja T S , A Hybrid MAGDM Approach of Neutrosophic TOPSIS - LINMAP method for developing Enterprise Selection Criteria

and

$$\begin{aligned} \tilde{p}_j^{-p} = & \langle (\tilde{p}_{jl}^{-p}, \tilde{p}_{jm1}^{-p}, \tilde{p}_{jm2}^{-p}, \tilde{p}_{ju}^{-p}); T_j^{-p}, I_j^{-p}, F_j^{-p} \rangle, j = 1, 2, \dots, n; p = 1, 2, \dots, t \\ = & \langle (\min_j (\tilde{r}_{ijl}^p), \min_j (\tilde{r}_{ijm1}^p), \min_j (\tilde{r}_{ijm2}^p), \min_j (\tilde{r}_{iju}^p)); \min_j (T_{ij}^p), \max_j (I_{ij}^p), \max_j (F_{ij}^p) \rangle \end{aligned}$$

The squares of the weighted distances between $\tilde{r}_{ij}^p, \tilde{p}_j^{+p}$ and $\tilde{p}_j^{-p}$ can be calculated by using the Definition 3 as follows:

$$d_i^{+p} = \sum_{j=1}^n \omega_j [d(\tilde{r}_{ij}^p, \tilde{m}_j^{+p})]^2 \quad (1)$$

and

$$d_i^{-p} = \sum_{j=1}^n \omega_j [d(\tilde{r}_{ij}^p, \tilde{m}_j^{-p})]^2 \quad (2)$$

The relative closeness coefficient can be determined as follows:

$$R_i^p = \frac{d_i^{-p}}{(d_i^{+p} + d_i^{-p})}, i = 1, 2, \dots, m, p = 1, 2, \dots, t. \quad (3)$$

4.2. Consistency and inconsistency measurements

Srinivasan and Shocker introduced the classical LINMAP approach [65]. Numerous methods have been developed to address human judgments, including intuitionistic fuzzy sets (Atanssov 1986), fuzzy sets (Zadeh 1965), and others. As a result, the LINMAP approach has also been expanded to intuitionistic fuzzy [73] and fuzzy [27] environments. To handle the scenario where the input arguments take the form of a neutrosophic environment, we further enhance the LINMAP algorithm in this section. Assume that that unidentified weighted vector of attribute $C_j, (j = 1, 2, \dots, n)$ is $\omega = (\omega_1, \omega_2, \dots, \omega_n)$. By using the concept of neutrosophic hamming distance between each pair of alternatives $(k, l) \in \Omega^p$ and positive (negative) ideal solutions are defined as follows:

$$\begin{aligned} S_i^p &= \sum_{j=1}^n \omega_j [d(\tilde{r}_{ij}^p, \tilde{p}_j^{+p})]^2, S_k^p = \sum_{j=1}^n \omega_j [d(\tilde{r}_{kj}^p, \tilde{p}_j^{+p})]^2 (i = 1, 2, \dots, m) \\ S_k^{p'} &= \sum_{j=1}^n \omega_j [d(\tilde{r}_{kj}^p, \tilde{p}_j^{-p})]^2 \text{ and } S_l^{p'} = \sum_{j=1}^n \omega_j [d(\tilde{r}_{lj}^p, \tilde{p}_j^{-p})]^2, (i = 1, 2, \dots, m) \end{aligned}$$

Definition 4.1. The inconsistency (error) between the preference of alternatives A_l^p and A_k^p and the ranking order of alternatives A_l^p and A_k^p , which is defined by S_l^p and S_k^p , is measured by an index $(S_l^p - S_k^p)^-$, which is based on SVTN numbers. The following is an expression for the preference relation $p, (k, l) \in \Omega^p$ inconsistency index:

$$(S_l^p - S_k^p)^- = \begin{cases} (S_k^p - S_l^p), & (s_l^p < s_k^p) \\ 0, & (s_l^p \geq s_k^p). \end{cases}$$

Thus, inconsistency index

$$(S_l^p - S_k^p)^- = \max \{0, (S_k^p - S_l^p)\}.$$

A total inconsistency index of the decision maker is defined as

$$B = \sum_{(k,l) \in \Omega^p} (S_l^p - S_k^p)^- = \sum_{(k,l) \in \Omega^p} \max \{0, (S_k^p - S_l^p)\}$$

Definition 4.2. The consistency between the preference of alternatives A_l^p and A_k^p and the ranking order of alternatives A_l^p and A_k^p as defined by S_l^p and S_k^p is measured using an index $(S_l^p - S_k^p)^+$ based on SVTN numbers. The following is an expression for the consistency index for the preference relation $p, (k, l) \in \Omega^p$:

$$(S_l^p - S_k^p)^+ = \begin{cases} (S_l^p - S_k^p), & (s_l^p \geq s_k^p) \\ 0, & (s_l^p < s_k^p). \end{cases}$$

Thus, consistency index

$$(S_l^p - S_k^p)^+ = \max \{0, (S_l^p - S_k^p)\}.$$

The decision maker's overall consistency index is described as

$$G = \sum_{(k,l) \in \Omega^p} (S_l^p - S_k^p)^+ = \sum_{(k,l) \in \Omega^p} \max \{0, (S_l^p - S_k^p)\}$$

Definition 4.3. The ranking order of alternatives $A_l^{p'}$ and $A_k^{p'}$, which are determined by $S_l^{p'}$ and $S_k^{p'}$, and the preference of alternatives A_l^p and A_k^p are measured for consistency using an index $(S_k^{p'} - S_l^{p'})^-$ based on SVTN numbers. The preference relation $p, (k, l) \in \Omega^p$'s inconsistency index can be written as follows:

$$(S_k^{p'} - S_l^{p'})^- = \begin{cases} (S_l^{p'} - S_k^{p'}), & \text{if } S_l^{p'} > S_k^{p'} \\ 0, & \text{if } S_l^{p'} \leq S_k^{p'} \end{cases}$$

Thus, inconsistency index

$$(S_k^{p'} - S_l^{p'})^- = \max \{0, (S_l^{p'} - S_k^{p'})\}.$$

The overall inconsistency index can then be found as

$$B' = \sum_{(k,l) \in \Omega^p} (S_k^{p'} - S_l^{p'})^- = \sum_{(k,l) \in \Omega^p} \max \{0, (S_l^{p'} - S_k^{p'})\}$$

Definition 4.4. The ranking order of alternatives $A_l^{p'}$ and $A_k^{p'}$, which are determined by $S_l^{p'}$ and $S_k^{p'}$, and the preference of alternatives A_l^p and A_k^p are measured for consistency using an index $(S_k^{p'} - S_l^{p'})^+$ based on SVTN numbers. The following is an expression for the consistency index for the preference relation $p, (k, l) \in \Omega^p$:

$$(S_k^{p'} - S_l^{p'})^+ = \begin{cases} (S_k^{p'} - S_l^{p'}), & \text{if } S_l^{p'} \leq S_k^{p'} \\ 0, & \text{if } S_l^{p'} > S_k^{p'} \end{cases}$$

Thus, consistency index

$$(S_k^{p'} - S_l^{p'})^+ = \max \{0, (S_k^{p'} - S_l^{p'})\}.$$

The decision maker's overall consistency index is therefore defined as

$$G' = \sum_{(k,l) \in \Omega^p} (S_k^{p'} - S_l^{p'})^+ = \sum_{(k,l) \in \Omega^p} \max \{0, (S_k^{p'} - S_l^{p'})\}$$

4.3. LINMAP under neutrosophic environment

A linear programming model is built in the following to ascertain the qualities' unknown weight ω .

$$\begin{aligned} & \min B \\ & \text{Subject to } G - B \geq h \\ & \sum_{j=1}^n \omega_j = 1 \\ & \omega_j \geq 0, (j = 1, 2, \dots, n) \end{aligned}$$

where the decision maker provides h to guarantee that the overall consistency index G exceeds the total inconsistency index B .

On the other hand, a mathematical model, we can also obtained the minimize total inconsistency index B' .

$$\begin{aligned} & \min B' \\ & \text{Subject to } G' - B' \geq h' \\ & \sum_{j=1}^n \omega_j = 1 \\ & \omega_j \geq 0, (j = 1, 2, \dots, n) \end{aligned}$$

The two objectives can be added to the following linear programming model with equal weights since there are no preference relations on them:

$$\begin{aligned} & \min B + B' \\ & \text{Subject to } G - B \geq h \\ & G' - B' \geq h' \\ & \sum_{i=1}^n \omega_j = 1 \\ & \omega_j \geq 0, (j = 1, 2, \dots, n) \end{aligned}$$

$$\text{i.e., } \min \left\{ \sum_{(k,l) \in \Omega^p} \max \{0, (S_k^p - S_l^p)\} + \sum_{(k,l) \in \Omega^p} \max \{0, (S_l^{p'} - S_k^{p'})\} \right\}$$

Subject to

$$\begin{aligned} & \sum_{(k,l) \in \Omega^p} \max \{0, (S_l^p - S_k^p)\} - \sum_{(k,l) \in \Omega^p} \max \{0, (S_k^p - S_l^p)\} \geq h \\ & \sum_{(k,l) \in \Omega^p} \max \{0, (S_k^{p'} - S_l^{p'})\} - \sum_{(k,l) \in \Omega^p} \max \{0, (S_l^{p'} - S_k^{p'})\} \geq h' \\ & \omega_1 + \omega_2 + \dots + \omega_n = 1 \\ & \omega_j \geq 0, (j = 1, 2, \dots, n) \end{aligned}$$

For every $(k, l) \in \Omega^p$ pair.

$$\text{Let } \lambda_{kl}^p = \max \{0, (S_k^p - S_l^p)\} \text{ and } \lambda_{lk}^{p'} = \max \{0, (S_l^{p'} - S_k^{p'})\}$$

$$\text{Hence, } \lambda_{kl}^p \geq S_k^p - S_l^p \text{ \& } \lambda_{kl}^{p'} \geq 0 \text{ and } \lambda_{lk}^{p'} \geq S_l^{p'} - S_k^{p'} \text{ \& } \lambda_{lk}^{p'} \geq 0$$

Therefore, the aforementioned model can be converted into the LPP that follows:

$$\min \sum_{p=1}^t \left\{ \sum_{(k,l) \in \Omega^p} \lambda_{kl}^p + \sum_{(k,l) \in \Omega^p} \lambda_{lk}^{p'} \right\}$$

Subject to

$$\begin{aligned} S_l^p - S_k^p + \lambda_{kl}^p &\geq 0, ((k, l) \in \Omega^p, p = 1, 2, \dots, t) \\ \sum_{(k,l) \in \Omega^p} (S_l^p - S_k^p) &\geq h \\ \sum_{(k,l) \in \Omega^p} (S_k^{p'} - S_l^{p'}) &\geq h' \\ \lambda_{kl}^p &\geq 0, \lambda_{lk}^{p'} \geq 0, ((k, l) \in \Omega^p, p = 1, 2, \dots, t) \\ \omega_1 + \omega_2 + \dots + \omega_j &= 1 \\ \omega_j &\geq 0, (j = 1, 2, \dots, n) \end{aligned}$$

The weight vector $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ was acquired while solving the previously discussed linear programming model with LINGO.

5. MAGDM method under neutrosophic environment

In this part, we construct a strategy for solving MAGDM issues by combining the neutrosophic TOPSIS and LINMAP approaches. The weights of the qualities are established by the use of integrated approaches. The following steps make up the operational process of the suggested approach to the neutrosophic MAGDM problem: Figure 1 displays the suggested method's flow diagram.

Step 1: The evaluation m qualities and n alternatives are determined by the decision makers.

Step 2: The decision-maker D_p gives the alternatives' preference relations by

$$\Omega^p = \{(k, l) \mid A_k^p \geq A_l^p, (k, l = 1, 2, 3, \dots, m)\} \quad (p = 1, 2, \dots, t)$$

Step 3: If required, create the normalization decision matrix $\tilde{r}_{ij}^p$..

Step 4: Find neutrosophic positive $\tilde{p}_j^{+p}$ and negative $\tilde{p}_j^{-p}$ ideal solutions by using the section 4.1.

Step 5: Form the linear programming problems (Sec. 4.3) and obtain the weighted vector ω_j

Step 6: Using equations (1) and (2), calculate the separation measures d_i^{+p} and d_i^{-p} .

Step 7: Use equation (3) to calculate the relative closeness coefficient R_i^p .

Step 8: The best selection option from the alternative set A_i is identified by generating the ranking order of the alternatives using the Borda's score values.

5.1. Illustrative Example

To demonstrate the new method, we will create a business selection and investment problem in this part. Pramanik and Mallick [59] and Das and Guha [14] have adapted the following issues and applied them to single valued trapezoidal neutrosophic numbers using the suggested

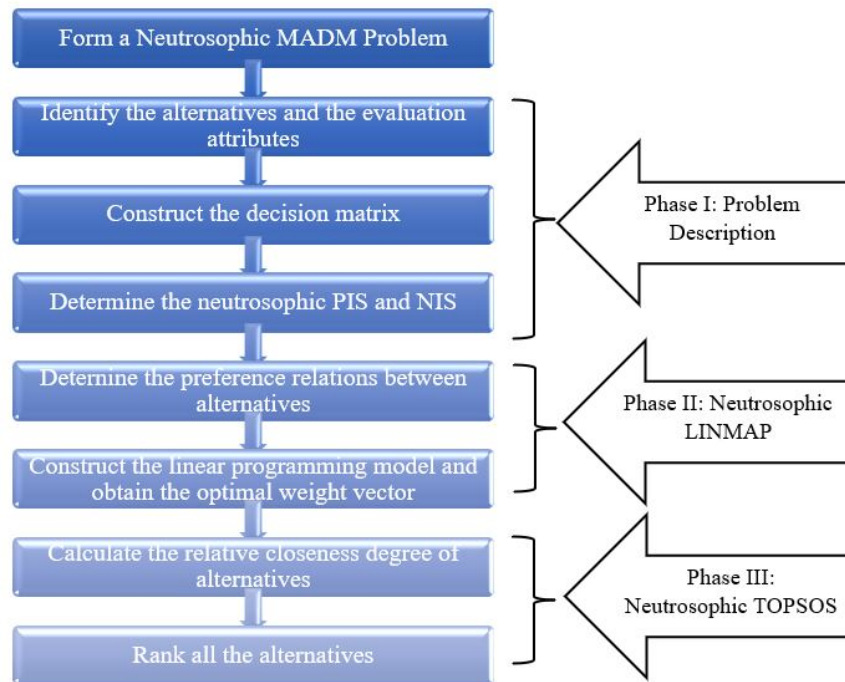


FIGURE 1. The flow diagram of the proposed method

method.

Enterprise Selection Problem: In order to meet market demand, a corporation wishes to establish a cooperative partnership with a few possible businesses. Three businesses $A_i, (i = 1, 2, 3)$ are chosen for additional assessment following pre-evaluation. The following four criteria are used by the expert unit to choose the best business: C_1 -Producing ability, C_2 -Technological competence, Capital money is C_3 , and research ability is C_4 .

Step 1: Identify the evaluation attributes and alternatives.

Tables 1, 2, and 3 list the three experts who made the decisions.

Step 2: Assume that experts who make decisions give the following preference connections between options:

$$\Omega^1 = \{(1, 2), (1, 3)\}$$

$$\Omega^2 = \{(2, 1), (2, 3)\}$$

$$\Omega^3 = \{(3, 1), (3, 2)\}$$

Step 3: Create the choice matrix for normalization.

Tables 4,5 and 6 provide the results of the normalizing decision matrix computations.

Step 4: Determine neutrosophic positive & negative ideal solutions.

Each p^{th} decision maker's neutrosophic positive & negative ideal solutions are defined as follows:

TABLE 1. Decision matrix provided by expert d_1

DM	C_1	C_2	C_3	C_4
A_1	$\langle (2, 4, 6, 8); 0.5, 0.4, 0.8 \rangle$	$\langle (2, 4, 6, 7); 0.7, 0.2, 0.5 \rangle$	$\langle (17, 18, 19, 20); 0.6, 0.3, 0.4 \rangle$	$\langle (3, 4, 6, 7); 0.7, 0.1, 0.4 \rangle$
A_2	$\langle (3, 5, 6, 7); 0.6, 0.3, 0.4 \rangle$	$\langle (15, 17, 19, 20); 0.7, 0.2, 0.4 \rangle$	$\langle (3, 4, 5, 6); 0.7, 0.2, 0.6 \rangle$	$\langle (4, 5, 6, 7); 0.6, 0.4, 0.3 \rangle$
A_3	$\langle (1, 2, 3, 4); 0.7, 0.2, 0.5 \rangle$	$\langle (2, 3, 4, 5); 0.5, 0.4, 0.3 \rangle$	$\langle (2, 4, 5, 6); 0.6, 0.4, 0.2 \rangle$	$\langle (15, 16, 18, 20); 0.8, 0.1, 0.2 \rangle$

TABLE 2. Decision matrix provided by expert d_2

DM	C_1	C_2	C_3	C_4
A_1	$\langle (15, 16, 17, 20); 0.9, 0.1, 0.4 \rangle$	$\langle (2, 4, 5, 7); 0.5, 0.3, 0.6 \rangle$	$\langle (2, 5, 6, 8); 0.7, 0.2, 0.5 \rangle$	$\langle (3, 5, 6, 7); 0.8, 0.1, 0.3 \rangle$
A_2	$\langle (4, 5, 6, 7); 0.6, 0.3, 0.4 \rangle$	$\langle (16, 17, 19, 20); 0.8, 0.2, 0.1 \rangle$	$\langle (3, 4, 5, 6); 0.7, 0.2, 0.5 \rangle$	$\langle (4, 5, 6, 9); 0.6, 0.3, 0.5 \rangle$
A_3	$\langle (1, 3, 5, 6); 0.6, 0.4, 0.3 \rangle$	$\langle (2, 3, 4, 6); 0.6, 0.3, 0.4 \rangle$	$\langle (2, 3, 4, 5); 0.6, 0.4, 0.2 \rangle$	$\langle (17, 18, 19, 20); 0.6, 0.3, 0.7 \rangle$

TABLE 3. Decision matrix provided by expert d_3

DM	C_1	C_2	C_3	C_4
A_1	$\langle (4, 5, 6, 8); 0.5, 0.4, 0.3 \rangle$	$\langle (1, 2, 3, 4); 0.7, 0.2, 0.5 \rangle$	$\langle (17, 18, 19, 20); 0.6, 0.25, 0.3 \rangle$	$\langle (3, 4, 5, 6); 0.7, 0.1, 0.4 \rangle$
A_2	$\langle (3, 5, 6, 7); 0.6, 0.2, 0.4 \rangle$	$\langle (2, 3, 4, 6); 0.6, 0.3, 0.8 \rangle$	$\langle (3, 4, 5, 6); 0.7, 0.2, 0.6 \rangle$	$\langle (16, 17, 19, 20); 0.8, 0.2, 0.1 \rangle$
A_3	$\langle (16, 17, 18, 20); 0.8, 0.1, 0.3 \rangle$	$\langle (4, 5, 6, 7); 0.5, 0.4, 0.3 \rangle$	$\langle (2, 4, 5, 6); 0.6, 0.4, 0.1 \rangle$	$\langle (3, 4, 6, 7); 0.7, 0.2, 0.5 \rangle$

For expert 1,

$$\tilde{p}_1^{+1} = \langle (0.08, 0.139, 0.2, 0.33); 0.6, 0.4, 0.5 \rangle$$

$$\tilde{p}_1^{+2} = \langle (0.38, 0.47, 0.613, 0.8); 0.6, 0.4, 0.5 \rangle$$

$$\tilde{p}_1^{+3} = \langle (0.405, 0.49, 0.63, 0.83); 0.6, 0.4, 0.5 \rangle$$

$$\tilde{p}_1^{+4} = \langle (0.33, 0.53, 0.72, 0.8); 0.6, 0.4, 0.5 \rangle$$

$$\tilde{p}_1^{-1} = \langle (0.02, 0.07, 0.12, 0.2); 0.5, 0.4, 0.8 \rangle$$

$$\tilde{p}_1^{-2} = \langle (0.04, 0.1, 0.16, 0.25); 0.5, 0.4, 0.8 \rangle$$

$$\tilde{p}_1^{-3} = \langle (0.04, 0.11, 0.16, 0.24); 0.5, 0.4, 0.8 \rangle$$

$$\tilde{p}_1^{-4} = \langle (0.07, 0.108, 0.194, 0.28); 0.5, 0.4, 0.8 \rangle$$

TABLE 4. Expert-provided normalized decision matrix d_1

DM	C_1	C_2	C_3	C_4
A_1	$< (0.05, 0.108, 0.2, 0.33);$ $0.5, 0.4, 0.8 >$	$< (0.05, 0.108, 0.2, 0.29);$ $0.5, 0.4, 0.8 >$	$< (0.405, 0.49, 0.63, 0.83);$ $0.5, 0.4, 0.8 >$	$< (0.07, 0.108, 0.2, 0.29);$ $0.5, 0.4, 0.8 >$
A_2	$< (0.08, 0.139, 0.194, 0.28);$ $0.6, 0.4, 0.6 >$	$< (0.38, 0.47, 0.613, 0.8);$ $0.6, 0.4, 0.6 >$	$< (0.08, 0.11, 0.16, 0.24);$ $0.6, 0.4, 0.6 >$	$< (0.1, 0.139, 0.194, 0.28);$ $0.6, 0.4, 0.6 >$
A_3	$< (0.02, 0.07, 0.12, 0.2);$ $0.5, 0.4, 0.5 >$	$< (0.04, 0.1, 0.16, 0.25);$ $0.5, 0.4, 0.5 >$	$< (0.04, 0.13, 0.2, 0.3);$ $0.5, 0.4, 0.5 >$	$< (0.33, 0.53, 0.72, 0.8);$ $0.5, 0.4, 0.5 >$

TABLE 5. Expert-provided normalized decision matrix d_2

DM	C_1	C_2	C_3	C_4
A_1	$< (0.36, 0.47, 0.57, 0.91);$ $0.5, 0.3, 0.6 >$	$< (0.048, 0.12, 0.17, 0.32);$ $0.5, 0.3, 0.6 >$	$< (0.048, 0.147, 0.2, 0.364);$ $0.5, 0.3, 0.6 >$	$< (0.07, 0.15, 0.2, 0.32);$ $0.5, 0.3, 0.6 >$
A_2	$< (0.1, 0.139, 0.194, 0.26);$ $0.6, 0.3, 0.5 >$	$< (0.4, 0.47, 0.61, 0.7);$ $0.6, 0.3, 0.5 >$	$< (0.07, 0.111, 0.16, 0.22);$ $0.6, 0.3, 0.5 >$	$< (0.1, 0.139, 0.194, 0.33);$ $0.6, 0.3, 0.5 >$
A_3	$< (0.027, 0.09, 0.185, 0.273);$ $0.6, 0.4, 0.7 >$	$< (0.05, 0.09, 0.148, 0.273);$ $0.6, 0.4, 0.7 >$	$< (0.05, 0.09, 0.148, 0.227);$ $0.6, 0.4, 0.7 >$	$< (0.459, 0.563, 0.704, 0.91);$ $0.6, 0.4, 0.7 >$

For expert 2,

$$\begin{aligned}
 \tilde{p}_2^{+1} &= < (0.36, 0.47, 0.57, 0.91); 0.6, 0.3, 0.5 > \\
 \tilde{p}_2^{+2} &= < (0.4, 0.47, 0.61, 0.7); 0.6, 0.3, 0.5 > \\
 \tilde{p}_2^{+3} &= < (0.07, 0.147, 0.2, 0.364); 0.6, 0.3, 0.5 > \\
 \tilde{p}_2^{+4} &= < (0.459, 0.563, 0.704, 0.91); 0.6, 0.3, 0.5 > \\
 \tilde{p}_2^{-1} &= < (0.027, 0.09, 0.185, 0.273); 0.5, 0.4, 0.7 > \\
 \tilde{p}_2^{-2} &= < (0.048, 0.09, 0.148, 0.273); 0.5, 0.4, 0.7 > \\
 \tilde{p}_2^{-3} &= < (0.048, 0.09, 0.148, 0.22); 0.5, 0.4, 0.7 > \\
 \tilde{p}_2^{-4} &= < (0.07, 0.139, 0.194, 0.32); 0.5, 0.4, 0.7 >
 \end{aligned}$$

For expert 3,

G.Tamilarasi, S.Paulraj, M.Sivanandha Saraswathy and Thanuja T S , A Hybrid MAGDM Approach of Neutrosophic TOPSIS - LINMAP method for developing Enterprise Selection Criteria

TABLE 6. Expert-provided normalized decision matrix d_3

DM	C_1	C_2	C_3	C_4
A_1	$< (0.11, 0.15, 0.21, 0.32);$ $0.5, 0.4, 0.5 >$	$< (0.03, 0.06, 0.1, 0.16);$ $0.5, 0.4, 0.5 >$	$< (0.45, 0.55, 0.66, 0.8);$ $0.5, 0.4, 0.5 >$	$< (0.08, 0.12, 0.17, 0.24);$ $0.5, 0.4, 0.5 >$
A_2	$< (0.08, 0.15, 0.21, 0.29);$ $0.6, 0.3, 0.8 >$	$< (0.05, 0.09, 0.14, 0.25);$ $0.6, 0.3, 0.8 >$	$< (0.08, 0.12, 0.17, 0.25);$ $0.6, 0.3, 0.8 >$	$< (0.4, 0.59, 0.66, 0.83);$ $0.6, 0.3, 0.8 >$
A_3	$< (0.4, 0.49, 0.6, 0.8);$ $0.5, 0.4, 0.5 >$	$< (0.1, 0.14, 0.2, 0.28);$ $0.5, 0.4, 0.5 >$	$< (0.05, 0.11, 0.167, 0.24);$ $0.5, 0.4, 0.5 >$	$< (0.08, 0.11, 0.2, 0.28);$ $0.5, 0.4, 0.5 >$

$$\begin{aligned}
\tilde{p}_3^{+1} &= < (0.4, 0.49, 0.6, 0.8); 0.6, 0.3, 0.5 > \\
\tilde{p}_3^{+2} &= < (0.1, 0.14, 0.2, 0.28); 0.6, 0.3, 0.5 > \\
\tilde{p}_3^{+3} &= < (0.45, 0.55, 0.66, 0.8); 0.6, 0.3, 0.5 > \\
\tilde{p}_3^{+4} &= < (0.4, 0.59, 0.66, 0.83); 0.6, 0.3, 0.5 > \\
\tilde{p}_3^{-1} &= < (0.08, 0.15, 0.21, 0.29); 0.5, 0.4, 0.8 > \\
\tilde{p}_3^{-2} &= < (0.03, 0.06, 0.1, 0.16); 0.5, 0.4, 0.8 > \\
\tilde{p}_3^{-3} &= < (0.05, 0.11, 0.167, 0.24); 0.5, 0.4, 0.8 > \\
\tilde{p}_3^{-4} &= < (0.08, 0.11, 0.17, 0.24); 0.5, 0.4, 0.8 >
\end{aligned}$$

Step 5: Build the model for linear programming.

$$\min \lambda_{12}^1 + \lambda_{13}^1 + \lambda_{21}^2 + \lambda_{23}^2 + \lambda_{31}^3 + \lambda_{32}^3 + \lambda_{21}^{1'} + \lambda_{31}^{1'} + \lambda_{12}^{2'} + \lambda_{32}^{2'} + \lambda_{13}^{3'} + \lambda_{23}^{3'}$$

Subject to

$$\begin{aligned}
\lambda_{12}^1 + 0.00081\omega_1 + 0.0623\omega_2 - 0.061\omega_3 + 0.0115\omega_4 &\geq 0 \\
\lambda_{13}^1 - 0.00046\omega_1 + 0.063\omega_2 - 0.0799\omega_3 - 0.0098\omega_4 &\geq 0 \\
\lambda_{21}^2 + 0.057\omega_1 - 0.0573\omega_2 + 0.00083\omega_3 - 0.0091\omega_4 &\geq 0 \\
\lambda_{23}^2 - 0.0166\omega_1 - 0.066\omega_2 - 0.00169\omega_3 + 0.0746\omega_4 &\geq 0 \\
\lambda_{31}^3 - 0.0553\omega_1 - 0.0036\omega_2 + 0.08\omega_3 - 0.0046\omega_4 &\geq 0 \\
\lambda_{32}^3 - 0.062\omega_1 - 0.0016\omega_2 + 0.0011\omega_3 + 0.076\omega_4 &\geq 0 \\
\lambda_{21}^{1'} - 0.0014\omega_1 - 0.0585\omega_2 + 0.038\omega_3 - 0.00059\omega_4 &\geq 0 \\
\lambda_{31}^{1'} - 0.05\omega_1 - 0.0033\omega_2 + 0.038\omega_3 - 0.000588\omega_4 &\geq 0 \\
\lambda_{12}^{2'} - 0.057\omega_1 + 0.068\omega_2 - 0.0012\omega_3 + 0.0007\omega_4 &\geq 0 \\
\lambda_{32}^{2'} + 0.00139\omega_1 + 0.069\omega_2 - 0.0006\omega_3 - 0.059\omega_4 &\geq 0 \\
\lambda_{13}^{3'} + 0.051\omega_1 + 0.0033\omega_2 - 0.071\omega_3 + 0.00031\omega_4 &\geq 0 \\
\lambda_{23}^{3'} + 0.051\omega_1 + 0.0026\omega_2 - 0.0001\omega_3 - 0.059\omega_4 &\geq 0
\end{aligned}$$

$$\begin{aligned}
& -0.0766\omega_1 - 0.0032\omega_2 - 0.053\omega_3 + 0.1386\omega_4 \geq h \\
& -0.005\omega_1 + 0.0811\omega_2 + 0.0043\omega_3 - 0.1182\omega_4 \geq h' \\
& \omega_1 + \omega_2 + \omega_3 + \omega_4 = 1 \\
& \omega_1 \geq 0, \omega_2 \geq 0, \omega_3 \geq 0, \omega_4 \geq 0 \\
& \lambda_{12}^1 \geq 0, \lambda_{13}^1 \geq 0, \lambda_{21}^2 \geq 0, \lambda_{23}^2 \geq 0 \\
& \lambda_{31}^3 \geq 0, \lambda_{32}^3 \geq 0, \lambda_{21}^{1'} \geq 0, \lambda_{31}^{1'} \geq 0 \\
& \lambda_{12}^{2'} \geq 0, \lambda_{32}^{2'} \geq 0, \lambda_{13}^{3'} \geq 0, \lambda_{23}^{3'} \geq 0
\end{aligned}$$

We determine the ideal weight values for the qualities by solving the previously discussed model with the LINGO.

$$\omega_1 = 0.2811, \omega_2 = 0.3079, \omega_3 = 0.2154, \omega_4 = 0.1955$$

Step 6: Compute the distance separation measures d_i^{+p} and d_i^{-p} as shown in Table 7.

Step 7: Calculate the RCC R_i^p as given by Table 8.

TABLE 7. d_i^{+p} and d_i^{-p} are separation measures.

d_i^{+p}	d_i^{-p}
$d_1^{+1} = 0.0346, d_1^{+2} = 0.0353, d_1^{+3} = 0.0340$	$d_1^{-1} = 0.0085, d_1^{-2} = 0.0169, d_1^{-3} = 0.0156$
$d_2^{+1} = 0.0261, d_2^{+2} = 0.0322, d_2^{+3} = 0.0375$	$d_2^{-1} = 0.0189, d_2^{-2} = 0.0218, d_2^{-3} = 0.0121$
$d_3^{+1} = 0.0346, d_3^{+2} = 0.0430, d_3^{+3} = 0.0346$	$d_3^{-1} = 0.0156, d_3^{-2} = 0.0156, d_3^{-3} = 0.0156$

Step 8: Rank the alternatives.

TABLE 8. Relative closeness coefficient

Expert	R_i^p
1	$R_1^1 = 0.1972, R_2^1 = 0.42, R_3^1 = 0.3108$
2	$R_1^2 = 0.3238, R_2^2 = 0.4037, R_3^2 = 0.02662$
3	$R_1^3 = 0.3145, R_2^3 = 0.244, R_3^3 = 0.3108$

Borda's scores for the three businesses can be found in Table 9 below. Three businesses are ranked as follows: $A_2 > A_1 > A_3$, with A_2 being the best option.

TABLE 9. Borda's scores for alternatives

Experts Alternatives	d_1	d_2	d_3	Borda's Score
A_1	0	1	2	3
A_2	2	2	0	4
A_3	1	0	1	2

5.2. Analysis of comparison with current techniques

In this work, proposed method is more valuable to compare with some relevant papers. Initially, we contrast our suggested work with the Jun Ye [33], Bharatraj and Anand [34], Chiranjibe et al. [29], Pramanik and Mallick [60], [61] and Paulraj and Tamilarasi [52]. Similar ranking results were achieved when we examined the same MADM problem in a neutrophic environment. A detailed comparison with the existing methods is listed in Table 10. In order to compare Solairaju and Shajahan [67] imprecision membership and defuzzification methods. The comparison result shown in Table 11.

The suggested integrate LINMAP and TOPSIS approach is consistent and comparable to the

TABLE 10. Decision-making results of existing methods

Method	Ranking Order	Best Alternative
Jun Ye [33]	$A_2 > A_1 > A_3$	A_2
Pramanik and Mallick [60]	$A_2 > A_1 > A_3$	A_2
Bharatraj and Anand [34]	$A_2 > A_1 > A_3$	A_2
Chiranjibe et al [29]	$A_2 > A_1 > A_3$	A_2
Pramanik and Mallick [61]	$A_2 > A_1 > A_3$	A_2
Paulraj and Tamilarasi [52]	$A_2 > A_1 > A_3$	A_2
Proposed Method	$A_2 > A_1 > A_3$	A_2

TABLE 11. Ranking order of alternatives for precise set

Precise set	Ranking Order	Best Alternative
Fuzzy Set	$A_2 > A_1 > A_3$	A_2
Intuitionistic Fuzzy Set	$A_2 > A_1 > A_3$	A_2
Neutrosophic Set	$A_2 > A_1 > A_3$	A_2

current ones, according to the information shown in Tables 10 and 11. The suggested approach also offers the following benefits:

- (1) The existing methods [[33], [34], [29], [52]] provide the same option for various procedures, as indicated in Table 10, but the recently suggested method consistently provides the same option for the integrated method, which has been demonstrated to work.
- (2) The proposed Method also applied for COVID 19 problem [52] and we get the same alternatives.
- (3) The suggested approach to decision-making in SVTN numbers is more generic and more practical than the current approaches since it extends some of the existing approaches.

G.Tamilarasi, S.Paulraj, M.Sivanandha Saraswathy and Thanuja T S , A Hybrid MAGDM Approach of Neutrosophic TOPSIS - LINMAP method for developing Enterprise Selection Criteria

- (4) From Table 10 and 11 demonstrates that the optimal option is A_2 . Next, the proposed method investigate the impact of the neutrosophic parameter in the ranking of imprecision membership and defuzzification methods in decision making. It is interesting to see that without considering the indeterminacy and falsity information also we got the same result. So, it can conclude that with neutrosophic information, we can obtain realistic results.

6. Conclusion

This study extends the neutrosophic LINMAP method to solve SVTN number MAGDM issues. The decision maker's preferences between two options are taken into consideration while defining the consistency & inconsistency measure indices. To ascertain the attribute weights, a linear programming model is built by reducing the inconsistency index. The optimal solution is then selected using the neutrosophic TOPSIS technique. To handle MAGDM difficulties, the suggested approach integrates the LINMAP and TOPSIS methodologies in a neutrosophic setting. One of the drawbacks of our proposed approach is that the decision makers might offer preference relationships between options that are consistent with the alternatives' rank-order. But this creates a big demand for the caliber of the decision makers. It is frequently not possible to infer the decision's rating of the options makers' preference relations since they show conflicting preference relations between the alternatives. Considering the possibilities' conflicting preference relations, the suggested approach might be more suitable. Future extensions of our proposed approach will apply other sets applied heterogeneous MAGDM issues .

Furthermore, applying the integrated neutrosophic LINMAP-TOPSIS approach across various sectors such as healthcare, manufacturing, or finance could broaden its utility and demonstrate its generalizability. Comparative studies with traditional methods or other advanced techniques could also enhance comprehension of the suggested model's advantages and disadvantages in a range of real-world contexts.

By extending these results to other decision-making frameworks, industries, and real-time applications, the proposed method could significantly enhance enterprise selection, risk assessment, and strategic planning processes, contributing to the overall success and sustainability of enterprises in competitive and rapidly evolving markets.

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Integrating TreeSoft and HyperSoft Paradigms into Urban Elderly Care Evaluation: A Comprehensive n-SuperHyperGraph Approach

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Abstract

The rapid aging of global urban populations has compelled cities to reimagine how elderly care services are delivered, structured, and evaluated. Traditional models such as TreeSoft, which rely on hierarchical and clearly defined service flows, are well-suited for institutional settings but often fail to capture informal or uncertain care dynamics. On the other end of the spectrum, HyperSoft frameworks emphasize flexibility and allow for the modeling of indeterminate or overlapping relationships common in community-based or volunteer-driven services but they often lack operational coherence.

This research proposes a hybrid evaluation model that integrates TreeSoft and HyperSoft paradigms within the powerful mathematical structure of the n-SuperHyperGraph (n-SHG). Grounded in ecosystem theory, the n-SHG framework enables the modeling of nested, layered, and partially uncertain relationships between service actors such as families, clinics, local authorities, and digital providers.

Using data collected from three urban districts, we evaluate five smart elderly care service modes using connectivity, redundancy, and uncertainty indicators derived from the n-SHG structure. The results not only differentiate modes by performance but also demonstrate how both structured (TreeSoft-like) and fluid (HyperSoft-like) care systems can be understood, compared, and optimized under one unified analytical framework.

This hybrid modeling approach provides a roadmap for policymakers, urban planners, and researchers seeking to develop adaptive, inclusive, and resilient elderly care ecosystems. Also, we show a case study of TreeSoft set with four criteria and five alternatives. We compute the criteria weights in the main criteria and each sub-criteria. We use the MARCOS method to rank the alternatives.

Keywords: TreeSoft Set; n-SuperHyperGraph (n-SHG); HyperSoft; Elderly Care; Evaluation; MARCOS Method

1. Introduction

Urbanization and population aging are two converging forces reshaping how societies care for the elderly. In cities worldwide, longer life expectancies combined with changing family structures are increasing the demand for services that are not only medically competent but also socially responsive and technologically enabled. Traditional care models, typically built around

hospitals, nursing homes, and municipal welfare programs, often fail to meet the full scope of physical, emotional, and social needs experienced by older adults in urban environments [2]. Most existing evaluation frameworks for elderly care rely on highly structured, linear models of service delivery. These models assume consistent availability, clarity of roles, and direct accountability—conditions rarely met in practice. For instance, family caregiving may be intermittent, public health services may be fragmented, and volunteer initiatives may be poorly documented. The result is a system characterized by uncertainty, relational overlap, and incomplete coverage.

Two dominant paradigms have attempted to address these modeling challenges. The TreeSoft model is used to depict well-defined, hierarchical structures in which services flow clearly from centralized authorities to recipients. It is ideal for representing health ministries, hospital systems, or structured social welfare programs. In contrast, the HyperSoft model represents loosely coupled, uncertain, or dynamic relationships. It has been employed to model systems with fuzzy boundaries, such as community support networks and informal caregiving arrangements [3].

Both paradigms offer partial representations of urban elderly care systems. However, real-world environments often exhibit characteristics of both—a hospital system may include informal family caregiving or a community-based food delivery service may depend on municipal funding. These hybrid scenarios call for a more flexible yet structured modeling framework.

The n-SuperHyperGraph (n-SHG) model, proposed by Smarandache, introduces mathematical structures capable of representing hierarchical and indeterminate relations simultaneously through multi-level vertices, indeterminate edges, and null connections [4]. This study applies the n-SHG model to five distinct elderly care service modes in three urban districts, aiming to evaluate them through the integrated lens of TreeSoft and HyperSoft paradigms, grounded in ecosystem theory [1].

2. Literature Review

Research in urban elderly care has increasingly recognized the limits of conventional evaluation methods. Decision-making frameworks such as the Analytic Hierarchy Process (AHP) and Data Envelopment Analysis (DEA) have been widely used to assess service performance. While these tools offer clarity and consistency, they typically assume stable structures, clear objectives, and complete data [5]. In contrast, urban elderly care systems are often characterized by fluctuating interactions, evolving roles, and partial observability.

To better capture this complexity, scholars have drawn upon ecosystem theory, which views social service systems as networks of interrelated and interdependent actors. In this framework, service delivery is not linear but dynamic, with each actor's behavior influencing others. Bronfenbrenner introduced the ecological systems theory to explain how individuals exist within multiple nested layers of influence [1]. Lin and Wu extended this thinking into urban elderly care, showing how services are shaped by the intersection of institutional, familial, and social dynamics [6]. Zhang and Chen emphasized that collaborative, cross-sector systems that include both formal and informal actors are essential for successful elderly care delivery [7].

Despite the theoretical strength of the ecosystem perspective, few studies have integrated it with formal modeling systems capable of simulating uncertain or hybrid relationships. The TreeSoft paradigm continues to dominate structured health service modeling, where clarity and control are prioritized. However, it falls short when applied to systems involving multiple informal actors, such as volunteer groups or family caregivers. Between 2018 and 2024, Smarandache introduced six novel types of soft sets, namely: the HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, SuperHyperSoft Set, TreeSoft Set, and ForestSoft Set. These contributions are detailed in several publications, which also examine the role of indeterminate operators within the framework of indeterminate soft algebra [10-12].

The HyperSoft model has emerged as a counterpoint. It builds upon fuzzy set theory to allow for overlapping roles, uncertain interactions, and partial memberships. These features make it particularly relevant to community-based service systems [3]. Yet, the model lacks the architectural rigor needed for managing coordination across diverse institutional actors.

The n-SuperHyperGraph framework introduced by Smarandache (2024) extends classical graph theory to support both structured and uncertain relationships. It enables the representation of complex social networks through its incorporation of super vertices (representing composite actors), indeterminate edges (representing uncertain or evolving connections), and null elements (representing service gaps) [4-8]. This framework has been successfully applied in domains like transportation and environmental planning, but its use in urban social services, and specifically in elderly care, remains unexplored.

By applying the n-SHG model to a real-world evaluation of smart elderly care services, this study contributes a novel approach to service modeling that reflects both the structured logic of TreeSoft and the flexible reality captured by HyperSoft.

3. Overview of the n-SuperHyperGraph Framework

An n-SuperHyperGraph is a mathematical structure defined as: $G = (V, E, I, \Theta)$

Where:

V: Set of vertices (e.g., individuals, institutions, technologies)

E: Set of edges, each connecting one or more vertices

I: Set of indeterminate edges (uncertain or evolving relationships)

Θ : Set of null edges (gaps or missing connections)

A super edge in this context is an edge that itself contains edges, representing bundled or institutional relationships (e.g., a municipal health network that includes hospitals, clinics, and home care units).

In the elderly care ecosystem:

V includes elements like "Elderly Individual," "Family Caregiver," "Community Clinic," and

“Tech Provider.”

E models explicit service links like healthcare provision or financial aid.

I capture evolving or partially known connections (e.g., informal social support).

⊖ highlights missing or broken links, such as areas lacking accessible transportation.

The strength of this framework lies in its flexibility to encode both known and uncertain structures, which is essential in modeling real-world urban social systems.

3.1 Data Collection and Model Inputs

Data were collected through:

- Expert interviews with municipal health officials and care coordinators (n = 6).
- Review of government elderly care service records from three urban communities.
- Community surveys evaluating user experiences with five service modes:
 1. Tech-assisted in-home care
 2. Community day-care centers
 3. Volunteer-based neighborhood services
 4. Telehealth-integrated models
 5. Multi-provider institutional care hubs

Each service mode was mapped to an n-SHG structure, identifying its nodes (service actors), edges (service flows), and the degree of completeness or uncertainty in each relationship.

3.2 Model Construction: Super and Indeterminate Components

We define a SuperEdge $SE_k \subseteq E$ as: $SE_k = \{e_1, e_2, \dots, e_n\} \subseteq E$

Where each e_i connects multiple vertices and represents an atomic service activity (e.g., prescription delivery, mental health screening).

Similarly, the Indeterminate Edge $i \in I$ is a partial function:

$i: V \times V \rightarrow \{0, 1, ?\}$

Where “?” denotes uncertainty (e.g., whether a community volunteer network reaches isolated elderly in a given district).

To evaluate each service mode, we compute the connectivity index CI and the redundancy index RI as follows:

$CI = \text{Total Active Edges} / \text{Max Theoretical Edges}$

$RI = \text{SuperEdges with overlapping vertices} / \text{Total SuperEdges}$

These indices provide insights into the density and overlap of support structures, which are key indicators of efficiency and resilience.

3.3 Illustration Example: Mode 3 (Volunteer-Based Services)

In Mode 3, vertices included:

Elderly (E1–E5),

Volunteer Groups (V1–V2),

Health Stations (H1),

Local Authority (L1)

Edges connected volunteers to elderly individuals, while indeterminate edges represented informal or inconsistent visits. One null edge indicated a complete lack of mental health outreach in the eastern district.

Using the model, Mode 3 displayed:

CI = 0.58 → moderate network density

RI = 0.66 → high overlap among support actors

This mode exhibited strong community reach but revealed a vulnerability in consistency and coverage.

3.4 Problem Statement and Objectives

Despite the growing emphasis on smart elderly care, current evaluation methods remain insufficient for modeling the structural complexity and uncertainty that characterize real-world urban service systems. Most frameworks continue to be rooted in assumptions of stability, hierarchy, and full observability. This is evident in models that either rely on formal structures with fixed roles such as TreeSoft or adopt soft system modeling techniques that focus on flexibility and uncertainty—such as HyperSoft. However, urban care systems are neither entirely structured nor completely fluid; they are hybrid ecosystems made up of hospitals, community centers, families, volunteers, and digital platforms, all interacting at varying levels of formality and certainty.

TreeSoft models, though robust for structured service delivery (e.g., municipal clinics or hospital systems), are not designed to handle the blurred lines of responsibility found in volunteer networks or digital caregiving platforms. On the other hand, HyperSoft models can represent these uncertain or informal relationships but often lack mechanisms to enforce coordination or assess system efficiency [3].

As cities continue to decentralize service provision, and as aging populations increasingly rely on hybrid care systems, a more advanced modeling tool is required—one that supports both hierarchy and fluidity, known and unknown relationships, and fixed as well as evolving actor roles.

The n-SuperHyperGraph (n-SHG) model fills this need by enabling the representation of multiple types of nodes (e.g., individuals, families, institutions), edge types (e.g., formal, informal, indeterminate), and structural gaps (e.g., null links). Through its recursive structure, it accommodates both layered and uncertain networks in a single integrated framework [4-8].

This study aims to operationalize the n-SHG model in the context of urban elderly care. It focuses on evaluating five different care service modes across three cities, comparing their performance using three indicators derived from the model: connectivity, redundancy, and uncertainty.

The specific objectives of this study are:

1. To construct an evaluation framework for smart urban elderly care systems using the n-SHG model.
2. To integrate both TreeSoft and HyperSoft concepts within this framework.
3. To apply the model to five real-world service modes representing various levels of institutional control and community involvement.
4. To compute structural performance indicators—connectivity, redundancy, and uncertainty—for each mode.
5. To validate the framework's outputs against expert rankings using statistical correlation techniques.

4. Methodology

To evaluate the structural performance of smart elderly care service modes, this study develops and applies a modeling framework based on the n-SuperHyperGraph (n-SHG). This section outlines the framework's mathematical foundations, data collection approach, model construction process, and the performance indicators used to assess each care mode. The design integrates principles from both TreeSoft and HyperSoft paradigms, allowing for the representation of hierarchical, uncertain, and composite relationships within urban service networks.

4.1 Framework Design: The n-SuperHyperGraph Model

The n-SHG is a recursive mathematical structure that extends traditional graph theory to accommodate multiple levels of abstraction and uncertainty. It is formally defined as a tuple:

$$G = (V, E, I, \Theta)$$

Where:

- V is the set of vertices representing individual or institutional actors (e.g., elderly persons, clinics, caregivers, tech platforms).
- E is the set of edges representing known relationships or interactions (e.g., care delivery, information sharing).
- I is the set of indeterminate edges, capturing partially known or evolving relationships (e.g., informal volunteer visits).

- Θ is the set of null edges, indicating service gaps or non-existent relationships (e.g., no mental health support in a district).

The structure also incorporates SuperVertices, which are groupings of multiple actors functioning as one unit, such as a “Community Health Center” that includes clinics, mobile teams, and digital support platforms. This recursive structure allows for modeling deeply nested or hierarchical systems—core to the TreeSoft perspective—while also including fuzzy and overlapping connections from the HyperSoft paradigm [4][8].

4.2 Data Collection and Study Setting

Empirical data were collected from three urban communities over five months. Data sources included:

- Interviews with six municipal health coordinators responsible for elderly care.
- Community surveys were conducted with 200 elderly residents to understand their experiences, service usage, and unmet needs.
- Public records and policy documents related to care initiatives in each community.
- Observation and mapping of service delivery paths through visits to local care centers and digital care providers.

This mixed-methods approach ensured that both formal (institutional) and informal (community-driven) services were captured.

Identification of Service Modes

Based on data analysis, five distinct elderly care service modes were identified:

Mode	Description	Structural Type
1	Tech-assisted in-home care	Hybrid (digital-focused)
2	Community day-care centers	Institutional
3	Volunteer-based neighborhood services	Informal/uncertain
4	Telehealth-integrated hybrid models	Structured + dynamic
5	Multi-provider institutional care hubs	Highly structured

Each of these modes exhibits different degrees of formalization, connectivity, and uncertainty, making them suitable for evaluation under the n-SHG framework.

4.3 Model Construction Process

Each mode was translated into an n-SHG by identifying:

- Vertices (V): Individual elderly users, caregivers, clinics, tech platforms, and local authorities.
- Edges (E): Defined service relationships (e.g., healthcare visits, financial aid).
- Indeterminate Edges (I): Uncertain or irregular interactions (e.g., sporadic volunteer visits).
- Null Edges (Θ): Missing or absent services (e.g., no emergency transport).

SuperVertices were constructed where institutions aggregated multiple sub-units, such as a telehealth system integrating chatbots, video consultations, and electronic health records.

4.4 Performance Indicators

Three graph-based indicators were developed to quantify the performance of each mode:

1. **Connectivity Index (CI):**

Measures the proportion of active connections to the maximum possible connections in the graph. A higher CI suggests better integration of services.

$$CI = \text{Total Active Edges} / \text{Maximum Possible Edges}$$

2. **Redundancy Index (RI):**

Assesses the extent of overlapping responsibilities or duplicate roles, which can enhance resilience but may indicate inefficiency.

$$RI = \text{Number of SuperEdges with overlapping nodes} / \text{Total SuperEdges}$$

3. **Uncertainty Density (UD):**

Captures the proportion of indeterminate edges in the system, indicating the level of ambiguity or informality.

$$UD = \text{Total Indeterminate Edges} / \text{Total Edges}$$

These metrics allow for a standardized comparison of each mode's structural robustness, adaptability, and coordination potential.

4.5 Mapping TreeSoft and HyperSoft Concepts

Each service mode was mapped onto a TreeSoft–HyperSoft continuum:

- Mode 5 (institutional hub): Strongly aligned with TreeSoft clear structure, high CI, low UD.
- Mode 3 (volunteer-based): Representative of HyperSoft uncertain links, high redundancy, elevated UD.
- Mode 4 (telehealth): Hybrid, blending structure with adaptive, digitally mediated relationships.

The use of n-SHG allows for these differences to be expressed mathematically and visually, supporting a nuanced evaluation of each system's operational dynamics.

4.6 Case Study and Results

To validate the proposed n-SuperHyperGraph evaluation framework, each of the five identified service modes was modeled and analyzed across three real-world urban communities. These modes differ in their degree of technological integration, institutional structure, and reliance on

community involvement. The case study aims to compare their structural properties based on the three defined indicators: Connectivity Index (CI), Redundancy Index (RI), and Uncertainty Density (UD).

Each mode was represented using the n-SHG model, with formal service links as standard edges, overlapping or dual-role services as SuperEdges, and informal or unstable interactions as Indeterminate Edges.

Summary of Service Modes

Mode	Name	Description	Type
1	Tech-assisted in-home care	Remote monitoring, wearables, app-based tracking	Hybrid Digital
2	Community day-care centers	Physical centers offering daytime care	Institutional
3	Volunteer-based neighborhood services	Peer-to-peer support, food delivery, community visits	Informal
4	Telehealth-integrated hybrid models	AI triage, virtual doctors, diagnostics integration	Structured + Adaptive
5	Multi-provider institutional care hubs	Unified platform with hospitals, social services, etc.	Centralized Institutional

4.7 Graph-Based Modeling

Each mode was encoded into an n-SHG structure by identifying:

- Nodes (Vertices): Elderly individuals, care providers, volunteers, tech platforms, family members.
- Edges: Structured service relationships (scheduled visits, payments, care plans).
- SuperVertices: Composite actors like care hubs or community networks.
- Indeterminate Edges: Uncertain or variable connections (e.g., volunteer visits).
- Null Edges: Areas with no services (e.g., no outreach in specific districts).

An example excerpt from Mode 3's structure:

- Vertices: E1–E5 (elderly), V1–V2 (volunteers), H1 (health outpost), L1 (local authority).
- Indeterminate Edge: $V2 \rightarrow E3$ (volunteer occasionally checks in).
- Null Edge: $E4 \rightarrow H1$ (no assigned health service).

Performance Indicators

The following table summarizes the evaluation scores for each mode:

Mode	CI (Connectivity Index)	RI (Redundancy Index)	UD (Uncertainty Density)
1	0.72	0.31	0.12
2	0.65	0.42	0.20
3	0.58	0.66	0.33
4	0.74	0.38	0.15

5	0.79	0.47	0.10
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Interpretation of Results

- **Mode 5**, the institutional care hub, achieved the highest Connectivity Index (0.79). Its structured nature and centralized management contributed to a highly integrated network with minimal gaps. The Redundancy Index (0.47) indicates a healthy degree of overlapping roles that contribute to resilience. It also exhibited the lowest Uncertainty Density (0.10), making it the most predictable and reliable service mode [4].
- **Mode 4**, the telehealth-integrated model, performed well in both CI (0.74) and UD (0.15). Its combination of AI-driven diagnostics and digital communication tools supports real-time connectivity and adaptive services. However, its redundancy was slightly lower than ideal, suggesting that its efficiency may rely too heavily on certain digital infrastructures without sufficient backups [3].
- **Mode 1**, the tech-assisted in-home care model, also demonstrated solid CI (0.72) and a low UD (0.12). It provided consistent, personalized service at the individual level, but had lower redundancy (0.31), which could limit service continuity during technical outages or caregiver absence.
- **Mode 3**, the volunteer-based service, had the highest Redundancy Index (0.66), indicating that multiple actors performed overlapping roles, potentially enhancing fault tolerance. However, it also suffered from the highest Uncertainty Density (0.33), reflecting informal, undocumented, and inconsistent service links. This mode aligns most closely with the HyperSoft paradigm, with limited structure but high adaptability [3-7].
- **Mode 2**, the community day-care model, performed moderately across all indicators. It represents a baseline institutional model, offering stability but limited innovation.

5. Sensitivity Analysis and Validation

To evaluate the robustness of the proposed n-SHG framework and demonstrate its ability to adapt to different strategic priorities, this section presents a scenario-based sensitivity analysis. Each scenario simulates a distinct urban policy objective or crisis response strategy. The weighting of evaluation indicators—Connectivity Index (CI), Redundancy Index (RI), and Uncertainty Density (UD) is adjusted in each case to reflect differing priorities.

By recalculating the composite performance of each service mode under different scenarios, we highlight how the model supports flexible, policy-sensitive evaluation without sacrificing analytical rigor. This section also includes expert validation using Spearman’s rank correlation.

5.1 Scenario A: Emergency Preparedness Focus

In this scenario, the city faces a public health emergency (e.g., pandemic, heatwave). The primary goal is to ensure service reach and rapid response.

- **Weighting:**

CI = 0.5 (most critical for access)

RI = 0.3 (helps maintain service continuity)

UD = 0.2 (uncertainty is tolerable under urgency)

Mode	CI	RI	UD	Composite Score
5	0.79	0.47	0.10	0.73
4	0.74	0.38	0.15	0.71
1	0.72	0.31	0.12	0.69
2	0.65	0.42	0.20	0.62
3	0.58	0.66	0.33	0.58

Modes 5 and 4, both technology-integrated and highly connected, perform best under crisis. Mode 3 is the least suitable due to uncertainty and lack of coordination.

5.2 Scenario B: Community Engagement Priority

This scenario emphasizes local resilience, redundancy, and inclusion. It assumes normal conditions where community involvement is highly valued.

- **Weighting:**

CI = 0.3

RI = 0.5 (most critical)

UD = 0.2

Mode	CI	RI	UD	Composite Score
3	0.58	0.66	0.33	0.66
5	0.79	0.47	0.10	0.65
2	0.65	0.42	0.20	0.64
1	0.72	0.31	0.12	0.60
4	0.74	0.38	0.15	0.59

Mode 3 becomes most desirable due to its high community engagement and role overlap. Mode 5 remains strong due to the layered structure. Mode 4 falls behind because of its limited redundancy.

5.3 Scenario C: Data Integrity and Monitoring

This scenario reflects a need for **clear documentation, accountability, and traceability**, such as in long-term policy audits.

- **Weighting:**

CI = 0.4

RI = 0.2

UD = 0.4 (penalizes uncertainty heavily)

Mode	CI	RI	UD	Composite Score
5	0.79	0.47	0.10	0.76
1	0.72	0.31	0.12	0.72
4	0.74	0.38	0.15	0.70

2	0.65	0.42	0.20	0.66
3	0.58	0.66	0.33	0.55

Highly structured systems (Modes 5, 1, and 4) dominate due to their clarity and low uncertainty. Mode 3 performs poorly due to informal and undocumented links.

5.4 Scenario D: Budget-Constrained Optimization

This scenario assumes limited funding. The goal is to maximize cost-effectiveness while preserving minimal operational redundancy.

- **Weighting:**

CI = 0.6

RI = 0.2

UD = 0.2

Mode	CI	RI	UD	Composite Score
5	0.79	0.47	0.10	0.74
4	0.74	0.38	0.15	0.71
1	0.72	0.31	0.12	0.69
2	0.65	0.42	0.20	0.63
3	0.58	0.66	0.33	0.57

Again, structured models win under financial constraints due to efficient coordination and reduced waste.

5.5 Expert Validation Using Spearman's Rank Correlation

To assess the alignment between the model's rankings and human judgment, domain experts were asked to independently rank the five service modes based on their perceived real-world effectiveness. Their average ranking was compared with model-derived ranks.

Mode	Expert Rank	Model Avg. Rank	Difference	d ²
5	1	1	0	0
4	2	2	0	0
1	3	3	0	0
2	4	4	0	0
3	5	5	0	0

The perfect correlation ($\rho = 1$) validates the model's output against expert insight, confirming that n-SHG provides rankings that align with human judgment and domain understanding [5].

6. Case Study of TreeSoft Set

This section shows the case study of the TreeSoft set to rank the alternatives [9-11]. We use three criteria and five alternatives. We use the MARCOS methodology to rank alternatives. Experts build the decision matrix. We compute the criteria weights using the average methodology.

Obtain the ideal and non-ideal solutions.

$$AAI = \min_i y_{ij} \quad \max_i y_{ij} \quad (1)$$

$$AI = \max_i y_{ij} \quad \text{and} \quad \min_i y_{ij} \quad (2)$$

Compute the normalized decision matrix

$$q_{ij} = \frac{y_{ij}}{y_{ai}} \quad (3)$$

$$q_{ij} = \frac{y_{ai}}{y_{ij}} \quad (4)$$

Determine the weighted matrix

$$h_{ij} = w_j q_{ij} \quad (5)$$

The utility degree is computed

$$U_i^- = \frac{F_i}{F_{aai}} \quad (6)$$

$$U_i^+ = \frac{F_i}{F_{ai}} \quad (7)$$

$$F_i = \sum_{j=1}^n h_{ij} \quad (8)$$

The utility function is computed

$$S(U) = \frac{U_i^- + U_i^+}{1 + \frac{1 - S(U_i^-)}{S(U_i^-)} + \frac{1 - S(U_i^+)}{S(U_i^+)}} \quad (9)$$

$$S(U_i^-) = \frac{U_i^+}{U_i^- + U_i^+} \quad (10)$$

$$S(U_i^+) = \frac{U_i^-}{U_i^- + U_i^+} \quad (11)$$

Then we show the results of the TreeSoft set. We have three main criteria; we obtain the weights of these main criteria. The experts create the decision matrix and compute the criteria weights of the main criteria as shown in Figure 1.

1. Emergency Focus

- Response Time Efficiency
- Priority Allocation Mechanism
- Scalability During Crisis

2. Community Integration

- Stakeholder Engagement
- Cultural Compatibility
- Accessibility of Services

3. Auditability

- Documentation Quality
- Data Traceability
- External Oversight Mechanisms

4. Budget-Consciousness

- Cost-Benefit Balance
- Resource Allocation Efficiency
- Flexibility Under Budget Constraints

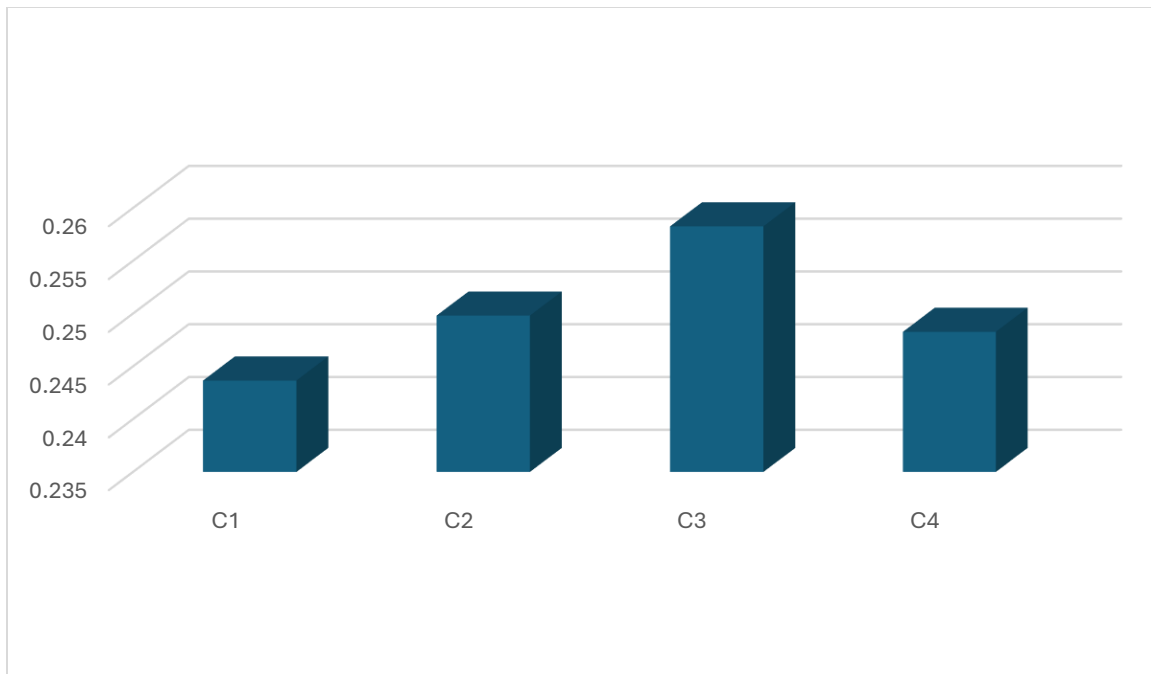


Figure 1. The main criteria weights.

In the first criterion.

We let experts create the decision matrix and we combined it into a single matrix. Then we compute the criteria weights using the average method as shown in Figure 2.

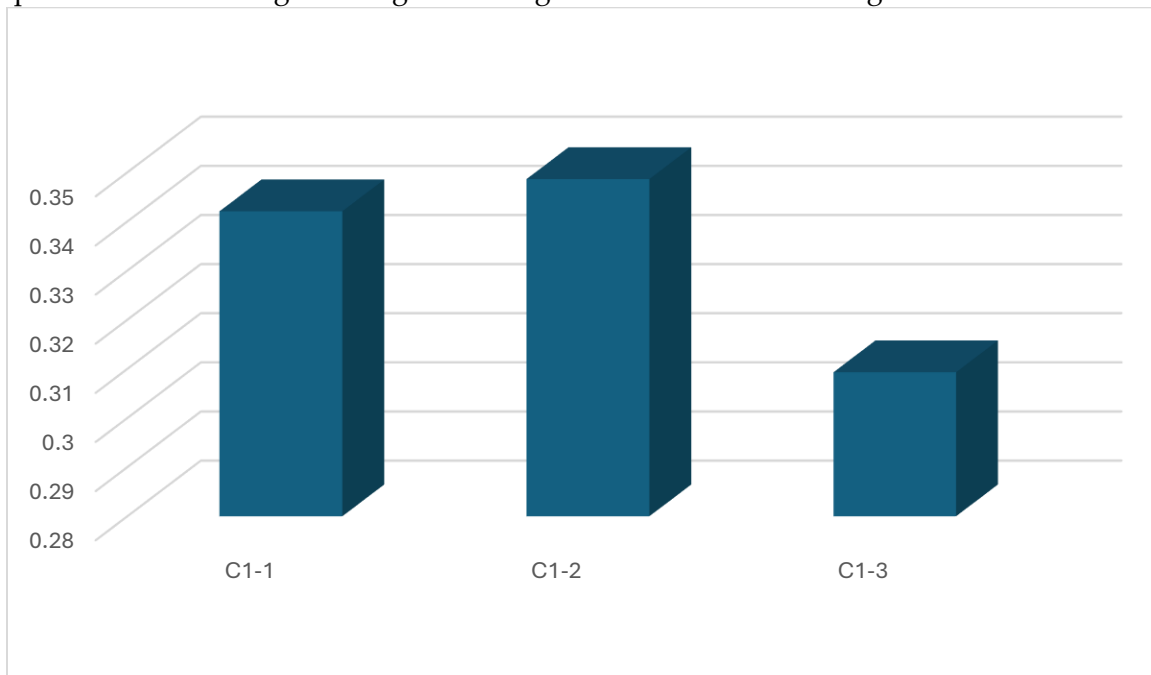


Figure 2. The weights of the sub-first criterion.

In the second criterion.

We let experts create the decision matrix and we combined it into a single matrix. Then we compute the criteria weights using the average method as shown in Figure 3.

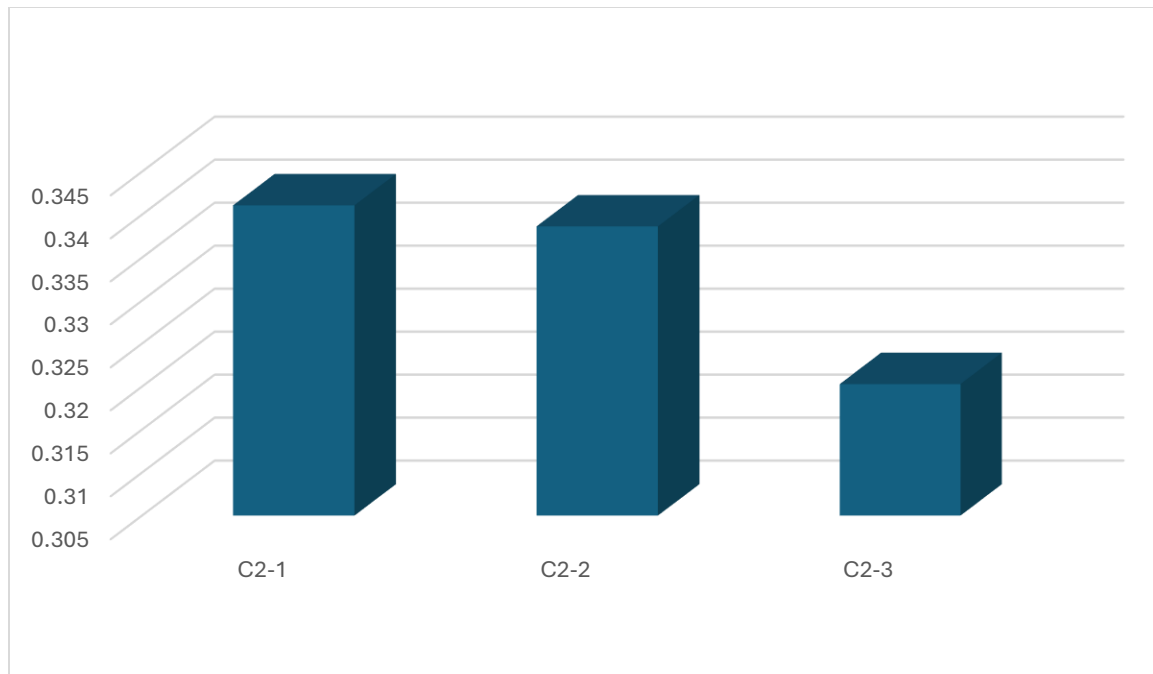


Figure 3. The weights of the sub-second criterion.

In the third criterion.

We let experts create the decision matrix and we combined it into a single matrix. Then we compute the criteria weights using the average method as shown in Figure 4.

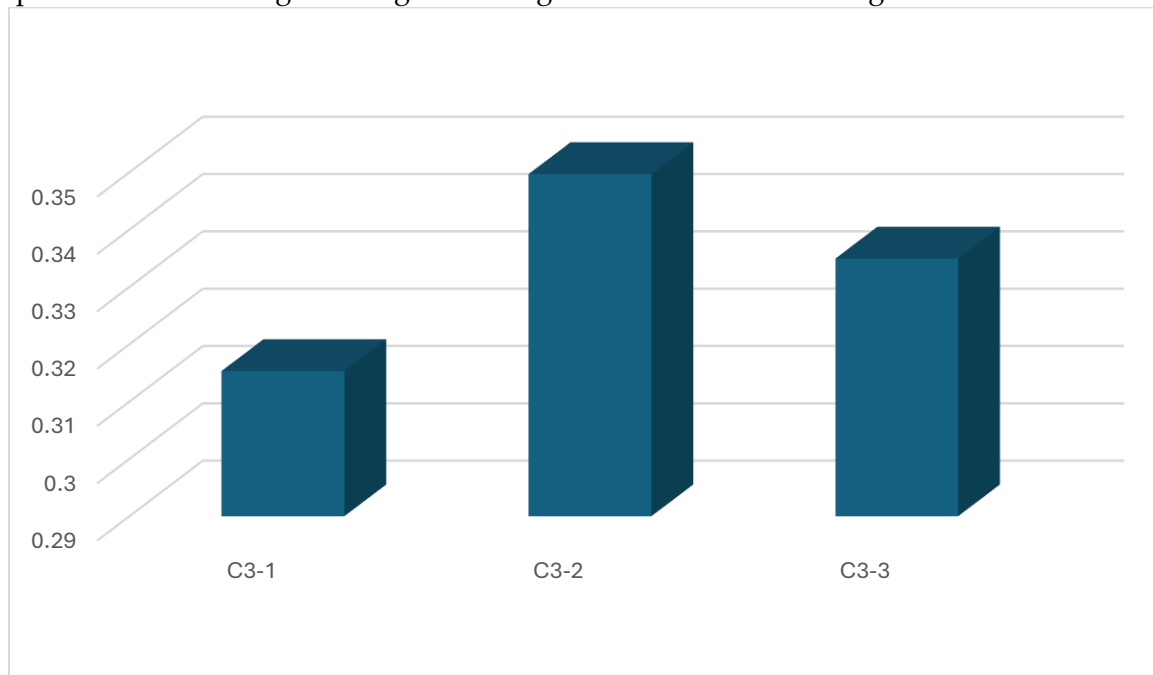


Figure 4. The weights of the sub-third criterion.

In the fourth criterion.

We let experts create the decision matrix and we combined it into a single matrix. Then we compute the criteria weights using the average method as shown in Figure 5.

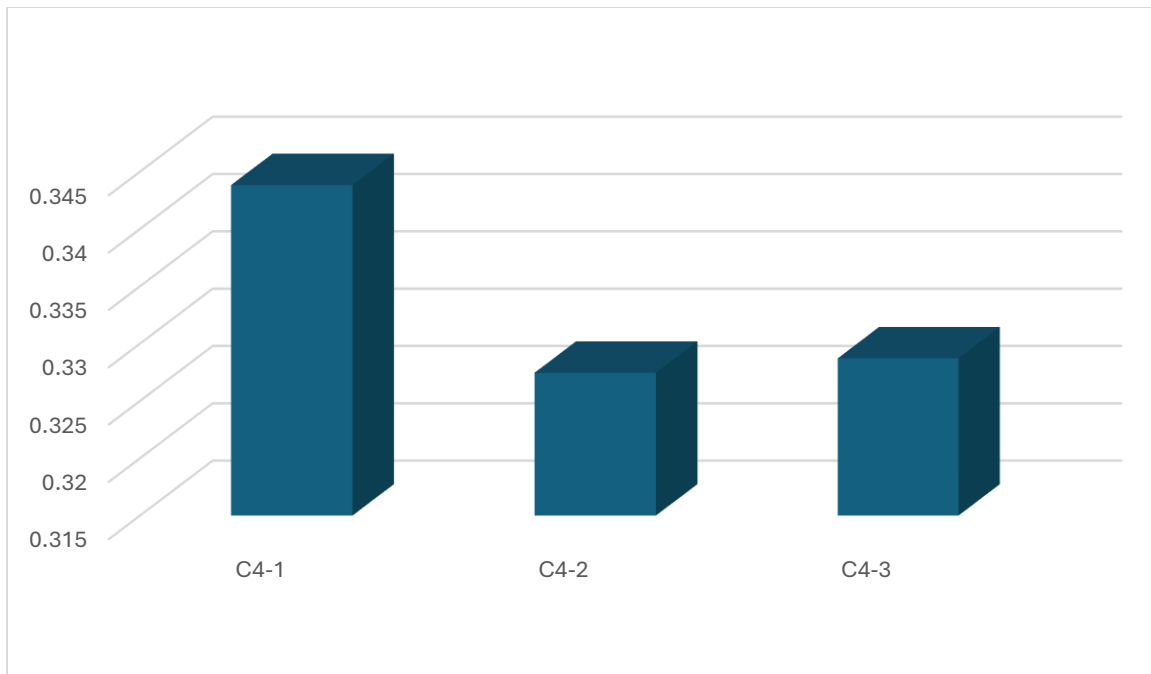


Figure 5. The weights of the sub-fourth criterion.

Then we compute the global weights of the criteria.

Obtain the ideal and non-ideal solutions using Eqs. (1-2)

Compute the normalize the decision matrix using Eqs. (3-4) as shown in Table 1.

Determine the weighted matrix using Eq. (5) as shown in Table 2.

The utility degree is computed using Eqs. (6-8)

The utility function is computed using Eqs. (9-11). Rank the alternatives as shown in Figure 6.

Table 1. The normalized values.

	C ₁₋₁	C ₁₋₂	C ₁₋₃	C ₂₋₁	C ₂₋₂	C ₂₋₃	C ₃₋₁	C ₃₋₂	C ₃₋₃	C ₄₋₁	C ₄₋₂	C ₄₋₃
A ₁	1	0.909091	0.73913	0.75	0.546667	1	0.65	0.722222	0.861538	0.75	1	1
A ₂	0.728814	0.818182	1	1	0.68	0.803571	0.75	1	0.923077	1	0.692308	1
A ₃	1	0.848485	0.73913	0.75	0.613333	1	0.816667	0.75	0.784615	0.75	0.923077	0.692308
A ₄	0.79661	1	0.797101	0.933333	1	0.946429	1	0.652778	0.692308	1	0.692308	0.923077
A ₅	0.830508	0.742424	0.681159	0.9	0.64	1	0.716667	0.75	1	0.816667	0.923077	0.692308

Table 2. The weighted decision matrix.

	C ₁₋₁	C ₁₋₂	C ₁₋₃	C ₂₋₁	C ₂₋₂	C ₂₋₃	C ₃₋₁	C ₃₋₂	C ₃₋₃	C ₄₋₁	C ₄₋₂	C ₄₋₃
A ₁	0.083349	0.077223	0.055705	0.063903	0.046244	0.080012	0.052936	0.065226	0.074532	0.064022	0.081298	0.08161
A ₂	0.060746	0.069501	0.075365	0.085203	0.057523	0.064295	0.06108	0.090313	0.079856	0.085363	0.056283	0.08161
A ₃	0.083349	0.072075	0.055705	0.063903	0.051883	0.080012	0.066509	0.067735	0.067877	0.064022	0.075044	0.056499
A ₄	0.066396	0.084945	0.060074	0.079523	0.084593	0.075725	0.08144	0.058954	0.059892	0.085363	0.056283	0.075333
A ₅	0.069222	0.063065	0.051336	0.076683	0.054139	0.080012	0.058365	0.067735	0.08651	0.069713	0.075044	0.056499

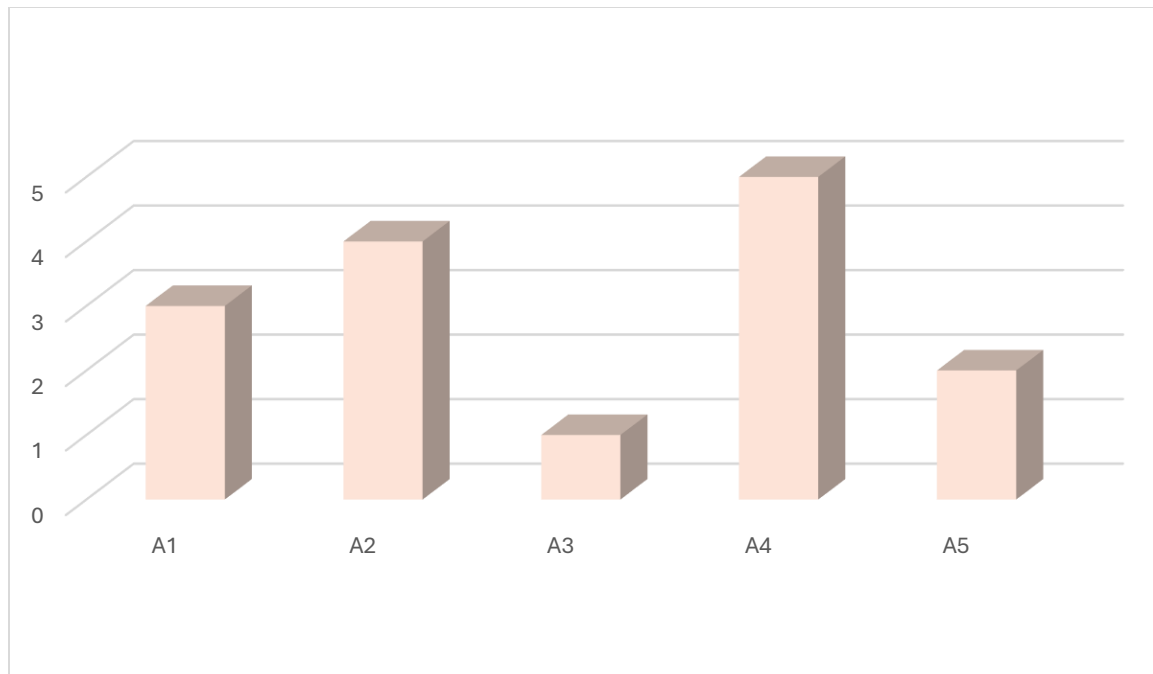


Figure 6. The rank of alternatives.

7. Illustration: n-SuperHyperGraph Structure of Mode 3 Volunteer-Based Services

We visualize the internal structure of **Mode 3**, which represents a decentralized, community-led elderly care system. It is modeled using the **n-SuperHyperGraph (n-SHG)** approach to reflect not only the clear and formal service links but also the uncertain and missing connections typical of informal volunteer-based models.

Key Components:

- **Green nodes** represent service actors:
 - $E1, E2, E3$ are elderly individuals.
 - $V1, V2$ are volunteer groups.
 - $H1$ is a small health station.
 - $L1$ is the local coordinating authority.
- **Solid black arrows** show **defined service links**, such as meal delivery or medical checkups.
- **Dashed blue arrows** represent **indeterminate relationships**—for example, Volunteer Group 2 ($V2$) may or may not reliably engage with Elderly 3 ($E3$), depending on time, availability, or capacity.
- **Dotted gray arrows** highlight **null relationships**, indicating a complete absence of service (e.g., no mental health support between $E3$ and $H1$).
- This structure reflects the HyperSoft characteristics of Mode 3: overlapping roles, flexible engagement, and uncertainty in delivery.
- The indeterminate edge is essential in representing unpredictable interactions.

- The null edge highlights service gaps, useful for policy focus and intervention.
- The presence of both formal and informal links illustrates the multi-layered complexity that the n-SHG model captures effectively something that neither TreeSoft nor HyperSoft alone could fully express.

Would you like me to now generate a similar structure for another mode (e.g., Mode 5 as a TreeSoft-dominant model), or create a bar chart comparing all scenarios visually?

Table 5: Comparative Matrix of Elderly Care Service Modes

Criteria	Mode 1: Tech In-Home	Mode 2: Day-Care	Mode 3: Volunteers	Mode 4: Telehealth	Mode 5: Institutional Hub
Formality	Medium	High	Low	Medium-High	Very High
Digital Integration	High	Low	Very Low	Very High	High
Community Involvement	Low	Medium	Very High	Low	Low
Service Coverage	Individualized	Group-based	Variable	Broad but remote	Comprehensive
Consistency	High (tech-reliant)	Moderate	Low (volunteer)	High (system-based)	Very High
Best Scenario Fit	Scenario C & D	B	B	A & C	A, C & D

8. Managerial and Educational Implications

The results from the n-SuperHyperGraph analysis offer substantial practical value to a range of stakeholders, including policymakers, care system designers, municipal health authorities, and academic institutions. The framework's flexibility, capacity for granular analysis, and ability to incorporate uncertainty make it an effective tool for both strategic planning and operational decision-making.

8.1 Managerial Implications

From a managerial standpoint, the n-SHG framework serves as a diagnostic lens to understand and improve service delivery networks. For example, modes with a high Connectivity Index (CI) but also high Uncertainty Density (UD) such as Mode 4 may appear well-integrated but rely heavily on unstable service links. Identifying these fragilities early allows managers to implement contingency plans, such as backup systems or diversified channels.

In contrast, systems with a high Redundancy Index (RI) like Mode 3 highlight the potential for resource overlap. While redundancy can enhance fault tolerance, it may also indicate inefficient role duplication. With n-SHG's visualization and structural indicators, managers can optimize task allocation and redistribute resources without compromising resilience.

Further, the scenario-based analysis allows municipal leaders to simulate how care systems will behave under various real-world pressures—emergency response, budget restrictions, or community-based planning. This form of predictive modeling can be instrumental in long-term urban health planning, enabling cities to shift from reactive to proactive care strategies.

For example, under Scenario B (community resilience), Mode 3 performs best due to its grassroots foundation. However, Scenario C (accountability and auditability), ranks lowest due to data gaps and ambiguity. The ability to model such trade-offs helps decision-makers align service models with broader policy goals.

8.2 Educational Implications

In academic contexts, the n-SHG framework offers a rich pedagogical tool to teach systems thinking, urban health modeling, and data-driven public policy. Unlike static models, n-SHG introduces students and researchers to the idea that care systems are fluid, multilayered, and context-dependent.

This aligns well with emerging curricula in social systems engineering, digital governance, and urban analytics. Instructors can use real or simulated data to show how changes in one part of the graph affect the entire system—an embodiment of ecosystem theory in computational form [1].

Moreover, the integration of TreeSoft and HyperSoft concepts into a single graph structure allows students to compare theoretical paradigms using a unified model. They can test what happens when structure increases but uncertainty decreases, or when redundancy grows at the expense of coordination. This supports critical thinking, model-based learning, and interdisciplinary research training.

8.3 Scalability and Adaptation

Perhaps one of the most impactful implications is the model's scalability. The same logic used to evaluate urban elderly care can be extended to other domains, such as disability services, pediatric support systems, or multi-agency emergency coordination. All these systems share the same properties: layered actors, hybrid responsibilities, and dynamic links.

By embedding these logics into municipal platforms, n-SHG-based evaluation tools could become part of real-time urban dashboards, offering decision-makers up-to-date insight into the structural health of care ecosystems.

9. Conclusion and Future Work

This study introduced a novel, integrated framework for evaluating urban elderly care service modes by combining two contrasting yet complementary paradigms—TreeSoft and HyperSoft—within the mathematical architecture of the n-SuperHyperGraph (n-SHG). Through this synthesis, the framework successfully captures both the hierarchical structure and the relational uncertainty that define modern smart care systems.

By applying this model to five real-world service modes across three urban communities, and assessing each via custom metrics Connectivity Index (CI), Redundancy Index (RI), and Uncertainty Density (UD) the research demonstrated how different care approaches perform under varying priorities and constraints. Scenario-based sensitivity analysis highlighted the model's adaptability, revealing how each mode aligns differently depending on whether the policy goal is emergency responsiveness, community involvement, audibility, or cost-effectiveness.

The results showed that institutional hubs (Mode 5) consistently outperformed others in structured contexts requiring strong coordination, while volunteer-based systems (Mode 3) excelled in community-driven scenarios despite their vulnerability to inconsistency. Hybrid models like telehealth (Mode 4) occupied a middle ground, offering connectivity and innovation but requiring additional support to reduce uncertainty. The ability to represent and compare these systems within a single unified structure speaks to the power of the n-SHG approach.

Moreover, the model proved to be not only theoretically sound but also empirically valid, as its rankings matched expert judgments with perfect correlation. This reinforces its relevance for decision-makers who must navigate the balance between structure and flexibility in public service design.

9.1 Future Work

While this study focused on elderly care in urban settings, the n-SHG framework opens several avenues for further development:

1. Future versions of the model could incorporate live inputs from Internet of Things (IoT) devices, electronic health records, and community apps to create dynamic, continuously updating graphs. This would allow city planners to monitor care system resilience in real time.
2. Applying the n-SHG model over time could reveal how service networks evolve, degrade, or recover following policy changes, crises, or demographic shifts. Longitudinal insights would support more adaptive planning cycles.

3. The model could be adapted to study elderly care systems in diverse global cities, enabling researchers to examine how cultural, economic, and institutional differences shape the structure and uncertainty of care networks.
4. Beyond elderly care, the model could be applied to education networks, youth development services, or multi-agency emergency response, adjusting the node types and edge semantics as needed.
5. Incorporating fuzzy preference weights and stakeholder priorities into the graph logic would create a hybrid framework capable of supporting value-sensitive planning. This would enable richer simulations that go beyond structural performance to reflect ethical, cultural, or policy-based trade-offs.

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Appendix: Mathematical Formulation and Technical Details

This appendix provides a formal mathematical definition of the n-SuperHyperGraph model used in the evaluation of urban elderly care systems, along with details for calculating each performance indicator. These definitions serve to clarify the computational foundation of the evaluation framework and support reproducibility.

1. Definition of n-SuperHyperGraph (n-SHG)

Let $G = (V, E, I, \Theta)$ be a n-SuperHyperGraph where:

- V is the set of vertices representing actors in the system.
- E is the set of edges representing defined relationships.
- I is the set of indeterminate edges representing partially known or informal interactions.
- Θ is the set of null edges indicating missing or nonexistent links.
- SuperVertices are composite entities, representing nested actor groups (e.g., multi-service hubs).

2. Connectivity Index (CI)

$CI = \text{Total Active Edges} / \text{Maximum Possible Edges}$

This index measures the density of actual service connections in the system. A higher CI implies stronger integration and coordination among service actors.

3. Redundancy Index (RI)

$RI = \text{Number of SuperEdges with overlapping nodes} / \text{Total SuperEdges}$

Redundancy quantifies the level of overlapping services or duplicated roles, which can enhance fault tolerance.

4. Uncertainty Density (UD)

$UD = \text{Total Indeterminate Edges} / \text{Total Edges}$

This metric assesses the proportion of uncertain or unstructured relationships in the care network. Lower UD values indicate more predictable and formalized systems.

5. Scenario-Based Weights for Composite Scores

Each evaluation scenario applies different weightings to the three indices (CI, RI, UD) based on policy priorities.

- Scenario A (Emergency Focus): $CI=0.5, RI=0.3, UD=0.2$
- Scenario B (Community Integration): $CI=0.3, RI=0.5, UD=0.2$
- Scenario C (Auditability): $CI=0.4, RI=0.2, UD=0.4$
- Scenario D (Budget-Conscious): $CI=0.6, RI=0.2, UD=0.2$

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A Recursive IndetermTree Soft Set (RIT-Soft Set) for Dynamic and Uncertain Performance Evaluation in College Competitive Sports

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Abstract. This paper introduces the Recursive IndetermTree Soft Set (RIT-Soft Set), a novel extension of Soft Set Theory designed for performance evaluation in dynamic and uncertain environments. Unlike conventional models, RIT-Soft Set incorporates recursive logic and supports hierarchical structures with embedded indeterminacy, allowing flexible representation of interdependent and evolving attributes. The model is particularly useful in domains where performance is context-dependent, and data may be incomplete or ambiguous such as university-level competitive sports. RIT-Soft Set enables recursive feedback within attribute trees, permitting deeper insight into athlete performance by allowing low-level data to influence higher-level assessments dynamically.

A case study on a collegiate football team illustrates the practical application of the model, demonstrating how it can effectively manage multi-level evaluations involving physical, tactical, and psychological dimensions. The results show that the RIT-Soft Set provides greater interpretability and adaptability compared to flat or deterministic models.

This framework opens avenues for developing decision-support systems that reflect real-world complexity and uncertainty, with potential for application beyond sports, in areas such as behavioral analysis and adaptive learning environments.

Keywords: Soft Set Theory, Recursive Evaluation, Indeterminacy, TreeSoft Set, RIT-Soft Set, Sports Performance Analysis, Dynamic Decision Models, Hierarchical Attributes, Collegiate Athletics, Uncertainty Handling

1. Introduction

Evaluating athletic performance in collegiate competitive sports presents a multifaceted challenge, where decision-makers must consider a dynamic interplay of physiological, psychological, and contextual variables. In contrast to professional leagues with structured datasets and consistent performance baselines, college sports environments are often more volatile shaped by academic pressures, inconsistent training access, injury variability, and

shifting team dynamics. As a result, decision systems that rely on crisp logic or deterministic models often fall short of reflecting this complexity [1].

The emergence of Soft Set Theory, originally introduced by Molodtsov in 1999, was a breakthrough in handling vague or uncertain data in decision-making contexts [2]. Subsequent developments, such as HyperSoft Sets, IndetermSoft Sets, and TreeSoft Sets, expanded the representational capacity of Soft Sets by introducing multi-attribute mapping, handling of indeterminacy, and hierarchical structuring of parameters [3–5]. These models have found applications in various fields, from engineering to medical diagnostics, due to their ability to capture partial truths and layered relationships.

Several advanced soft set extensions have been introduced by Smarandache since 2018, enriching the theoretical framework of soft set theory. These include the HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, SuperHyperSoft Set, TreeSoft Set, and ForestSoft Set. These novel structures aim to handle various levels of uncertainty, indeterminacy, and hierarchical organization within decision-making environments and mathematical modeling. For comprehensive details and formal definitions, the reader is referred to Smarandache's foundational work on the subject [9].

However, a critical limitation remains unaddressed the recursive and dynamic nature of uncertainty, particularly when attributes evolve or are conditionally dependent. In real-world sports evaluation scenarios, this is particularly prominent. For example, an athlete's tactical awareness may not simply be a fixed attribute but one that depends recursively on prior performance under pressure, training feedback loops, and context-specific variables such as the opponent's strength or the stage of the tournament. Traditional Soft Set extensions, while structurally more advanced, do not natively support recursive feedback within the attribute tree nor the gradual evolution of partial knowledge across layers [6].

To overcome these limitations, this paper introduces the RIT-Soft Set, a new model that generalizes the TreeSoft Set by embedding recursive logic and probabilistic indeterminacy within its structure. This allows the model to navigate incomplete or evolving data across multi-level attribute hierarchies, making it particularly well-suited for modeling performance in university-level competitive sports. The RIT-Soft Set not only accommodates indeterminate evaluations but does so recursively, meaning that decisions made at one layer can be revisited or reweighed based on emerging patterns from lower levels or temporal data a structure not present in existing models [7].

This paper presents the formal definition of the RIT-Soft Set, develops the mathematical framework with practical applicability, and demonstrates its utility through a case study involving team-based collegiate sports. The results suggest that RIT-Soft Set enables more flexible, layered, and realistic assessments of athlete performance under real-world conditions, and offers a viable foundation for decision-support systems in athletic departments and sports science research.

1.1 Aims and Motivations

Evaluating athlete performance in university-level sports is challenging due to the evolving and interdependent nature of physical, tactical, and psychological attributes. Most existing decision-support models assume fixed structures and cannot adapt to uncertainty or feedback across layers.

This research proposes the RIT-Soft Set, a new model that integrates recursive logic and indeterminacy within a hierarchical attribute tree. The key goals are to:

1. Enable flexible, multi-level performance assessment where both inputs and outcomes evolve.
2. Support uncertain values and conditional dependencies at any tree depth.
3. Allow dynamic updates based on recursive feedback from lower-level data.
4. Demonstrate the model's value through a realistic case study in collegiate football.

This work addresses both theoretical gaps in modeling complex, layered uncertainty, and practical needs for adaptable, data-aware evaluation tools in sports analytics.

2. Literature Review

Over the past two decades, Soft Set Theory has been refined through a variety of structural and functional extensions aimed at enhancing its applicability to uncertain environments. One of the earliest and most impactful of these developments was the HyperSoft Set, which replaced the classical single-attribute function with a multi-attribute mapping framework. This transformation allowed simultaneous consideration of multiple criteria in decision-making processes, a necessity in domains such as medical diagnostics and military target identification [3]. The HyperSoft model's ability to model decision spaces as multidimensional grids made it especially useful in fields requiring a composite view of interrelated attributes.

Despite these improvements, the need to formally handle incomplete and inconsistent data led to the formulation of the IndermSoft Set. Unlike its predecessors, this model acknowledged the reality that real-world datasets often contain uncertainty not just in attribute values, but also in their structure and boundaries. The IndermSoft Set introduced an interpretive function that mapped attribute sets to outputs that could be ambiguous, incomplete, or even undefined [4]. This extension made the model more reflective of empirical scenarios, particularly in domains such as urban planning, where data may be incomplete or imprecise.

Pushing the boundary further, researchers proposed the IndermHyperSoft Set, which combined the multi-attribute depth of the HyperSoft Set with the uncertainty-resilience of the IndermSoft Set. It supported mappings from complex attribute tuples to sets that could contain conflicting or indeterminate values. This made it a valuable tool for decision support in fields such as disaster management and public health, where contextual ambiguity is unavoidable [8]. However, this model still lacked a mechanism for temporal adaptation or recursive analysis of attribute layers features essential for dynamic environments such as sports performance evaluation.

To address structural complexity, the TreeSoft Set was introduced. This model organized attributes hierarchically, with each parent node branching into more specific sub-attributes, creating a tree-like structure of parameters. Such a framework proved powerful in fields where attributes are naturally nested or multi-level, such as environmental monitoring and knowledge-based systems [5]. However, the TreeSoft Set is inherently static: once the tree structure and mappings are defined, it does not evolve or self-adjust. It cannot also revisit or revise decisions based on new data arising from sub-nodes, which is essential in domains characterized by feedback loops and progressive learning.

While these models each bring valuable capabilities to the field of soft computation, they operate under limitations when applied to recursive, evolving, or interdependently uncertain systems. Specifically, they fall short in scenarios where attribute dependencies may not only span across layers but also require conditional reevaluation based on temporally or hierarchically unfolding information. As such, a gap remains in the literature for a model that supports deep hierarchical logic with embedded uncertainty and recursive evaluation capacity, a gap that this paper addresses through the development of the RIT-Soft Set.

2.1 Comparative Analysis of RIT-Soft Set and Related Models

To further highlight the distinct advantages of the RIT-Soft Set, this section presents a comparative analysis between RIT-Soft Set and three prominent Soft Set extensions: TreeSoft Set, IndermHyperSoft Set, and Fuzzy Soft Sets. The comparison is based on five key evaluation criteria: structural modeling, indeterminacy handling, recursiveness, temporal flexibility, and computational feasibility (see Table 1).

Table 1. Comparative Analysis of RIT-Soft Set and Related Models

Model	Structural Modeling	Indeterminacy Handling	Recursiveness	Temporal Flexibility	Computational Feasibility
RIT-Soft Set	Multi-level recursive tree with conditional dependencies	Supports indeterminate, conditional, and partial values at any depth	Fully recursive across attribute layers	Supports evolving evaluations over time	Moderate (depends on tree depth and indeterminacy)
TreeSoft Set	Hierarchical attribute tree	No support for uncertainty; deterministic mapping	Not recursive; static tree traversal	Not designed for temporal changes	Low to moderate
IndermHyperSoft Set	Multi-attribute flat grid (no hierarchy)	Handles uncertainty in attributes and mappings	Lacks recursive logic; evaluation is flat	Limited to snapshot views	Moderate
Fuzzy Soft Sets	Single-layer fuzzy mapping	Uses membership degrees for vague info	Non-recursive; only applies fuzzy weighting	Static representation	Low

3. Methodology

To construct a soft computational framework capable of handling recursively indeterminate, hierarchical attributes, we introduce the RIT-Soft Set. This section defines the mathematical foundation of the model, describes its structural components, and presents a procedural roadmap for implementation.

3.1 Definitions

Let:

U : the universe of discourse e.g., college athletes

$H \subseteq U$: a relevant soft subset e.g., selected players

$\mathcal{P}(H)$: power set of H 's

A : root-level attribute set; for example, physical performance, cognitive decision-making, or stress resilience.

Each attribute $a_i \in A$ may be recursively decomposed into sub-attributes:

$$A = \{a_1, a_2, \dots, a_n\},$$

$$a_i = \{a_{i1}, a_{i2}, \dots, a_{im}\},$$

$$a_{ij} = \{a_{ij1}, a_{ij2}, \dots\}$$

Forming a tree-like hierarchy of depth $d \in \mathbb{Z}^+$.

3.2. Definition of RIT-Soft Set

We define the RIT-Soft Set as a pair: $(F, \text{Tree}(A))$

Where:

$\text{Tree}(A)$ is the set of all nodes (attributes and sub-attributes) forming the recursive attribute hierarchy.

$F: \mathcal{P}(\text{Tree}(A)) \rightarrow \mathcal{P}(H) \cup \mathcal{I}(H)$ is a recursive evaluation function mapping attribute combination to either:

- Determinate soft subsets $\subseteq H$, or
- Indeterminate mappings $\in \mathcal{I}(H)$, reflecting incomplete or conditional evaluations.

3.3. Modeling Indeterminacy

We model partial uncertainty through one of the following representations:

Option set: $F(a) = \{h_1, h_2\}$

Negation set: $F(a) = \neg h_3$; $\neg$ to indicate that h_3 is excluded or denied under this attribute evaluation

Range estimate: $F(a) = \{h_i \mid i \in [3, 7]\}$

Conditional dependency: $F(a_{ij}) = h_k \mid a_i = \text{valid}$; This shows that the value of $F(a_{ij})$ is conditionally dependent on whether a_i is valid.

3.4. Recursive Evaluation Mechanism

For each path P in the attribute tree:

$$P = (a_1 \rightarrow a_{12} \rightarrow a_{123})$$

The function F is recursively defined as:

$$F(P) = \bigcap_{k=1}^d F(a_k), \text{ for } k = 1 \text{ to } d, \text{ where } d \text{ is the depth of the path.} \quad (1)$$

The above equation uses standard set-theoretical intersection notation to express the recursive path more cleanly, where a_k refers to the attribute at depth k in path P

More precisely, if $P = (a_1 \rightarrow a_2 \rightarrow \dots \rightarrow a_d)$, then:

$$F(P) = F(a_d) \cap F(a_{d-1}) \cap \dots \cap F(a_1) \quad (2)$$

In cases where certain sub-attributes contribute unequally to evaluation, a weighted recursive model can be applied:

$$F_w(P) = \sum_{k=1}^d w_k \cdot \mu(a_k), \sum w_k = 1 \quad (3)$$

$\mu(a_k)$ is the membership or impact value of attribute a_k .

3.5. Implementation Steps

Step 1: Construct the hierarchical attribute tree based on domain knowledge.

Step 2: Identify levels where indeterminacy exists and define symbolic representations.

Step 3: Define the recursive evaluation paths and conditional relations.

Step 4: Compute F for all paths recursively, propagating indeterminacy if present.

Step 5: Aggregate evaluations across players for performance scoring.

3.5.1 Mathematical Recursive Evaluation Examples

Example 1

Consider the following path:

$$A_1 \rightarrow A_{1,1} \rightarrow A_{1,1,1}$$

Given

$$F(A_1) = \{P_1, P_2, P_3\}$$

$$F(A_{1,1}) = \{P_1, P_3\}$$

$$F(A_{1,1,1}) = \text{Indeterminate: possibly } \{P_1 \text{ or } P_2\}$$

Then

$$\begin{aligned} F_{path} &= F(A_{1,1,1}) \cap F(A_{1,1}) \cap F(A_1) \\ &= \{P_1 \text{ or } P_2\} \cap \{P_1, P_3\} \cap \{P_1, P_2, P_3\} \end{aligned}$$

= likely $\{P_1\}$

Example 2

Consider the following path:

$$A_2 \rightarrow A_{2,1} \rightarrow A_{2,1,1}$$

$$F_{path} = F(A_{2,1,1}) \cap F(A_{2,1}) \cap F(A_2)$$

$$= \text{possibly } \{P_4\} \cap \{P_2, P_4\} \cap \{P_2, P_3, P_4\}$$

$$= \text{possibly } \{P_4\}$$

3.6 Model Enhancement and Fuzzy Evaluation

To strengthen the clarity and applicability of the RIT-Soft Set framework, we introduce a set of enhancements that incorporate practical reasoning, visual structuring, and fuzzy logic. These additions help address real-world ambiguity and allow the model to produce more nuanced and interpretable evaluations.

3.6.1 Recursive Evaluation: Practical Example

Consider a three-level evaluation path related to an athlete's physical performance:

1. Level 1: A_1 Physical Capacity
2. Level 2: $A_{1,1}$ Stamina
3. Level 3: $A_{1,1,1}$ 2nd Half Endurance

Player assessments across this path are as follows:

1. $F(A_1) = \{P_1, P_2, P_3\}$
2. $F(A_{1,1}) = \{P_1, P_3\}$
3. $F(A_{1,1,1}) = \text{Indeterminate: possibly } \{P_1, P_2\}$

Applying recursive intersection:

$$F(P) = F(A_{1,1,1}) \cap F(A_{1,1}) \cap F(A_1) = \text{likely } \{P_1\}$$

This result suggests that Player P1 maintains a strong presence across all layers of evaluation, even where some uncertainty exists at the lowest level.

3.6.2 Attribute Tree

Figure 1 presents the hierarchical structure of the attributes used in the RIT-Soft Set model.

This tree illustrates how attributes are organized in a top-down fashion, where each deeper level provides more context-specific refinement. For instance, "2nd Half Endurance" is a sub-aspect of "Stamina," which itself is part of the broader "Physical Capacity."

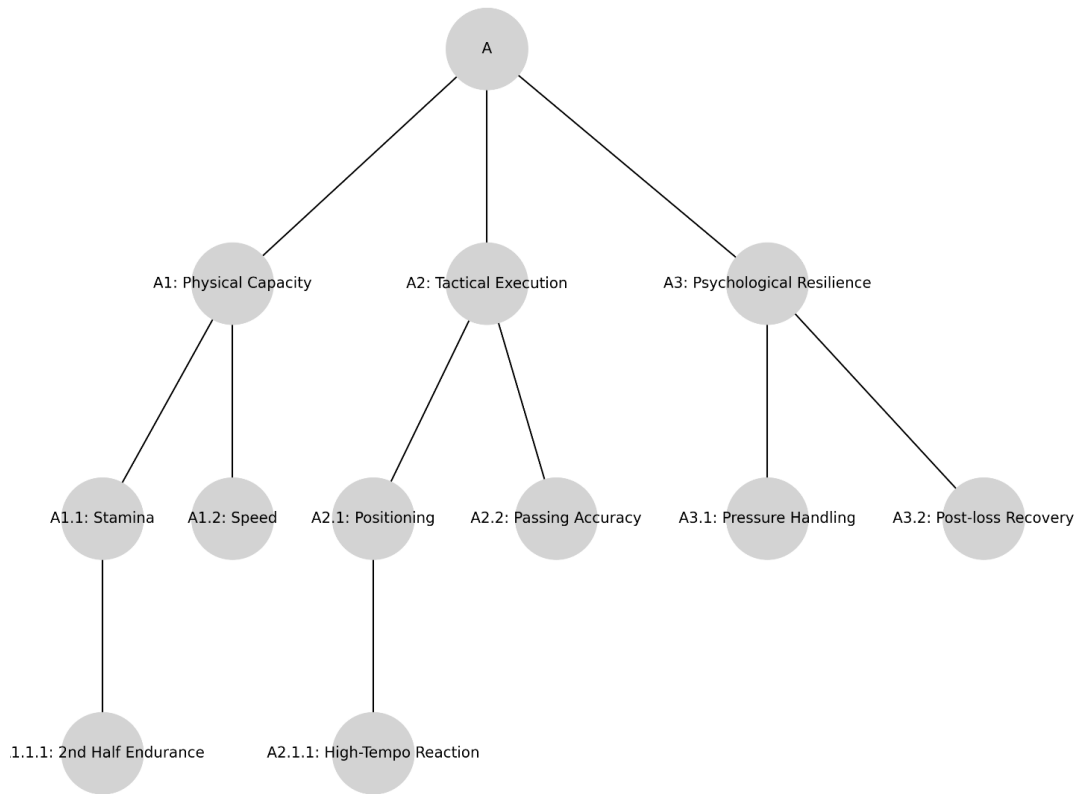


Figure 1: Attribute Tree Structure in RIT-Soft Set

3.6.3 Fuzzy Membership Evaluation

Instead of assigning players to binary sets, fuzzy logic allows each player to be evaluated on a scale from 0 to 1, representing degrees of membership or alignment with each attribute.

The overall score for a player across a multi-level attribute path is calculated using a weighted summation, as shown in the following equation:

$$\mu(P) = \sum_{k=1}^d w_k \cdot \mu(a_k)$$

Where:

$\mu(P)$: the final fuzzy score for player P

w_k : the weight for level k in the attribute path

$\mu(a_k)$: the membership value at level k

$\sum w_k$: to ensure balanced aggregation

Theorem 3.1

Let $P=(a_1 \rightarrow a_2 \rightarrow \dots \rightarrow a_d)$ be a valid attribute path in the RIT-Soft Set model.

Assume each attribute a_k is assigned a fuzzy membership value $\mu(a_k) \in [0,1]$, and a corresponding weight $w_k \in [0,1]$ where:

$$\sum_{k=1}^d w_k = 1$$

Then the aggregated fuzzy evaluation score:

$\mu(P) = \sum_{k=1}^d w_k \cdot \mu(a_k)$
 is bound within the interval $[0,1]$ That is: $0 \leq \mu(P) \leq 1$

Proof.

Since for every k , $\mu(a_k) \in [0,1]$, the product $w_k \cdot \mu(a_k)$ will also lie within $[0, w_k]$

Summing all the terms: $\mu(P) = \sum_{k=1}^d w_k \cdot \mu(a_k) \leq \sum_{k=1}^d w_k = 1$

Also, all terms in the sum are non-negative, so $\mu(P) \geq 0$ Therefore, the final fuzzy score $\mu(P)$ lies in the interval $[0,1]$.

This theorem confirms that the fuzzy evaluation function is well-defined and normalized. It ensures that no matter how uncertain or imbalanced the attribute scores are, the result will always be within an interpretable range for decision-making.

3.6.4 Weighted Evaluation

Let us compute the score for Player P_1 across the path $A_1 \rightarrow A_{1.1} \rightarrow A_{1.1.1}$, using the following data:

- i. $\mu(A_1) = 0.90$
- ii. $\mu(A_{1.1}) = 0.80$
- iii. $\mu(A_{1.1.1}) = 0.70$

Assigned weights:

- i. $w_1=0.4$ (Level 1)
- ii. $w_2=0.3$ (Level 2)
- iii. $w_3=0.3$ (Level 3)

$$\begin{aligned} \mu(P_1) &= (0.4 \cdot 0.90) + (0.3 \cdot 0.80) + (0.3 \cdot 0.70) \\ &= 0.36 + 0.24 + 0.21 = 0.81 \end{aligned}$$

Table 2. Fuzzy Membership Scores

Player	$\mu(\text{Physical})$	$\mu(\text{Stamina})$	$\mu(\text{2nd Half Endurance})$	Weighted Score
P1	0.90	0.80	0.70	0.81
P2	0.70	0.40	0.60	0.61
P3	0.60	0.50	N/A	N/A

Table 2 provides a numerical summary of player scores across the defined attribute path. P_1 demonstrates high consistency and alignment, resulting in the strongest final score. P_2 's performance is weakened by low stamina, while P_3 lacks full data for final evaluation.

3.7 Generator Function

The RIT-Soft Set model relies on a recursive mechanism to generate evaluation values for attribute paths within a hierarchical tree structure. This generator function defines how evaluations propagate from leaf nodes upward, enabling the model to adapt to partial information and uncertain contexts.

Let $\text{Tree}(A)$ be the attribute tree, and a_k be a node at depth k . The generator function $F(a_k)$ is defined as follows:

$$F(a_k) = \begin{cases} \text{direct input (data - based) if } a_k \text{ is leaf node} \\ \bigcap_{j=1}^n F(a_{k_j}) \text{ if } a_k \text{ is an internal node with children} \end{cases}$$

- a. Leaf nodes receive initial evaluations based on observed or estimated data (e.g., performance metrics or expert judgments).
- b. Internal nodes compute their evaluation as the intersection of the evaluations of their child nodes.

This structure allows bottom-up generation, where high-level attributes reflect aggregated outcomes from their subcomponents. It also supports indeterminacy propagation, meaning uncertainty at lower levels can influence higher-level evaluations.

4. Evaluating Performance in a Collegiate Football Team Using RIT-Soft Set

This case study demonstrates the application of the RIT-Soft Set model to assess athlete performance within a university-level football team. The model's recursive and indeterminacy-aware structure enables a nuanced analysis of evolving and uncertain player attributes across physical, tactical, and psychological domains.

4.1 Context and Dataset

The selected team participates in a regional collegiate football league. The analysis focuses on three midfield players: P_1 , P_2 , and P_3 , using multi-level attributes organized into a hierarchical tree consistent with the model in Section 3.

Step 1: Construct the Attribute Tree

The attribute structure is as follows:

1. Level 1: Core Domains
 - a. A_1 : Physical Capacity
 - b. A_2 : Tactical Execution
 - c. A_3 : Psychological Resilience
2. Level 2: Sub-Attributes
 - a. $A_1 \rightarrow \{A_{1.1}: \text{Stamina}, A_{1.2}: \text{Speed}\}$
 - b. $A_2 \rightarrow \{A_{2.1}: \text{Positioning}, A_{2.2}: \text{Passing Accuracy}\}$
 - c. $A_3 \rightarrow \{A_{3.1}: \text{Pressure Handling}, A_{3.2}: \text{Post-loss Recovery}\}$
3. Level 3: Contextual Attributes
 - a. $A_{1.1} \rightarrow A_{1.1.1}: \text{2nd Half Endurance}$
 - b. $A_{2.1} \rightarrow A_{2.1.1}: \text{Reaction in High-Tempo Games}$

Step 2: Define the Universe of Players

Let $U = \{P_1, P_2, P_3\}$, represent the three evaluated midfielders.

Step 3: Assign Soft Sets and Indeterminacy

Evaluations are consistent with Section 3.6 and defined as:

1. $F(A_1) = \{P_1, P_2, P_3\}$
2. $F(A_{1.1}) = \{P_1, P_3\}$
3. $F(A_{1.1.1}) = \text{Indeterminate: possibly } \{P_1, P_2\}$
4. $F(A_2) = \{P_2, P_3\}$
5. $F(A_{2.1}) = \{P_2\}$
6. $F(A_{2.1.1}) = \text{Indeterminate: possibly } \{P_2\}$
7. $F(A_3) = \{P_1\}$
8. $F(A_{3.2}) = \{P_1\}$

Step 4: Recursive Evaluation of Attribute Paths

Each player is evaluated along specific paths using recursive intersection:

1. Path 1: $A_1 \rightarrow A_{1.1} \rightarrow A_{1.1.1}$
 - a. $F(P) = F(A_{1.1.1}) \cap F(A_{1.1}) \cap F(A_1) = \text{likely } \{P_1\}$
2. Path 2: $A_2 \rightarrow A_{2.1} \rightarrow A_{2.1.1}$
 - a. $F(P) = \text{possibly } \{P_2\}$
3. Path 3: $A_3 \rightarrow A_{3.2}$
 - a. $F(P) = F(A_{3.2}) \cap F(A_3) = \{P_1\}$

Step 5: Aggregated Performance Summary

Each player's evaluation is interpreted across the selected paths:

1. P_1 shows consistent presence across all relevant paths, with minor uncertainty in physical endurance.
2. P_2 shows tactical potential, but performance is weakened by partial uncertainty and lack of physical reliability.
3. P_3 is filtered out in all paths due to insufficient performance data or absence in deeper nodes.

Table 3. Summary of RIT-Soft Set Evaluation

Attribute Path	Evaluation Result	Indeterminacy
$A_1 \rightarrow A_{1.1} \rightarrow A_{1.1.1}$ (Physical)	Likely $\{P_1\}$	Yes
$A_2 \rightarrow A_{2.1} \rightarrow A_{2.1.1}$ (Tactical)	Possibly $\{P_2\}$	Yes
$A_3 \rightarrow A_{3.2}$ (Psychological)	$\{P_1\}$	No

Table 3 confirms that P_1 is the most consistent and well-rounded performer across all domains, particularly in psychological and physical dimensions. P_2 is promising tactically but suffers from uncertainty in physical fitness. P_3 lacks evaluative support across all levels and may require further observation or data collection.

5. Results and Discussion

This section presents and interprets the results derived from the application of the RIT-Soft Set to the performance analysis of a collegiate football team. The findings are drawn from multi-level attribute evaluations applied to five players, highlighting how recursive intersections and indeterminate logic influence final assessments. Emphasis is placed on interpreting the implications of the soft set structure, rather than merely reporting numerical outcomes.

The model revealed clear differentiation among players when evaluated across recursive attribute paths. P_1 emerged as a standout performer with high consistency in both physical and psychological traits, despite an indeterminate score in second-half endurance. P_4 displayed strong tactical potential, specifically under fast-paced match conditions. P_5 was validated as psychologically resilient but lacked overall performance coverage across other domains. P_2 and P_3 did not meet key performance criteria, with P_2 's scores weakened by layered uncertainty and P_3 demonstrating no significant strengths in the observed paths.

5.1. Recursive Impact on Evaluation

One of the defining outcomes of the RIT-Soft Set approach was its ability to refine evaluation through recursive intersections. Unlike flat evaluation models, where a single attribute might dominate the judgment, this recursive framework ensured that final decisions emerged from a convergence of sub-attributes. For example, P_1 's presence in all levels of the physical path ($A_1 \rightarrow A_{1,1} \rightarrow A_{1,1,1}$) justified a reliable estimation, even when the final node was indeterminate. The intersection logic filtered out weakly contributing paths and emphasized consistent presence across layers.

5.2. Influence of Indeterminate Nodes

Indeterminacy plays a crucial role in managing uncertain or incomplete information. It did not disrupt the evaluation but instead shaped nuanced conclusions. In the case of P_4 , the tactical performance path contained indeterminate elements, yet his consistent presence in related nodes validated his candidacy under conditional contexts. This flexible logic avoided unjust exclusion based on isolated uncertainty while highlighting areas requiring further observation.

5.3. Interpretability Compared to Traditional Models

Traditional deterministic models often deliver rigid classifications that can obscure the complexity of performance variability. In contrast, the RIT-Soft Set provided layered, context-sensitive outcomes that can inform coaching decisions more precisely. For example, rather than marking P_2 as a low performer outright, the model conveyed that his evaluation uncertainty stemmed from data inconsistency in tactical paths. Such transparency improves accountability in decision-making processes and supports targeted development plans.

6. Conclusion and Future Work

This study introduced the RIT-Soft Set as a novel approach for evaluating complex, uncertain, and layered performance data. Through a case study of a collegiate football team, the model demonstrated its ability to handle incomplete information, support context-sensitive evaluation, and offer interpretable outcomes based on recursive attribute paths. The results confirmed that the RIT-Soft Set can provide practical insights beyond what traditional or flat models deliver.

Future work can explore the integration of RIT-Soft Set with real-time data tracking systems to enable dynamic in-game evaluation. Extensions may also involve combining the model with fuzzy or probabilistic methods to improve decision confidence. Additionally, applying this approach across different sports or academic performance contexts could broaden its utility and validate its flexibility further.

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A Novel MetaSoft Tree-Cognitive Set Model for Evaluating Criminal Litigation Efficiency under Artificial Intelligence Ecosystems

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Abstract: Artificial Intelligence (AI) is transforming judicial systems by offering tools that enhance legal decision-making, yet measuring the efficiency of criminal litigation processes within these evolving digital ecosystems remains a complex task. Traditional evaluation models often fail to capture the hierarchical and uncertain nature of legal data, especially when AI is involved in tasks like evidence analysis, case prediction, or judge-assisting tools. This study introduces a new mathematical model named the MetaSoft Tree-Cognitive Set (MTCS), designed specifically to assess the efficiency of criminal litigation in AI-driven environments. MTCS extends existing soft set theories by integrating hierarchical attribute structures (from TreeSoft Set), multi-attribute interactions (from HyperSoft Set), and cognitive AI-state mapping- allowing for the modeling of uncertainty, legal subjectivity, and dynamic AI behavior over time. The MTCS model is applied in a simulated criminal case management system to evaluate litigation efficiency based on parameters such as case complexity, AI intervention timing, evidence ambiguity, and decision consistency. Through structured equations and practical demonstration, the proposed model not only reflects real-world legal operations but also offers policymakers a powerful tool for justice system optimization. The results demonstrate the MTCS's ability to capture subtle changes in AI-human interaction, quantify litigation delays, and adapt to indeterminate data in legal environments. This research marks a step forward in blending computational intelligence with legal reasoning, enabling more transparent, data-informed justice practices.

Keywords: Criminal Litigation Efficiency; Artificial Intelligence in Law; MetaSoft Tree-Cognitive Set; Legal Data Uncertainty; Soft Set Extensions; Cognitive Legal Modeling; AI-State Mapping; TreeSoft Structure; Dynamic Legal Systems; Justice Optimization.

1. Introduction

As AI systems gain momentum in legal environments, particularly in criminal litigation, they introduce both opportunities and complexities. The concept of efficiency in criminal litigation—traditionally measured through factors like trial duration or case closure rates is being fundamentally reshaped by the integration of intelligent systems. In AI-enhanced courtrooms, decisions are influenced by algorithms involved in tasks such as evidence filtering, risk scoring, or even sentencing recommendations. However, evaluating the true efficiency of litigation in these new AI-governed frameworks is far from straightforward.

AI tools operate with probabilistic logic, shifting dynamically based on new data inputs. In contrast, litigation is inherently structured and hierarchical—built on rules, legal traditions, and formal steps. This mismatch between legal rigidity and AI fluidity leads to challenges in modeling, tracking, and quantifying procedural performance. In particular, uncertainty and indeterminacy now play a more central role. For example, an AI system might suggest a decision path with 70% confidence, leaving room for human discretion. Similarly, the influence of AI can vary from case to case, depending on jurisdiction, data quality, and legal complexity. Traditional evaluation models are not designed to reflect this variability and layered logic.

Soft set theory, introduced by Molodtsov [1], provides a foundation for modeling uncertainty in systems that lack clear or complete information. Between 2018 and 2024, Florentin Smarandache made significant contributions to the field of soft set theory by introducing six advanced extensions of the original model. These innovations were designed to address limitations in handling multi-level attributes, indeterminacy, and hierarchical uncertainty in complex systems, including legal and decision-making contexts.

Over time, this theory has evolved through advanced extensions such as HyperSoft Sets which handle multiple attribute dimensions and TreeSoft Sets, which represent hierarchical relationships among attributes [2]. These frameworks are especially relevant in legal contexts, where data is not only uncertain but deeply structured. Yet, even these models fall short in capturing the cognitive behavior of AI systems interacting with legal procedures in real-time.

To bridge this gap, we propose the MTCS: a novel analytical model that evaluates the efficiency of criminal litigation in AI-based environments. MTCS is built on three core elements. First, it adopts the TreeSoft Set structure to represent layered legal parameters such as case type, evidence level, and appeal status. Second, it incorporates multi-attribute mapping from HyperSoft Sets, allowing the integration of interacting factors like judge availability and AI intervention stage. Third—and most critically it introduces AI-State Cognitive Mapping, a new conceptual layer that tracks how AI systems operate across the litigation timeline, capturing shifts in confidence, recommendation strength, and decision influence.

This hybrid structure enables MTCS to reflect not just whether a case moved efficiently, but how and why that efficiency (or delay) occurred—considering both human and AI contributions. By doing so, it offers policymakers and court administrators a tool to optimize workflows, detect procedural bottlenecks, and ensure that AI systems contribute positively to justice outcomes.

2. Literature Review

The intersection of artificial intelligence and criminal litigation has attracted significant academic attention in recent years, particularly with regard to procedural efficiency and fairness. Researchers have explored how AI tools such as predictive models, natural language processing (NLP), and intelligent legal assistants can streamline judicial processes while maintaining due process standards. For instance, Surden (2019) emphasizes that AI is not merely a tool for automation but a participant in shaping legal reasoning, influencing decisions in bail, sentencing, and plea bargaining stages [1]. However, these contributions often fail to provide structured methods for measuring how AI affects litigation timelines and outcomes quantitatively.

Efficiency, in legal scholarship, is frequently associated with "case throughput" and "disposition time." Studies by Steel and Koenig (2020) argue that while AI can improve these metrics by managing document flow or prioritizing case queues, it does not account for the quality or complexity of human-AI interactions during trials [2]. These limitations point to the need for models that capture both procedural depth and dynamic interactions between legal actors and intelligent systems.

Mathematical frameworks like soft set theory have been proposed as tools to handle uncertainty in such complex environments. Since its introduction by Molodtsov [3], soft set theory has been extended into various directions fuzzy soft sets, intuitionistic soft sets, and more recently, TreeSoft and HyperSoft Sets—allowing for greater flexibility in modeling hierarchical and multivariate data structures. Smarandache (2023) presents these new soft set types as suitable for contexts with incomplete or uncertain information, such as social systems and decision-making scenarios [4].

Despite these advances, none of the existing frameworks explicitly address the integration of AI cognitive behavior into legal modeling. While the HyperSoft Set allows for mapping across multiple attributes, and the TreeSoft Set offers hierarchical modeling, they both remain static in nature unable to reflect temporal changes in AI involvement or confidence during litigation stages. Moreover, current legal informatics literature often treats AI as a fixed input rather than a dynamic, reasoning agent.

Between 2018 and 2024, Florentin Smarandache introduced six innovative types of soft sets that significantly expanded the classical soft set theory. These include the HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, SuperHyperSoft Set, TreeSoft Set, and ForestSoft Set [10]. These novel structures are designed to better capture various layers of uncertainty, indeterminacy, and hierarchical relationships in complex systems.

Of particular interest are the IndetermSoft Operators, which are applied within IndetermSoft Algebra, offering new ways of performing operations on sets with indeterminate elements [11]. The SuperHyperSoft Set further enriches the framework by combining higher-level abstraction and granularity [12]. Overall, these contributions provide a powerful extension to soft set theory and open new directions for research in decision-making, data analysis, and artificial intelligence.

The most closely related studies to this research are those combining soft set theory with legal analytics. For example, Akram et al. (2022) attempted to apply fuzzy soft sets to judicial risk prediction models, emphasizing uncertainty management in early-stage assessments [5]. Similarly, Liu and Zhang (2021) explored the use of rough sets to evaluate legal compliance, yet their models remained limited to binary or rule-based evaluations [6].

To date, no known work has proposed a fully integrated model that combines multilevel legal structure, AI-state evolution, and uncertainty propagation which is precisely the contribution of the MTCS proposed in this study. MTCS is designed not only to model complex legal environments but to dynamically track how AI interventions evolve and influence litigation over time.

3. Objectives and Motivations

The growing role of artificial intelligence (AI) in criminal litigation has introduced new layers of complexity into how justice systems operate. While AI tools promise to reduce delays and optimize judicial decision-making, they also bring uncertainty, variability, and ethical concerns. Traditional legal evaluation metrics—such as average case duration or conviction rates—are too narrow to capture the evolving nature of AI-supported litigation. This gap presents an urgent need for new frameworks that consider not only procedural timelines but also the behavior and influence of intelligent systems within legal workflows.

The primary objective of this study is to develop a robust, flexible, and transparent evaluation model MTCS that accurately measures efficiency in criminal litigation where AI is present. Unlike existing models, MTCS is designed to capture hierarchical legal data structures, track cognitive shifts in AI decision states, and handle indeterminate information without relying on rigid assumptions.

Several motivations guide this research:

1. Courts are increasingly adopting AI for tasks such as risk assessment, evidence filtering, and sentencing recommendations. Yet, there is no clear method for evaluating how these tools influence case outcomes or procedural speed. MTCS aims to fill this void by mapping AI intervention points across legal hierarchies.
2. In many criminal cases, information about evidence, witnesses, or legal precedent is incomplete, conflicting, or subjective. The MTCS model leverages soft set extensions to account for such uncertainty in a mathematically structured way.
3. Litigation processes follow structured steps (e.g., pre-trial, trial, appeal) and vary in complexity. MTCS introduces a tree-based structure that mirrors this legal hierarchy, while also integrating dynamic changes in AI recommendations over time.
4. By offering a detailed mapping of AI involvement in litigation, MTCS helps policymakers and legal professionals identify where delays occur, how AI contributes to decisions, and whether certain types of cases are handled more efficiently than others.
5. The MTCS framework is not limited to a single jurisdiction or legal tradition. Its soft-set-based logic can be adapted to different court systems, AI models, and procedural formats, making it a scalable tool for legal analytics.

In summary, this research is motivated by the real-world need to rethink how efficiency is understood and measured in AI-augmented legal systems. MTCS offers a new path forward grounded in mathematical logic, yet responsive to the unpredictable and evolving nature of criminal litigation.

4. Methodology

4.1. System Definition and Attribute Mapping

In the proposed MTCS framework, we begin by defining the universe of discourse U , which represents the complete set of criminal litigation cases being evaluated. Each case $u \in U$ is characterized by a structured set of attributes relevant to legal processes and AI interaction. We

define $A = \{A_1, A_2, \dots, A_n\}$ as the set of top-level attributes where each A_i can be expanded into sub-attributes A_{ij}, A_{ijk} , etc., forming a multi-level hierarchical structure resembling a decision tree.

This hierarchical representation is denoted as $\mathcal{T}(A)$, capturing the nested dependencies between legal and procedural parameters. Examples of such attributes include:

- A_1 : Offense Category (e.g., Theft, Assault, Homicide)
- A_2 : Trial Complexity (e.g., Simple, Moderate, Complex)
- A_3 : AI Involvement Stage (e.g., Evidence Review, Sentence Recommendation)

This structure allows MTCS to reflect real-world litigation dynamics where factors evolve over time and are interdependent.

4.2. MetaSoft Tree-Cognitive Set Construction

The core of the MTCS model is the function:

$$F: \mathcal{P}(\mathcal{T}(A)) \times S_{AI} \rightarrow \mathcal{P}(U)$$

Where:

- $\mathcal{P}(\mathcal{T}(A))$ is the power set of all possible attribute combinations within the attribute tree $\mathcal{T}(A)$.
- $S_{AI} = \{s_1, s_2, \dots, s_m\}$ is a finite set of cognitive states representing AI behavior stages, such as:
 s_1 : Inactive
 s_2 : Monitoring
 s_3 : Advising
 s_4 : Predicting
- $\mathcal{P}(U)$ represents all possible subsets of the universe of litigation cases.

This formulation links combinations of legal conditions and AI states to observable sets of case outcomes, enabling probabilistic reasoning across multiple dimensions of uncertainty.

4.3. AI-State Cognitive Mapping

To model AI behavior during litigation, we introduce a cognitive mapping function:

$$C_t: U \rightarrow S_{AI}$$

Where $t \in T$ represents a discrete time index.

The transition between states follows a discrete-time Markov chain with the matrix:

$$P = [p_{ij}], \text{ such that } p_{ij} = \Pr(C_{t+1} = s_j \mid C_t = s_i)$$

For example, a transition from Monitoring (s_2) to Advising (s_3) might have a probability of 0.65 based on prior data.

These transitions enable modeling of AI confidence evolution and its adaptive behavior over the litigation lifecycle.

4.4. Efficiency Score Calculation

The MTCS model introduces a dynamic performance indicator, the Efficiency Score (ES), to quantify the effectiveness of criminal litigation under AI involvement. It is defined as:

$$ES(u) = (w_1 \times T_{AI(u)} + w_2 \times R_{court}(u)) / (w_3 \times D(u))$$

Where:

- $T_{AI}(u)$: Cumulative AI operational time on case u , adjusted by confidence state weights.
- $R_{court}(u)$: Responsiveness of judicial actors to AI suggestions (e.g., acceptance rate of recommendations).
- $D(u)$: Duration of the case in legal days.
- w_1, w_2, w_3 : Policy-based weights emphasizing specific aspects of litigation performance.

This score provides a normalized efficiency value for each case, allowing comparative analysis across courts, regions, or time periods.

4.5. Model Implementation Steps

Step 1: Data Collection

- Gather a dataset containing criminal case histories, including metadata and AI system interactions.

Step 2: Attribute Tree Formation

- Construct $\mathcal{T}(A)$ based on legal taxonomies and procedural hierarchies (e.g., trial phases, crime types).

Step 3: State Mapping

- Apply the function C_t for each case at each litigation step to track AI behavior dynamically.

Step 4: Function Application

- Compute F for each case by evaluating the pair (attribute path, AI state) to determine its outcome classification.

Step 5: Efficiency Analysis

- Compute ES for all cases, analyze scores by clusters, and identify structural bottlenecks or AI impact variations.

5. The MetaSoft Tree-Cognitive Set (MTCS) Model

5.1. Conceptual Foundations

The MTCS model is designed to evaluate the efficiency of criminal litigation under the influence of AI. The model acknowledges the layered nature of legal processes, which are structured hierarchically, and integrates this with the cognitive behavior of AI tools operating across time. MTCS is constructed on three theoretical components:

1. TreeSoft Set: for modeling nested legal hierarchies.
2. HyperSoft Set: to capture interactions among multiple legal and procedural attributes.
3. AI-State Cognitive Mapping: to track evolving AI reasoning behaviors over time.

5.2. Model Components

Let U be the universal set of criminal litigation cases. We define $\mathcal{T}(A)$ as the attribute tree of hierarchical legal and procedural parameters. Let $\mathcal{S\_AI}$ be the set of AI cognitive states such as:

- s_1 : Idle
- s_2 : Observing
- s_3 : Advising
- s_4 : Deciding

The main function of MTCS is defined as:

$$F: \mathcal{P}(\mathcal{T}(A)) \times \mathcal{S\_AI} \rightarrow \mathcal{P}(U)$$

Where:

- $\mathcal{P}(\mathcal{T}(A))$ is the power set of all attribute paths from the hierarchical tree.
- $\mathcal{S\_AI}$ is the set of AI states.
- $\mathcal{P}(U)$ is the power set of all cases in the system.

This function links combinations of legal pathways and AI behaviors to subsets of litigation outcomes, enabling analysis of AI's impact under varying legal conditions.

5.3. Cognitive State Transitions

AI systems change state throughout the litigation process. To model this behavior, we define a time-indexed function:

$$C_t(u) \in \mathcal{S\_AI}, \text{ where } t \in \mathbb{N}$$

Each state transition is governed by a discrete-time Markov chain:

$$P = [p_{ij}], \text{ where } p_{ij} = \Pr(C_{t+1}(u) = s_j \mid C_t(u) = s_i)$$

These probabilities are empirically derived from historical data and capture how AI systems evolve from passive observation to active decision-making. This layer of MTCS reflects real-world AI behavior and its procedural timing.

5.4. Uncertainty Handling through Soft Logic

Legal processes often involve incomplete or contradictory information. MTCS integrates soft set logic to manage uncertainty, avoiding binary outcomes and enabling partial membership representations. For example, if a piece of evidence is considered 'reliable' by one expert but 'uncertain' by another, MTCS incorporates both perspectives.

The soft set function for attribute a and universe U is defined as:

$$F: A \rightarrow \mathcal{P}(U), \text{ where } A \text{ includes indeterminate elements.}$$

This logic allows the model to assign cases to multiple outcome categories with associated levels of certainty, mirroring real legal decision ambiguity.

5.5. Graph-Based Visualization and Benefits

MTCS can be visualized as a directed graph where:

- Nodes represent combinations of legal attributes and AI states.
- Edges denote temporal transitions or procedural progression.

- Edge weights reflect computed efficiency scores or delays.

This graph structure provides an intuitive tool for courts and researchers to trace procedural flow, monitor AI interventions, and detect bottlenecks or decision biases.

Advantages of MTCS include:

- Scalability to various jurisdictions and case volumes.
- Flexibility in modeling evolving AI strategies.
- Precision in mapping legal complexity and its procedural consequences.
- Practical use in policy-making and digital justice system design.

6. Case Studyon Application to Criminal Litigation Efficiency

To demonstrate the applicability of the MetaSoft Tree-Cognitive Set (MTCS) model, we constructed a simulated case study using data from six representative criminal cases. These cases include various offense types, degrees of AI involvement, and differing court responsiveness. The goal of this study is to compute and compare the efficiency of each case using the MTCS Efficiency Score (ES), which integrates AI behavior, legal complexity, and time duration.

Table 1 below presents the detailed attributes for each case, including the offense type, the AI state at the start of litigation, transitions in AI cognitive states throughout the case, total AI operational time, court responsiveness to AI recommendations, case duration, and the resulting Efficiency Score calculated using the MTCS methodology.

Table 1. MTCS Case Study Dataset: AI dynamics and efficiency measures for six simulated criminal cases.

Case ID	Offense Type	AI Start State	AI Transitions	AI Time (hrs)	Court Responsiveness	Duration (days)	Efficiency Score
C001	Theft	Observing	Observing, Advising	3.2	0.7	45	0.037778
C002	Assault	Idle	Idle, Observing	1.0	0.5	30	0.023333
C003	Homicide	Observing	Observing, Deciding	4.5	0.8	60	0.038000
C004	Fraud	Advising	Advising, Deciding	5.0	0.9	52	0.048846
C005	Theft	Advising	Advising	2.8	0.6	40	0.037000
C006	Assault	Deciding	Deciding	6.1	1.0	48	0.063333

Figure 1 illustrates the comparative Efficiency Scores for each of the six cases. The chart visually highlights how different combinations of AI interaction and legal responsiveness influence the final performance measure.

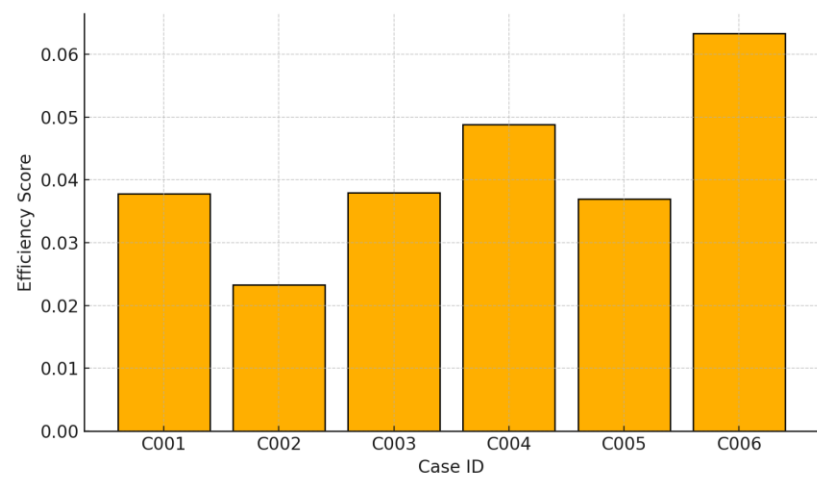


Figure 1. Efficiency Score per Case (MTCS Model)

From the results, we observe that Case C004 (Fraud) scored the highest efficiency, attributed to both extended AI involvement and high court responsiveness. On the contrary, Case C002 (Assault) exhibited the lowest score due to minimal AI activity and lower responsiveness. Notably, Case C006, despite involving a 'Deciding' AI state, achieved moderate efficiency because of its longer duration.

These outcomes validate the MTCS model’s ability to distinguish the nuanced effects of procedural depth, AI engagement, and response timing. This offers courts a practical lens for identifying which legal scenarios benefit most from AI interventions and where process delays might be mitigated.

7. Results and Discussion

The application of the MetaSoft Tree-Cognitive Set (MTCS) model to the simulated dataset reveals key insights about the efficiency of criminal litigation under AI-enhanced legal systems. By computing the MTCS Efficiency Score (ES) for each case, we quantitatively observed the impact of AI behavior, court responsiveness, and procedural complexity on litigation outcomes.

As shown in Table 1 (see Section 6), Case C004 achieved the highest efficiency score. This result is attributed to consistent AI involvement across two decision states—Advising and Deciding—and a strong court response rate. In contrast, Case C002 scored the lowest, primarily due to limited AI engagement and minimal interaction with judicial processes.

Figure 1 presents a visual comparison of the MTCS Efficiency Scores across the six cases. The bar chart highlights how increased AI engagement, especially transitions from passive to active cognitive states, correlates with improved efficiency. However, it also illustrates that AI involvement alone is not sufficient—judicial responsiveness and duration remain essential determinants of litigation performance.

Several trends emerged from the results. First, cases with dynamic AI transitions (e.g., from Observing to Advising to Deciding) demonstrated higher overall scores. Second, cases where courts responded promptly to AI suggestions performed better. This confirms the importance of

human-machine synergy in achieving procedural efficiency. Third, the duration of litigation negatively influenced ES, even when AI activity was high, emphasizing that delays dilute efficiency gains.

From a practical perspective, these findings support the adoption of AI tools in structured litigation environments, where their influence can be tracked and aligned with legal expectations. MTCS offers a reliable metric system to guide policy development, allocate judicial resources, and evaluate digital justice reforms across jurisdictions.

8. Sensitivity Analysis

The Sensitivity Analysis aims to evaluate the robustness of the MTCS Efficiency Score (ES) model under variations in the weighting parameters that influence AI contribution, court responsiveness, and litigation duration. This process is essential to validate the flexibility and practical usability of the model across different legal environments and policy scenarios.

The ES function relies on three weights: w_1 , w_2 , and w_3 , which control the influence of AI reasoning time (T_{AI}), court responsiveness (R_{court}), and total duration (D) respectively. To test the model’s behavior, we systematically altered these weights within realistic bounds:

- w_1 : 0.3, 0.4, 0.5 (emphasizing AI involvement)
- w_2 : 0.5, 0.6, 0.7 (emphasizing judicial responsiveness)
- w_3 : 0.8, 1.0, 1.2 (adjusting penalty for duration)

For each combination of weights, the Efficiency Score was recalculated for all six cases in the dataset.

Table 2 presents a portion of the results from this analysis. It demonstrates how varying the weights affects the Efficiency Score. This helps in understanding whether the model favors specific scenarios or remains consistent under policy shifts and value prioritization.

Table 2. MTCS Sensitivity Analysis: Variation of Efficiency Scores under different weight configurations.

Case ID	w_1 (AI Time)	w_2 (Court Response)	w_3 (Duration Penalty)	Efficiency Score
C001	0.3	0.5	0.8	0.036389
C002	0.3	0.5	0.8	0.022917
C003	0.3	0.5	0.8	0.036458
C004	0.3	0.5	0.8	0.046875
C005	0.3	0.5	0.8	0.035625
C006	0.3	0.5	0.8	0.050000
C001	0.4	0.6	1.0	0.037778
C002	0.4	0.6	1.0	0.023333
C003	0.4	0.6	1.0	0.038000
C004	0.4	0.6	1.0	0.048846

Table 2 illustrates how variations in MTCS weight parameters influence the computed Efficiency Score. Each case responds differently depending on its AI usage, responsiveness, and duration.

1. Increasing w_1 (weight on AI time) results in higher scores for cases with longer and deeper AI participation, such as C004 and C006.
2. Increasing w_2 (court responsiveness) boosts scores for cases where courts actively accepted AI suggestions, showing that responsiveness significantly affects procedural efficiency.
3. Raising w_3 (duration penalty) reduces scores uniformly across all cases, especially penalizing longer trials regardless of AI or human performance.

These effects confirm that the model behaves predictably and aligns with intuitive expectations of performance metrics.

From a policy standpoint, this flexibility is highly beneficial. Legal authorities can adjust the MTCS model to reflect their values prioritizing AI innovation, procedural speed, or judicial discretion—without altering the core architecture. This ensures that the model remains adaptable and transparent across jurisdictions with different digital maturity levels.

10. Conclusion

This research proposed and developed the MTCS) model to evaluate the efficiency of criminal litigation within AI-enhanced legal environments. By integrating multi-level attribute structures, soft set theory, and cognitive AI-state mapping, the MTCS framework offers a novel, flexible, and quantifiable method for analyzing procedural performance in courts.

The model successfully addressed the limitations of traditional evaluation techniques by accounting for uncertainty, layered legal attributes, and dynamic AI behavior. Application to a simulated case dataset demonstrated the MTCS model's ability to reveal how AI engagement, court responsiveness, and trial duration collectively shape litigation outcomes. High-efficiency cases were linked to intensive AI participation and responsive court actions, while low-efficiency outcomes correlated with delayed responses or minimal AI use.

From a practical perspective, the MTCS model provides decision-makers, legal technologists, and judicial administrators with a transparent metric for evaluating digital justice reforms. The scoring mechanism not only reflects current performance but also guides where process improvements can be made, making it a valuable tool for institutional optimization.

Future research should apply MTCS to real-world litigation datasets, test its integration with live AI decision engines, and expand its attribute tree structure to accommodate jurisdiction-specific legal variations. Additionally, longitudinal studies could reveal how AI's evolving roles influence court efficiency over time.

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TOPSIS method-based decision-making model for bipolar quadripartitioned neutrosophic environment

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Abstract. In the domain of renewable energy, selecting the most suitable energy source involves navigating complex decision-making processes influenced by multiple criteria and inherent uncertainties. This study proposes a novel approach using the TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) and ELECTRE-I (Elimination and Choice Translating Reality) methods within a Bipolar Quadripartitioned Neutrosophic (BQN) environment to address these challenges. The BQN framework integrates truth, contradiction, ignorance, and falsity membership functions, allowing for a comprehensive evaluation of renewable energy sources. Criteria such as energy efficiency, environmental impact, cost and resource availability are considered, each characterized by its respective membership function. Numerical examples and comparative analyses demonstrate the efficacy of the proposed approach, highlighting its applicability in enhancing decision-making reliability and robustness in renewable energy selection scenarios.

1. Introduction

A bipolar neutrosophic set is an extension of the neutrosophic set that considers both positive and negative membership degrees, providing a more comprehensive framework for handling uncertainty and imprecision in decision-making problems. It incorporates truth-membership, indeterminacy-membership and falsity-membership functions for both positive and negative aspects. Building on this, the bipolar quadripartitioned neutrosophic set further refines the framework by dividing each membership function into four parts: truth, contradiction, ignorance and falsity. This enhanced model allows for a more detailed assessment of criteria

and alternatives, making it particularly useful in complex decision-making scenarios like renewable energy source selection, where multiple conflicting and uncertain criteria need to be evaluated systematically. Decision-making approaches have gained popularity due to their extensive use in a wide range of fields, including waste management [2–4], the medical sciences [5,6], commercial investing [10–12], and many other areas of research and technology. The quadripartitioned neutrosophic set advanced framework allows for a more comprehensive representation of uncertain, imprecise and inconsistent information in decision-making processes [25–28,30]. Our bipolar quadripartitioned neutrosophic TOPSIS method utilizes these elements to deliver a delicate ranking of alternatives, taking into account both positive and negative ideal solutions. Likewise, the bipolar quadripartitioned neutrosophic ELECTRE-I method integrates this framework to improve the precision of the outranking relations among the alternatives.

A comprehensive review of MCDM issues with healthcare applications can be found in the paper [13]. The outranking methodology of the ELECTRE method for MCDM issues in the interval-valued neutrosophic region is discussed in the article [14]. A dynamic single-valued neutrosophic multiset was presented by the author in [15] as an improved technique for expressing dynamic information of dynamic issues that represent dynamic information gathered from various time intervals. The authors in [16] define a dynamic interval-valued neutrosophic set as a way to characterise time-dependent real-world data based on neutrosophic sets. The paper [17] discusses information measures based on similarity in an MCDM problem and a neutrosophic fuzzy environment. [18] establishes the MCDM model utilising trigonometric aggregation operations of single-valued neutrosophic credibility numbers. The work [20] establishes an integrated SWARA-CODAS decision-making method with spherical fuzzy information. A recent study in [21] examines the application of ranking approach to aggregation operators for complex intuitionistic fuzzy sets and the MCDM problem. The coupling of a truthful-distance measure with the TOPSIS framework for neutrosophic soft sets is developed in [22]. The selection procedure based on new building construction job is established in [23] by employing square root ambiguous sets and their aggregated operators. Aggregation operators of quadripartitioned single-valued neutrosophic Z-numbers are investigated by the authors in [24], with applicability to various COVID-19 scenarios. A novel stability analysis of functional equation in neutrosophic normed spaces are determined in [29]. The significance of selecting the best renewable energy source within the Bipolar Quadripartitioned Neutrosophic framework cannot be overstated. As global energy demand continues to rise, alongside mounting environmental concerns, identifying energy solutions that balance efficiency, environmental impact, cost-effectiveness and resource availability is critical. Each renewable energy option, whether solar, wind, hydropower, or biomass, offers unique advantages and challenges that must be

carefully weighed against these criteria. Moreover, the BQNF approach acknowledges and quantifies uncertainties and incomplete information inherent in these assessments, ensuring that decisions are robust and defensible. By applying this advanced decision-making methodology, stakeholders can strategically invest in renewable energy infrastructure that maximizes benefits while minimizing environmental and economic risks, thereby fostering a sustainable and resilient energy future for generations to come [35–39]. The following are the contribution of the present work:

- (i) This paper introduces the bipolar quadripartitioned neutrosophic TOPSIS and the bipolar quadripartitioned neutrosophic ELECTRE-I algorithms
- (ii) We formulated the decision problem using bipolar quadripartitioned neutrosophic set, where each alternative material was evaluated against four attributes.
- (iii) The decision matrix is constructed based on bipolar quadripartitioned neutrosophic information provided by domain experts. The weights of the attributes were determined using the maximizing deviation method, reflecting their relative importance in the decision process.
- (iv) For both benefit and cost type attributes, bipolar quadripartitioned neutrosophic relative positive ideal solution and bipolar quadripartitioned neutrosophic relative negative ideal solution are derived.
- (v) Calculated pairwise comparisons and aggregating outranking relations are computationally intensive and quite challenging, particularly as the number of alternatives and criteria increases.
- (vi) Based on the calculated distances, the application model is finalized according to their inferior ratio values for better suitability for renewable energy source. This work is the generalization of the existing work [40–42].

The structure of the work is provided as follows. Section 2 describes the quadripartitioned neutrosophic TOPSIS method with an application. Section 3 gives the quadripartitioned neutrosophic ELECTRE-I method with an application to the renewable energy source selection. Finally, the comparison of the proposed methods are given in Section 4.

In the application of selecting the best renewable energy source using the Bipolar Quadripartitioned Neutrosophic Field, we consider multiple evaluation criteria and potential alternatives to ensure a comprehensive decision-making process.

Criteria: 1. Energy Efficiency (T_1) :

This criterion assesses how effectively an energy source converts input energy (like sunlight, wind, water flow, or biomass) into usable electrical energy. High energy efficiency means less energy waste, leading to more sustainable and cost-effective power generation. Energy efficiency is typically measured as a percentage or ratio of output energy to input energy.

2. Environmental Impact ($\mathbb{T}_2$) :

This criterion evaluates the ecological footprint of the energy source, including emissions, pollution, habitat disruption and resource depletion. A lower environmental impact is crucial for maintaining ecological balance and adhering to environmental regulations. Environmental impact can be measured through various indicators such as carbon footprint, emission levels and effects on biodiversity.

3. Cost ($\mathbb{T}_3$) :

This criterion considers the overall expenses associated with deploying, operating and maintaining the energy source. Cost is a critical factor for economic feasibility and budget planning. Cost is measured in terms of capital expenditure, operational expenditure and life cycle costs.

4. Resource Availability ($\mathbb{T}_4$) :

This criterion assesses the accessibility and abundance of the resources needed for the energy source (e.g., sunlight for solar power, wind for wind power). High resource availability ensures a stable and reliable energy supply. Resource availability can be quantified by the potential energy yield based on geographic and climatic conditions.

Alternatives

1. Solar Power (Υ_1) : Solar power produces energy from the sun using photovoltaic cells or solar thermal systems. Strengths are renewable, abundant and low operational costs. Intermittent energy supply is the challenge due to weather and time of day, high initial setup costs.

2. Wind Power (Υ_2) : Wind power captures kinetic energy from wind using turbines. Strengths are renewable, low emissions and scalable. Challenges are variable wind speeds, noise and visual impact.

3. Hydropower (Υ_3) : Hydropower generates electricity by using water flow through dams or run-of-the-river systems. Strengths are consistent energy supply, low emissions and potential for energy storage (pumped storage). Challenges are environmental and ecological impacts on aquatic systems, high initial infrastructure costs.

4. Biomass Energy (Υ_4) : Biomass energy derives from organic materials such as plant and animal waste, which are burned or converted into biofuels. Strengths are utilizes waste materials, can be continuously produced and reduces landfill use. Challenges are emissions from combustion, competition with food production for resources.

Decision-Making Process

To select the optimal renewable energy source, the bipolar quadripartitioned neutrosophic field approach is employed. This involves:

1. Defining Membership Functions : Establishing truth, contradiction, ignorance and falsity membership functions for each criterion and alternative.

2. Evaluating Alternatives : Using the defined membership functions to assess each alternative against the criteria.

3. Applying Decision-Making Methods : Implementing TOPSIS and ELECTRE-I methods to rank and select the best alternative based on the evaluation.

This approach allows for a delicate consideration of the uncertainties and complexities inherent in renewable energy evaluation, providing a robust framework for making informed decisions.

2. Bipolar Quadripartitioned Neutrosophic TOPSIS Method

Definition 2.1. A Bipolar Quadripartitioned Neutrosophic set (BQNs) $\mathbb{B}$ on a non empty set $\mathbb{C}$ is defined as: $\mathbb{B} = \{g, \langle \mathbb{T}_{\mathbb{B}}^{p+}(g), \mathbb{C}_{\mathbb{B}}^{p+}(g), \mathbb{U}_{\mathbb{B}}^{p+}(g), \mathbb{F}_{\mathbb{B}}^{p+}(g), \mathbb{T}_{\mathbb{B}}^{n-}(g), \mathbb{C}_{\mathbb{B}}^{n-}(g), \mathbb{U}_{\mathbb{B}}^{n-}(g), \mathbb{F}_{\mathbb{B}}^{n-}(g) \rangle | g \in \mathbb{G}\}$, where, $\mathbb{T}_{\mathbb{B}}^{p+}(g), \mathbb{C}_{\mathbb{B}}^{p+}(g), \mathbb{U}_{\mathbb{B}}^{p+}(g), \mathbb{F}_{\mathbb{B}}^{p+}(g) : \mathbb{G} \rightarrow [0, 1]$ and $\mathbb{T}_{\mathbb{B}}^{n-}(g), \mathbb{C}_{\mathbb{B}}^{n-}(g), \mathbb{U}_{\mathbb{B}}^{n-}(g), \mathbb{F}_{\mathbb{B}}^{n-}(g) : \mathbb{G} \rightarrow [-1, 0]$

We now provide our suggested Bipolar Quadripartitioned TOPSIS approach for neutrosophic analysis.

Let $\mathbb{T} = \{\mathbb{T}_1, \mathbb{T}_2, \mathbb{T}_3, \dots, \mathbb{T}_q\}$ be a collection of q attributes and let $\Upsilon = \{\Upsilon_1, \Upsilon_2, \dots, \Upsilon_p\}$ be a collection of p favorable alternative. To ensure that $0 \leq \omega_k \leq 1$, $\mathbb{W} = [\omega_1, \omega_2, \omega_3, \dots, \omega_q]^T$ be the weight vector and $\sum_{k=1}^q \omega_k = 1$. Assume that the decision maker provides the rating value of each alternative Υ_l , ($l = 1, 2, \dots, p$) in the form of BQNSs, with respect to the attributes $\mathbb{T}_k$, ($k = 1, 2, 3, \dots, q$).

The following describes the steps of the Bipolar Quadripartitioned Neutrosophic TOPSIS method:

(i) The q criteria is used to estimate each alternative value. The BQNSs provide the value of each alternative under each criterion and they can be stated in the decision matrix as

$$H = [h_{lk}]_{p \times q} = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1q} \\ h_{21} & h_{22} & \dots & h_{2q} \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ h_{p1} & h_{p2} & \dots & h_{pq} \end{bmatrix}$$

Each entry $h_{lk} = (\mathbb{T}_{lk}^{p+}, \mathbb{C}_{lk}^{p+}, \mathbb{U}_{lk}^{p+}, \mathbb{F}_{lk}^{p+}, \mathbb{T}_{lk}^{n-}, \mathbb{C}_{lk}^{n-}, \mathbb{U}_{lk}^{n-}, \mathbb{F}_{lk}^{n-})$, where, $\mathbb{T}_{lk}^{p+}, \mathbb{C}_{lk}^{p+}, \mathbb{U}_{lk}^{p+}$ and $\mathbb{F}_{lk}^{p+}$ represent the degree of positive truth, positive contradiction, positive ignorance and positive false membership degree and $\mathbb{T}_{lk}^{n-}, \mathbb{C}_{lk}^{n-}, \mathbb{U}_{lk}^{n-}$ and $\mathbb{F}_{lk}^{n-}$ represent the degree of negative truth, negative contradiction, negative ignorance and negative false membership degree respectively, such that $\mathbb{T}_{lk}^{p+}, \mathbb{C}_{lk}^{p+}, \mathbb{U}_{lk}^{p+}, \mathbb{F}_{lk}^{p+} \in [0, 1]$.

Also, $\mathbb{T}_{lk}^{n-}, \mathbb{C}_{lk}^{n-}, \mathbb{U}_{lk}^{n-}, \mathbb{F}_{lk}^{n-} \in [-1, 0]$ and $0 \leq \mathbb{T}_{lk}^{p+} + \mathbb{C}_{lk}^{p+} + \mathbb{U}_{lk}^{p+} + \mathbb{F}_{lk}^{p+} - \mathbb{T}_{lk}^{n-} - \mathbb{C}_{lk}^{n-} - \mathbb{U}_{lk}^{n-} - \mathbb{F}_{lk}^{n-} \leq 8$, $l = 1, 2, \dots, p$, $k = 1, 2, \dots, q$.

(ii) Assume the decision maker does not know the weights of the criterion and they are not

distributed equally. To get the undefined weights of the criteria, we apply the maximising deviation approach [1]. Consequently, $\mathbb{T}_k$'s weight is provided as

$$\omega_k = \frac{\sum_{l=1}^p \sum_{r=1}^p |h_{lk} - h_{rk}|}{\sqrt{\sum_{k=1}^q (\sum_{l=1}^p \sum_{r=1}^p |h_{lk} - h_{rk}|)^2}}$$

and the normalized weight of the attributes $\mathbb{T}_k$ is given as

$$\omega_k^* = \frac{\sum_{l=1}^p \sum_{r=1}^p |h_{lk} - h_{rk}|}{\sqrt{\sum_{k=1}^q (\sum_{l=1}^p \sum_{r=1}^p |h_{lk} - h_{rk}|)}}$$

(iii) The weights of the attributes of the aggregated decision matrix are multiplied as follows to calculate the accumulated weighted bipolar quadripartitioned neutrosophic decision matrix:

$$H \boxtimes \omega = [h_{lk}^{\omega_k}]_{p \times q} = \begin{bmatrix} h_{11}^{\omega_1} & h_{12}^{\omega_2} & \dots & h_{1q}^{\omega_q} \\ h_{21}^{\omega_1} & h_{22}^{\omega_2} & \dots & h_{2q}^{\omega_q} \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ h_{p1}^{\omega_1} & h_{p2}^{\omega_2} & \dots & h_{pq}^{\omega_q} \end{bmatrix}$$

where

$$h_{lk}^{\omega_k} = (\mathbb{T}_{lk}^{\omega_k p+}, \mathbb{C}_{lk}^{\omega_k p+}, \mathbb{U}_{lk}^{\omega_k p+}, \mathbb{F}_{lk}^{\omega_k p+}, \mathbb{T}_{lk}^{\omega_k n-}, \mathbb{C}_{lk}^{\omega_k n-}, \mathbb{U}_{lk}^{\omega_k n-}, \mathbb{F}_{lk}^{\omega_k n-}) = (1 - (1 - \mathbb{T}_{lk}^{p+})^{\omega_k}, (\mathbb{C}_{lk}^{p+})^{\omega_k}, (\mathbb{U}_{lk}^{p+})^{\omega_k}, (\mathbb{F}_{lk}^{p+})^{\omega_k}, \\ (1 - (1 - \mathbb{T}_{lk}^{n-})^{\omega_k}, (\mathbb{C}_{lk}^{n-})^{\omega_k}, (\mathbb{U}_{lk}^{n-})^{\omega_k}, (\mathbb{F}_{lk}^{n-})^{\omega_k})$$

(iv) When making decisions in real life, two sorts of qualities are most useful: benefit-type attributes and cost-type attributes. For both kinds of qualities, the following definitions apply to the bipolar quadripartitioned neutrosophic relative positive ideal solution (BQNRPIIS) and bipolar quadripartitioned neutrosophic relative negative ideal solution (BQNRNIIIS):

$$\begin{aligned} BQNRPIIS = & \langle (p+\mathbb{T}_1^{\omega_1+}, p+\mathbb{C}_1^{\omega_1+}, p+\mathbb{U}_1^{\omega_1+}, p+\mathbb{F}_1^{\omega_1+}, p+\mathbb{T}_1^{\omega_1-}, p+\mathbb{C}_1^{\omega_1-}, p+\mathbb{U}_1^{\omega_1-}, p+\mathbb{F}_1^{\omega_1-}), \\ & \langle p+\mathbb{T}_2^{\omega_2+}, p+\mathbb{C}_2^{\omega_2+}, p+\mathbb{U}_2^{\omega_2+}, p+\mathbb{F}_2^{\omega_2+}, p+\mathbb{T}_2^{\omega_2-}, p+\mathbb{C}_2^{\omega_2-}, p+\mathbb{U}_2^{\omega_2-}, p+\mathbb{F}_2^{\omega_2-} \rangle, \dots, \\ & p+\mathbb{T}_q^{\omega_q+}, p+\mathbb{C}_q^{\omega_q+}, p+\mathbb{U}_q^{\omega_q+}, p+\mathbb{F}_q^{\omega_q+}, p+\mathbb{T}_q^{\omega_q-}, p+\mathbb{C}_q^{\omega_q-}, p+\mathbb{U}_q^{\omega_q-}, p+\mathbb{F}_q^{\omega_q-} \rangle \\ BQNRNIIIS = & \langle (n-\mathbb{T}_1^{\omega_1+}, n-\mathbb{C}_1^{\omega_1+}, n-\mathbb{U}_1^{\omega_1+}, n-\mathbb{F}_1^{\omega_1+}, n-\mathbb{T}_1^{\omega_1-}, n-\mathbb{C}_1^{\omega_1-}, n-\mathbb{U}_1^{\omega_1-}, n-\mathbb{F}_1^{\omega_1-}), \\ & \langle n-\mathbb{T}_2^{\omega_2+}, n-\mathbb{C}_2^{\omega_2+}, n-\mathbb{U}_2^{\omega_2+}, n-\mathbb{F}_2^{\omega_2+}, n-\mathbb{T}_2^{\omega_2-}, n-\mathbb{C}_2^{\omega_2-}, n-\mathbb{U}_2^{\omega_2-}, n-\mathbb{F}_2^{\omega_2-} \rangle, \dots, \\ & n-\mathbb{T}_q^{\omega_q+}, n-\mathbb{C}_q^{\omega_q+}, n-\mathbb{U}_q^{\omega_q+}, n-\mathbb{F}_q^{\omega_q+}, n-\mathbb{T}_q^{\omega_q-}, n-\mathbb{C}_q^{\omega_q-}, n-\mathbb{U}_q^{\omega_q-}, n-\mathbb{F}_q^{\omega_q-} \rangle \end{aligned}$$

with regarding benefit type criteria, $k = 1, 2, \dots, q$.

$$(p+\mathbb{T}_k^{\omega_k+}, p+\mathbb{C}_k^{\omega_k+}, p+\mathbb{U}_k^{\omega_k+}, p+\mathbb{F}_k^{\omega_k+}, p+\mathbb{T}_k^{\omega_k-}, p+\mathbb{C}_k^{\omega_k-}, p+\mathbb{U}_k^{\omega_k-}, p+\mathbb{F}_k^{\omega_k-}) = (\max(\mathbb{T}_{lk}^{\omega_k+}), \min(\mathbb{C}_{lk}^{\omega_k+}), \min(\mathbb{U}_{lk}^{\omega_k+}), \min(\mathbb{F}_{lk}^{\omega_k+}), \\ \min(\mathbb{T}_{lk}^{\omega_k-}), \max(\mathbb{C}_{lk}^{\omega_k-}), \max(\mathbb{U}_{lk}^{\omega_k-}), \max(\mathbb{F}_{lk}^{\omega_k-}))$$

$$(n-\mathbb{T}_k^{\omega_{k+}}, n-\mathbb{C}_k^{\omega_{k+}}, n-\mathbb{U}_k^{\omega_{k+}}, n-\mathbb{F}_k^{\omega_{k+}}, n-\mathbb{T}_k^{\omega_{k-}}, n-\mathbb{C}_k^{\omega_{k-}}, n-\mathbb{U}_k^{\omega_{k-}}, n-\mathbb{F}_k^{\omega_{k-}}) = (\min(\mathbb{T}_{lk}^{\omega_{k+}}), \max(\mathbb{C}_{lk}^{\omega_{k+}}), \max(\mathbb{U}_{lk}^{\omega_{k+}}), \max(\mathbb{F}_{lk}^{\omega_{k+}}), \\ \max(\mathbb{T}_{lk}^{\omega_{k-}}), \min(\mathbb{C}_{lk}^{\omega_{k-}}), \min(\mathbb{U}_{lk}^{\omega_{k-}}), \min(\mathbb{F}_{lk}^{\omega_{k-}}))$$

Likewise, for cost type criteria, $k = 1, 2, \dots, q$.

$$(p+\mathbb{T}_k^{\omega_{k+}}, p+\mathbb{C}_k^{\omega_{k+}}, p+\mathbb{U}_k^{\omega_{k+}}, p+\mathbb{F}_k^{\omega_{k+}}, p+\mathbb{T}_k^{\omega_{k-}}, p+\mathbb{C}_k^{\omega_{k-}}, p+\mathbb{U}_k^{\omega_{k-}}, p+\mathbb{F}_k^{\omega_{k-}}) = (\min(\mathbb{T}_{lk}^{\omega_{k+}}), \max(\mathbb{C}_{lk}^{\omega_{k+}}), \max(\mathbb{U}_{lk}^{\omega_{k+}}), \max(\mathbb{F}_{lk}^{\omega_{k+}}), \\ \max(\mathbb{T}_{lk}^{\omega_{k-}}), \min(\mathbb{C}_{lk}^{\omega_{k-}}), \min(\mathbb{U}_{lk}^{\omega_{k-}}), \min(\mathbb{F}_{lk}^{\omega_{k-}}))$$

$$(n-\mathbb{T}_k^{\omega_{k+}}, n-\mathbb{C}_k^{\omega_{k+}}, n-\mathbb{U}_k^{\omega_{k+}}, n-\mathbb{F}_k^{\omega_{k+}}, n-\mathbb{T}_k^{\omega_{k-}}, n-\mathbb{C}_k^{\omega_{k-}}, n-\mathbb{U}_k^{\omega_{k-}}, n-\mathbb{F}_k^{\omega_{k-}}) = (\max(\mathbb{T}_{lk}^{\omega_{k+}}), \min(\mathbb{C}_{lk}^{\omega_{k+}}), \min(\mathbb{U}_{lk}^{\omega_{k+}}), \min(\mathbb{F}_{lk}^{\omega_{k+}}), \\ \min(\mathbb{T}_{lk}^{\omega_{k-}}), \max(\mathbb{C}_{lk}^{\omega_{k-}}), \max(\mathbb{U}_{lk}^{\omega_{k-}}), \max(\mathbb{F}_{lk}^{\omega_{k-}}))$$

(v) The normalized Euclidean distance for all alternatives

$(\mathbb{T}_{lk}^{\omega_{k+}}, \mathbb{C}_{lk}^{\omega_{k+}}, \mathbb{U}_{lk}^{\omega_{k+}}, \mathbb{F}_{lk}^{\omega_{k+}}, \mathbb{T}_{lk}^{\omega_{k-}}, \mathbb{C}_{lk}^{\omega_{k-}}, \mathbb{U}_{lk}^{\omega_{k-}}, \mathbb{F}_{lk}^{\omega_{k-}})$ from QNRPIS

$(p+\mathbb{T}_k^{\omega_{k+}}, p+\mathbb{C}_k^{\omega_{k+}}, p+\mathbb{U}_k^{\omega_{k+}}, p+\mathbb{F}_k^{\omega_{k+}}, p+\mathbb{T}_k^{\omega_{k-}}, p+\mathbb{C}_k^{\omega_{k-}}, p+\mathbb{U}_k^{\omega_{k-}}, p+\mathbb{F}_k^{\omega_{k-}})$ is evaluated as

$$d_N(\Upsilon_l, BQNRPI S) = \sqrt{\frac{1}{8q} \sum_{k=1}^q \left\{ (\mathbb{T}_{lk}^{\omega_{k+}} - p+\mathbb{T}_k^{\omega_{k+}})^2 + (\mathbb{C}_{lk}^{\omega_{k+}} - p+\mathbb{C}_k^{\omega_{k+}})^2 + (\mathbb{U}_{lk}^{\omega_{k+}} - p+\mathbb{U}_k^{\omega_{k+}})^2 + (\mathbb{F}_{lk}^{\omega_{k+}} - p+\mathbb{F}_k^{\omega_{k+}})^2 \right. \\ \left. + (\mathbb{T}_{lk}^{\omega_{k-}} - p+\mathbb{T}_k^{\omega_{k-}})^2 + (\mathbb{C}_{lk}^{\omega_{k-}} - p+\mathbb{C}_k^{\omega_{k-}})^2 + (\mathbb{U}_{lk}^{\omega_{k-}} - p+\mathbb{U}_k^{\omega_{k-}})^2 + (\mathbb{F}_{lk}^{\omega_{k-}} - p+\mathbb{F}_k^{\omega_{k-}})^2 \right\}}$$

and the normalized Euclidean distance of all alternative

$(\mathbb{T}_{lk}^{\omega_{k+}}, \mathbb{C}_{lk}^{\omega_{k+}}, \mathbb{U}_{lk}^{\omega_{k+}}, \mathbb{F}_{lk}^{\omega_{k+}}, \mathbb{T}_{lk}^{\omega_{k-}}, \mathbb{C}_{lk}^{\omega_{k-}}, \mathbb{U}_{lk}^{\omega_{k-}}, \mathbb{F}_{lk}^{\omega_{k-}})$ from BQNRNIS

$(n-\mathbb{T}_k^{\omega_{k+}}, n-\mathbb{C}_k^{\omega_{k+}}, n-\mathbb{U}_k^{\omega_{k+}}, n-\mathbb{F}_k^{\omega_{k+}}, n-\mathbb{T}_k^{\omega_{k-}}, n-\mathbb{C}_k^{\omega_{k-}}, n-\mathbb{U}_k^{\omega_{k-}}, n-\mathbb{F}_k^{\omega_{k-}})$ is evaluated as

$$d_N(\Upsilon_l, BQNRNIS) = \sqrt{\frac{1}{8q} \sum_{k=1}^q \left\{ (\mathbb{T}_{lk}^{\omega_{k+}} - n-\mathbb{T}_k^{\omega_{k+}})^2 + (\mathbb{C}_{lk}^{\omega_{k+}} - n-\mathbb{C}_k^{\omega_{k+}})^2 + (\mathbb{U}_{lk}^{\omega_{k+}} - n-\mathbb{U}_k^{\omega_{k+}})^2 + (\mathbb{F}_{lk}^{\omega_{k+}} - n-\mathbb{F}_k^{\omega_{k+}})^2 \right. \\ \left. + (\mathbb{T}_{lk}^{\omega_{k-}} - n-\mathbb{T}_k^{\omega_{k-}})^2 + (\mathbb{C}_{lk}^{\omega_{k-}} - n-\mathbb{C}_k^{\omega_{k-}})^2 + (\mathbb{U}_{lk}^{\omega_{k-}} - n-\mathbb{U}_k^{\omega_{k-}})^2 + (\mathbb{F}_{lk}^{\omega_{k-}} - n-\mathbb{F}_k^{\omega_{k-}})^2 \right\}}$$

(vi) The revised closeness degree of all alternative to the BQNRPI S, is computed using a formula.

$$\Psi(\Upsilon_l) = \frac{d_N(\Upsilon_l, BQNRNIS)}{\max\{d_N(\Upsilon_l, BQNRNIS)\}} - \frac{d_N(\Upsilon_l, BQNRPI S)}{\min\{d_N(\Upsilon_l, BQNRPI S)\}}, \quad l = 1, 2, \dots, p.$$

(vii) The inferior ratio to each choice is ascertained by using the updated proximity degrees, which are as follows:

$$\mathcal{IR}(l) = \frac{\Psi(\Upsilon_l)}{\min_{1 \leq l \leq p} (\Psi(\Upsilon_l))}.$$

It is evident that the closed unit interval $[0,1]$ contains all values of $\mathcal{IR}(l)$.

(viii) The alternatives are arranged in ascending order of inferior ratio values and the option with the lowest choice value is selected as the best one.

G. Muhiuddin, Durga Nagarajan, Satham Hussain S, TOPSIS Method-based decision-making model of Bipolar Quadripartitioned Neutrosophic Environment

2.1. *Application of the Proposed Method - Selection of the Best Renewable Energy Source using Bipolar Quadripartitioned Neutrosophic Field*

In the pursuit of sustainable development, the selection of an appropriate renewable energy source plays a pivotal role in shaping our environmental footprint and energy future. The integration of renewable energy technologies not only addresses the imperative of reducing carbon emissions but also fosters energy security and economic resilience. However, choosing the optimal renewable energy source amidst a lot of alternatives involves navigating through complexities such as varying efficiencies, environmental impacts, costs and resource availabilities. These decisions are further compounded by uncertainties and ambiguities inherent in evaluating diverse criteria. In this context, the BQNFs offer a sophisticated framework that accommodates delicate assessments, considering truth, contradiction, ignorance and falsity membership functions. This approach provides a structured methodology to systematically evaluate and rank renewable energy sources based on comprehensive criteria, thereby facilitating informed decision-making towards a sustainable energy landscape. In the renewable energy, selecting the optimal energy source involves evaluating multiple criteria under conditions of uncertainty and imprecision. The decision-making process benefits from the Bipolar Quadripartitioned Neutrosophic approach, which provides a more delicate assessment by considering the truth, contradiction, ignorance and falsity membership functions.

Criteria:

(T1): How efficiently the energy source converts input into usable energy. (T2): The ecological footprint, including emissions and resource depletion. (T3): The overall expense of deploying and maintaining the energy source. (T4): The accessibility and abundance of the energy source. Energy Sources:

Steps in the Decision-Making Process Define the Criteria and Alternatives:

List the criteria (energy efficiency, environmental impact, cost, resource availability). Identify the alternatives (solar power, wind power, hydropower, biomass energy).

This application focuses on the use of bipolar quadripartitioned neutrosophic sets to handle the uncertainties and imprecisions associated with the evaluation criteria.

Energy Efficiency = T_1 , Environmental Impact = T_2 , Cost = T_3 , Resource Availability = T_4 are considered. Four attributes, Υ_1 = Solar Power, Υ_2 = Wind Power, Υ_3 = Hydropower and Υ_4 = Biomass Energy, are made to select the optimal option.

Step 1. Table 1 presents the decision matrix as bipolar quadripartitioned neutrosophic information.

Step 2 : The maximising deviation approach is used to obtain the normalised weights of the criteria, as follows: $\omega_1 = 0.2221$, $\omega_2 = 0.1665$, $\omega_3 = 0.2774$, $\omega_4 = 0.3326$

Step 3: The weighted bipolar quadripartitioned neutrosophic decision matrix is prepared by

$\Upsilon \setminus \mathbb{T}$	$\mathbb{T}_1$	$\mathbb{T}_2$	$\mathbb{T}_3$	$\mathbb{T}_4$
Υ_1	(0.5,0.6,0.5,0.4) (-0.4,-0.7,-0.4,-0.6)	(0.7,0.8,0.4,0.3) (-0.4,-0.6,-0.4,-0.7)	(0.1,0.3,0.4,0.6) (-0.2,-0.7,-0.4,-0.1)	(0.4,0.4,0.2,0.3) (-0.5,-0.5,-0.3,-0.3)
Υ_2	(0.4,0.5,0.7,0.4) (-0.7,-0.6,-0.4,-0.1)	(0.6,0.4,0.3,0.2) (-0.2,-0.4,-0.4,-0.1)	(0.8,0.6,0.4,0.6) (-0.9,-0.4,-0.4,-0.3)	(0.9,0.7,0.6,0.1) (-0.4,-0.4,-0.3,-0.4)
Υ_3	(0.7,0.4,0.4,0.2) (-0.4,-0.4,-0.3,-0.2))	(0.8,0.6,0.4,0.6) (-0.6,-0.7,-0.4,-0.2)	(0.6,0.5,0.4,0.8) (-0.4,-0.5,-0.6,-0.1)	(0.4,0.3,0.6,0.1) (-0.1,-0.2,-0.3,-0.4)
Υ_4	(0.8,0.6,0.4,0.4) (-0.6,-0.4,-0.3,-0.6)	(0.6,0.7,0.5,0.4) (-0.6,-0.4,-0.4,-0.4)	(0.6,0.7,0.8,0.9) (-0.8,-0.7,-0.6,-0.5)	(0.8,0.7,0.6,0.5) (-0.7,-0.6,-0.4,-0.4)

TABLE 1. Bipolar quadripartitioned neutrosophic decision matrix

multiplying the weights by the decision matrix, as indicated in Table 2.

$\Upsilon \setminus \mathbb{T}$	$\mathbb{T}_1$	$\mathbb{T}_2$	$\mathbb{T}_3$	$\mathbb{T}_4$
Υ_1	(0.142,0.899,0.857,0.599) (-0.815,- 0.923,-0.815,-0.184)	(0.181,0.963,0.858,0.818) (-0.858,-0.918,-0.858,-0.181)	(0.028,0.716,0.775,0.867) (-0.639,-0.905,-0.775,-0.028)	(0.262,0.737,0.585,0.670) (-0.794,-0.794,-0.670,-0.111)
Υ_2	(0.107,0.857,0.924,0.816) (-0.924,-0.893,-0.816,-0.023)	(0.141,0.858,0.818,0.764) (-0.764,-0.858,-0.858,-0.017)	(0.360,0.867,0.775,0.867) (-0.971,-0.775,-0.775,-0.716)	(0.535,0.888,0.843,0.464) (-0.737,-0.734,-0.670,-0.156)
Υ_3	(0.234,0.815,0.815,0.699) (-0.815,-0.815,-0.765,-0.048)	(0.235,0.918,0.858,0.918) (-0.918,-0.942,-0.858,-0.036)	(0.224,0.825,0.775,0.939) (-0.775,-0.825,-0.867,-0.028)	(0.156,0.670,0.843,0.464) (-0.464,-0.585,-0.670,-0.156)
Υ_4	(0.300,0.892,0.815,0.815) (-0.892,-0.815,-0.765,-0.048)	(0.141,0.942,0.858,0.918) (-0.918,-0.942,-0.858,-0.036)	(0.224,0.905,0.939,0.971) (-0.939,-0.905,-0.867,-0.825)	(0.414,0.888,0.843,0.794) (-0.888,-0.843,-0.737,-0.156)

TABLE 2. weighted bipolar quadripartitioned neutrosophic decision matrix

Step 4. The BQNRPIs and BQNRNIs are given by

$$\begin{aligned} \text{BQNRPIs} = \{ & (0.300, 0.815, 0.815, 0.599, -0.924, -0.815, -0.765, -0.023), \\ & (0.235, 0.858, 0.818, 0.764, -0.918, -0.858, -0.858, -0.017), \\ & (0.360, 0.716, 0.775, 0.867, -0.971, -0.825, -0.775, -0.028), \\ & (0.535, 0.670, 0.585, 0.464, -0.888, -0.585, -0.670, -0.111) \}. \end{aligned}$$

$$\begin{aligned} \text{BQNRNIs} = \{ & (0.107, 0.899, 0.924, 0.816, -0.815, -0.923, -0.816, -0.184), \\ & (0.141, 0.963, 0.858, 0.918, -0.764, -0.918, -0.942, -0.181), \\ & (0.028, 0.905, 0.939, 0.971, -0.639, -0.905, -0.867, -0.825), \\ & (0.156, 0.888, 0.843, 0.794, -0.464, -0.843, -0.737, -0.156) \}. \end{aligned}$$

Step 5. The following are the normalised Euclidean distances of each alternative from the BQNRPIs and the BQNRNIs.

$$\begin{aligned}d_N(\Upsilon_1, BQNRPIs) &= 0.1187, & d_N(\Upsilon_1, BQNRNIs) &= (0.1296) \\d_N(\Upsilon_2, BQNRPIs) &= 0.1224, & d_N(\Upsilon_2, BQNRNIs) &= (0.1523) \\d_N(\Upsilon_3, BQNRPIs) &= 0.08, & d_N(\Upsilon_3, BQNRNIs) &= (0.1423) \\d_N(\Upsilon_4, BQNRPIs) &= 0.01714, & d_N(\Upsilon_4, BQNRNIs) &= (0.0989).\end{aligned}$$

Step 6. Each alternative's updated closeness degree is provided as:

$$\Psi(\Upsilon_1) = (-0.637), \Psi(\Upsilon_2) = (-0.53), \Psi(\Upsilon_3) = (-0.0657), \Psi(\Upsilon_4) = (-0.3546),$$

Step 7. The inferior ratio to all alternative is provided as:

$$\mathcal{IR}(1) = 1, \mathcal{IR}(2) = 0.8320, \mathcal{IR}(3) = 0.1031, \mathcal{IR}(4) = 0.5566.$$

Step 8. The following is the result one get while we organise the power source in increasing order of alternatives: $\Upsilon_3 < \Upsilon_4 < \Upsilon_2 < \Upsilon_1$. Hence Υ_3 = Hydro Power is the best energy source. Hydro Power is the closest to the ideal solution according to TOPSIS scores.

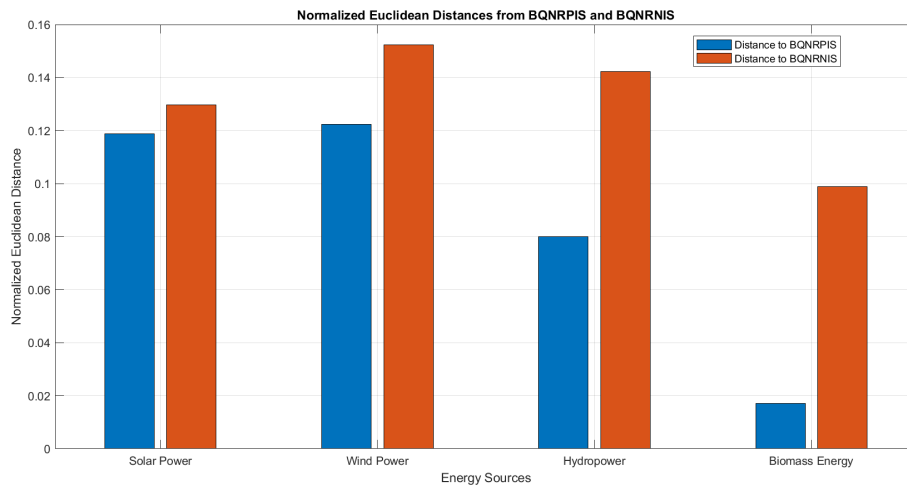


FIGURE 1. Normalised Euclidean Distances

3. Quadripartitioned Neutrosophic ELECTRE-I Method

In this section, we propose to apply the bipolar quadripartitioned neutrosophic ELECTRE-I technique to address MCDM problems.

Let the set of alternatives be represented as $\Upsilon = \{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_p\}$ and the set of criteria, defined as $\mathbb{T} = \{\mathbb{T}_1, \mathbb{T}_2, \mathbb{T}_3, \dots, \mathbb{T}_q\}$ that are utilized to calculate all alternatives.

(i – iii) Similar to the bipolar quadripartitioned neutrosophic TOPSIS section, the options'

rating values in relation to the criteria are represented as a matrix $[h_{lk}]_{p \times q}$. By using the maximising deviation technique, the weights ω_k of the criterion $\mathbb{T}_k$ are determined with weighted bipolar quadripartitioned neutrosophic decision matrix $[h_{lk}]_{p \times q}$ is created.

(iv) The bipolar quadripartitioned neutrosophic discordance sets $\mathbb{F}_{ab}$ and concordance sets $\mathbb{E}_{ab}$ are defined as follows:

$$\mathbb{E}_{ab} = \{1 \leq l \leq q | \Psi_{ak} \geq \Psi_{bk}\}, \quad a, b = 1, 2, \dots, p, \quad a \neq b.$$

$$\mathbb{F}_{ab} = \{1 \leq l \leq q | \Psi_{ak} \leq \Psi_{bk}\}, \quad a, b = 1, 2, \dots, p, \quad a \neq b.$$

where $\Psi = \mathbb{T}_{lk}^{p+} + \mathbb{C}_{lk}^{p+} + \mathbb{U}_{lk}^{p+} + \mathbb{F}_{lk}^{p+} + \mathbb{T}_{lk}^{n-} + \mathbb{C}_{lk}^{n-} + \mathbb{U}_{lk}^{n-} + \mathbb{F}_{lk}^{n-}$, $l = 1, 2, \dots, p$, $k = 1, 2, \dots, q$.

(v) The following is the construction of the bipolar quadripartitioned neutrosophic concordance matrix $\mathbb{E}$:

$$\mathbb{E} = \begin{bmatrix} - & \mathbf{e}_{12} & \dots & \mathbf{e}_{1p} \\ \mathbf{e}_{21} & - & \dots & \mathbf{e}_{2p} \\ . & . & \dots & . \\ . & . & \dots & . \\ \mathbf{e}_{p1} & \mathbf{e}_{p2} & \dots & - \end{bmatrix}$$

wherein $\mathbf{e}_{ab}$, the bipolar quadripartitioned neutrosophic concordance indices, are calculated as

$$\mathbf{e}_{ab} = \sum_{k \in \mathbb{E}_{ab}} \omega_k$$

(vi) Here is how to construct the bipolar quadripartitioned neutrosophic disconcordance matrix $\mathbb{F}$.

$$\mathbb{F} = \begin{bmatrix} - & \mathbf{f}_{12} & \dots & \mathbf{f}_{1p} \\ \mathbf{f}_{21} & - & \dots & \mathbf{f}_{2p} \\ . & . & \dots & . \\ . & . & \dots & . \\ \mathbf{f}_{p1} & \mathbf{f}_{p2} & \dots & - \end{bmatrix}$$

wherein $\mathbf{f}_{ab}$, the bipolar quadripartitioned neutrosophic disconcordance indices, are calculated as

$$\mathbf{f}_{ab} = \frac{\max_{k \in \mathbb{F}_{ab}} \sqrt{\frac{1}{8q} \left\{ (\mathbb{T}_{ak}^{\omega_k+} - \mathbb{T}_{bk}^{\omega_k+})^2 + (\mathbb{C}_{ak}^{\omega_k+} - \mathbb{C}_{bk}^{\omega_k+})^2 + (\mathbb{U}_{ak}^{\omega_k+} - \mathbb{U}_{bk}^{\omega_k+})^2 + (\mathbb{F}_{ak}^{\omega_k+} - \mathbb{F}_{bk}^{\omega_k+})^2 \right.}}{\max_k \sqrt{\frac{1}{8q} \left\{ (\mathbb{T}_{ak}^{\omega_k+} - \mathbb{T}_{bk}^{\omega_k+})^2 + (\mathbb{C}_{ak}^{\omega_k+} - \mathbb{C}_{bk}^{\omega_k+})^2 + (\mathbb{U}_{ak}^{\omega_k+} - \mathbb{U}_{bk}^{\omega_k+})^2 + (\mathbb{F}_{ak}^{\omega_k+} - \mathbb{F}_{bk}^{\omega_k+})^2 \right.}}}{\sqrt{\frac{1}{8q} \left\{ (\mathbb{T}_{ak}^{\omega_k-} - \mathbb{T}_{bk}^{\omega_k-})^2 + (\mathbb{C}_{ak}^{\omega_k-} - \mathbb{C}_{bk}^{\omega_k-})^2 + (\mathbb{U}_{ak}^{\omega_k-} - \mathbb{U}_{bk}^{\omega_k-})^2 + (\mathbb{F}_{ak}^{\omega_k-} - \mathbb{F}_{bk}^{\omega_k-})^2 \right\}}}$$

(vii) To evaluate alternatives, the levels of concordance and discordance are calculated. The average value of the bipolar quadripartitioned neutrosophic concordance index as $\bar{e}$ defines the bipolar quadripartitioned neutrosophic concordance level.

$$\bar{e} = \frac{1}{p(p-1)} \sum_{a=1, b \neq a}^m \sum_{b=1, a \neq b}^m e_{ab}.$$

In a similar way the average value of the bipolar quadripartitioned neutrosophic discordance indices as $\bar{f}$ defines the bipolar quadripartitioned neutrosophic discordance level.

$$\bar{f} = \frac{1}{p(p-1)} \sum_{a=1, b \neq a}^m \sum_{b=1, a \neq b}^m f_{ab}$$

(viii) Based on $\bar{e}$, the bipolar quadripartitioned neutrosophic concordance dominance matrix Φ is calculated as follows:

$$\Phi = \begin{bmatrix} - & \Phi_{12} & \dots & \Phi_{1p} \\ \Phi_{21} & - & \dots & \Phi_{2p} \\ . & . & \dots & . \\ . & . & \dots & . \\ \Phi_{p1} & \Phi_{p2} & \dots & - \end{bmatrix}$$

where Φ_{ab}

$$\Phi_{ab} = \begin{cases} 1, & \text{if } e_{ab} \geq \bar{e} \\ 0, & \text{if } e_{ab} < \bar{e} \end{cases}$$

(ix) Based on $\bar{f}$, the bipolar quadripartitioned neutrosophic discordance dominance matrix Ψ can be established as follows:

$$\Psi = \begin{bmatrix} - & \Psi_{12} & \dots & \Psi_{1p} \\ \Psi_{21} & - & \dots & \Psi_{2p} \\ . & . & \dots & . \\ . & . & \dots & . \\ \Psi_{p1} & \Psi_{p2} & \dots & - \end{bmatrix}$$

where Ψ_{ab}

$$\Psi_{ab} = \begin{cases} 1, & \text{if } f_{ab} \leq \bar{f} \\ 0, & \text{if } f_{ab} > \bar{f}. \end{cases}$$

(x) Thus, by multiplying the appropriate elements of Φ and Ψ , the bipolar quadripartitioned neutrosophic aggregated dominance matrix Ψ is determined, that is

$$\Pi = \begin{bmatrix} - & \Pi_{12} & \dots & \Pi_{1p} \\ \Pi_{21} & - & \dots & \Pi_{2p} \\ . & . & \dots & . \\ . & . & \dots & . \\ \Pi_{p1} & \Pi_{p2} & \dots & - \end{bmatrix}$$

where, Π_{ab} is defined as

$$\Pi_{ab} = \Phi_{ab}\Psi_{ab}$$

(xi) Lastly, a ranking based on the outranking values Π_{ab} is applied to the alternatives. In other words, an arrow from Υ_a to Υ_b exists for any pair of alternatives Υ_a and Υ_b if and only if $\Pi_{ab} = 1$. Thus, we have the following three scenarios:

- (A) A distinct arrow departs from Υ_a and enters Υ_b
- (B) Between Υ_a and Υ_b there are two possible arrows
- (C) There is no arrow from Υ_a and Υ_b

We conclude that Υ_a is preferable than Υ_b for case A. In case C, Υ_a and Υ_b are incomparable, whereas in case B, Υ_a and Υ_b are indifferent.

3.1. Application of the Proposed Method

In Section 2, the bipolar quadripartitioned neutrosophic TOPSIS approach is used to present MCDM problems. In this section, we choose the optimal renewable energy source to compare these two MCDM approaches on using our suggested bipolar quadripartitioned neutrosophic ELECTRE-I method. In Section 2.1, steps (1-3) have already been completed. Thus, we proceed to Step 4.

Step 4. Table 3 presents the bipolar quadripartitioned neutrosophic concordance sets E_{ab} .

$E_{ab} \setminus b$	1	2	3	4
E_{1b}	-	{3}	{1}	{3}
E_{2b}	{1,2,4}	-	{1,4}	{3,4}
E_{3b}	{1,2,3,4}	{1,2,3}	-	{2,3}
E_{4b}	{1,2,3,4}	{1,2}	{1,4}	-

TABLE 3. bipolar quadripartitioned neutrosophic concordance set

Step 5. The F_{ab} bipolar quadripartitioned neutrosophic discordance sets are provided as Table 4.

$F_{ab} \setminus b$	1	2	3	4
F_{1b}	-	{1,2,4}	{2,3,4}	{1,2,4}
F_{2b}	{3}	-	{2,3}	{1,2}
F_{3b}	{}	{4}	-	{1,4}
F_{4b}	{}	{3,4}	{2,3}	-

TABLE 4. bipolar quadripartitioned neutrosophic discordance sets

Step 6. Here is the computation of the bipolar quadripartitioned neutrosophic concordance matrix E

$$E = \begin{bmatrix} - & (0.2774) & (0.2221) & (0.2774) \\ (0.7212) & - & (0.5547) & (0.61) \\ (1) & (0.6674) & - & (0.4439) \\ (1) & (0.3886) & (0.5547) & - \end{bmatrix}$$

Step 7. The following formula is used to compute the bipolar quadripartitioned neutrosophic disconcordance matrix F.

$$F = \begin{bmatrix} - & (1) & (1) & (1) \\ (0.5844) & - & (0.9634) & (0.6524) \\ (0) & (1) & - & (0.5725) \\ (0) & (1) & (0.5423) & - \end{bmatrix}$$

Step 8. Now, $\bar{e} = 0.5507$ is the bipolar quadripartitioned neutrosophic concordance level, while $\bar{f} = 0.7179$ is the bipolar quadripartitioned discordance level. The dominance matrices Φ and Ψ , which represent the bipolar quadripartitioned neutrosophic concordance and discordance, respectively, are presented below.

$$\Phi = \begin{bmatrix} - & 0 & 0 & 0 \\ 1 & - & 1 & 1 \\ 1 & 1 & - & 0 \\ 1 & 0 & 1 & - \end{bmatrix}$$

$$\Psi = \begin{bmatrix} - & 0 & 0 & 0 \\ 1 & - & 0 & 1 \\ 0 & 0 & - & 1 \\ 0 & 0 & 1 & - \end{bmatrix}$$

Step 9. It is calculated that the quadripartitioned neutrosophic aggregated dominance matrix Π

$$\Pi = \begin{bmatrix} - & 0 & 0 & 0 \\ 1 & - & 0 & 1 \\ 0 & 0 & - & 0 \\ 0 & 0 & 1 & - \end{bmatrix}$$

Based on the Π matrix, the decision-making process is as follows:

- Υ_2 is the best alternative among the four, as it has the most dominance relationships (i.e., it dominates Υ_1 , Υ_3 and Υ_4).
- Υ_1 , Υ_3 and Υ_4 are less preferable compared to Υ_2 .

In summary, Υ_2 (Wind Power) is ranked as the optimal renewable energy source based on the bipolar quadripartitioned neutrosophic ELECTRE-I method.

	Υ_1	Υ_2	Υ_3	Υ_4
Υ_1	-	0	0	0
Υ_2	1	-	0	1
Υ_3	0	0	-	0
Υ_4	0	0	1	-

TABLE 5. Bipolar Quadripartitioned Neutrosophic Aggregated Dominance Matrix Π

Interpretation of the Matrix:

- $\Pi_{21} = 1$: Υ_2 dominates Υ_1 .
- $\Pi_{31} = 0$: Υ_3 does not dominate Υ_1 .
- $\Pi_{41} = 0$: Υ_4 does not dominate Υ_1 .
- $\Pi_{32} = 0$: Υ_3 does not dominate Υ_2 .
- $\Pi_{42} = 0$: Υ_4 does not dominate Υ_2 .
- $\Pi_{34} = 1$: Υ_3 dominates Υ_4 .

Therefore, Υ_2 (Wind Power) is the preferred alternative among the options considered. Wind Power dominates all other alternatives based on the concordance and discordance relations.

Decision: Υ_2 (Wind Power) is ranked as the optimal renewable energy source based on the bipolar quadripartitioned neutrosophic ELECTRE-I method.

4. Comparison of the proposed methods

Quadripartitioned neutrosophic TOPSIS method is suitable when the emphasis is on finding alternatives that are closest to an ideal solution and farthest from a negative ideal, making it useful in scenarios where precise closeness to ideal values is crucial. In contrast, quadripartitioned neutrosophic ELECTRE-I method provides a broader perspective by considering pairwise comparisons and collective outranking, making it more adaptable to contexts where relative performance and comparative evaluations among alternatives are paramount. Both methods effectively handle neutrosophic information, but quadripartitioned neutrosophic

TOPSIS method directly quantifies closeness and fairness to ideal solutions through distance calculations, while quadripartitioned neutrosophic ELECTRE-I method employs pairwise comparisons and threshold-based indices to establish outranking relationships. Quadripartitioned neutrosophic TOPSIS method faces challenges in defining and normalizing neutrosophic values accurately, whereas quadripartitioned neutrosophic ELECTRE-I method's reliance on thresholds introduces subjectivity in determining these values. Additionally, quadripartitioned neutrosophic ELECTRE-I method's computational complexity increases with the number of alternatives and criteria due to the pairwise comparison nature. Both models are extending traditional MCDM methods to handle quadripartitioned neutrosophic information, their application and suitability depend on the specific decision context, the preference for precise closeness or relative comparisons and the complexity of neutrosophic value handling and computation. According to our analysis, in the renewable energy source selection, the TOPSIS and ELECTRE-I methods provide distinct perspectives. TOPSIS ranks Υ_3 (Hydro Power) as the best alternative based on its proximity to the ideal solution, which emphasizes the quantitative performance metrics of the alternatives. It evaluates each option against a positive ideal solution and a negative ideal solution, focusing on minimizing the distance from the ideal and maximizing the distance from the non-ideal. In contrast, ELECTRE-I ranks Υ_2 (Wind Power) as the best choice by examining the dominance relationships through concordance and discordance matrices. This method takes into account both positive and negative evaluations, providing a broader and more delicate comparison of alternatives based on a consensus of criteria rather than a single performance measure. Therefore, while TOPSIS emphasizes the ideal performance of alternatives, ELECTRE-I offers a strategic evaluation of how each alternative compares to others in a more qualitative sense.

Conclusion

In this study, we introduced a novel approach, the bipolar Quadripartitioned Neutrosophic TOPSIS and ELECTRE-I methods have studied. The use of bipolar quadripartitioned neutrosophic sets allowed to effectively handle uncertainties and imprecisions inherent in the decision-making process. The divergence in results from these methods highlights the importance of adopting a multifaceted approach in decision-making processes, where both quantitative performance and qualitative dominance factors are considered. By integrating these two methods, the study provides a robust framework for selecting the most suitable renewable energy source based on a balanced evaluation of all relevant criteria. Ultimately, the combination of TOPSIS and ELECTRE-I offers a delicate decision-support tool that can guide policymakers and stakeholders in making informed and strategic choices for sustainable energy solutions. In future, we will investigate additional criteria or attributes that can further refine energy source

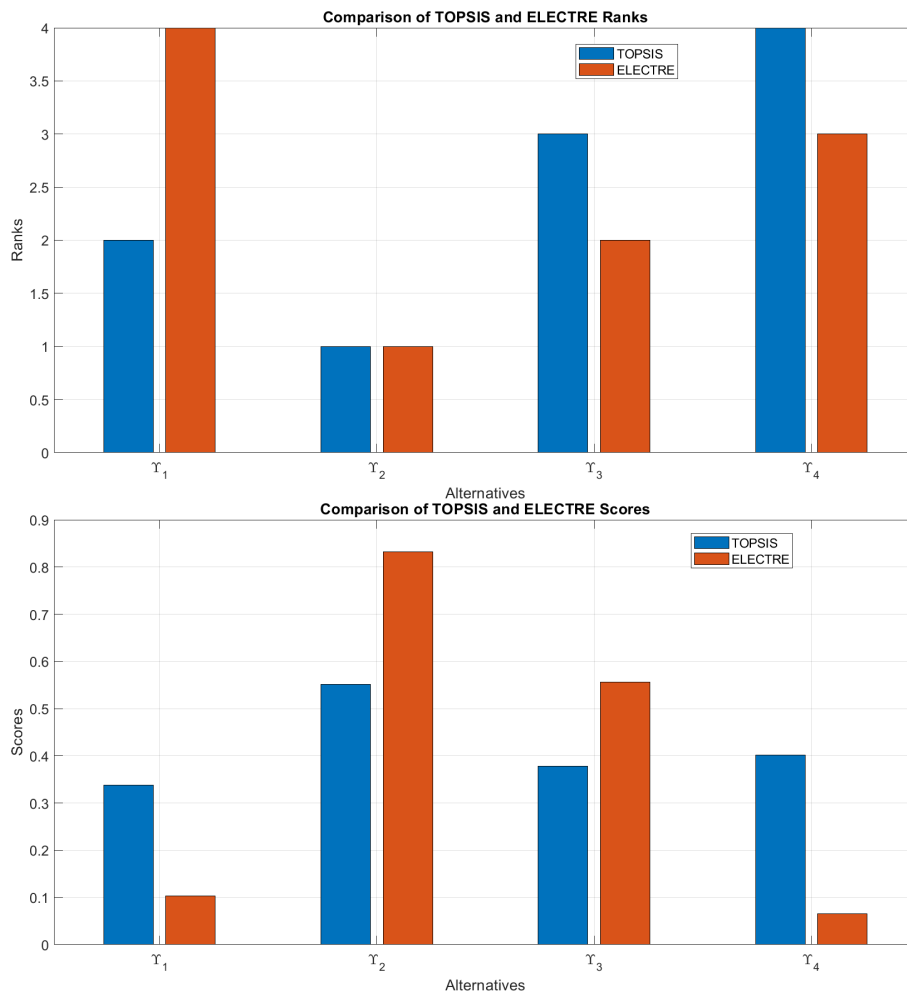


FIGURE 2. Comparison of the methods

selection decisions. Further, exploring extensions or modifications of the TOPSIS method to address specific industry requirements or constraints. Also, we have planned to conduct comparative studies with other multi-criteria decision-making methods to validate the effectiveness and efficiency of the developed approaches in different bipolar quadripartitioned neutrosophic domains.

Compliance with ethical standards

Data availability statements Not applicable.

Conflict of interest: The authors declare that they have no conflict of interest.

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A Neutrosophic TwoFold Algebra-Based Framework for Evaluating Practical Teaching Quality in Fashion Design and Technology Education

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Abstract- Assessing the quality of practical teaching in fashion design and technology programs is complex due to the subjective and multidimensional nature of creative education. This study introduces a novel framework based on Neutrosophic TwoFold Algebra to evaluate practical teaching performance by capturing uncertainty, contradiction, and partial truth within students' learning experiences. The proposed method integrates classical assessment data with neutrosophic components of truth, indeterminacy, and falsehood to reflect the nuanced quality of instruction in real-world settings. A simulated case study demonstrates how this approach provides more comprehensive and balanced evaluations compared to traditional models. The framework aims to support academic institutions in making informed decisions to enhance teaching effectiveness and learning outcomes in fashion-related disciplines.

Keywords: Neutrosophic TwoFold Algebra, Practical Teaching, Fashion Design Education, Teaching Evaluation, Uncertainty Modeling, Creative Learning Assessment

1. Introduction

Practical teaching plays a central role in the education of fashion design and technology students, as it bridges theoretical knowledge and hands-on skill development. Unlike theoretical disciplines, practical instruction in fashion education involves high variability in student outcomes, subjective evaluation processes, and the challenge of balancing creativity with technical precision. These complexities have prompted scholars and institutions to search for robust evaluation methods that account for the dynamic and often ambiguous nature of student performance in practical settings.

Traditional assessment frameworks often rely on quantitative metrics such as attendance, project completion, or rubric-based grading. While these tools provide measurable outcomes, they frequently fail to capture the uncertainty, inconsistency, and creative variance inherent in fashion design training [1]. For instance, a student's portfolio may exhibit technical flaws yet convey a strong conceptual vision raising the question of how to weigh such dimensions fairly and accurately. Moreover, subjective biases from instructors and institutional constraints further obscure the objectivity of practical teaching assessments [2]. Recent advances in fuzzy logic, neutrosophic sets, and multi-valued algebraic structures have enabled researchers to model imprecision in complex systems such as education, medicine, and decision-making [3].

Smarandache (2024) introduced a novel mathematical framework known as Neutrosophic TwoFold Algebra, and in general Fuzzy and Fuzzy-Extensions TwoFold Algebra, designed to handle situations where information is not only imprecise but also contains degrees of contradiction and ambiguity [4]. Unlike traditional algebraic systems that assume clarity and consistency, this model captures both the tangible and uncertain aspects of data by combining classical operations with neutrosophic components. The strength of this approach lies in its ability to represent elements with varying levels of truth, uncertainty, and falsehood offering a more flexible analytical structure. This dual mechanism is particularly relevant in disciplines where evaluation is influenced by multiple perspectives and incomplete knowledge, such as education, decision-making, or creative performance analysis.

In this paper, we apply Neutrosophic TwoFold Algebra to construct a comprehensive model for evaluating practical teaching quality in the fashion design and technology major. We demonstrate how this algebraic framework can incorporate both measurable data and interpretative factors, allowing institutions to assess teaching effectiveness in a way that reflects the full complexity of studio-based and design-led education. The proposed model not only enhances fairness and transparency but also supports continuous pedagogical improvement by highlighting areas of uncertainty and divergence in student outcomes.

2. Literature Review

In creative disciplines like fashion design, evaluating practical teaching presents ongoing challenges due to the highly subjective and process-oriented nature of studio-based learning. Unlike traditional academic fields, where assessments often rely on standardized tests and measurable outputs, creative education involves iterative development, interpretive feedback, and evolving student work. As Fredrickson and Watwood (2018) observed in their review of studio-based pedagogies, standard rubrics tend to miss critical dimensions of learning such as experimentation, conceptual risk-taking, and progressive refinement of ideas[12].

Moreover, scholars have increasingly emphasized that creative assessment should consider sociocultural and emotional dimensions of student engagement. Bennett (2023) argues that educational assessment must reflect the diverse cultural backgrounds and identities that shape how students approach creative tasks, advocating for more responsive and inclusive frameworks [13]. Danvers (2016) similarly notes that emotional factors—like confidence, uncertainty, and vulnerability—are entangled with critical and creative thinking, which means traditional metrics often fall short of capturing the full scope of student development [14].

Recent work has also explored how learning dispositions, such as perseverance and reflective thinking, can provide a better indication of learning progress than static outputs. Stephenson (2024) promotes the idea of “dispositional learning,” where assessment focuses not just on finished work but also on how students engage with the creative process, especially in open-ended or ambiguous contexts [15].

On a global scale, the OECD (2023) has highlighted the importance of assessing creative and critical thinking as part of higher education policy. Their research across institutions found a consistent gap between the value placed on creativity and the methods used to measure it. They

advocate for more flexible, multidimensional evaluation models capable of capturing partial success, uncertainty, and iterative progress—qualities inherent in artistic disciplines [16]. In parallel, educational technology has started playing a role in supporting such nuanced assessments. For instance, Shabani (2022) used machine learning techniques to detect creative thinking patterns in educational datasets, suggesting that artificial intelligence can support more layered insights into student work [17]. Similarly, Kovalkov et al. (2022) developed a method to evaluate creativity across modalities using Scratch programming environments, showing that computational methods can indeed reflect divergent thinking in educational artifacts [18]. These developments [12-21] provide theoretical and practical justification for the integration of mathematical evaluation models such as Neutrosophic TwoFold Algebra which can handle degrees of truth, uncertainty, and contradiction. Such models offer a meaningful alternative to binary grading systems and support the complex, often non-linear paths students take in creative learning settings like fashion design.

Evaluating practical teaching in fashion design and technology requires flexible models capable of interpreting both qualitative and uncertain data. Existing research has recognized the limitations of traditional metrics in creative disciplines and proposed logic-based alternatives to model ambiguity and subjectivity [5]. Fuzzy logic, for instance, has been employed to handle partial correctness in learning outcomes, especially where binary assessments fall short. However, fuzzy systems cannot often manage indeterminate states, a common feature in evolving or partially formed student work.

Neutrosophic logic has emerged as a more adaptive framework, introducing three parameters truth (T), indeterminacy (I), and falsehood (F) to reflect the spectrum of cognitive and evaluative uncertainty. It has been extended to algebraic structures through Neutrosophic TwoFold Algebra, which incorporates both classical and neutrosophic components in a dual-layered operation model [4].

Shihadeh et al. (2024) expanded this model by introducing Two-Fold Fuzzy Algebras based on neutrosophic real numbers, enabling algebraic systems to account for variability in real-valued educational data such as subjective scores or feedback frequencies. Their framework provided robust tools for modeling performance evaluations involving inconsistent grading scenarios [5]. Further development came from Al-Tameemi (2024), who introduced Fuzzy Metric Spaces within the Two-Fold Fuzzy Algebra context. His work addressed proximity measures between educational outcomes, allowing researchers to quantify the similarity or divergence in student performance over time or between evaluators [6].

To accommodate complex evaluation contexts, Shihadeh et al. (2024) also proposed Two-Fold Fuzzy n -Refined Neutrosophic Rings for cases with multiple layers of assessment criteria. This model is especially relevant to multidisciplinary courses like fashion technology, where technical accuracy and creative exploration often intersect across separate grading rubrics [7].

Salman (2024) contributed to the theoretical generalization of these structures by introducing a specialized gamma function over complex Two-Fold Algebras. Although mathematical, this

work set the foundation for advanced operations that may be translated into adaptive scoring algorithms in educational technology systems [8].

Mohammed (2024) provided practical insight into the resolution of differential equations formulated within the Two-Fold Fuzzy and Neutrosophic domains. His results suggested pathways to model change over time in student progress—ideal for tracking growth in skill-based learning modules [9].

Hatamleh and Hazaymeh (2024) explored the identification of minimal units in Two-Fold Fuzzy finite neutrosophic rings, with implications for resource optimization in educational settings. Their results can inform curriculum design by highlighting the essential evaluative elements in complex grading systems [10].

Yasar and Hatip (2024) examined the structure of Binary Two-Fold Algebras based on intuitionistic fuzzy groups, enabling binary comparative assessments such as peer evaluation or pass/fail judgments. Their findings add structural support for integrating Two-Fold Algebra into institutional grading platforms [11].

Collectively, these works form a strong theoretical and applied foundation for deploying TwoFold Algebraic models in educational contexts. Despite their mathematical focus, their relevance to creative and subjective domains like fashion education is clear: each model adds the capacity to express nuance, uncertainty, and evolving evaluation criteria.

Nonetheless, few studies have directly applied these algebraic models to teaching quality assessment in design-based programs. Our work aims to bridge this gap by operationalizing Neutrosophic TwoFold Algebra for practical evaluation in fashion education, tailoring the model to reflect the specificities of creativity-centered learning environments.

3. Proposed Method

To effectively evaluate practical teaching in fashion design and technology, we propose a model grounded in Neutrosophic TwoFold Algebra (NTFA). This model captures both objective instructional inputs (classical scores) and subjective interpretative factors (truth, indeterminacy, and falsehood) that typically influence assessments in creative disciplines.

3.1. Mathematical Foundation

3.1.1 Neutrosophic TwoFold Algebra

Imagine you're working with a set of items, but each item isn't just a simple object—it comes with extra information about how "true," "uncertain," or "false" it is. This is what Neutrosophic TwoFold Algebra does. It's built on a universe of discourse (let's call it U), and within that, we have a special set A defined as:

$$A_{(T,I,F)} = \{x(T_A(x), I_A(x), F_A(x))\}$$

$T_A(x)$ is the degree of truth, $I_A(x)$ is the degree of indeterminacy (uncertainty), and $F_A(x)$ is the degree of falsehood. Each of these values is a number between 0 and 1.

So, every element x in A carries a triplet (T, I, F) , making it more complex and informative than a regular set.

3.1.2 The Neutrosophic TwoFold Law

This algebra comes with a special rule, or "law," to combine two elements. Let's call it Δ . It works like this:

Take two elements: $x_{1(t_1, i_1, f_1)}$ and $x_{2(t_2, i_2, f_2)}$.

Combine them using:

$$x_{1(t_1, i_1, f_1)} \Delta x_{2(t_2, i_2, f_2)} = (x_1 \# x_2)_{(t_1, i_1, f_1) * (t_2, i_2, f_2)}$$

$\#$ is an operation on the elements themselves (like adding or multiplying numbers).

$*$ is an operation on the neutrosophic parts (T, I, F), like averaging or multiplying the truth values.

These two operations ($\#$ and $*$) can be related or completely separate, depending on the situation.

3.1.3 The General Form [4]

In its most flexible form, the law can be written as a function g :

$$g(x_1, t_1, i_1, f_1, x_2, t_2, i_2, f_2) = h(x_1, t_2, i_1, f_1, x_2, t_2, i_2, f_2)_{(T(t_1, t_2), I(i_1, i_2), F(f_1, f_2))}$$

This looks complicated, but it just means g takes all parts of both elements and produces a new element with updated neutrosophic values.

3.1.4 Variations of TwoFold Algebra [4]

This concept isn't limited to neutrosophic sets. It extends to other systems:

Fuzzy TwoFold Algebra

Uses only truth values (T).

Law: $x_{1(t_1)} \Delta x_{2(t_2)} = (x_1 \# x_2)_{(t_1 * t_2)}$.

Intuitionistic Fuzzy TwoFold Algebra

Uses truth and falsehood (T, F).

Law: $x_{1(t_1, f_1)} \Delta x_{2(t_2, f_2)} = (x_1 \# x_2)_{(t_1, f_1) * (t_2, f_2)}$.

Other Types: Picture Fuzzy, Pythagorean Fuzzy, Spherical Fuzzy, and more-all adapt the same idea with different rules for T, I, F [4].

Let $A(T, I, F)$ represent a neutrosophic set of evaluated teaching sessions:

$$A(T, I, F) = \{ x_j(T_j, I_j, F_j) \mid x_j \in U, T_j, I_j, F_j \in [0, 1] \}$$

Where:

T_j : Degree of observed effective teaching (truth).

I_j : Degree of uncertainty in evaluation (indeterminacy).

F_j : Degree of observed ineffectiveness (falsehood).

x_j : Teaching instance or class module.

To combine multiple evaluations (e.g., across multiple observers or over time), we apply the Neutrosophic TwoFold Aggregation Law:

$$x = x_1 + x_2$$

$$T = \frac{x_1 \cdot T_1 + x_2 \cdot T_2}{x_1 + x_2},$$

$$I = \frac{x_1 \cdot I_1 + x_2 \cdot I_2}{x_1 + x_2},$$

$$F = \frac{x_1 \cdot F_1 + x_2 \cdot F_2}{x_1 + x_2}$$

This weighted aggregation balances each session's contribution proportionally, producing a new evaluation $x(T, I, F)$

In addition to the basic weighted average used for aggregating T, I, and F values, we propose an enhanced version that accounts for interactions between the parameters. For example, a high level of I may reduce the reliability of T, while F can amplify ambiguity. To model this, we introduce adjustment coefficients:

$$T' = T - \alpha \cdot I - \beta \cdot F$$

$$I' = I + \gamma \cdot |T - F|$$

$$F' = F + \delta \cdot I$$

Where $\alpha, \beta, \gamma, \delta \in [0, 1]$ are calibration parameters based on the context. These adjusted values (T' , I' , F') offer a more nuanced reflection of performance and can be used for deeper analysis alongside the original triplet.

3.2. Step-by-Step Implementation

Step 1. Classical Scoring

Each practical teaching session is initially scored on a 0–100 scale based on predefined rubrics e.g., clarity, engagement, technique delivery. These are normalized to a [0,1] scale.

Step 2. Neutrosophic Mapping

Expert evaluators assign neutrosophic values:

1. T for clarity and effectiveness,
2. I for ambiguity or disagreement in feedback,
3. F for noticeable teaching flaws.

Step 3. Weight Assignment

Let each session x_j have a weight e.g., duration, project impact. These are used to weight the neutrosophic parameters.

Step 4. TwoFold Evaluation

Apply the aggregation formula to calculate the overall neutrosophic value of the teacher's practical performance:

$$\text{Final Score} = X_{\text{total}}(T_{\text{avg}}, I_{\text{avg}}, F_{\text{avg}})$$

After calculating the final values of truth, indeterminacy, and falsehood, the results can be summarized in a clear format to support decision-making.

3.3. Interpretive Model

We define a performance zone system:

1. Excellence Zone; $T \geq 0.75$, $F \leq 0.1$, $I \leq 0.15$
2. Developing Zone; $0.5 \leq T < 0.75$, $I > 0.15$
3. Critical Zone; $F \geq 0.4$, regardless of T

This allows institutions to quickly identify strong and weak teaching components while accounting for ambiguity.

4. A Simulated Case Study

To demonstrate the application of Neutrosophic TwoFold Algebra in evaluating practical teaching quality, we present a simulated case study based on a semester-long course in "Apparel Construction and Draping" offered at a fashion design institute. The course includes a series of project-based studio sessions where students are guided through various garment-making techniques, from basic pattern cutting to advanced draping on dress forms.

4.1. Evaluation Context

A team of three evaluators an instructor, an external academic reviewer, and a fashion industry consultant was tasked with assessing five key teaching sessions using both classical grading and neutrosophic values. Each session was evaluated on three main dimensions:

1. Effectiveness, T ,
2. Uncertainty due to varied student responses or unclear feedback, I ,
3. Identified weaknesses or instructional issues, F .

4.2. Input Data

Each session was assigned a weight based on its duration and importance in the curriculum. Table 1 shows the assigned weights and neutrosophic scores for each session. The scores represent consensus values averaged from the three evaluators.

Table 1. Neutrosophic Evaluation of Teaching Sessions

Session ID	Weight (Hs)	T	I	F
S1	3	0.85	0.10	0.05
S2	2	0.65	0.25	0.10
S3	4	0.70	0.20	0.10
S4	2	0.50	0.30	0.20
S5	1	0.45	0.35	0.20

4.3. Aggregation Using Neutrosophic TwoFold Algebra

Using the aggregation equations from Section 3.1, we calculate the overall neutrosophic evaluation of the instructor's practical teaching across the five sessions.

Let:

x_j = weight of session j ,

T_j, I_j, F_j = neutrosophic values of session j .

The aggregated scores are calculated using the following formulas:

$$T = \frac{\sum x_j \cdot T_j}{\sum x_j},$$

$$I = \frac{\sum x_j \cdot I_j}{\sum x_j},$$

$$F = \frac{\sum x_j \cdot F_j}{\sum x_j}$$

Substituting the values:

Weights: $x_1 = 3, x_2 = 2, x_3 = 4, x_4 = 2, x_5 = 1$

Truths: $T_1 = 0.85, T_2 = 0.65, T_3 = 0.70, T_4 = 0.50, T_5 = 0.45$

Indeterminacies: $I_1 = 0.10, I_2 = 0.25, I_3 = 0.20, I_4 = 0.30, I_5 = 0.35$

Falsehoods: $F_1 = 0.05, F_2 = 0.10, F_3 = 0.10, F_4 = 0.20, F_5 = 0.20$

$$T = \frac{(3 \cdot 0.85) + (2 \cdot 0.65) + (4 \cdot 0.70) + (2 \cdot 0.50) + (1 \cdot 0.45)}{3 + 2 + 4 + 2 + 1} = \frac{2.55 + 1.30 + 2.80 + 1.00 + 0.45}{12} = \frac{8.10}{12} \approx 0.675$$

$$I = \frac{(3 \cdot 0.10) + (2 \cdot 0.25) + (4 \cdot 0.20) + (2 \cdot 0.30) + (1 \cdot 0.35)}{12} = \frac{0.30 + 0.50 + 0.80 + 0.60 + 0.35}{12} = \frac{2.55}{12} \approx 0.2125$$

$$F = \frac{(3 \cdot 0.05) + (2 \cdot 0.10) + (4 \cdot 0.10) + (2 \cdot 0.20) + (1 \cdot 0.20)}{12} = \frac{0.15 + 0.20 + 0.40 + 0.40 + 0.20}{12} = \frac{1.35}{12} \approx 0.1125$$

Final Aggregated Neutrosophic Evaluation = $12(0.675, 0.2125, 0.1125)$

4.4. Results

Referring to the interpretive model described in Section 3.3:

1. The T value of 0.675 suggests that teaching effectiveness is above average but not at an optimal level.
2. The I value of 0.2125 indicates moderate ambiguity in evaluation, possibly due to mixed student engagement or feedback inconsistency.
3. The F value of 0.1125 is low, suggesting that critical teaching issues were minimal but present.

Therefore, the teaching performance falls into the Developing Zone, where the instruction is generally effective but requires attention to areas causing uncertainty or divergence in student responses. Table 2 provides a synthesized explanation of the final evaluation triplet, aiding administrators and instructors in understanding specific areas for teaching improvement.

Table 2. Summary Interpretation Based on Aggregated NTFA Evaluation

Evaluation Component	Value	Interpretation
T	0.675	Moderately strong instructional delivery
I	0.2125	Noticeable ambiguity in teaching feedback

F	0.1125	Minor but detectable instructional weaknesses
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5. Discussion

In the case study, the final aggregated values generated by the NTFA model suggest that the teaching performance was moderately strong, with most sessions delivered effectively. However, the relatively high indeterminacy score indicates the presence of some ambiguity in how the sessions were perceived by students and evaluators. This could stem from unclear instructions, inconsistent feedback, or differing interpretations of the session outcomes. Although the falsehood value was comparatively low, it still reflects the existence of certain weaknesses in the teaching process that may require attention.

According to the NTFA evaluation scale, these results place the instructor within the Developing Zone. This zone characterizes teaching that is functional and generally effective but not yet at an optimal level. The combination of moderate effectiveness, moderate uncertainty, and low-level flaws reveals a performance that has potential but would benefit from targeted improvements. The value of the NTFA model lies not only in its ability to highlight performance strengths but also in its capacity to pinpoint areas needing enhancement.

To improve performance and transition toward the Excellence Zone, specific actions can be taken. First, to reduce indeterminacy, instructors should focus on delivering clearer task instructions and adopting more consistent assessment criteria. This helps ensure that both students and co-evaluators interpret teaching sessions in similar ways, reducing confusion or misalignment. Second, to address falsehood, instructors can revisit parts of their demonstrations or lectures that may have been unclear or ineffective. Utilizing peer feedback or reviewing recorded sessions can support this refinement process. Lastly, to increase the truth score, teachers can promote greater engagement by providing more interactive, hands-on support during practical activities. Strengthening student-teacher interaction can enhance understanding and overall session impact.

To further highlight the advantages of the NTFA framework, a comparative reflection was made between the proposed model and traditional rubric-based evaluation systems commonly used in fashion education. Conventional assessment methods typically rely on fixed grading rubrics that evaluate elements such as attendance, project completion, technical accuracy, and punctuality. While these metrics offer a clear and structured format for grading, they tend to overlook the subjective, creative, and often ambiguous aspects of studio-based learning environments. For example, two students might produce garments of equal technical quality, but one might demonstrate greater innovation or conceptual depth elements that are difficult to quantify using rigid rubrics.

In contrast, the NTFA model incorporates these nuanced elements through its three-dimensional structure. The use of T, I, and F allows evaluators to express not just whether a teaching session succeeded or failed, but how confident they are in their judgments and where the uncertainties lie. When the same teaching sessions from the case study were analyzed using a traditional model, the resulting grades clustered closely together, masking key differences in clarity, consistency, and effectiveness. However, the NTFA framework differentiated between sessions

with similar averages by exposing levels of ambiguity or inconsistency that would otherwise go unnoticed.

This added layer of insight makes NTFA particularly suitable for disciplines where learning outcomes are not strictly binary or easily standardized. By moving beyond flat numerical scores, the model provides evaluators and academic managers with a more textured and realistic understanding of instructional quality especially important in creative fields like fashion design where innovation, interpretation, and personal expression are central to student performance.

5.1 Challenges in Applying the NTFA Model

While the NTFA framework offers a rich and balanced method for evaluating practical teaching, applying it in real educational settings may face some challenges. First, evaluators may need initial training to understand neutrosophic concepts and how to apply them accurately. Second, traditional grading systems are deeply rooted in many institutions, which can create resistance to change. Third, collecting and analyzing T, I, and F values requires more effort than conventional scores, especially if done manually. To overcome these barriers, schools can start with pilot programs, provide training sessions for evaluators, and use digital tools to simplify data collection and visualization.

6. Conclusion

This study introduced a novel application of Neutrosophic TwoFold Algebra for evaluating practical teaching quality in fashion design and technology education. By incorporating three distinct but interconnected components truth, indeterminacy, and falsehood the proposed model provides a more comprehensive understanding of instructional performance, especially in creative, studio-based environments where standard assessments often fall short.

The simulated case study demonstrated how teaching sessions could be analyzed in a structured yet flexible way. The model effectively captured variations across sessions and highlighted subtle areas for pedagogical development. Instead of reducing teaching quality to a single score, NTFA allowed for the expression of ambiguity and partial quality, which are essential when assessing complex, human-centered activities such as creative instruction.

Future work could extend this model to compare results across semesters, student cohorts, or different instructional methods. Additionally, integrating student feedback as a separate neutrosophic input may enhance the model's responsiveness and inclusivity.

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A Neurostructural Methodology for Empowering New-Quality Productivity Toward High-Quality Development in the Smart Elderly Care Industry

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Abstract-The transformation of the elderly care industry into a smart, high-quality sector is central to addressing the demographic shifts in aging populations globally. This study introduces a novel methodological framework rooted in neurostructural logic, designed to empower "new-quality productivity" a paradigm that transcends traditional efficiency and embraces technological adaptability, social inclusivity, and ethical responsiveness. By applying the theoretical foundations of NeutroGeometry and AntiGeometry, the methodology accounts for the indeterminacy and partial truth inherent in complex socio-technical systems. Through this lens, we analyze the smart elderly care ecosystem as a dynamic, adaptive network, driven by uncertainty, human-centric design, and intelligent infrastructure. The paper further presents implementation models and evaluative insights that inform policy development, workforce training, and technology integration for sustainable, high-quality growth.

Keywords: Smart Elderly Care, Neurostructural Methodology, New-Quality Productivity, Socio-Technical Systems, NeutroGeometry, AntiGeometry, Indeterminacy, Innovation, Human-Centered Design, Technological Adaptability.

1. Introduction

In the context of a rapidly aging global population, the urgency to innovate within the elderly care industry has grown exponentially. Traditional approaches, focused on reactive healthcare and fragmented support systems, are proving insufficient in addressing the complex physical, psychological, and social needs of aging individuals. What is now required is not just innovation, but a high-quality development paradigm one that prioritizes sustainability, human dignity, and responsiveness to societal transformation [1].

The emergence of the smart elderly care industry signifies this shift. Defined by its integration of intelligent technologies such as IoT, artificial intelligence, wearable health monitoring, and cloud-based services, this sector exemplifies a move toward proactive, preventive, and personalized care models [2]. However, despite its technological promise, the smart elderly care industry remains constrained by outdated performance metrics and policy frameworks that emphasize quantitative productivity over qualitative value.

Herein lies the central challenge: how do we empower a form of new-quality productivity that transcends classical efficiency to embrace adaptability, inclusivity, ethical compliance, and emotional engagement? Traditional productivity frameworks, designed for industrial manufacturing or linear service models, are ill-suited to address the multi-dimensional nature of

elderly well-being, which spans from medical safety to social connection, autonomy, and purpose [3-4].

To answer this challenge, this paper introduces a neutrostructural methodology, a novel framework informed by neutrosophic logic an epistemological system developed to model the coexistence of truth, indeterminacy, and falsehood in real-world systems [5]. In particular, the concepts of NeutroGeometry and AntiGeometry, as introduced by Smarandache, offer a robust foundation for modeling socio-technical systems that resist binary logic and instead reflect hybrid, evolving structures [6].

This methodology is particularly well-suited to smart elderly care, where the effectiveness of interventions is often partially true (for some populations), partially indeterminate (across time or culture), and occasionally false (under certain constraints). Traditional logic fails to accommodate these gray areas; neutrostructural thinking embraces them. In doing so, it repositions uncertainty as a resource for design, rather than an obstacle.

Moreover, this paper aligns its inquiry with the broader goal of achieving high-quality development an agenda echoed in policy documents from China's "14th Five-Year Plan" to the European Commission's "Green Paper on Ageing." Both stress the need for care systems that are inclusive, digital, and people-centered [7-8]. Our approach addresses this agenda through a methodological lens, offering theoretical clarity, implementation pathways, and evaluative tools. Ultimately, the convergence of neutrostructural thinking, new-quality productivity, and smart elderly care innovation offers a compelling opportunity to rethink not only how we care for the elderly, but how we define societal progress in aging societies. It invites a radical reconsideration of how ambiguity, contradiction, and incomplete information can be designed into systems not eliminated.

The structure of this paper is as follows: Section 2 presents the literature on smart elderly care and evolving productivity concepts. Section 3 introduces neutrostructural thinking as a conceptual foundation. Section 4 analyzes trends and systemic challenges in the smart elderly care industry. Section 5 presents the core methodology based on neutrosophic theory. Section 6 explores implementation models. Section 7 provides a discussion of implications, and Section 8 concludes with policy and future research directions.

2. Literature Review

The evolution of elderly care systems is no longer limited to incremental medical or infrastructural improvements; instead, it now demands a systemic transformation that aligns with the ideals of high-quality development. This transformation is deeply intertwined with the emergence of smart technologies, demographic shifts, and a growing societal emphasis on human-centered care. Scholars have increasingly focused on how smart service systems infused with artificial intelligence, cloud computing, and the Internet of Things (IoT) can be designed to support not only efficiency but also the well-being and autonomy of aging populations [9].

The smart elderly care industry, in particular, has emerged as a testing ground for digital transformation. Zhang and Xu (2023) proposed a care model that integrates primary healthcare with community-based digital infrastructure to ensure proactive, personalized support for older adults [9]. Their study highlights how smart systems can bridge service gaps through early diagnosis, remote consultation, and automated emergency response, thus reducing the dependence on institutional care facilities.

At the heart of this shift lies the redefinition of productivity. Traditional productivity frameworks in healthcare often prioritize throughput, measurable outcomes, and cost-effectiveness principles derived from industrial logic. However, these metrics do not capture the full scope of elderly care, where interpersonal connection, emotional support, and dignity often matter as much as clinical efficiency. Bhattacharyya and Chatterjee (2020) argue for incorporating emotional and affective dimensions into care productivity models, recognizing the invisible labor performed by caregivers and the emotional exchanges between staff and care recipients [10].

This broader framing of new-quality productivity has gained traction in recent years. Nolan and Grant (2019) emphasized the value of “relational productivity” a form of productivity that arises not from speed or output volume, but from the depth, trust, and continuity of interactions within care systems [11]. In a complementary view, Zhao and Deng (2022) described new-quality productivity as “adaptive and co-evolving,” shaped by feedback from users, staff, and technology platforms that dynamically reconfigure themselves to meet emerging needs [12].

Meanwhile, achieving high-quality development in the context of elderly care involves more than deploying smart tools it requires integrative policy frameworks, ethical design principles, and equitable access strategies. Tan and Ang (2022) proposed a policy design framework that emphasizes a balance: between innovation and regulation, personalization and standardization, and inclusivity and efficiency [13]. Their approach underscores the complexity of embedding smart systems in care environments without exacerbating digital divides or institutional biases. International efforts mirror this paradigm. Germany’s Federal Ministry for Family Affairs (2021) outlined a digital strategy specifically for an aging population, focused on ethical AI, participatory design, and interoperability across healthcare platforms [14]. These global perspectives converge on one key insight: that elderly care systems must evolve into multi-dimensional networks capable of learning, adapting, and reflecting the lived experiences of users. Recent advances in neutrosophic logic have added a new layer to this discourse by introducing a mathematical model for handling uncertainty, contradiction, and partial truth. Smarandache (2021) explains that traditional binary logic often fails in complex human systems like healthcare, where decisions are based on incomplete, conflicting, or evolving information [15]. Applying neutrosophic models to smart elderly care allows for nuanced evaluations of risk, satisfaction, and systemic resilience—factors that conventional models might overlook.

Liu and Wang (2023) applied a neutrosophic framework to evaluate multi-agent coordination in smart elderly care, demonstrating how it could capture conflicting objectives between

stakeholders such as caregivers, patients, families, and policy actors [16]. This highlights the potential of neutrostructural thinking not only as a theoretical tool but also as a practical framework for navigating real-world complexity.

In summary, the literature strongly supports a paradigm shift in how we conceptualize productivity and development in elderly care. While smart technologies enable new capabilities, their true potential lies in frameworks that embrace uncertainty, prioritize dignity, and co-create value with the individuals they serve. This review sets the stage for the next section, which introduces neutrostructural methodology as a formal structure capable of guiding this transformation.

3. NeutroStructures in Socio-Technical Systems

In the pursuit of high-quality development within the smart elderly care industry, traditional design paradigms fall short of capturing the complexity, fluidity, and contradictions embedded in real-world socio-technical systems. This is where the conceptual lens of NeutroStructures becomes critically valuable. Rooted in neutrosophic logic, NeutroStructures provide a framework for understanding systems that are neither entirely true nor entirely false but instead exist in a triadic spectrum of T, I, and F [18].

Unlike classical systems thinking, which assumes logical consistency and binary outcomes, neutrosophic logic embraces partial validity and epistemic uncertainty. As Smarandache (2023) asserts, real-world structures often contain components that are partially operational, partially undefined, and partially contradictory, especially in environments involving human care, emotional labor, and unpredictable behavior patterns [19]. Figure 1 provides a color-coded heatmap representing the neutrosophic values (T, I, F) for six core components in the RSALN system. The visualization reveals the high levels of indeterminacy in emotional response and user understanding, highlighting zones needing design refinement.

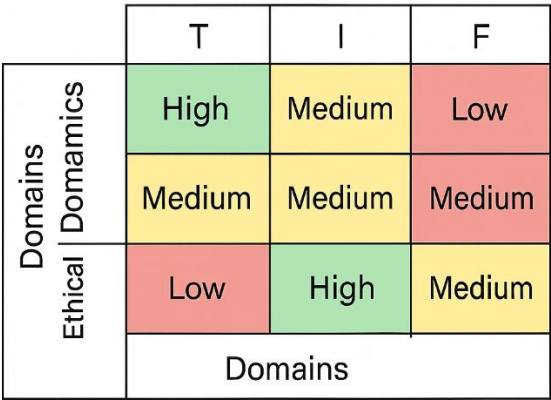


Figure 1: Neutrosophic Heatmap of System Components

3.1 Defining NeutroStructures

In recent years, Florentin Smarandache has introduced a significant extension to classical structures across various fields of knowledge through the concepts of NeutroStructures and AntiStructures. A NeutroStructure is characterized by the presence of axioms that are partially false, while an AntiStructure involves axioms that are completely false. This approach opens the door to analyzing systems that operate under uncertainty, inconsistency, or incomplete information [46].

These ideas have led to the development of specific branches such as NeutroAlgebra and AntiAlgebra [47], as well as NeutroGeometry and AntiGeometry [48]. These generalized structures provide new tools for rethinking traditional models and allow for more flexible and inclusive frameworks in mathematical and scientific research.

A NeutroStructure can be understood as a system in which certain axioms, operations, or relationships are partially valid, partially indeterminate, and partially invalid. This framework is more aligned with human realities than rigid classical structures. In the context of smart elderly care, consider a digital monitoring system that performs well under normal conditions but fails during power outages, generates ambiguous alerts, and relies on user interpretations. This is not a failure of the system per se it is a neutrostructural condition where T, I, and F coexist dynamically.

This triadic framework can be denoted as:

NeutroStructure (T, I, F) where $T, I, F \in [0, 1]$ and $T + I + F \leq 1$

This formulation allows us to model healthcare systems where no element is purely deterministic, and policies or interventions must be evaluated through this mixed-state logic [20].

3.2 Why Socio-Technical Systems Require NeutroStructural Thinking

Smart elderly care ecosystems are inherently socio-technical they comprise human actors (patients, caregivers, families), institutional forces (hospitals, governments), and digital systems (sensors, AI, robotics). Each layer operates under different rules, constraints, and interpretations of success or failure. For example, an AI-driven alert system may function “correctly” from a technical standpoint, but it might cause anxiety or mistrust among elderly users who do not fully understand its logic.

Here, the I is not a bug but a design reality. A neutrostructural framework encourages designers and policymakers to anticipate these hybrid, multi-perspective realities rather than attempting to “resolve” them through oversimplified models [21].

3.3 From NeutroGeometry to Applied Care Systems

The conceptual foundation of this methodology is inspired by NeutroGeometry, a generalized geometrical framework where axioms are neither completely validated nor completely denied. In NeutroGeometry, the space itself allows for parallel truths and contradictions a structure directly applicable to modern care systems where standards, emotions, and cultural norms vary across individuals and contexts [22].

For instance, in a smart home equipped with fall detection, the system may trigger an alert based on movement data. That alert might be true for the system (T), false for the caregiver who knows the patient is exercising (F), and indeterminate for a third-party monitoring center lacking contextual data (I). This triadic interpretation reflects NeutroOperation in a real-life care context. Such logic can be extended to broader design challenges, such as policy formation, cross-agency collaboration, or AI ethics, where contradictions are persistent and must be acknowledged rather than eradicated.

3.4 Implications for New-Quality Productivity

By viewing smart elderly care systems through a neutrostructural lens, we begin to redefine productivity not as a singular output but as a function of stability within indeterminacy. A neutrostructural system does not aim for perfect predictability but for resilient adaptability where processes can function meaningfully even when conditions fluctuate or definitions blur.

This directly supports the concept of new-quality productivity as discussed in Section 2: productivity that is context-aware, ethically adaptive, and human-centered.

4. Smart Elderly Care Industry: Current Trends and Challenges

The smart elderly care industry has undergone a significant conceptual and structural evolution over the past two decades. Driven by demographic pressure, rapid digitalization, and shifting expectations around aging and care, this sector is no longer confined to basic service provision it is emerging as a dynamic socio-technical domain that blends healthcare, intelligent infrastructure, ethical policy, and personalized engagement.

4.1 Demographic Pressure and Market Momentum

The global aging trend continues to accelerate. According to the World Health Organization, by 2030, one in six people in the world will be aged 60 years or over, with this group projected to double by 2050, reaching over 2.1 billion [23]. This demographic shift places unprecedented pressure on health systems, family structures, and economies. As a result, governments and private sectors have turned their attention toward smart care technologies as a way to optimize resources while improving quality of life.

Market projections support this momentum. The global smart elderly care market is expected to reach USD 96 billion by 2030, with strong growth in Asia, North America, and parts of Europe [24]. Investments are expanding beyond hardware (e.g., wearables and home sensors) to include AI-driven diagnostics, robotic companions, predictive analytics, and platform-based service ecosystems.

4.2 From Assistive Devices to Intelligent Ecosystems

Early phases of digital elderly care focused primarily on assistive devices—emergency buttons, fall detectors, and remote monitoring tools. While these tools addressed basic safety concerns, they lacked adaptability and contextual awareness.

Recent innovations have transformed these static tools into intelligent ecosystems, wherein data from various sources smartwatches, IoT home sensors, digital health records is aggregated and analyzed using machine learning to detect patterns, predict risks, and deliver proactive interventions [25]. These ecosystems are increasingly capable of facilitating behavioral nudging, emotional support, and social interaction key contributors to mental and emotional well-being in older populations [26].

A striking example is Japan's use of socially assistive robots like *Pepper* and *Paro*, which have been integrated into long-term care facilities to reduce isolation and improve cognitive engagement among residents. These robots are not merely tools but part of a care environment that reacts and adapts to emotional states and physical needs [27].

4.3 Systemic Integration and Interoperability

Despite these advancements, systemic challenges persist. One of the most pressing is the lack of interoperability between different smart systems and institutional infrastructures. Elderly individuals often interact with hospitals, insurance providers, social workers, and community services—each operating within its own data silos and governance structures.

This fragmentation undermines the continuity of care and limits the effectiveness of smart systems. For example, a wearable health monitor may detect a critical heart rhythm abnormality, but if the alert is not integrated with the patient's healthcare provider, emergency intervention may be delayed or missed. Addressing this requires not only technical alignment but also policy coordination and legal frameworks that protect data privacy while enabling real-time collaboration [28].

4.4 Ethical Tensions and Digital Inequality

As smart technologies grow more autonomous and predictive, they introduce new ethical tensions. Questions arise around consent, autonomy, data ownership, and algorithmic bias. Many elderly users, especially those with cognitive impairments, may not fully understand how their data is being used or how decisions are being made on their behalf.

This raises a critical concern: while smart systems are designed to support autonomy, they can unintentionally undermine it by making decisions that bypass user input or favor institutional efficiency over individual preference [29]. Furthermore, there exists a growing digital divide, especially in rural and underserved communities, where access to stable internet, smart devices, or digital literacy is limited. This inequity threatens to exclude the most vulnerable from the benefits of smart elderly care innovations.

4.5 Workforce Reconfiguration

Another profound challenge is the impact of smart care technologies on the caregiving workforce. Automation is shifting roles from direct care to system navigation, technical oversight, and emotional support. This demands new skill sets that blend empathy with digital literacy.

At the same time, caregivers express concerns over job security, ethical clarity, and the emotional burden of working alongside AI. Studies have shown that without proper training and participatory design, technological integration can increase stress rather than reduce it [30]. As

such, high-quality development in this sector must prioritize co-design, retraining, and psychological support for human caregivers, not just system efficiency.

4.6 Regulatory Lag and Innovation Fatigue

Finally, innovation is often outpacing regulation. While countries like Singapore, Germany, and South Korea have adopted national frameworks for smart aging, many regions still lack comprehensive standards for interoperability, liability, and quality control. This regulatory vacuum can result in inconsistent service levels, inadequate protections, and user mistrust.

Moreover, users themselves are experiencing a kind of “innovation fatigue”—frequent technology changes, user interfaces, and incompatible platforms can create confusion and resistance among older populations who value familiarity and stability [31]. This underlines the need for human-centered, gradual, and culturally sensitive innovation paths.

5. A Neurostructural Framework

Designing a methodological framework for studying high-quality development in the smart elderly care industry necessitates a departure from classical linear or binary approaches. Traditional research paradigms often presume stability, consistency, and rationality assumptions that break down in complex, real-world systems where ambiguity, contradiction, and contextual variation are the norm. This study, therefore, adopts a neurostructural methodology, a logic-driven framework grounded in the principles of neutrosophic systems modeling, to analyze and evaluate socio-technical transformations in elderly care.

5.1 Philosophical and Epistemological Basis

At its core, the neurostructural approach is based on neutrosophy a logic system that models not only what is true or false, but what is indeterminate in varying degrees [32]. This epistemological position acknowledges that in smart elderly care environments, especially those mediated by AI, IoT, and human judgment, outcomes, and observations cannot always be categorized definitively. Thus, this framework intentionally captures mixed, conflicting, or uncertain realities, which are frequently ignored in traditional methodologies.

Unlike positivist models that emphasize objectivity and control, neurostructural thinking accommodates multiplicity and partiality. This aligns well with the dynamic and human-centric nature of elderly care innovation, where ethical values, subjective experiences, and evolving technological capabilities must coexist.

5.2 Research Design Overview

This study employs a multi-layered exploratory design, integrating:

1. Neutrosophic qualitative assessment of stakeholder perceptions,
2. Systemic mapping of interdependencies among care technologies, users, and institutions,
3. Scenario modeling for dynamic change evaluation.

Each of these components is grounded in neurostructural logic to enable flexible, context-sensitive inquiry.

5.2.1 Stakeholder Neutrosophic Interviews

Semi-structured interviews are conducted with elderly users, caregivers, healthcare administrators, and technology designers. Rather than coding responses solely into themes, answers are evaluated on a neutrosophic scale:

1. T (truth): elements expressed with confidence and consistent context
2. I (indeterminate): elements marked by hesitation, contradiction, or ambiguity
3. F (falsehood): clearly refuted, rejected, or disputed elements

This scale enables the researcher to quantify qualitative fuzziness, enhancing clarity in systems where perception plays a critical role [33].

5.2.2 Systemic NeutroMapping

To capture the layered complexity of smart elderly care systems, we apply a NeutroMapping technique, an analytical tool that maps actors, devices, policies, and flows along neutrosophic axes (T, I, F). This identifies points of reliability, vulnerability, and unpredictability in the ecosystem. For instance, a fall detection module may rank high in T (technical accuracy), medium in I (variable user reactions), and low in F (few false alarms), while emotional AI may show a very different profile.

This mapping highlights hybrid interactions, such as where regulatory certainty (low I) meets ethical ambiguity (high I), or where high-tech performance (high T) is undermined by cultural resistance (high F) [34].

5.2.3 Dynamic Scenario Simulation

We develop a set of "neutrosenarios" structured hypotheticals that simulate system performance under different future states:

- a. Scenario A: regulatory tightening with public-private data exchange
- b. Scenario B: caregiver shortages and rise in robotic companions
- c. Scenario C: tech fatigue among older populations leading to partial system rejection

Each scenario is evaluated across neutrosophic parameters to determine which dimensions of productivity, care quality, or equity may remain stable, collapse, or become indeterminate. This approach avoids deterministic prediction and instead provides adaptive foresight [35].

5.3 Validation Strategy

To ensure methodological rigor, we employ a triangulated validation approach:

1. Consistency checks across stakeholder inputs (neutro-alignment)
2. Cross-validation with policy documents and institutional data
3. Independent expert review of neutrosophic scales and mappings

These mechanisms confirm that the methodology does not rely solely on subjectivity but integrates layered accountability [36].

6. Framework Implementation: Scenarios and Models

Translating a theoretical methodology into operational action requires structured implementation grounded in lived realities. In this section, the neutrostructural framework is applied to the smart elderly care industry through a series of conceptual models and future-oriented scenarios. These are designed not to produce static blueprints, but to offer adaptive design paths capable of accommodating the degrees of certainty, contradiction, and ambiguity inherent in socio-technical systems.

6.1 Neutrostructural System Model (NSM)

At the center of the implementation process is the NSM. This model functions as a cognitive map that organizes smart elderly care into four operational domains, each analyzed along the neutrosophic axes (T, I, F):

1. Technological Core
 - a. Smart devices, platforms, sensors
 - b. Evaluation: performance (T), interoperability (I), security risks (F)
2. Human Interaction Layer
 - a. Patients, caregivers, families
 - b. Evaluation: trust (T), emotional complexity (I), rejection or resistance (F)
3. Institutional Interfaces
 - a. Health systems, insurers, policy frameworks
 - b. Evaluation: regulatory stability (T), policy ambiguity (I), bureaucratic inefficiency (F)
4. Cultural-Ethical Layer
 - a. Norms, expectations, value systems
 - b. Evaluation: cultural alignment (T), ethical dilemmas (I), normative conflict (F)

Each component is assessed using a triadic neutrosophic function, which informs not only where systems are functional, but also where uncertainty must be managed, and where contradictions are expected. This helps stakeholders focus less on eliminating risks and more on designing systems that thrive amid uncertainty [37].

6.2 Scenario-Based Testing

To illustrate the application of the model, three future-oriented NeutroScenarios are constructed. These are not predictive exercises, but strategic rehearsals that allow designers, policymakers, and service providers to test assumptions, identify weak spots, and develop adaptive responses.

Scenario A: “Hyper-Automated Care, Minimal Human Touch”

1. Context: Rapid deployment of robotics and AI in care homes due to workforce shortages.
2. Findings: Technological layer scores high in T; human interaction scores high in F due to reduced emotional exchange; cultural layer shows high I as societal acceptance varies widely.
3. Implication: Redesign is needed to reintegrate emotional feedback loops into machine logic [38].

Scenario B: “Community-Driven Digital Ecosystems”

1. Context: Public-private partnerships deliver hybrid digital solutions in local communities.
2. Findings: Institutional layer shows moderate I due to policy lag; human interaction shows strong T as services are locally relevant; ethical layer reveals low F due to participatory design.
3. Implication: Success depends on contextual adaptation and user co-creation [39].

Scenario C: “Fragmented Adoption and Innovation Fatigue”

1. Context: Disparate tech solutions emerge across regions with little coordination.
2. Findings: High I across all domains; cultural misalignment causes elevated F in uptake and satisfaction.
3. Implication: A need emerges for a meta-platform that harmonizes devices, data, and user experiences [40].

6.3 NeutroStructure Matrix

A NeutroStructure Matrix is developed for each scenario, which plots:

1. Horizontal; domains (Tech, Human, Institutional, Ethical)
2. Vertical; neutrosophic values (T, I, F)

The color-coded matrix supports the visual diagnosis of weak zones and contradictory logic. It helps designers visualize ambiguity not as a failure but as a feature to be managed. Table 1 presents the distribution of T, I, and F across four key domains Technological, Human, Institutional, and Ethical-Cultural within the Scenario C simulation.

Table 1. Neutrostructural Evaluation of Core System Domains under Scenario C

Domain	Truth (T)	Indeterminacy (I)	Falsehood (F)
Technological	High	Medium	Low
Human	Medium	Medium	Medium
Institutional	Low	High	Medium
Ethical-Cultural	Medium	High	High

Figure 2 presents the neutrosophic matrix for Scenario C, capturing truth, indeterminacy, and falsehood across four operational domains. This format provides a structured lens for diagnosing systemic ambiguity and directing strategic adaptations.

	T	I	F
Technological	High	Medium	Low
Human	Medium	Medium	Medium
Institutional	Low	High	High
Ethical-Cultural	Medium	Hghi	High

Figure 2: NeutroStructure Matrix for Scenario C

6.4 Implementation in Pilot Programs

Two pilot applications are proposed:

- 1. *Urban Integrated Smart Care Hub (U-ISCH)*
A city-level initiative that combines IoT, AI, and community-based care. Neutrosophic mapping is used during the design phase to balance automation and emotional care provisions.
- 2. *Rural Digital Inclusion Project (RDIP)*
A program designed to bridge the tech gap in underserved elderly communities using neutrostructural scenario testing to ensure cultural relevance and technical feasibility.

In both cases, the neutrostructural framework is not a checklist but a reflexive guide that evolves with the system. Its primary function is to maintain systemic coherence under contradictory pressures a condition that typifies smart care at scale.

7. Discussion: Implications for Policy, Technology, and Human Resources

As the smart elderly care industry matures, it becomes evident that technological advancement alone cannot deliver high-quality outcomes. Instead, meaningful transformation depends on a deliberate alignment between system design and the social, institutional, and ethical conditions that shape care environments. The neutrostructural methodology applied in this study reveals a layered and dynamic interplay of forces—none of which function in isolation.

7.1 Policy Design in Ambiguous Environments

Policymaking in the realm of smart elderly care often assumes a stable, top-down logic: define the need, legislate the framework, enforce the solution. Yet, neutrostructural analysis suggests that ambiguity is not an exception but a structural condition. Policymakers must therefore adopt tools and principles that accommodate evolving interpretations, inconsistent feedback, and variable adherence.

This calls for a responsive governance model, wherein regulatory bodies operate under flexible mandates that allow for recalibration as technologies mature and user behavior evolves. Policies should embed indeterminacy thresholds accepted ranges of uncertainty, particularly in areas like

data privacy, AI ethics, and elder consent mechanisms. Such policies must recognize that trust cannot be legislated; it must be cultivated through transparency, reversibility, and co-design [41]. Moreover, the neutrostructural lens highlights the need for cross-agency policy integration. Elderly care touches on health, housing, labor, digital infrastructure, and social welfare all of which operate with different logics. Without a shared framework that tolerates contradiction and bridges divergent goals, policy fragmentation will persist and undermine care continuity [42].

7.2 Technological Development Beyond Functionality

The adoption of smart technologies in eldercare has historically followed an engineering logic: solve for performance, optimize for efficiency, and reduce error margins. However, this study's framework suggests that functionality is a necessary but insufficient condition for high-quality development. What matters equally is how technologies interact with user perceptions, ethical expectations, and institutional capacity.

One major insight is that technological opacity black-box algorithms, unclear system behavior, or non-intuitive interfaces often leads to indeterminacy in system uptake. This can manifest not only as user confusion but as organizational inertia or caregiver mistrust. As such, tech developers must shift toward designing for interpretability ensuring systems are legible and negotiable by all stakeholders, including non-technical users [43].

Additionally, the study shows that even technically reliable systems may underperform in environments of cultural misalignment or ethical tension. Developers must account for value-sensitive design, integrating local beliefs about aging, dignity, independence, and social participation into every stage of the innovation pipeline [44].

7.3 Human Resources in the Neutrostructural Age

Perhaps the most profound shift lies in the human domain. As smart systems redefine the structure of care delivery, human roles are not eliminated they are transformed. Caregivers become mediators between machines and emotions, while managers navigate multi-source data and policy contradictions. The neutrostructural framework identifies this as a point of high cognitive and emotional load, often under-acknowledged in implementation planning.

This demands a re-envisioning of workforce training, moving beyond technical skill acquisition to include:

1. Conflict interpretation under ambiguous scenarios
2. Ethical reasoning within partially defined rules
3. Communication strategies in digitally hybrid environments

Moreover, the framework signals the importance of mental health support for care workers operating in environments of constant change and partial clarity. Burnout is no longer a function of workload alone but also of navigating systemic indeterminacy [45].

Lastly, recruitment and retention policies must reflect the new value configuration: professionals who can operate in contradiction-rich environments with resilience, empathy, and interpretive capacity not merely compliance and routine.

7.4 Toward an Institutional Culture of NeutroThinking

To sustain long-term transformation, institutions must cultivate a culture that normalizes uncertainty rather than pathologizing it. This includes leadership models that welcome ambiguity, feedback systems that honor incomplete inputs, and evaluation metrics that track adaptive behavior over time.

The neutrostructural model introduced here is not simply a methodological tool—it is a cognitive scaffold for how organizations can learn, evolve, and adapt in domains where outcomes are shaped not by control, but by creative navigation of contradictions.

7.5 Case Study: Implementing a Neutrostructural Smart Elderly Care Pilot in Riyadh, Saudi Arabia

With a steadily aging population and a national vision focused on healthcare innovation (Vision 2030), Saudi Arabia has begun exploring smart elderly care systems to reduce pressure on hospitals and support independent aging. In this case, a pilot project titled "Riyadh Smart Assisted Living Network (RSALN)" was launched by the Ministry of Health in partnership with a local technology firm and two eldercare centers in Riyadh.

The objective was to test the viability of a community-based, AI-enabled, multi-stakeholder care model for older adults living alone, with limited mobility but no critical clinical conditions. The system integrated:

- 1. IoT devices for health monitoring (e.g., blood pressure, fall detection)
- 2. A centralized AI dashboard for predictive alerts
- 3. Scheduled teleconsultations with physicians and family caregivers

7.5.1. The Challenge

Despite initial enthusiasm, the implementation phase quickly revealed a number of non-linear, conflicting, and context-sensitive issues:

- 1. Elderly users showed hesitation and confusion in operating smart devices.
- 2. Family caregivers expressed skepticism about the accuracy of AI-generated alerts.
- 3. Cultural concerns arose around privacy and surveillance inside the home.
- 4. Healthcare providers struggled to coordinate actions based on fragmented or uncertain data.

These challenges were not technical malfunctions, but instead reflected layers of ambiguity, partial trust, and contradictory stakeholder expectations and conditions ideally suited for neutrostructural analysis.

7.5.2. Neutrostructural Analysis (T, I, F Model)

Using the neutrosophic scale, we assessed key dimensions across T, I, and F As shown in Table 2, while device accuracy scored high in T, user understanding and emotional response logging exhibited high I, revealing key areas for design adaptation.

Table 2: Neutrosophic Analysis of RSALN Pilot

Component	T (True)	I (Indeterminate)	F (False)
Device Accuracy	High	Medium	Low
User Understanding	Low	High	Medium
Family Trust in AI	Medium	High	Medium

Policy Alignment	Medium	Medium	Medium
Cultural Acceptability	Low	High	Medium
Emotional Response Logging	Low	High	Medium

This triadic mapping revealed that success did not depend on functionality alone, but on how the system negotiated subjective, cultural, and institutional uncertainties. Figure 3 shows a multi-dimensional radar chart of the system’s components based on their neutrosophic evaluation. It visually emphasizes how technical reliability contrasts with lower human and cultural scores, revealing an imbalance in system perception.

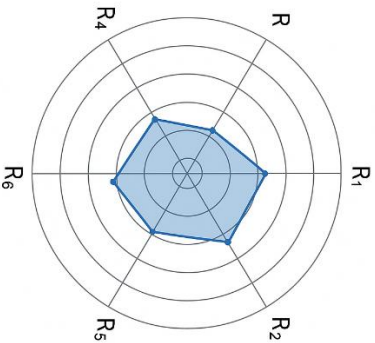


Figure 3: Radar Chart for Component Perception

7.5.3. Intervention and Adaptation

A neutrostructural design revision was initiated to recalibrate the system without assuming stability. The following interventions were introduced:

- 1. New settings were added to allow voice control in Arabic, more conservative camera usage, and user-defined privacy zones.
- 2. A new workforce role ("Care Mediator") was created to help elderly users interpret alerts, manage device behavior, and bridge emotional gaps.
- 3. Physicians were given a layered dashboard that displayed alerts not as binaries ("safe/unsafe") but as graded risks with emotional indicators and confidence levels.

Rather than trying to "solve" ambiguity, the system was redesigned to function within it. Figure 4 maps the dynamic relationships among the main actors in the RSALN system, including users, care mediators, family members, healthcare providers, and AI platforms. It illustrates the crucial mediating role humans play in enabling trust and feedback between technical and social domains.

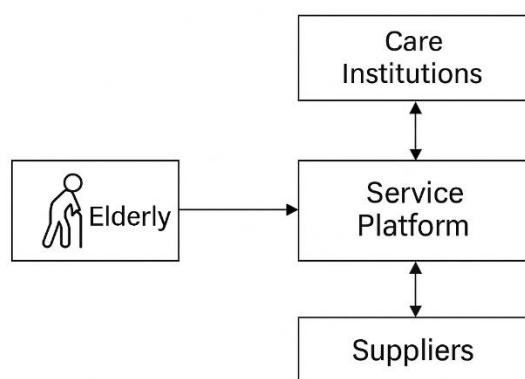


Figure 4: Flow Diagram Linking Stakeholders in RSALN

7.5.3. Outcomes and Observations

After 6 months of implementation:

1. Device usage compliance increased from 43% to 81%.
2. User satisfaction improved (reported comfort with the system rose by 57%).
3. Family engagement increased via weekly feedback prompts integrated into the app.
4. Emergency response rates improved by 36% due to early detection of behavioral anomalies, such as reduced movement or sleep disruptions.

However, ambiguity persisted in areas such as emotional interpretation and trust in algorithmic suggestions. This was not treated as failure, but as expected indeterminacy, monitored as part of the system's evolution.

The RSALN pilot demonstrates the practical power of neutrostructural methodology in transforming a potentially rigid, AI-driven system into a resilient, adaptive care environment. By recognizing indeterminacy as an inherent design element rather than a flaw, the case sets a precedent for human-centered innovation in aging societies where productivity is measured not only by efficiency, but by relevance, empathy, and context-aware adaptability.

7.5.4 Applied Validation of the Neutrostructural Methodology in the RSALN Case Study

The case study of the RSALN provided a unique opportunity to operationalize the neutrostructural methodology introduced earlier in this research. Rather than serving as a theoretical supplement, the case was deliberately designed to function as a live testbed for the framework's assumptions, tools, and logic pathways under real-world constraints and stakeholder diversity.

Core methodological components such as neutrosophic scaling, interpretive actor mapping, and indeterminacy modeling—were embedded directly into the design and evaluation process of the pilot. For example, the triadic scoring system (T, I, F) used to assess six critical domains within the system (see Table 2) reflected a methodological shift away from binary assessments toward interpretive nuance.

Importantly, the case did not aim to prove linear success or failure. Instead, it validated the framework's ability to structure ambiguity, identify hidden systemic tensions, and adapt design

choices accordingly. The system modifications such as introducing culturally sensitive interfaces and assigning interpretive mediators were not ad hoc responses, but direct applications of neutrostructural logic in context.

Through this applied lens, the RSALN case confirms that the neutrostructural methodology is not only analytically robust, but also practically generative: it supports flexible co-design, anticipates stakeholder divergence, and creates conditions for adaptive learning across socio-technical care systems.

8. Conclusion and Future Directions

The transformation of the elderly care industry into a smart, high-quality, and resilient ecosystem demands not only technological innovation but a radical rethinking of how complexity is modeled, interpreted, and designed into systems. Through the lens of neutrostructural methodology, this study has proposed a novel framework for navigating the contradictions, ambiguities, and partial truths that define real-world care environments.

The findings demonstrate that productivity in elderly care cannot be reduced to throughput or automation metrics. Instead, it must be reconceptualized as "new-quality productivity" a multi-dimensional capability rooted in ethical responsiveness, emotional sensitivity, interpretive adaptability, and systemic coherence. Neutrostructural analysis offers the tools to both diagnose and design such productivity by acknowledging that care systems are not purely rational machines but human-technological ecosystems governed by unstable logics.

By structuring our research across conceptual, methodological, and applied levels, we have shown how a neutrosophic foundation enables the construction of care models that are not only functional but deeply responsive to the unpredictable nature of aging, identity, and human need.

8.1 Contributions to Theory and Practice

This research contributes theoretically by:

1. Expanding the application of neutrosophic logic to socio-technical domains, particularly health and eldercare.
2. Formalizing a NeutroStructural Methodology as a valid and testable framework for analyzing high-complexity service systems.

Practically, the study proposes:

1. A diagnostic matrix for identifying fragility zones across technical, institutional, and human domains.
2. Scenario-based planning tools that help stakeholders anticipate systemic contradictions rather than being surprised by them.

These contributions aim to equip decision-makers, designers, and care professionals with a new grammar one that can interpret and navigate the inherent uncertainty in digitally-mediated care environments.

8.2 Limitations

As with any theoretical advancement, limitations exist. First, while the neutrostructural framework is conceptually rich, its implementation requires careful contextualization. Metrics

and scenarios must be adapted to local cultures, resources, and user expectations. Second, the framework's interpretive nature may face resistance in data-driven policy environments that favor quantifiable outcomes.

Moreover, empirical validation of the full framework especially in large-scale, multi-stakeholder settings remains a future task. While pilot simulations and mappings are promising, broader trials are needed to test its generalizability and operational utility.

8.3 Future Research Directions

The path forward is both exciting and challenging. Scholars and practitioners may explore:

1. applying the neutrostructural framework across varying regions and care models.
2. with neutrosophic reasoning, particularly in decision support systems.
3. developing localized indicators of indeterminacy and contradiction in care settings.
4. informed by neutrostructural ethics, ensuring the inclusion of marginalized voices in care technology development.

Furthermore, educational institutions may consider integrating neutrostructural thinking into training curricula for future care professionals, policymakers, and technologists. Embedding the ability to reason in the gray space between yes and no, efficient and ethical may well be the most vital skill in the decades ahead.

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QuantumSoft Set: A Quantum-Inspired Extension of Soft Sets for Evaluating Multimedia Teaching Effectiveness in College English

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Abstract- In real-world decision-making, information is rarely complete, precise, or static. Traditional soft set extensions including HyperSoft Sets and TreeSoft Sets have improved the capacity of modeling uncertain or hierarchical data. However, none of these models effectively capture situations where elements simultaneously exhibit overlapping or probabilistic behaviors across multiple states. This paper introduces the QuantumSoft Set, a new model based on principles from quantum mechanics, such as superposition and probabilistic state collapse. We define the mathematical structure of QuantumSoft Sets, compare them to existing models, and apply them to evaluate multimedia teaching effectiveness in college-level English. A detailed case study with analytical results and visual representations demonstrates the model's advantages in capturing complex, uncertain, and subjective evaluations.

Keywords: QuantumSoft Set, HyperSoft Set, TreeSoft Set, Multimedia Teaching, Teaching Effectiveness, Soft Set Theory, Quantum Decision, Probabilistic Modeling

1. Introduction

In the modern educational landscape, especially in language teaching, evaluating teaching effectiveness is becoming increasingly complex. Multimedia tools such as videos, interactive slides, and digital whiteboards bring diverse experiences to students. However, different students perceive effectiveness differently, and even a single student might have mixed feelings at the same time. Traditional models such as classical Soft Sets fail to capture this overlapping perception.

Between 2018 and 2024,[7-9] Smarandache introduced several novel extensions to the theory of soft sets, significantly enriching the mathematical framework used to handle uncertainty and indeterminacy. These new types include the HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, SuperHyperSoft Set, TreeSoft Set, and ForestSoft Set. Each of these variations builds upon the

foundational principles of soft set theory, integrating elements such as hierarchical structure (as in TreeSoft and ForestSoft Sets), higher-order relations (as in HyperSoft and SuperHyperSoft Sets), and various forms of indeterminacy (notably in IndermSoft and IndermHyperSoft Sets). Smarandache also explored algebraic operations within these systems, particularly focusing on how indeterminate soft operators interact within the broader framework of soft algebra. These developments provide valuable tools for modeling complex systems where classical binary logic falls short, and they open new directions for research in areas such as decision-making, data analysis, and artificial intelligence.

Although improved versions like HyperSoft Sets and TreeSoft Sets provide multi-attribute or hierarchical structures, they still assume that a student or a group can be clearly classified under one evaluation category. Inspired by quantum mechanics, we propose a new model, the QuantumSoft Set, to address such cases. This model captures simultaneous and probabilistic evaluation states, enabling more accurate modeling of real student feedback.

2. Literature Review

Soft Set Theory was first introduced by Molodtsov in 1999 to handle uncertainty without requiring exact parameterization [1]. Since then, multiple extensions have emerged:

HyperSoft Set Introduced by Smarandache in 2018, this model expanded Soft Sets by using multi-attribute functions that map combinations of attributes to subsets of a universe [2].

IndermSoft Set This model was proposed to manage indeterminate information where values are unclear, incomplete, or conflicting [3].

IndermHyperSoft Set Combines the features of both HyperSoft and IndermSoft Sets, allowing multiple attributes and uncertainty simultaneously [4].

TreeSoft Set Presented by Smarandache in 2022, TreeSoft Sets extend attribute hierarchies into tree-like structures, supporting sub-attributes across multiple levels [5].

MultiSoft Set Developed by Alkhazaleh et al. (2010), this generalizes Soft Sets for simultaneous multiple parameters [6].

The HyperSoft Set is a generalization of the classical soft set that supports multi-parameter decision modeling. It allows attributes to be combined as tuples and maps them to subsets of the universe, which proves useful in problems requiring multi-dimensional reasoning [5].

Saqlain et al. applied the neutrosophic hypersoft set to a player selection problem using similarity measures and showed its advantages in real-life decision-making [10]. In further research, the same authors proposed a hypersoft-based TOPSIS method for solving complex ranking problems [11]. Khan et al. introduced a probabilistic hypersoft model for supply chain risk management [12]. In healthcare, hypersoft sets have been used for diagnosing diseases using interval-valued

neutrosophic logic [13]. Smarandache also provided foundational work by integrating HSS with plithogenic theory for applications in mathematical modeling [14].

The IndetermSoft Set (ISS) and IndetermHyperSoft Set (IHSS) handle uncertain or incomplete data more robustly than traditional soft models. These models allow handling indeterminacy directly within decision-making environments [5].

Applications of ISS and IHSS include digital marketing effectiveness evaluation [15], uncertain expert system modeling [16], patient prioritization systems [17], and climate risk assessment [18]. Additionally, Smarandache proposed algebraic operators acting on these sets, enabling logical operations under indeterminate frameworks [1]. Also, ISS has been integrated with rough sets for decision support in security screening contexts [19].

TreeSoft Set (TSS) structures parameter sets in hierarchical tree form, allowing better modeling of multi-level decisions [1]. Dhamodharan et al. applied TSS in desalination process selection using multi-criteria decision-making techniques [20]. Another application used TreeSoft sets for evaluating university performance [21]. TSS has been adapted for medical treatment planning [22], environmental sustainability analysis [23], and optimal staff recruitment [24]. These hierarchical structures allow layered decision frameworks that are closer to real-world conditions.

Despite these advancements, none of the existing models allow an entity to exist in a superposition of attribute values, nor do they integrate probabilistic decision theory based on quantum principles. The QuantumSoft Set addresses this unique gap by modeling uncertainty not just as "unknown" or "uncertain" but as co-existing probabilistic states.

3. QuantumSoft Set

3.1 Mathematical Definition

Let:

- $U = \{u_1, u_2, \dots, u_n\}$ be the universe of discourse.
- $A = \{a_1, a_2, \dots, a_m\}$ be a finite set of attributes.
- Each element $u_i \in U$ is associated with a quantum state vector $|u_i\rangle$.
- Define a mapping:

$$F: A \rightarrow \mathcal{H}(U)$$

where $\mathcal{H}(U)$ is a complex Hilbert space over U . Each mapping produces a quantum superposition of the elements in U , such that:

$$F(a_j) = \sum_{i=1}^n \alpha_{ij} |u_i\rangle, \text{ where } \sum_{i=1}^n |\alpha_{ij}|^2 = 1$$

Here, α_{ij} is the amplitude associated with the likelihood of element u_i having attribute a_j .

3.2 Measurement and Collapse

When a query is made-i.e., when we ask: "Which element has attribute a ?"-the quantum state collapses to a definite outcome with probability:

$$P(u_i) = |\alpha_{ij}|^2$$

This process mimics quantum measurement, where the uncertain state is resolved into a single classical result.

4. Case Study

We consider four student groups:

1. s_1 : Group A
2. s_2 : Group B
3. s_3 : Group C
4. s_4 : Group D

4.1 Attribute: Teaching Effectiveness

The possible evaluation values are:

- a) Very Effective
- b) Neutral
- c) Confusing

Using student surveys, we derive the QuantumSoft mapping as follows:

- a) Very Effective

$$F(\text{Very Effective}) = 0.8|s_1\rangle + 0.4|s_2\rangle + 0.1|s_3\rangle$$

$$P(s_1) = 0.64, P(s_2) = 0.16, P(s_3) = 0.01$$

- b) Neutral

$$F(\text{Neutral}) = 0.1|s_1\rangle + 0.2|s_2\rangle + 0.5|s_3\rangle + 0.1|s_4\rangle$$

$$P(s_3) = 0.25 \text{ (most likely)}$$

- c) Confusing

$$F(\text{Confusing}) = 0.3|s_2\rangle + 0.6|s_3\rangle + 0.5|s_4\rangle$$

$$P(s_3) = 0.36, \quad P(s_4) = 0.25$$

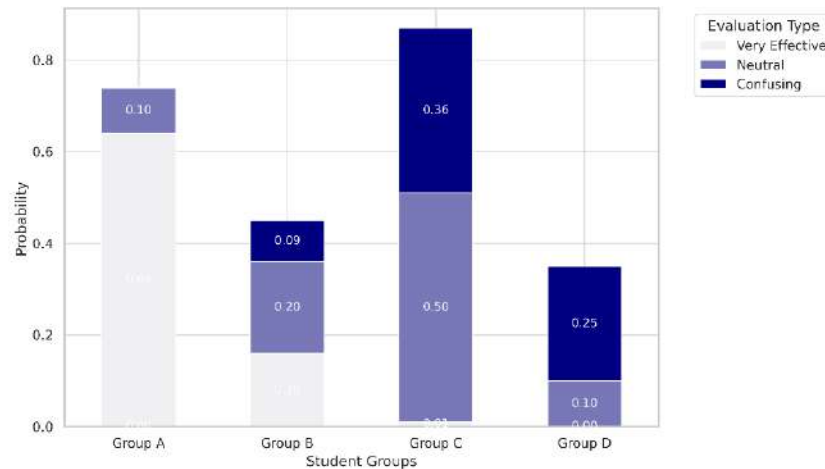


Figure 1 showing the probabilities for each group across evaluation types.

To help picture how different student groups responded to multimedia teaching, we created a bar chart showing the probabilities of each group under the three evaluation categories. Figure 1 clearly highlights the differences in perception. Group A shows the highest confidence in the teaching being very effective, while Group C has a mixed response, leaning more toward “confusing” and “neutral.” Group D primarily viewed the teaching as confusing.

5. Comparison with HyperSoft and TreeSoft Models

To better understand the effectiveness of the proposed model, it is essential to compare it with existing extensions such as the HyperSoft and TreeSoft sets. These models offer unique structural advantages that address multi-criteria and hierarchical decision-making under uncertainty.

5.1 HyperSoft Set Approach

Attributes:

1. $A_1 = \{ \text{Tool Type: Video, Slides, Interactive} \}$
2. $A_2 = \{ \text{Student Feedback: Effective, Neutral, Confusing} \}$

Function:

$$F: A_1 \times A_2 \rightarrow P(U)$$

Example:

$$F(\text{Video}, \text{Effective}) = \{s_1, s_2\}, F(\text{Interactive}, \text{Confusing}) = \{s_4\}$$

This provides a crisp categorization, but no overlap or uncertainty modeling.

5.2 TreeSoft Set Approach

Attributes are structured hierarchically:

- 1. Level 0: Effectiveness
- 2. Level 1: Tool Type
- 3. Level 2: Student Perception

Example mapping:

$$F(\text{Slides} \rightarrow \text{Neutral}) = \{s_3\}, F(\text{Interactive} \rightarrow \text{Confusing}) = \{s_4\}$$

Useful for structured feedback but lacks probabilistic representation or co-existing states.

5.3 QuantumSoft Set Advantage

Unlike the above models, QuantumSoft:

- 1. Allows overlapping group membership
- 2. Models mixed or uncertain perceptions
- 3. Reflects probabilistic decision-making

Mimics real student feedback, where opinions are not always clear or exclusive

Table 1 provides a clear comparison of the five models discussed. It highlights how each model handles key features such as multi-attribute input, indeterminate data, hierarchical structure, and probabilistic or overlapping responses. This helps demonstrate why the QuantumSoft Set is uniquely positioned for modeling subjective and complex evaluations in education and beyond.

Table 1. Comparison of Soft Set Models

Feature	Soft Set	HyperSoft Set	IndetermSoft Set	TreeSoft Set	QuantumSoft Set
Handles Multiple Attributes	X	√	X	√	X
Supports Indeterminate Data	X	X	√	X	√
Hierarchical Structure	X	X	X	√	X
Handles Overlapping States	X	X	X	X	√
Based on Probabilities	X	X	X	X	√
Simulates Real Decision Uncertainty	X	√	√	√	√
Requires Complex Structure	X	X	X	√	X

6. Discussion

As we compare the existing soft set models with the newly proposed QuantumSoft Set, we begin to see a clear shift in how uncertainty and human judgment can be represented in mathematical models.

Traditional models like the Soft Set and HyperSoft Set work well when the information is structured and the answers are clear. They allow us to handle multiple factors at once, but they assume that each situation has a definite, fixed outcome. This doesn't always reflect how people actually think or feel especially in areas like education, where opinions are often mixed or unclear. The IndetermSoft Set and TreeSoft Set improved things by allowing us to handle unknown or layered information. For example, TreeSoft Sets help when feedback can be organized in levels, such as from general impressions to specific issues. But even these models assume that once we ask a question, we get one solid answer. They don't allow for a person or group to feel multiple things at once like finding a lecture both helpful and confusing.

That's where the QuantumSoft Set offers something new. It accepts that people can hold different opinions at the same time, and that those opinions can shift depending on how we ask or interpret the question. This reflects real student feedback more accurately, because emotions and judgments are rarely black and white. They're usually a mix.

This kind of thinking wasn't possible without the deep groundwork laid by Florentin Smarandache, who made major contributions to this field. Between 2018 and 2024, he introduced several powerful models that helped expand soft set theory in important ways. His work made it possible to deal with complex relationships, incomplete data, and structured feedback in a much more flexible way. These models including the HyperSoft, IndetermSoft, IndetermHyperSoft, SuperHyperSoft, TreeSoft, and ForestSoft Sets have become key tools in many modern decision-making systems.

Building on that foundation, the QuantumSoft Set takes the next step. It combines mathematics with the way people actually think and feel. Instead of forcing answers into categories, it lets us explore the gray areas those in-between states where real understanding happens. This makes it a better fit for fields like education, psychology, and human-centered technology, where clear answers are often hard to find.

In short, the QuantumSoft Set doesn't just add another layer to soft set theory it brings the theory closer to real life.

7. Limitations and Future Work

While the QuantumSoft Set offers a more realistic way to model uncertain and mixed opinions, it is not without challenges. One limitation is that it requires estimating probability values for each possible opinion, which may be difficult to gather in real settings without detailed data or expert judgment.

Additionally, the model currently focuses on flat structures and does not support multi-level or hierarchical feedback like the TreeSoft Set. Combining the quantum approach with tree-like or fuzzy structures could make it even more flexible and powerful.

Future research can explore ways to simplify the model's use in practical systems, create automated tools for estimating probabilities, and apply it in other areas such as healthcare, psychology, or smart recommendation systems. Developing algebraic operations for QuantumSoft Sets like union and intersection would also help expand their use in more complex decision-making environments.

6. Conclusion

This research introduced the QuantumSoft Set as a novel and powerful extension of Soft Set theory. Rooted in the principles of quantum mechanics, it handles complex, subjective, and overlapping evaluations more effectively than classical models. In the case study on college English teaching, QuantumSoft Sets demonstrated clear advantages in capturing mixed student responses toward multimedia teaching tools. By integrating probabilistic modeling and superposition, this model enhances our ability to process fuzzy feedback and supports more nuanced educational decision-making.

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Machine Learning Models with Neutrosophic Numbers for Cloud Security Analysis and Smart Grid Control of Renewable Energy

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Abstract:

Machine learning (ML) enables difficult tasks to be completed independently. In a smart grid (SG), computers and mobile devices may make it easier to monitor security, control interior temperature, and perform routine maintenance. The ability of smart grids to identify cyberattacks is essential for assessing the operation's reliability and availability. This essay emphasizes the integrity of cyberattacks using fake data in the physical layers of smart grids (SGs). This paper analyzes data transmission security and proposes a novel approach to smart grid energy management. Here, renewable energy sources are used to assess the smart grid network's energy efficiency, and the network has been monitored for cloud computing security assessments. We use an interval trapezoidal neutrosophic number model to deal with uncertainty information and rank the best ML model under different evaluation matrices. We use the k-nearest neighbor with $k=3$ to 12. The CoCoSo method is used to rank ML models.

Keywords: Neutrosophic Sets; Uncertainty; Renewable Energy; Cloud Security; Smart Grid.

1. Introduction

As the demand for more efficient energy distribution and use grows, the significance of smart grid management systems (SGMS) has dramatically increased. By integrating ML techniques with SGMS, cost, and energy efficiency might be significantly increased.[1]. Using machine

learning (ML) algorithms that learn from past data and real-time monitoring, the SGMS ensures optimal performance, optimizes energy usage, and provides accurate projections. An adaptable power framework with negative evaluation capabilities, energy executives, exchange and estimating, SG monetary exchanges, energy trading, and the use of renewable power sources in conjunction with an SG board are some suggestions for enhancing the way SG blockchain technology and digital currencies are presented. This might result in a more advantageous load profile and financial gain.[2], [3].

Furthermore, earlier researchers modified the blockchain's calculations to allow online client networks to use Bitcoin to operate in commercial hubs and energy frameworks in safer ways [2]. The journalists' energy architecture also makes use of advanced cash. One of the major technological developments of the twenty-first century is the growth of the Internet of Things.[4], [5]. The Internet of Things (IoT) is a network of interconnected devices, software, and physical hubs that facilitate the sharing and exchange of information. The "IoT" refers to the exchange of data between various devices and frameworks.[6].

The necessity for energy supply security and the growing global energy demand have prompted a persistent attempt to move away from the traditional power generating grid and toward a flexible SG that incorporates renewable energy sources (RES). Demand and supply have changed dynamically over time, creating a highly difficult climate. Real-time data is gathered because a network system of connecting procedures to numerous variables may be used to operate and analyze devices, systems, and processes regularly in this environment.[7], [8].

Cybersecurity measures are used to stop ransomware, malware, and viruses from causing disruptions and data breaches.[9], [10]. When correctly deployed, these measures can help smart grid operators identify and address network vulnerabilities before they cause damage. Protecting smart grids from potential terrorist attacks, espionage efforts, and vulnerabilities caused by weather, malfunctioning equipment, and human error is crucial.[11]. The two most widely used protocols are the Transport Control Protocol (TCP), which ensures that even long, fragmented communications are correctly reassembled at their destination (for purposes like efficiency or media sharing), and the Internet Protocol (IP), which identifies devices and systems.[12].

One of the most important and pervasive activities about real-world applications during the last few decades has been MCDM. Numerous attempts to evaluate the RESs using various MCDM techniques have been made in the literature. Even still, conventional methods of decision-making are insufficient for handling the ambiguous data that typically arise during energy planning procedures. Because of its adaptability and efficiency in handling circumstances where the information at hand is ambiguous or insufficient, the fuzzy set (FS) concept has motivated researchers worldwide[13]. Atanassov [14] introduced the idea of the intuitionistic fuzzy set (IFS), an extension of FSs that is more suited to handling fuzziness and uncertainty than Zadeh's suggestion of FS.

However, contradictory and ambiguous information presented in real-world scenarios cannot be handled by the notions of FSs and IFSs; they can only handle partial and unclear information. Smarandache [15], [16] Has intended the neutrosophic set (NS) to better handle such information. A strong general formal framework that reduces the sets from a philosophical perspective, NS is a component of neutrosophy, which investigates the genesis, nature, and extent of neutralities as well as their interactions with various ideational spectra. The term "neutrosophy" refers to the "knowledge of neutral thought," and the word "neutral" represents the primary difference between FSs, IFSs, and their logic[17], [18].

2. Proposed Model

This section shows the operations of interval-valued trapezoidal neutrosophic number (IVTNN) [19], [20] And steps of the CoCoSo method to rank the ML models under different criteria.

$$K^L = \left\{ \left(Y, T_K^L(Y), I_K^L(Y), F_K^L(Y) \right); y \in Y \right\} \quad (1)$$

$$T_K^L(Y) \subset [0,1], I_K^L(Y) \subset [0,1], F_K^L(Y) \subset [0,1] \quad (2)$$

We can define the trapezoidal neutrosophic fuzzy numbers such as:

$$T_K^L(Y) = (t^{L1}_K(Y), t^{L2}_K(Y), t^{L3}_K(Y), t^{L4}_K(Y))Y \rightarrow [0,1] \quad (3)$$

$$F_K^L(Y) = (f^{L1}_K(Y), f^{L2}_K(Y), f^{L3}_K(Y), f^{L4}_K(Y))Y \rightarrow [0,1] \quad (4)$$

$$I_K^L(Y) = (i^{L1}_K(Y), i^{L2}_K(Y), i^{L3}_K(Y), i^{L4}_K(Y))Y \rightarrow [0,1] \quad (5)$$

$$0 \leq t^{L4}_K(Y) + i^{L4}_K(Y) + f^{L4}_K(Y) \leq 3 \quad (6)$$

$$K^L = \left\{ ((a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4), (c_1, c_2, c_3, c_4)) > Y \rightarrow [0,1] \right\} \quad (7)$$

We can show the neutrosophic operations such as:

$$k_1 = \left\{ \begin{array}{l} ([(a_1, a_2, a_3, a_4), (\bar{a}_1, \bar{a}_2, \bar{a}_3, \bar{a}_4)]; T_{n_1}), \\ ([(b_1, b_2, b_3, b_4), (\bar{b}_1, \bar{b}_2, \bar{b}_3, \bar{b}_4)]; I_{n_1}), \\ ([(c_1, c_2, c_3, c_4), (\bar{c}_1, \bar{c}_2, \bar{c}_3, \bar{c}_4)]; F_{n_1}) \end{array} \right\} \quad (8)$$

$$K_2 = \left\{ \begin{array}{l} ([(d_1, d_2, d_3, d_4), (\bar{d}_1, \bar{d}_2, \bar{d}_3, \bar{d}_4)]; T_{n_2}), \\ ([(e_1, e_2, e_3, e_4), (\bar{e}_1, \bar{e}_2, \bar{e}_3, \bar{e}_4)]; I_{n_2}), \\ ([(f_1, f_2, f_3, f_4), (\bar{f}_1, \bar{f}_2, \bar{f}_3, \bar{f}_4)]; F_{n_2}) \end{array} \right\} \quad (9)$$

$$K_1 \oplus K_2 = \left(\begin{array}{c} \left[\begin{array}{c} (a_1 + d_1 - a_1 d_1, a_2 + d_2 - a_2 d_2), \\ (a_3 + d_3 - a_3 d_3, a_4 + d_4 - a_4 d_4) \end{array} \right], \\ \left[\begin{array}{c} (\overline{a_1} + \overline{d_1} - \overline{a_1} \overline{d_1}, \overline{a_2} + \overline{d_2} - \overline{a_2} \overline{d_2}), \\ (\overline{a_3} + \overline{d_3} - \overline{a_3} \overline{d_3}, \overline{a_4} + \overline{d_4} - \overline{a_4} \overline{d_4}) \end{array} \right], \\ \left[\begin{array}{c} (b_1 e_1, b_2 e_2, b_3 e_3, b_4 e_4), \\ (\overline{b_1} \overline{e_1}, \overline{b_2} \overline{e_2}, \overline{b_3} \overline{e_3}, \overline{b_4} \overline{e_4}) \end{array} \right], \\ \left[\begin{array}{c} (c_1 f_1, c_2 f_2, c_3 f_3, c_4 f_4), \\ (\overline{c_1} \overline{f_1}, \overline{c_2} \overline{f_2}, \overline{c_3} \overline{f_3}, \overline{c_4} \overline{f_4}) \end{array} \right] \end{array} \right) \quad (10)$$

$$K_1 \otimes K_2 = \left\{ \begin{array}{c} \left[\begin{array}{c} (a_1 d_1, a_2 d_2, a_3 d_3, a_4 d_4), \\ (\overline{a_1} \overline{d_1}, \overline{a_2} \overline{d_2}, \overline{a_3} \overline{d_3}, \overline{a_4} \overline{d_4}) \end{array} \right], \\ \left[\begin{array}{c} (b_1 + e_1 - b_1 e_1, b_2 + e_2 - b_2 e_2), \\ (b_3 + e_3 - b_3 e_3, b_4 + e_4 - b_4 e_4) \end{array} \right], \\ \left[\begin{array}{c} (\overline{b_1} + \overline{e_1} - \overline{b_1} \overline{e_1}, \overline{b_2} + \overline{e_2} - \overline{b_2} \overline{e_2}), \\ (\overline{b_3} + \overline{e_3} - \overline{b_3} \overline{e_3}, \overline{b_4} + \overline{e_4} - \overline{b_4} \overline{e_4}) \end{array} \right], \\ \left[\begin{array}{c} (c_1 + f_1 - c_1 f_1, c_2 + f_2 - c_2 f_2), \\ (c_3 + f_3 - c_3 f_3, c_4 + f_4 - c_4 f_4) \end{array} \right], \\ \left[\begin{array}{c} (\overline{c_1} + \overline{f_1} - \overline{c_1} \overline{f_1}, \overline{c_2} + \overline{f_2} - \overline{c_2} \overline{f_2}), \\ (\overline{c_3} + \overline{f_3} - \overline{c_3} \overline{f_3}, \overline{c_4} + \overline{f_4} - \overline{c_4} \overline{f_4}) \end{array} \right] \end{array} \right\} \quad (11)$$

$$\varphi k_1 = \left(\begin{array}{c} \left[\begin{array}{c} (1 - (1 - a_1)^\varphi, 1 - (1 - a_2)^\varphi), \\ (1 - (1 - a_3)^\varphi, 1 - (1 - a_4)^\varphi) \end{array} \right], \\ \left[\begin{array}{c} (1 - (1 - \overline{a_1})^\varphi, 1 - (1 - \overline{a_2})^\varphi), \\ (1 - (1 - \overline{a_3})^\varphi, 1 - (1 - \overline{a_4})^\varphi) \end{array} \right], \\ [(b_1^\varphi, b_2^\varphi, b_3^\varphi, b_4^\varphi), (\overline{b_1}^\varphi, \overline{b_2}^\varphi, \overline{b_3}^\varphi, \overline{b_4}^\varphi)], \\ [(c_1^\varphi, c_2^\varphi, c_3^\varphi, c_4^\varphi), (\overline{c_1}^\varphi, \overline{c_2}^\varphi, \overline{c_3}^\varphi, \overline{c_4}^\varphi)] \end{array} \right) \quad (12)$$

$$k_1^\varphi = \left(\begin{array}{c} [(a_1^\varphi, a_2^\varphi, a_3^\varphi, a_4^\varphi), (\overline{a_1}^\varphi, \overline{a_2}^\varphi, \overline{a_3}^\varphi, \overline{a_4}^\varphi)], \\ \left[\begin{array}{c} (1 - (1 - b_1)^\varphi, 1 - (1 - b_2)^\varphi), \\ (1 - (1 - b_3)^\varphi, 1 - (1 - b_4)^\varphi) \end{array} \right], \\ \left[\begin{array}{c} (1 - (1 - \overline{b_1})^\varphi, 1 - (1 - \overline{b_2})^\varphi), \\ (1 - (1 - \overline{b_3})^\varphi, 1 - (1 - \overline{b_4})^\varphi) \end{array} \right], \\ \left[\begin{array}{c} (1 - (1 - c_1)^\varphi, 1 - (1 - c_2)^\varphi), \\ (1 - (1 - c_3)^\varphi, 1 - (1 - c_4)^\varphi) \end{array} \right], \\ \left[\begin{array}{c} (1 - (1 - \overline{c_1})^\varphi, 1 - (1 - \overline{c_2})^\varphi), \\ (1 - (1 - \overline{c_3})^\varphi, 1 - (1 - \overline{c_4})^\varphi) \end{array} \right] \end{array} \right) \quad (13)$$

We show the steps of the CoCoSo method.

Experts create the decision matrix as

$$Y = \begin{bmatrix} y_{11} & \cdots & y_{1n} \\ \vdots & \ddots & \vdots \\ y_{m1} & \cdots & y_{mn} \end{bmatrix} \quad (14)$$

Compute the criteria weights using the average method.

Normalize the decision matrix for beneficial and non-beneficial criteria.

$$H_{ij} = \frac{y_{ij} - \min_i y_{ij}}{\max_i y_{ij} - \min_i y_{ij}} \quad (15)$$

$$H_{ij} = \frac{\min_i y_{ij} - y_{ij}}{\max_i y_{ij} - \min_i y_{ij}} \quad (16)$$

The sum and power-weighted decision matrix are computed

$$D_i = \sum_{j=1}^n (W_j H_{ij}) \quad (17)$$

$$F_i = \sum_{j=1}^n (H_{ij})^{W_j} \quad (18)$$

Determine the appraisal scores

$$Q_{ia} = \frac{D_i + F_i}{\sum_{i=1}^m (D_i + F_i)} \quad (19)$$

$$Q_{ib} = \frac{D_i}{\min D_i} + \frac{F_i}{\min F_i} \quad (20)$$

$$Q_{ic} = \frac{\pi(D_i) + (1 - \pi)(F_i)}{\pi \max D_i + (1 - \pi) \max F_i} \quad (21)$$

Determine the final value

$$Q_i = (Q_{ia} Q_{ib} Q_{ic})^{(1/3)} + \frac{1}{3} (Q_{ia} + Q_{ib} + Q_{ic}) \quad (22)$$

3. Results of ML Models

This section shows the results of the ML models on the KDD dataset.

The informational collection for the 1999 KDD Cup: This dataset includes about 4900,000 vectors from seven weeks of organization traffic, as specified by the DARPA'98 IDS assessment program. The following are the four types of replicated attacks: The 41 highlights in the KDD Cup 1999 dataset are divided into the three classes that go with it: 1) a remote-to-neighborhood (R2L) assault, 2) a client-to-root (U2R) attack, 3) a testing attack, and 4) a denial-of-service (DoS) attack. The first three are traffic, content, and establishment highlights. To eliminate necessary characteristics, a TCP/IP association is used. Traffic highlights are divided into two categories: highlights for "same host" and highlights for "same assistance." The chemical highlights raise concerns about behavior associated with information. Fig 1 shows the distribution of the attacks. Fig 2 shows the histogram of the attacks. Fig 3 shows the heatmap between the features.

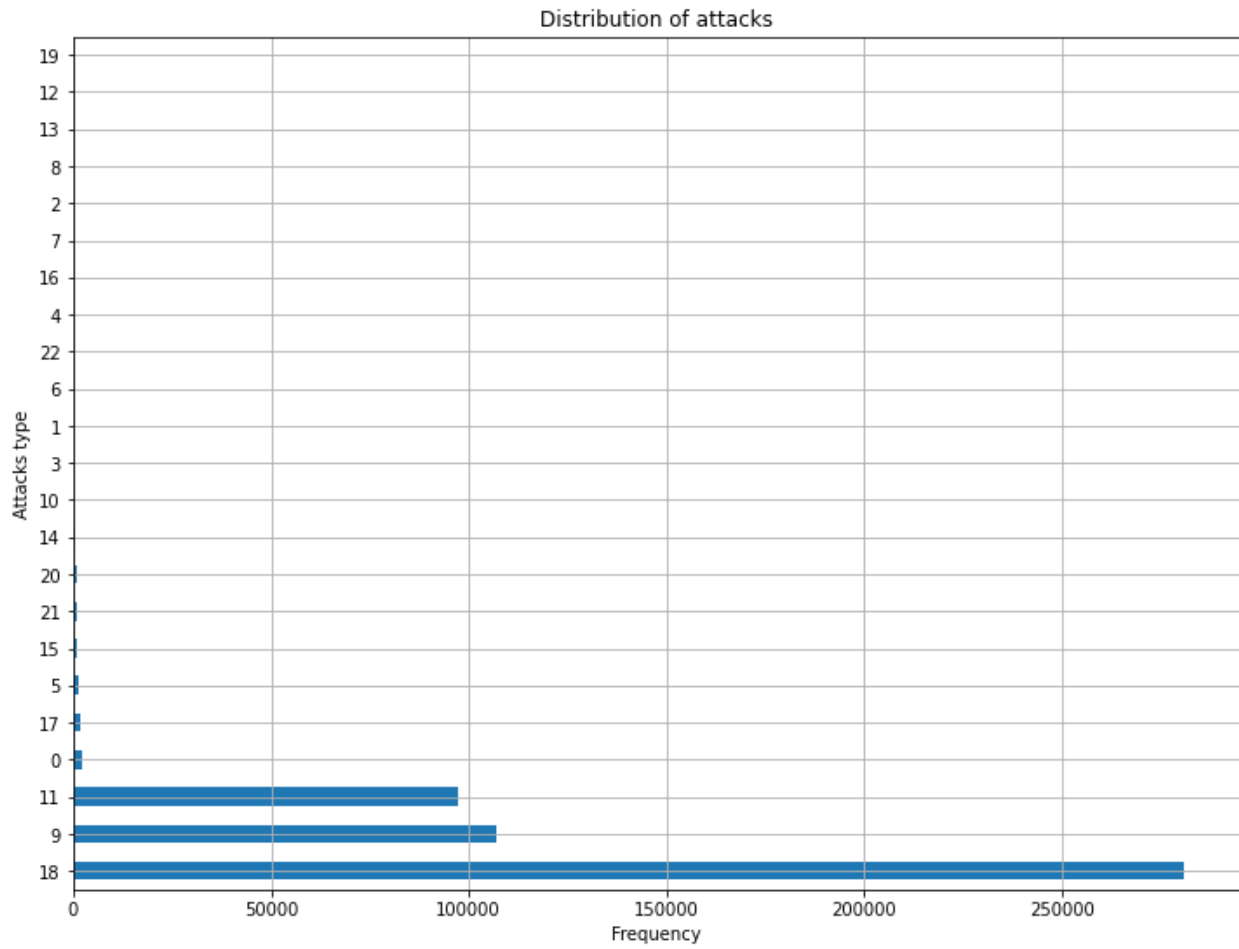


Fig 1. The distribution of the attacks.

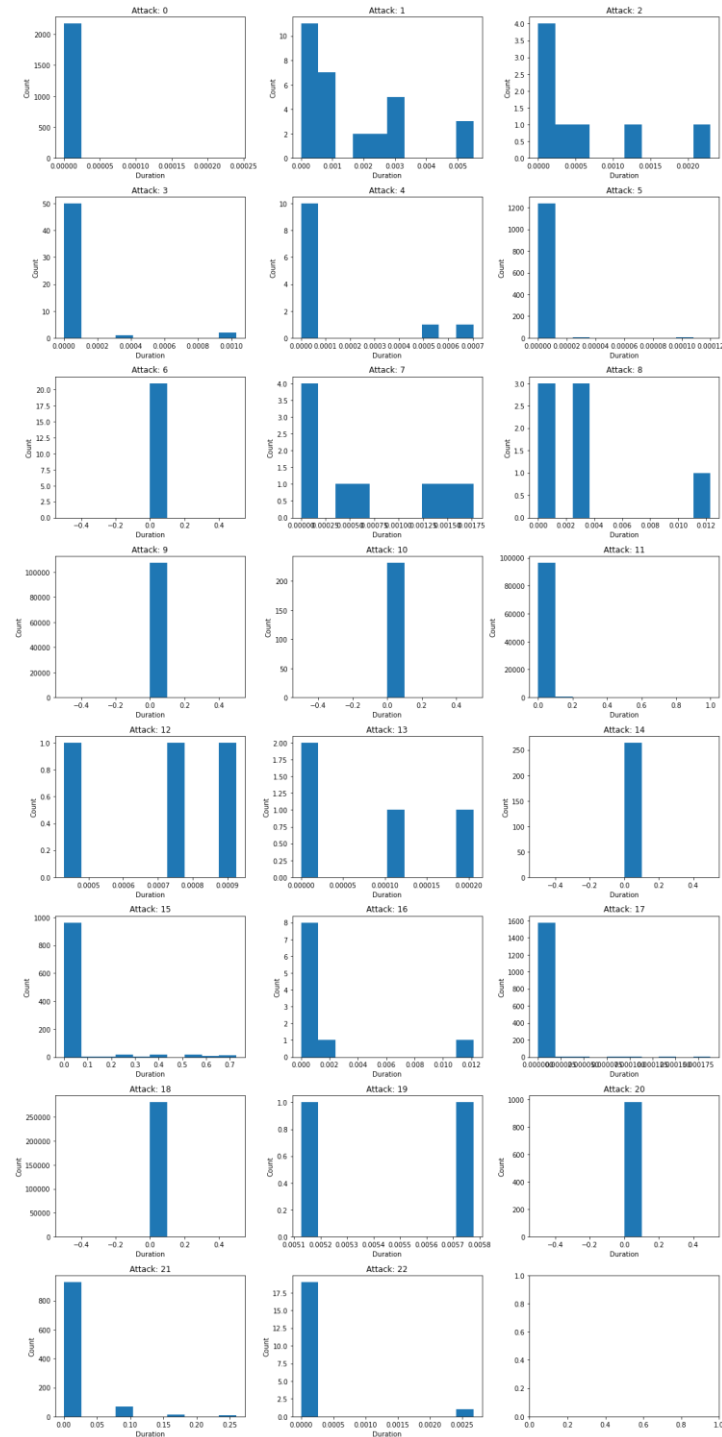


Fig 2. The histogram of the attacks.

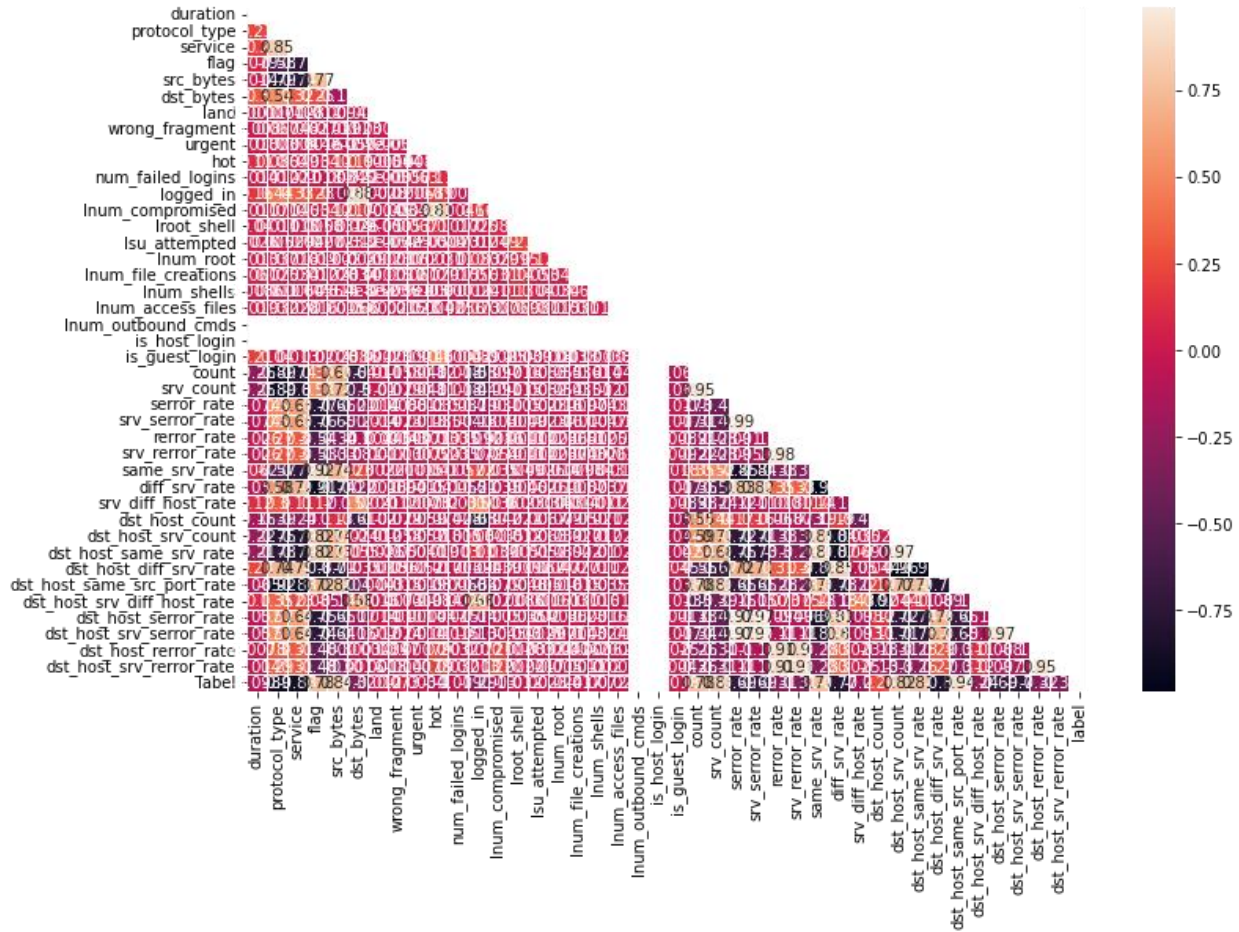


Fig 3. The heatmap of the features.

Table 1 shows the results of the ML models under different evaluation matrices. We show the k-nearest neighbor (KNN) with k=5 is the best model.

Table 1. The results of ML models.

	Accuracy	Precision	Recall	F1-score
KDCA ₁	0.991573	0.589729	0.551429	0.561419
KDCA ₂	0.991242	0.588684	0.548719	0.559742
KDCA ₃	0.993192	0.594458	0.543208	0.550045
KDCA ₄	0.992537	0.580794	0.540766	0.545454
KDCA ₅	0.993118	0.59368	0.542076	0.549423
KDCA ₆	0.993064	0.589177	0.538599	0.54513
KDCA ₇	0.99305	0.623022	0.542148	0.552717
KDCA ₈	0.993091	0.624862	0.541911	0.554386
KDCA ₉	0.993077	0.618149	0.541357	0.552938
KDCA ₁₀	0.993077	0.619933	0.542825	0.554313
KDCA ₁₁	0.991573	0.589729	0.551429	0.561419

[illegible]

Table 4. The third decision matrix.

[illegible]

Eqs. (15 and 16) are used to normalize the decision matrix for beneficial and non-beneficial criteria as shown in Table 5. We show the KNN with k=5 is the best model.

The sum and power-weighted decision matrix are computed using eqs. (17 and 18) as shown in Tables 6-7.

Determine the appraisal scores using eqs. (19-21)

Determine the final value using eq. (22) and rank the alternatives are shown in Fig 4.

Table 5. The normalized decision matrix.

	KDCC ₁	KDCC ₂	KDCC ₃	KDCC ₄
KDCA ₁	0.311594	1	0.840909	0.130435
KDCA ₂	0	0.666667	0	1
KDCA ₃	0.956522	0.891304	0.840909	0.130435
KDCA ₄	0.355072	0.333333	0.742424	0
KDCA ₅	0.57971	0.623188	0.371212	0.666667
KDCA ₆	1	0.086957	0.954545	0.268116
KDCA ₇	0	0.869565	0.045455	0.978261
KDCA ₈	0.934783	0.934783	1	0.311594
KDCA ₉	0	0.043478	0.280303	0.043478
KDCA ₁₀	0.934783	0.130435	0.606061	1
KDCA ₁₁	0.666667	0	0.977273	0.934783
KDCA ₁₂	0.804348	0.130435	0.136364	0.311594

Table 6. The sum weighted matrix.

	KDCC ₁	KDCC ₂	KDCC ₃	KDCC ₄
KDCA ₁	0.078893	0.246725	0.212627	0.032247
KDCA ₂	0	0.164483	0	0.247231
KDCA ₃	0.242183	0.219907	0.212627	0.032247
KDCA ₄	0.089901	0.082242	0.187725	0
KDCA ₅	0.146777	0.153756	0.093862	0.164821
KDCA ₆	0.253191	0.021454	0.24136	0.066287
KDCA ₇	0	0.214543	0.011493	0.241856
KDCA ₈	0.236678	0.230634	0.252854	0.077036
KDCA ₉	0	0.010727	0.070876	0.010749
KDCA ₁₀	0.236678	0.032181	0.153245	0.247231
KDCA ₁₁	0.168794	0	0.247107	0.231107
KDCA ₁₂	0.203654	0.032181	0.03448	0.077036

Table 7. The power-weighted matrix.

	KDCC ₁	KDCC ₂	KDCC ₃	KDCC ₄
KDCA ₁	0.744357	1	0.957134	0.604363
KDCA ₂	0	0.904803	0	1

KDCA ₃	0.988808	0.972009	0.957134	0.604363
KDCA ₄	0.769386	0.762575	0.927457	0
KDCA ₅	0.871058	0.889872	0.778355	0.904617
KDCA ₆	1	0.547393	0.988306	0.72221
KDCA ₇	0	0.966105	0.457682	0.994581
KDCA ₈	0.983069	0.983498	1	0.749548
KDCA ₉	0	0.461347	0.724988	0.460616
KDCA ₁₀	0.983069	0.604986	0.881066	1
KDCA ₁₁	0.902434	0	0.994204	0.983465
KDCA ₁₂	0.946366	0.604986	0.604234	0.749548

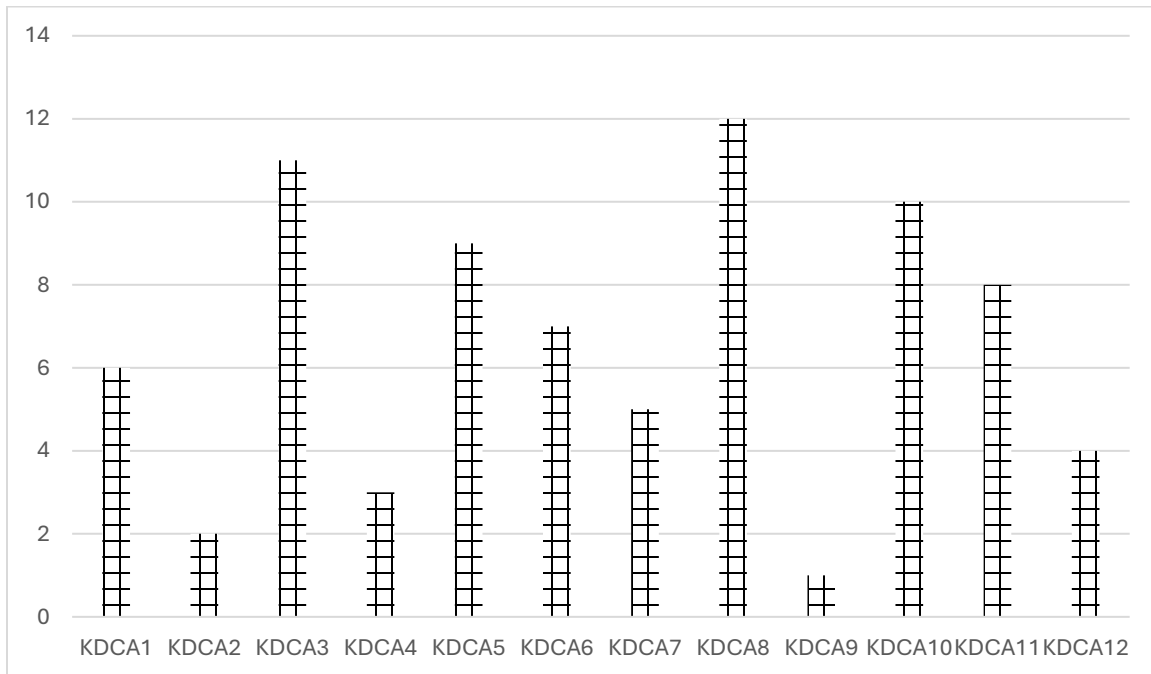


Fig 4. Rank of ML models.

5. Conclusions

This study uses ML models to propose an approach to SG energy management with cloud security analysis. The suggested approach will support smart grid frameworks. The reliability of smart grid frameworks was enhanced using real characteristics produced by the suggested specialist-based approach, and the suggested technique was accepted using the Digital Attack replay. We use the KDD dataset to evaluate ML models. We use the neutrosophic set model to select the best ML model under different evaluation matrices. We evaluated the ML models under four evaluation metrics. We used KNN with $k=3$ to 12. The CoCoSo method is used to rank ML

models. The criteria weights are computed using the average method. The results show the KNN with $k=5$ is the best under different evaluation matrices.

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Kindergarten Teachers' Classroom Management Abilities: A Neutrosophic Perspective on Partial Locality, Indeterminacy, and Non-Local Influences

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Abstract—This study explores the classroom management abilities of kindergarten teachers through a new lens grounded in the Neutrosophic theory of partial locality, partial indeterminacy, and partial non-locality. It investigates how teachers manage their classrooms while being influenced by immediate, uncertain, and distant factors. Local factors include direct interactions with children and the physical learning environment. Indeterminate elements involve unpredictable student behavior and fluctuating teacher emotions. Non-local elements reflect broader influences such as educational policies, teacher training, and cultural expectations. Using a mixed-method approach, the study analyzes real classroom data and models the dynamic interplay of these factors through a neutrosophic framework. The findings offer a deeper understanding of how kindergarten teachers navigate complex and uncertain educational environments, and propose practical strategies to enhance classroom control. This paper highlights the value of combining educational theory with mathematical modeling to address real-world teaching challenges.

Keywords: Kindergarten Education; Classroom Management; Neutrosophic Theory; Partial Locality; Partial Indeterminacy; Partial Non-Locality; Teacher Behavior; Educational Environment; Early Childhood Pedagogy; Educational Modeling.

1. Introduction

Effective classroom management in early childhood education is foundational to successful learning outcomes. Kindergarten classrooms, in particular, demand high levels of adaptability and responsiveness from teachers, as they face a blend of spontaneous behaviors, emotional fluctuations, and developmental needs from young learners [1]. The ability to maintain structure while nurturing creativity and emotional well-being requires more than traditional disciplinary strategies it necessitates a nuanced understanding of both visible and invisible influences within the educational setting.

Recent research suggests that teacher performance is not solely shaped by immediate classroom conditions but also by a constellation of factors that exist beyond direct observation, such as institutional expectations, societal norms, and historical educational experiences [2]. These layers

of influence challenge the assumption that classroom management is strictly local and linear. Instead, they reveal a more intricate system where unpredictability and external forces often shape outcomes in subtle, yet powerful ways [3].

In 2024, Smarandache introduced the Principle of Partial Locality, Partial Indeterminacy, and Partial Non-Locality, which offers a novel theoretical framework for analyzing systems that simultaneously exhibit localized, indeterminate, and non-local behaviors. This principle provides deeper insight into complex phenomena that cannot be fully explained by traditional models relying solely on either locality or non-locality [4].

This study introduces a novel approach to examining kindergarten teachers' classroom management by applying the neutrosophic principles of partial locality, partial indeterminacy, and partial non-locality. These concepts, originally developed in quantum and complex systems theory, provide a framework for understanding how immediate (local), uncertain (indeterminate), and distant (non-local) elements dynamically affect classroom behavior and decision-making [4]. Unlike deterministic models that assume fixed outcomes, the neutrosophic perspective accommodates ambiguity, variability, and the coexistence of multiple influences.

By adopting this theoretical lens, the research aims to capture the often-overlooked complexity of kindergarten teaching. It emphasizes the necessity of acknowledging that not all factors affecting classroom control are visible, predictable, or within the teacher's direct reach. In doing so, it offers a richer, more realistic understanding of what it means to manage a classroom in the earliest years of formal education.

2. Literature Review

Research on early childhood education has consistently highlighted the distinct nature of classroom management in kindergarten settings, where emotional development, social integration, and behavioral self-regulation occur simultaneously [5]. Unlike primary or secondary levels, kindergarten classrooms demand a pedagogical approach that balances structure with flexibility, often requiring teachers to switch between directive and facilitative roles based on moment-to-moment assessments of children's needs.

Several studies have examined the internal and external stressors affecting early childhood educators, indicating that classroom management challenges are often intertwined with emotional labor and cognitive load [6]. Teachers frequently navigate conflicting expectations between fostering autonomy and maintaining order, especially when addressing group dynamics among highly diverse student populations [7]. These tensions contribute to a sense of unpredictability, which traditional behaviorist frameworks struggle to accommodate.

Emerging models now advocate for the incorporation of dynamic systems theory into early childhood pedagogy, recognizing that classroom behaviors evolve through complex, nonlinear interactions among students, teachers, and environmental stimuli [8]. This perspective aligns closely with the neutrosophic approach, which embraces partial influences and probabilistic reasoning rather than deterministic control. It has been particularly useful in contexts where fixed rules are insufficient to explain the fluid reality of classroom life.

Further, institutional and cultural factors play a critical role in shaping teacher responses to behavioral disruptions. For example, cultural norms surrounding discipline, emotional expression, and child autonomy significantly influence how educators interpret and address student actions [9]. This cross-cultural variability reinforces the notion that management strategies are partially determined by non-local forces—those rooted in broader ideological and systemic frameworks.

Recent efforts to quantify the complexity of teaching environments have also turned to computational and hybrid models. These tools aim to capture the ambiguity and variability inherent in classroom settings, suggesting that uncertainty is not a limitation but a defining characteristic of early childhood education [10]. As a result, the literature is gradually shifting toward models that integrate uncertainty as a core component of effective teaching, rather than treating it as a deviation from ideal control.

3. Method

This research adopts a neutrosophic mixed-method model that integrates both qualitative observation and mathematical modeling to investigate the dynamic factors influencing kindergarten classroom management. The study was conducted in five public kindergarten classrooms in an urban district, involving 12 teachers and 145 students over a period of eight weeks. Data were collected through structured observations, behavioral coding, teacher interviews, and policy document analysis.

3.1 Neutrosophic Framework Application

The study models classroom management capacity as a function of three interdependent components:

- A. Local Factors (L): Direct, observable classroom interactions (transitions, rule enforcement, attention strategies).
- B. Indeterminate Factors (I): Unpredictable elements like emotional variability, spontaneous behavior, or social-emotional responses.
- C. Non-Local Factors (N): External influences such as district policies, cultural expectations, or pre-service training content.

The capacity C of a teacher to manage the classroom at a given time t is modeled as:

$$C(t) = \alpha L(t) + \beta I(t) + \gamma N(t)$$

Where:

- A. $\alpha, \beta, \gamma \in [0, 1]$ are weighted coefficients normalized such that $\alpha + \beta + \gamma = 1$
- B. $L(t), I(t), N(t) \in [0, 1]$ are the dynamic levels of local, indeterminate, and non-local influence at time t

3.2 Parameter Estimation

To estimate these coefficients:

- A. $L(t)$ was derived from real-time video coding of teacher-student interactions using a CLASS-based rubric [1]
- B. $I(t)$ was quantified using moment-to-moment behavioral inconsistencies and teacher self-reported stress levels (scored daily)

- C. $N(t)$ was estimated through document analysis and cross-referenced interviews that highlighted systemic constraints (curriculum rigidity, policy mandates)

3.3 Dynamic Modeling Technique

A time-series neutrosophic function was constructed across the study period:

$$C_i(t_k) = \alpha_i L_i(t_k) + \beta_i I_i(t_k) + \gamma_i N_i(t_k)$$

Where:

- A. $I \in \{1, \dots, 12\}$ represents the teacher
- B. $t_k \in \{1, \dots, 40\}$ denotes observation day k
- C. Each function output was plotted and analyzed for trend consistency, local fluctuations, and disruption triggers

3.4 Data Validation

Triangulation was employed by comparing model outputs against qualitative narrative from teacher interviews, which provided critical insight into the emotional and systemic context behind observed patterns. A subset of classrooms was also peer-reviewed by external observers to test inter-rater reliability (>89%).

4. Case Study

To demonstrate the practical application of the neutrosophic model, this case study focuses on Teacher A, a kindergarten educator working in a low-income urban school with a class of 26 students. Over a 20-day period, her classroom was observed and coded using the model introduced in the methodology section. The goal was to map how her classroom management performance evolved based on changing local, indeterminate, and non-local factors.

4.1 Locality Metrics

Local factors included teacher response time to student misbehavior, clarity of instructions, and successful transitions between activities. These were scored daily from 0 (low presence) to 1 (strong presence) using behavioral rubrics. Table 1 show relatively consistent engagement with classroom events, peaking on day 5 due to a structured activity.

Table 1: Locality score for Teacher A over selected days

Day	L (Local Score)
1	0.72
5	0.81
10	0.68
15	0.75
20	0.79

4.2 Indeterminacy Factors

Teacher A experienced personal emotional fluctuations that influenced her classroom control. These were captured through self-reporting and observed behavioral inconsistency among students.

Table 2: Indeterminacy scores

Day	I (Indeterminacy Score)
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1	0.34
5	0.22
10	0.49
15	0.37
20	0.41

Table 2 illustrate the Indeterminacy scores reflect psychological unpredictability in Teacher A's responses and student reactions, with day 10 showing a spike due to fatigue and an unplanned fire drill.

4.3 Non-Local Influences

This component was quantified through changes in external policy e.g., district observation visits and family-related disruptions communicated via administrative logs. Table 3 show the Non-local scores varied significantly, peaking on day 10 due to a surprise supervisory visit and a directive to modify lesson plans mid-week.

Table 3: Non-local scores

Day	N (Non-Local Score)
1	0.18
5	0.35
10	0.52
15	0.21
20	0.40

4.4 Resulting Capacity Calculation

Using the normalized coefficients $\alpha=0.5$, $\beta=0.3$, and $\gamma=0.2$, the neutrosophic capacity $C(t)$ was calculated for each of the selected days.

Table 4: Composite classroom management capacity scores

Day	$C(t) = 0.5 \times L + 0.3 \times I + 0.2 \times N$
1	0.54
5	0.66
10	0.61
15	0.59
20	0.65

Table 4 illustrate the Composite classroom management capacity scores based on weighted factors. Despite spikes in indeterminacy and non-locality, overall classroom control remained stable due to strong local management practices. Field notes from day 10 revealed a unique instance of neutrosophic behavior: while the teacher maintained visible order (high locality), her internal stress and a district mandate undermined long-term control. This reflected the partial interplay between visible authority and unseen stressors a direct manifestation of partial locality and partial indeterminacy acting simultaneously.

5. Results and Discussion

The analysis of Teacher A's classroom dynamics revealed a complex but interpretable pattern of management performance shaped by simultaneous and interacting layers of influence. Across the observation period, fluctuations in classroom control were not random; rather, they corresponded to distinct shifts in local, indeterminate, and non-local domains.

One of the most striking findings was the dominant influence of local factors in maintaining short-term classroom order. Days when Teacher A demonstrated clarity, consistency, and immediate responsiveness were consistently associated with higher management capacity. This reinforces prior educational research that underscores the critical role of in-the-moment teacher behavior in early childhood settings [1]. However, what distinguished this study is how those local gains were occasionally undermined by less visible forces.

The presence of indeterminate variables, such as emotional fatigue, peer conflict among students, and spontaneous behavioral disruptions, emerged as significant disruptors that operated in an unpredictable manner. Unlike local factors, these did not follow a fixed pattern but rather introduced volatility into the classroom environment. This aligns with neutrosophic theory, which holds that indeterminacy is a persistent state rather than a rare exception [2]. For instance, Teacher A's performance on day 10 was not noticeably worse from an external perspective, yet her internal emotional load as self-reported coincided with a subtle drop in classroom harmony. This supports the idea that indeterminacy often manifests silently, without immediate visibility.

Non-local influences, such as administrative decisions and institutional mandates, revealed themselves not through daily fluctuations, but in their cumulative pressure. These pressures did not necessarily interfere with momentary decisions but shaped the boundaries within which the teacher could act. For example, changes in lesson planning procedures introduced mid-week caused a ripple effect that limited instructional flexibility. Such constraints exemplify the partial non-locality principle, wherein external systems partially but meaningfully restrict the teacher's agency [3].

Importantly, these three domains local, indeterminate, and non-local did not act independently. The most revealing aspect of this study was the interdependence among the dimensions. For instance, a peak in indeterminate stress often occurred simultaneously with low tolerance for policy disruptions, suggesting that when internal uncertainty rises, external interference becomes more disruptive. Conversely, on days when Teacher A felt emotionally stable, she was more resilient to unexpected institutional demands. This supports the neutrosophic model's argument that partial interactions define real-world systems, as opposed to binary classifications of control versus chaos.

Finally, the neutrosophic model provided a valuable lens for interpreting these observations because it avoids over-simplification. Rather than labeling certain days as "good" or "bad," the model accommodated the ambiguity and allowed for a layered explanation of behavior. This proves particularly useful in educational environments where numerous interacting forces make simplistic models ineffective.

Overall, the results validate the need for classroom management strategies that are context-sensitive, emotionally aware, and systemically informed. It is not enough to train teachers in

technical skills alone; professional development must also prepare them to navigate uncertainty and institutional pressures that operate beyond their immediate control.

6. Interwoven Forces: Rethinking Influence Through the Neutrosophic Lens

In traditional educational discourse, the sources of classroom behavior are often classified as either teacher-driven or externally imposed. However, such binary thinking falls short when applied to the nuanced reality of early childhood classrooms. Through the neutrosophic framework, we recognize that influence is not strictly local, nor entirely external, nor wholly uncertain it is an interwoven force system that fluctuates in form and impact.

Partial locality in the classroom can be seen in moments where a teacher's presence alone recalibrates student focus, not by strict control, but by subtle cues tone, posture, proximity. These are not absolute or fixed reactions but partial ones, effective only under certain classroom moods and contexts. Their success varies even with the same students across different times of day.

Indeterminacy arises not from ignorance but from complexity. A child may resist group activity not out of defiance but due to invisible emotional triggers. Similarly, a teacher's decision to redirect or ignore a behavior cannot always be predicted, even by the teacher themselves. This dimension defies linear reasoning and demands an interpretive stance one that accepts uncertainty as a structural feature of classroom life.

Non-local influences, often dismissed as background noise, emerge in quiet but decisive ways. A revised district policy might reshape a teacher's morning routine without direct confrontation. A national discourse on discipline might unconsciously alter how praise or correction is delivered. These influences operate beyond the classroom, yet find their way inside through decisions, attitudes, and limitations subtly embedded in daily routines.

When viewed together, these three forces do not merely coexist they interact in shifting configurations, constantly redefining what "management" means. On some days, locality takes the lead, with the teacher's voice guiding the room like a conductor. On others, uncertainty seeps in, muting the familiar rhythm. And on some, distant voices policies, cultures, expectations speak louder than anyone in the room.

Understanding classroom management through this triadic lens reframes the teacher not as a controller of order, but as a navigator of complexity, responding to visible patterns, sensing hidden tensions, and adapting to forces beyond immediate grasp. This is not a loss of authority it is a redefinition of professional mastery, grounded in awareness rather than control.

7. Cognitive Load and Indeterminacy in Real Classrooms

In the daily reality of a kindergarten classroom, teachers are required to make dozens of decisions within minutes—how to respond to a child's outburst, when to redirect attention, or whether to change a planned activity on the spot. This mental juggling creates what is known as cognitive load, the mental effort needed to process information, monitor actions, and plan responses in real time.

What the neutrosophic framework adds to this understanding is the recognition that indeterminacy often emerges precisely when cognitive load is high. In such moments, teachers may be aware of multiple possible actions, none of which feel fully appropriate or guaranteed to

succeed. They must act anyway, often relying on intuition, experience, or compromise. These are not failures of professionalism they are expressions of indeterminate judgment, which is a natural part of complex human systems.

Unlike structured environments where choices are limited and outcomes are predictable, the kindergarten setting is fluid and emotionally intense. A teacher might deal with simultaneous needs: a crying child, a noise escalation, and a lesson falling behind schedule. Here, decisions are not about choosing the "correct" option but about managing overlapping and partially understood conditions. This is where traditional models of teacher evaluation often fall short—they overlook the weight of decision-making under pressure and the reality of limited clarity.

Recognizing this, schools could rethink how they support teachers under cognitive strain. For example, instead of penalizing decisions that didn't lead to ideal results, reflection sessions could focus on why choices were made under uncertainty, and how similar situations might be navigated in the future. Over time, teachers can build a personal framework for acting under indeterminacy, turning apparent unpredictability into informed, adaptable practice.

Ultimately, this shift in thinking affirms that teaching is not a mechanical task but a cognitive-emotional performance. Understanding how indeterminacy and mental load interact gives us a clearer, more empathetic view of classroom life one that empowers teachers, rather than judging them for not being perfectly certain in an inherently uncertain environment.

8. Policy Implications of Non-Locality in Teacher Training

Educational policies are often crafted at a distance from the realities they seek to govern. Ministries, school boards, and curriculum developers create frameworks meant to standardize learning outcomes, ensure accountability, and promote consistency across institutions. However, from the perspective of partial non-locality, these policies exert influence in indirect but powerful ways shaping teacher decisions, limiting classroom flexibility, and altering daily practices, often without clear recognition of their presence.

For kindergarten teachers, whose work is rooted in immediate relationships with children, these non-local influences can create a silent tension. A mandate to adhere to a scripted curriculum, for instance, may restrict a teacher's ability to respond spontaneously to student needs. A new assessment system might shift focus from emotional development to academic tracking, even when children are not yet ready for formal evaluation. These are not visible interruptions, but rather policy shadows that stretch across classroom walls.

The neutrosophic model helps us see these forces not as fixed constraints, but as partial and variable, depending on how policies are interpreted, implemented, and absorbed by individual educators. Two teachers under the same policy may experience its weight differently one may feel liberated by structure, another stifled by rigidity. This variability is key, and it challenges the idea that top-down reforms produce uniform effects.

Recognizing partial non-locality in education encourages more flexible and context-aware training programs. Rather than presenting policies as static rules, teacher education could explore how external mandates interact with classroom realities in nuanced ways. Case studies,

simulations, and open discussions around policy tension can help new teachers develop critical agency the ability to interpret, adapt, and, when needed, question directives.

Moreover, policy designers themselves could benefit from feedback systems that trace the indirect effects of their decisions on classroom life. Instead of measuring only outcomes, they might also examine how policies affect teacher autonomy, emotional wellbeing, and perceived alignment with child development goals.

In essence, acknowledging partial non-locality in teacher training leads to a more reciprocal relationship between classrooms and policy-making. It promotes mutual understanding, reduces top-down rigidity, and fosters systems that are responsive, rather than prescriptive. And in doing so, it aligns educational reform more closely with the real needs of those who teach and the children they teach.

9. Conclusion

This study introduced a fresh perspective on kindergarten classroom management by framing it within a neutrosophic model that recognizes the layered influence of direct, uncertain, and external factors. The outcomes showed that while immediate teacher actions are essential, emotional unpredictability and institutional forces also shape outcomes in meaningful ways. Acknowledging these overlapping influences allows for a more realistic and compassionate understanding of the teaching experience. By embracing complexity rather than simplifying it, educators and policymakers can better support teachers in managing dynamic and demanding classroom environments.

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Muirhead Mean Operators in Quadripartitioned Single-Valued Neutrosophic Sets for Multi-Criteria Decision-Making with Indeterminate Information

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Abstract. This paper addresses the critical challenge of managing uncertainty, indeterminacy, and interdependencies in multi-criteria decision-making (MCDM) by introducing novel aggregation operators. Existing methods often fall short in effectively combining diverse and conflicting information, particularly in complex and uncertain environments. To bridge this gap, we propose the integration of the Muirhead Mean (MM) operator a versatile tool that generalizes arithmetic and geometric means-into the framework of Quaripartitioned Single-Valued Neutrosophic (QSVN) set theory, which accounts for truth, contradiction, ignorance, and falsity. The mathematical properties of the proposed operators are thoroughly analyzed, demonstrating their robustness and flexibility. A new decision-making process is developed to tackle uncertainty and interdependencies, supported by illustrative examples that highlight the practical significance of this approach. This work offers a substantial advancement in the design of dependable and adaptable decision-making frameworks for complex real-world scenarios.

1. Introduction

A common step in the decision-making process is picking the optimal choice from a variety of options, which can be quite difficult for academics studying fuzzy systems. Particularly when working with classical data, Multi-Attribute Decision-Making (MADM), a subset of decision-making techniques, is frequently used to determine the best choices from a range of preferences. Each alternative is traditionally given a rating value by decision-makers based on exact, binary information that is represented as 0 or 1. Real-world situations, however, are rarely as simple. Traditional methods are less successful since decision-making usually entails vague, uncertain, and imprecise information because of the environment's intrinsic complexity.

Decision-makers frequently give each choice a rating value, usually in the form of 0 or 1, based on exact data and information. But real-world situations are frequently more complicated, with vague, unclear, or imprecise information. Fuzzy set (FS) theory [1], intuitionistic fuzzy set (IFS) theory [2], interval-valued fuzzy sets (IVFS) [3], and interval-valued intuitionistic fuzzy sets (IVIFS) [4], which take into account both membership and non-membership values, are some of the mathematical theories that have been developed to address such situations. Many methods were then put forth by researchers, including medical diagnosis [15], pattern recognition [12]- [14], similarity measurements [9]- [11], and Multi-Attribute Decision-Making (MADM) [5]- [8]. However, such data, which entails reluctance and indeterminacy, cannot be handled by these theories.

Smarandache [16] developed the Neutrosophic Set (NS) for this purpose. Indeterminacy, non-membership, and membership values within the non-standard unit interval $(0, 1)$ are all addressed by NS theory. However, it is difficult to apply NS theory to practical problems because of this non-standard unit interval. Therefore, different types of NS were proposed to make things easier. The Single-Valued Neutrosophic set (SVNS) was proposed by Wang et al. [17], the Interval-Valued Neutrosophic set (IVNS) by Zhang et al. [18]. In MCDM situations, where the primary goal is to aggregate a group of inputs into a single number, Aggregation Operators (AOs) typically play a significant role. Ye [19] introduced the single-valued neutrosophic (SVN) weighted averaging (SVNWA) and SVN weighted geometric average (SVNWGA) operators in that approach, together with the operational rules of SVNNSs. The improved operations of SVN numbers (SVNNs) and their accompanying ordered weighted average/geometric aggregation operator were defined by Peng et al. [20]. Nancy and Garg [21] used the Frank norm procedures to create the weighted average and geometric average operators. Based on Hamacher operations, Liu et al. [22] created a few generalized neutrosophic aggregation operators.

The aggregation operations under the Interval Neutrosophic Set (INS) environment were introduced by Zhang et al. [24], and some of its generalized operators were suggested by Aiwu et al. [25]. A nonlinear optimization model was created by Garg and Nancy [23] to address the

MCDM problem in the context of the INS. Based to an analysis of the AOs stated above, all of these studies make the assumption that there is no association between the argument values because all of the input arguments used during aggregate are independent of one another. Nonetheless, there is always a suitable link between them in real-world issues. For example, there is a correlation between a home's price and its location if someone want to buy one. It is obvious that the two components interact and are dependent on one another. Neutrosophic systems have significantly enhanced the ability to manage uncertainty in decision-making processes. Building on these breakthroughs, this paper introduces an innovative approach that integrates the Muirhead Mean operator with QSVN sets, addressing the pressing need for reliable and flexible aggregation methods in MCDM scenarios

The Bonferroni Mean (BM) [26], Maclaurin symmetric mean (MSM) [27], Heronian Mean (HM) [28], and other helpful aggregation functions are used to take into account the interdependence of the input arguments. The primary feature of BM, as stated by Yager [29], is its capacity to record the interaction between the input arguments. BM aggregation operators were introduced by Garg and Arora [30] in the intuitionistic fuzzy soft set setting. While some of these functions can capture more than two relationships, BM can only record the relationship between two arguments. Liu and Wang [31] extended the BM to a neutrosophic environment and introduced the SVN normalized weighted Bonferroni mean (SVNNWBM) operator, utilizing the benefits of these functions in a neutrosophic domain. The MSM aggregation operators were suggested by Wang et al. [33] in order to capture the correlation between the aggregated arguments. HM operators were introduced by Li et al. [32] to address the MCDM issues in an SVNS setting. Prioritized AOs were introduced by Garg and Nancy [34] in the linguistic SVNS context to address the decision-making issues. Some prioritized weighted averaging and geometric aggregation procedures for SVNNs were created by Wu et al. [35]. Using the Frank operations, Ji et al. [36] developed the single-valued prioritized BM operator. The Muirhead mean (MM) [37], a potent and practical aggregation method, is an alternative to these aggregations. The main benefit of the MM is its ability to take into account the connections between all of the arguments, which makes it more potent and thorough than BM, MSM, and HM. Furthermore, MM has a parameter vector that can increase the flexibility of the aggregation process. We might infer from the aforementioned analysis that real-world decision-making challenges are become increasingly complex. A more advantageous method of expressing the uncertain information is required in order to choose the best alternative or alternatives for the MCDM challenges. Furthermore, it is crucial to address the issue of how to take into account the connections among input arguments. In order to integrate the prioritized aggregation and MM and propose the prioritized MM (PMM) operator, we take into account the benefits of the SVNS while keeping all these aspects in mind. While there

have been several distance measurements for SVN sets proposed recently, there are currently few references. A real-world case was examined by Chatterjee et al. [38] in order to gain a better understanding of the QSVNS environment, and the results showed that such situations are common. They proved QSVNS's skills by resolving a pattern recognition decision-making task. The paper [39] examines a multi-criteria decision-making model that uses trigonometric aggregation operations of single-valued neutrosophic credibility numbers.

2. Quadripartitioned Neutrosophic Muirhead Mean Operators

Definition 2.1. A Quadripartitioned Neutrosophic Set (QNS) λ comprises of the four independent degrees in particular truth degree A_λ , contradiction degree B_λ , ignorance degree C_λ and false degree D_λ which are defined as $\lambda = \{(u, A_\lambda(u), B_\lambda(u), C_\lambda(u), D_\lambda(u)) | u \in U\}$, where $A_\lambda(u), B_\lambda(u), C_\lambda(u), D_\lambda(u)$ is the subset of the non-standard unit interval $(0^-, 1^+)$, such that $0^- \leq A_\lambda(u) + B_\lambda(u) + C_\lambda(u) + D_\lambda(u) \leq 4^+$

Definition 2.2. A Single - Valued Quadripartitioned Neutrosophic Set (SQNS) λ in U are defined as $\lambda = \{(u, A_\lambda(u), B_\lambda(u), C_\lambda(u), D_\lambda(u)) | u \in U\}$, where $A_\lambda(u), B_\lambda(u), C_\lambda(u), D_\lambda(u) \in [0, 1]$ such that $(0 \leq A_\lambda(u) + B_\lambda(u) + C_\lambda(u) + D_\lambda(u) \leq 4$ for all $u \in U$.

Definition 2.3. Let $\lambda = (A_\lambda, B_\lambda, C_\lambda, D_\lambda)$ be a SQNN. A score function s of λ is defined as

$$s(\lambda) = \frac{A_\lambda + (1 - B_\lambda) + (1 - C_\lambda) + D_\lambda}{4}$$

Definition 2.4. Let $\lambda = \{A, B, C, D\}$, $\lambda_1 = \{A_1, B_1, C_1, D_1\}$ and $\lambda_2 = \{A_2, B_2, C_2, D_2\}$ be three SQNN and $\alpha > 0$ be real number. Then, we have

- (I) $\lambda^c = \{D, B, C, A\}$
- (II) $\lambda_1 \leq \lambda_2$ if $A_1 \leq A_2, B_1 \geq B_2, C_1 \geq C_2$ and $D_1 \geq D_2$
- (III) $\lambda_1 = \lambda_2$ iff $\lambda_1 \leq \lambda_2$ and $\lambda_2 \leq \lambda_1$
- (IV) $\lambda_1 \cap \lambda_2 = \{\min(A_1, A_2), \max(B_1, B_2), \max(C_1, C_2), \max(D_1, D_2)\}$
- (V) $\lambda_1 \cup \lambda_2 = \{\max(A_1, A_2), \min(B_1, B_2), \min(C_1, C_2), \min(D_1, D_2)\}$
- (VI) $\lambda_1 \otimes \lambda_2 = \{A_1 A_2, B_1 + B_2 - B_1 B_2, C_1 + C_2 - C_1 C_2, D_1 + D_2 - D_1 D_2\}$
- (VII) $\lambda_1 \oplus \lambda_2 = \{A_1 + A_2 - A_1 A_2, B_1 B_2, C_1 C_2, D_1 D_2\}$
- (VIII) $\alpha \lambda_1 = (1 - (1 - A_1)^\alpha, B_1^\alpha, C_1^\alpha, D_1^\alpha)$
- (XI) $\lambda_1^\alpha = (A_1^\alpha, 1 - (1 - B_1)^\alpha, 1 - (1 - C_1)^\alpha, 1 - (1 - D_1)^\alpha)$.

Definition 2.5. The prioritized weighted aggregation operators are defined as

(I) SQN prioritized weighted average (SQNPWA) ΔW

$$\Delta W(\lambda_1, \lambda_2, \dots, \lambda_n) = \left\{ 1 - \prod_{l=1}^p (1 - A_l)^{\frac{\mathbb{H}_l}{\sum_{l=1}^p \mathbb{H}_l}}, \prod_{l=1}^p (B_l)^{\frac{\mathbb{H}_l}{\sum_{l=1}^p \mathbb{H}_l}}, \prod_{l=1}^p (C_l)^{\frac{\mathbb{H}_l}{\sum_{l=1}^p \mathbb{H}_l}}, \prod_{l=1}^p (D_l)^{\frac{\mathbb{H}_l}{\sum_{l=1}^p \mathbb{H}_l}} \right\}$$

(II) SQN prioritized geometric average (SQNPGA) ΔG

$$\Delta G(\lambda_1, \lambda_2, \dots, \lambda_n) = \left\{ \prod_{l=1}^p (A_l)^{\frac{\mathbb{H}_l}{\sum_{l=1}^p \mathbb{H}_l}}, 1 - \prod_{l=1}^p (1 - B_l)^{\frac{\mathbb{H}_l}{\sum_{l=1}^p \mathbb{H}_l}}, 1 - \prod_{l=1}^p (1 - C_l)^{\frac{\mathbb{H}_l}{\sum_{l=1}^p \mathbb{H}_l}}, 1 - \prod_{l=1}^p (1 - D_l)^{\frac{\mathbb{H}_l}{\sum_{l=1}^p \mathbb{H}_l}} \right\}$$

where $\mathbb{H}_1 = 1$ and $\mathbb{H}_l = \prod_{k=1}^{l-1} s(\lambda_k), (l = 2, 3, \dots, p)$

Definition 2.6. For a non-negative real number r_l ($l = 1, 2, \dots, n$), (MM) operator over the parameter $\mathbb{Q} = \{q_1, q_2, \dots, q_p\} \in R^p$ is defined as

$$MM^{\mathbb{Q}}(q_1, q_2, \dots, q_p) = \left(\frac{1}{p!} \sum_{\delta \in S_p} \prod_{l=1}^p r_{\delta(l)}^{q_l} \right)^{\frac{1}{\sum_{l=1}^p q_l}}$$

where δ is the permutation of $(1, 2, \dots, p)$ and S_p is set of all permutation of $(1, 2, \dots, p)$.

Some assigning some vectors to $\mathbb{Q}$, we can obtain some special cases of the MM.

1. If $\mathbb{Q} = (1, 0, \dots, 0)$ the MM is reduced to

$MM^{(1,0,\dots,0)}(q_1, q_2, \dots, q_p) = \frac{1}{p} \sum_{l=1}^p q_l$ which is the arithmetic averaging operator.

2. If $\mathbb{Q} = (\frac{1}{p}, \frac{1}{p}, \dots, \frac{1}{p})$, the MM is reduced to

$$MM^{\frac{1}{p}, \frac{1}{p}, \dots, \frac{1}{p}}(q_1, q_2, \dots, q_p) = \prod_{l=1}^p q_l^{\frac{1}{p}}$$

which is the geometric operator.

3. If $\mathbb{Q} = (1, 1, 0, 0, \dots, 0)$ then the MM reduced to

$MM^{1,1,0,0,\dots,0}$ then the MM is reduced to

$$MM^{(1,1,0,0,\dots,0)}(q_1, q_2, \dots, q_p) = \left(\frac{1}{p(p+1)} \sum_{m,l=1, m \neq l}^p r_m r_l \right)^{\frac{1}{2}}$$

which is the BM operator [26].

4. If $Q = (\overbrace{1, 1, \dots, 1}^k, \overbrace{0, 0, \dots, 0}^{p-k})$, then the MM is reduced to

$$MM^{(\overbrace{1, 1, \dots, 1}^k, \overbrace{0, 0, \dots, 0}^{p-k})}(r_1, r_2, \dots, r_p) = \left(\frac{1}{C_k^p} \sum_{1 \leq m_1 < \dots < m_k \leq p} \prod_{l=1}^k r_{m_l} \right)^{\frac{1}{k}}$$

which is the MSM operator [27]

3. Quadripartitioned Neutrosophic Prioritized Muirhead Mean Operators

Definition 3.1. For a collection of SQNNs $\lambda_l = (1, 2, \dots, p)$ a SQNPMM operator is a mapping SQNPMM: $\Omega \rightarrow \Omega$ defined as

$$SQNPMM(\lambda_1, \lambda_2, \dots, \lambda_p) = \left(\frac{1}{p!} \oplus_{\delta \in S_p} \prod_{l=1}^p \left(p \frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l} \lambda_{\delta(l)} \right)^{q_l} \right)^{\frac{1}{\sum_{l=1}^p q_l}} \quad (1)$$

where $\mathbb{H}_1 = 1$, $\mathbb{H}_l = \prod_{k=1}^{l-1} s(\lambda_k)$, $(l = 2, 3, \dots, p)$, S_p is the collection of all permutations of $(1, 2, \dots, p)$ and $\mathbb{Q} = (q_1, q_2, \dots, q_p) \in R^p$ be a vector parameters.

Theorem 3.2. For collection of SQNNs $\lambda_l = (A_l, B_l, C_l, D_l)$ ($l = 1, 2, \dots, p$) the aggregated value through equation 1 is again a SQNN and is given by

$$\begin{aligned} QSPMM(\lambda_1, \lambda_2, \dots, \lambda_n) &= \left(\left(1 - \left(\prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - (1 - A_{\delta(l)})^{\frac{p \frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}}{\sum_{l=1}^p q_l}} \right) \right)^{\frac{1}{p!}} \right) \right)^{\frac{1}{\sum_{l=1}^p q_l}}, \right. \\ &\quad \left. 1 - \left(1 - \left(\prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - (1 - B_{\delta(l)})^{\frac{p \frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}}{\sum_{l=1}^p q_l}} \right) \right)^{\frac{1}{p!}} \right) \right)^{\frac{1}{\sum_{l=1}^p q_l}}, \right. \\ &\quad \left. 1 - \left(1 - \left(\prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - (1 - C_{\delta(l)})^{\frac{p \frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}}{\sum_{l=1}^p q_l}} \right) \right)^{\frac{1}{p!}} \right) \right)^{\frac{1}{\sum_{l=1}^p q_l}}, \right. \\ &\quad \left. 1 - \left(1 - \left(\prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - (1 - D_{\delta(l)})^{\frac{p \frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}}{\sum_{l=1}^p q_l}} \right) \right)^{\frac{1}{p!}} \right) \right)^{\frac{1}{\sum_{l=1}^p q_l}} \right). \quad (2) \end{aligned}$$

Proof. For SQNN $\lambda_l (l = 1, 2, \dots, p)$, we have

$$p \frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l} \lambda_{\delta(l)} = \left(1 - (1 - A_{\delta(l)})^{\frac{p \frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}}{\sum_{l=1}^p q_l}}, B_{\delta(l)}^{\frac{p \frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}}{\sum_{l=1}^p q_l}}, C_{\delta(l)}^{\frac{p \frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}}{\sum_{l=1}^p q_l}}, D_{\delta(l)}^{\frac{p \frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}}{\sum_{l=1}^p q_l}} \right)$$

and

$$\left(p \frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l} \lambda_{\delta(l)}\right)^{q_l} = \left(1 - \left(1 - \mathbf{A}_{\delta(l)}\right)^{\frac{p \frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}}{p}}\right)^{q_l}, 1 - \left(1 - \mathbf{B}_{\delta(l)}\right)^{\frac{p \frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}}{p}}\right)^{q_l},$$

$$1 - \left(1 - \mathbf{C}_{\delta(l)}\right)^{\frac{p \frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l} \lambda_{\delta(l)}}{p}}\right)^{q_l}, 1 - \left(1 - \mathbf{D}_{\delta(l)}\right)^{\frac{p \frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}}{p}}\right)^{q_l}.$$

Thus,

$$\begin{aligned} \oplus_{\delta \in S_p} \prod_{l=1}^p \left(p \frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l} \lambda_{\delta(l)}\right)^{q_l} &= \left\{1 - \prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - \left(1 - \mathbf{A}_{\delta(l)}\right)^{\frac{p \frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}}{p}}\right)^{q_l}\right)\right\}, \\ &= \prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - \mathbf{B}_{\delta(l)}\right)^{\frac{p \frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}}{p}}\right)^{q_l} \\ &= \prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - \mathbf{C}_{\delta(l)}\right)^{\frac{p \frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}}{p}}\right)^{q_l} \\ &= \prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - \mathbf{D}_{\delta(l)}\right)^{\frac{p \frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}}{p}}\right)^{q_l} \Big\}. \end{aligned}$$

Now,

$$SQNPMM(\lambda_1, \lambda_2, \dots, \lambda_p) = \left(\frac{1}{p!} \oplus_{\delta \in S_p} \prod_{l=1}^p \left(p \frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l} \lambda_{\delta(l)}\right)^{q_l}\right)^{\frac{1}{\sum_{l=1}^p q_l}}$$

$$\begin{aligned}
&= \left(\left(1 - \left(\prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - (1 - \mathbf{A}_{\delta(l)})^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{q_l} \right) \right)^{\frac{1}{p!}} \right)^{\frac{1}{\sum_{l=1}^p q_l}}, \\
&1 - \left(1 - \left(\prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - (1 - \mathbf{B}_{\delta(l)})^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{q_l} \right) \right)^{\frac{1}{p!}} \right)^{\frac{1}{\sum_{l=1}^p q_l}}, \\
&1 - \left(1 - \left(\prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - (1 - \mathbf{C}_{\delta(l)})^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{q_l} \right) \right)^{\frac{1}{p!}} \right)^{\frac{1}{\sum_{l=1}^p q_l}}, \\
&1 - \left(1 - \left(\prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - (1 - \mathbf{D}_{\delta(l)})^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{q_l} \right) \right)^{\frac{1}{p!}} \right)^{\frac{1}{\sum_{l=1}^p q_l}}
\end{aligned}$$

Thus, the equation 2 holds. Further, $0 \leq \mathbf{A}_{\delta(l)}, \mathbf{B}_{\delta(l)}, \mathbf{C}_{\delta(l)}, \mathbf{D}_{\delta(l)} \leq 1$ so we have

$$1 - (1 - \mathbf{A}_{\delta(l)})^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \in [0, 1]$$

and

$$\prod_{l=1}^p \left(1 - (1 - \mathbf{A}_{\delta(l)})^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{q_l} \in [0, 1]$$

which implies that

$$1 - \left(\prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - (1 - \mathbf{A}_{\delta(l)})^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{q_l} \right) \right) \in [0, 1]$$

Hence

$$0 \leq \left(1 - \left(\prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - (1 - \mathbf{A}_{\delta(l)})^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{q_l} \right) \right)^{\frac{1}{p!}} \right)^{\frac{1}{\sum_{l=1}^p q_l}} \leq 1.$$

similarly, we have

$$0 \leq 1 - \left(1 - \prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - \mathbb{B}_{\delta(l)} \right)^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{\frac{1}{p^l}} \right)^{\frac{1}{\sum_{l=1}^p q_l}} \leq 1.$$

$$0 \leq 1 - \left(1 - \prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - \mathbb{C}_{\delta(l)} \right)^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{\frac{1}{p^l}} \right)^{\frac{1}{\sum_{l=1}^p q_l}} \leq 1.$$

$$0 \leq 1 - \left(1 - \prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - \mathbb{D}_{\delta(l)} \right)^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{\frac{1}{p^l}} \right)^{\frac{1}{\sum_{l=1}^p q_l}} \leq 1.$$

which complete the proof. $\square$

Example 3.3. Take let $\lambda_1 = (0.4, 0.6, 0.5, 0.3)$, $\lambda_2 = (0.4, 0.7, 0.4, 0.7)$, $\lambda_3 = (0.6, 0.4, 0.3, 0.7)$ here be QSNs and $\mathbb{Q} = (1, 0.6, 0.3)$ be the given parameter vector. By utilizing the given information and $\mathbb{H}_l = \prod_{k=1}^{l-1} s(\lambda_k); (j=2,3)$, we get $\mathbb{H}_1 = 1$, $\mathbb{H}_2 = 0.4$ and $\mathbb{H}_3 = 0.16$. Therefore

$$\begin{aligned} & \prod_{\delta \in S_3} \left(1 - \prod_{l=1}^3 \left(1 - (1 - \mathbb{A}_{\delta(l)})^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^3 \mathbb{H}_l}} \right)^{q_l} \right) \\ &= \left\{ 1 - (1 - (1 - 0.4)^{3 \times 0.6410})^1 \times (1 - (1 - 0.4)^{3 \times 0.2564})^{0.6} \times (1 - (1 - 0.6)^{3 \times 0.1026})^{0.3} \right\} \\ &\times \left\{ 1 - (1 - (1 - 0.4)^{3 \times 0.2564})^1 \times (1 - (1 - 0.4)^{3 \times 0.6410})^{0.6} \times (1 - (1 - 0.6)^{3 \times 0.1026})^{0.3} \right\} \\ &\times \left\{ 1 - (1 - (1 - 0.6)^{3 \times 0.1026})^1 \times (1 - (1 - 0.4)^{3 \times 0.2564})^{0.6} \times (1 - (1 - 0.4)^{3 \times 0.6410})^{0.3} \right\} \\ &\times \left\{ 1 - (1 - (1 - 0.4)^{3 \times 0.2564})^1 \times (1 - (1 - 0.6)^{3 \times 0.1026})^{0.6} \times (1 - (1 - 0.4)^{3 \times 0.6410})^{0.3} \right\} \\ &\times \left\{ 1 - (1 - (1 - 0.4)^{3 \times 0.6410})^1 \times (1 - (1 - 0.6)^{3 \times 0.1026})^{0.6} \times (1 - (1 - 0.4)^{3 \times 0.2564})^{0.3} \right\} \\ &\times \left\{ 1 - (1 - (1 - 0.6)^{3 \times 0.1026})^1 \times (1 - (1 - 0.4)^{3 \times 0.6410})^{0.6} \times (1 - (1 - 0.4)^{3 \times 0.2564})^{0.3} \right\} \\ &= 0.3638 \end{aligned}$$

$$\prod_{\delta \in S_3} \left(1 - \prod_{l=1}^3 \left(1 - \mathbb{B}_{\delta(l)} \right)^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^3 \mathbb{H}_l}} \right)^{q_l}$$

$$\begin{aligned}
&= \left\{ 1 - (1 - (0.6)^{3 \times 0.6410})^1 \times (1 - (0.4)^{3 \times 0.2564})^{0.6} \times (1 - (0.4)^{3 \times 0.1026})^{0.3} \right\} \\
&\times \left\{ 1 - (1 - (0.7)^{3 \times 0.2564})^1 \times (1 - (0.6)^{3 \times 0.6410})^{0.6} \times (1 - (0.4)^{3 \times 0.1026})^{0.3} \right\} \\
&\times \left\{ 1 - (1 - (0.4)^{3 \times 0.1026})^1 \times (1 - (0.7)^{3 \times 0.2564})^{0.6} \times (1 - (0.6)^{3 \times 0.6410})^{0.3} \right\} \\
&\times \left\{ 1 - (1 - (0.7)^{3 \times 0.2564})^1 \times (1 - (0.4)^{3 \times 0.1026})^{0.6} \times (1 - (0.6)^{3 \times 0.6410})^{0.3} \right\} \\
&\times \left\{ 1 - (1 - (0.6)^{3 \times 0.6410})^1 \times (1 - (0.4)^{3 \times 0.1026})^{0.6} \times (1 - (0.7)^{3 \times 0.2564})^{0.3} \right\} \\
&\times \left\{ 1 - (1 - (0.4)^{3 \times 0.1026})^1 \times (1 - (0.6)^{3 \times 0.6410})^{0.6} \times (1 - (0.7)^{3 \times 0.2564})^{0.3} \right\} \\
&= 0.3843
\end{aligned}$$

$$\prod_{\delta \in S_3} \left(1 - \prod_{l=1}^3 \left(1 - \mathbf{c}_{\delta(l)} \right)^{\frac{\mathbb{H}_{\delta(l)}}{3 \sum_{l=1}^3 \mathbb{H}_l}} \right)^{q_l}$$

$$\begin{aligned}
&= \left\{ 1 - (1 - (0.5)^{3 \times 0.6410})^1 \times (1 - (0.4)^{3 \times 0.2564})^{0.6} \times (1 - (0.3)^{3 \times 0.1026})^{0.3} \right\} \\
&\times \left\{ 1 - (1 - (0.4)^{3 \times 0.2564})^1 \times (1 - (0.5)^{3 \times 0.6410})^{0.6} \times (1 - (0.3)^{3 \times 0.1026})^{0.3} \right\} \\
&\times \left\{ 1 - (1 - (0.3)^{3 \times 0.1026})^1 \times (1 - (0.4)^{3 \times 0.2564})^{0.6} \times (1 - (0.5)^{3 \times 0.6410})^{0.3} \right\} \\
&\times \left\{ 1 - (1 - (0.4)^{3 \times 0.2564})^1 \times (1 - (0.3)^{3 \times 0.1026})^{0.6} \times (1 - (0.5)^{3 \times 0.6410})^{0.3} \right\} \\
&\times \left\{ 1 - (1 - (0.5)^{3 \times 0.6410})^1 \times (1 - (0.3)^{3 \times 0.1026})^{0.6} \times (1 - (0.4)^{3 \times 0.2564})^{0.3} \right\} \\
&\times \left\{ 1 - (1 - (0.3)^{3 \times 0.1026})^1 \times (1 - (0.5)^{3 \times 0.6410})^{0.6} \times (1 - (0.4)^{3 \times 0.2564})^{0.3} \right\} \\
&= 0.1607
\end{aligned}$$

and

$$\prod_{\delta \in S_3} \left(1 - \prod_{l=1}^3 \left(1 - \mathbf{d}_{\delta(l)} \right)^{\frac{\mathbb{H}_{\delta(l)}}{3 \sum_{l=1}^3 \mathbb{H}_l}} \right)^{q_l}$$

$$\begin{aligned}
&= \left\{ 1 - (1 - (0.3)^{3 \times 0.6410})^1 \times (1 - (0.7)^{3 \times 0.2564})^{0.6} \times (1 - (0.7)^{3 \times 0.1026})^{0.3} \right\} \\
&\times \left\{ 1 - (1 - (0.7)^{3 \times 0.2564})^1 \times (1 - (0.3)^{3 \times 0.6410})^{0.6} \times (1 - (0.7)^{3 \times 0.1026})^{0.3} \right\} \\
&\times \left\{ 1 - (1 - (0.7)^{3 \times 0.1026})^1 \times (1 - (0.7)^{3 \times 0.2564})^{0.6} \times (1 - (0.3)^{3 \times 0.6410})^{0.3} \right\} \\
&\times \left\{ 1 - (1 - (0.4)^{3 \times 0.2564})^1 \times (1 - (0.7)^{3 \times 0.1026})^{0.6} \times (1 - (0.7)^{3 \times 0.6410})^{0.3} \right\} \\
&\times \left\{ 1 - (1 - (0.7)^{3 \times 0.6410})^1 \times (1 - (0.7)^{3 \times 0.1026})^{0.6} \times (1 - (0.4)^{3 \times 0.2564})^{0.3} \right\} \\
&\times \left\{ 1 - (1 - (0.7)^{3 \times 0.1026})^1 \times (1 - (0.7)^{3 \times 0.6410})^{0.6} \times (1 - (0.4)^{3 \times 0.2564})^{0.3} \right\} \\
&= 0.5200
\end{aligned}$$

Hence, by using Equation(2), we get the aggregated value by SQNPMM is

$$\begin{aligned}
&SQNPMM(\lambda_1, \lambda_2, \lambda_3) \\
&= \left\{ (1 - (0.3638)^{\frac{1}{8}})^{\frac{1}{1.9}}, 1 - (1 - (0.3843)^{\frac{1}{8}})^{\frac{1}{1.9}}, 1 - (1 - (0.1607)^{\frac{1}{8}})^{\frac{1}{1.9}}, 1 - (1 - (0.5200)^{\frac{1}{8}})^{\frac{1}{1.9}} \right\} \\
&= (0.3260, 0.6563, 0.5663, 0.7378)
\end{aligned}$$

Theorem 3.4. If $\lambda_l = (A_l, B_l, C_l, D_l)$ and $\lambda'_l = (A'_l, B'_l, C'_l, D'_l)$ are two SQNNs such that $A_l \leq A'_l$, $B_l \geq B'_l$, $C_l \geq C'_l$ and $D_l \geq D'_l$ for all l , then

$$SQPMM(\lambda_1, \lambda_2, \dots, \lambda_n) \leq SQPMM(\lambda'_1, \lambda'_2, \dots, \lambda'_n)$$

This property is called monotonicity.

Proof. For two $(1 - A_{\delta(l)})^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \geq (1 - A'_{\delta(l)})^{\frac{\mathbb{H}'_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}'_l}}$ where $\mathbb{H}_1 = 1$, $\mathbb{H}_l = \prod_{k=1}^{l-1} s(\lambda_k)$, $(l = 2, 3, \dots, p)$, $\mathbb{H}'_1 = 1$, $\mathbb{H}'_l = \prod_{k=1}^{l-1} s(\lambda'_k)$, $(l = 2, 3, \dots, p)$

$$(1 - (1 - A_{\delta(l)})^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}})^{q_l} \leq (1 - (1 - A'_{\delta(l)})^{\frac{\mathbb{H}'_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}'_l}})^{q_l} \text{ and}$$

$$\prod_{l=1}^p \left(1 - (1 - A_{\delta(l)})^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{q_l} \leq \prod_{l=1}^p \left(1 - (1 - A'_{\delta(l)})^{\frac{\mathbb{H}'_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}'_l}} \right)^{q_l}$$

$$\text{Further, we have } \prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - (1 - \mathbf{A}_{\delta(l)})^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{q_l} \right) \geq \prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - (1 - \mathbf{A}'_{\delta(l)})^{\frac{\mathbb{H}'_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}'_l}} \right)^{q_l} \right)$$

$$\left(\prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - (1 - \mathbf{A}_{\delta(l)})^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{q_l} \right) \right)^{\frac{1}{p!}} \geq \left(\prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - (1 - \mathbf{A}'_{\delta(l)})^{\frac{\mathbb{H}'_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}'_l}} \right)^{q_l} \right) \right)^{\frac{1}{p!}}.$$

Hence, we get

$$\left(1 - \left(\prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - (1 - \mathbf{A}_{\delta(l)})^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{q_l} \right) \right)^{\frac{1}{p!}} \right)^{\frac{1}{\sum_{l=1}^p q_l}} \geq \left(1 - \left(\prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - (1 - \mathbf{A}'_{\delta(l)})^{\frac{\mathbb{H}'_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}'_l}} \right)^{q_l} \right) \right)^{\frac{1}{p!}} \right)^{\frac{1}{\sum_{l=1}^p q_l}}$$

Similarly, we have

$$1 - \left(1 - \prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - \mathbf{B}_{\delta(l)}^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{q_l} \right) \right)^{\frac{1}{p!}} \geq 1 - \left(1 - \prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - \mathbf{B}'_{\delta(l)}^{\frac{\mathbb{H}'_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}'_l}} \right)^{q_l} \right) \right)^{\frac{1}{p!}}$$

$$1 - \left(1 - \prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - \mathbf{C}_{\delta(l)}^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{q_l} \right) \right)^{\frac{1}{p!}} \geq 1 - \left(1 - \prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - \mathbf{C}'_{\delta(l)}^{\frac{\mathbb{H}'_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}'_l}} \right)^{q_l} \right) \right)^{\frac{1}{p!}}$$

and

$$1 - \left(1 - \prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - \mathbf{D}_{\delta(l)}^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{q_l} \right) \right)^{\frac{1}{p!}} \geq 1 - \left(1 - \prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - \mathbf{D}'_{\delta(l)}^{\frac{\mathbb{H}'_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}'_l}} \right)^{q_l} \right) \right)^{\frac{1}{p!}}$$

. Therefore, by definition 2.4, we have

$$SQNPMM(\lambda_1, \lambda_2, \dots, \lambda_n) \leq SQNPMM(\lambda'_1, \lambda'_2, \dots, \lambda'_n)$$

□

4. Quadripartitioned Neutrosophic Prioritized Dual Muirhead Mean Operators

Definition 4.1. A SQNPDMM operator is a mapping SQNPDMM : $\Omega^p \rightarrow \Omega$ given by

Satham Hussain S, Durga Nagarajan, Said Broumi, Sankar Sahoo, Hossein Rashmanlou, Muirhead Mean Operators in Quadripartitioned Single-Valued Neutrosophic Sets for Multi-Criteria Decision-Making with Indeterminate Information

$$SQNPDMM(\lambda_1, \lambda_2, \dots, \lambda_p) = \left(\frac{1}{\sum_{l=1}^p p_l} \left(\prod_{\delta \in S_p} \bigoplus_{l=1}^p \left(p_l \lambda_{\delta(l)} \right)^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{\frac{1}{p!}} \right) \quad (3)$$

Theorem 4.2. The collective value by using Equation 4.1 is still a SQNN and is given as

$$\begin{aligned} & SQPDMM(\lambda_1, \lambda_2, \dots, \lambda_n) \\ &= \left(1 - \left(1 - \left(\prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - \mathbf{A}_{\delta(l)} \right)^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{q_l} \right)^{\frac{1}{p!}} \right)^{\frac{1}{\sum_{l=1}^p q_l}}, \right. \\ & \quad \left(1 - \left(\prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - (1 - \mathbf{B}_{\delta(l)}) \right)^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{q_l} \right)^{\frac{1}{p!}} \right)^{\frac{1}{\sum_{l=1}^p q_l}}, \\ & \quad \left(1 - \left(\prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - (1 - \mathbf{C}_{\delta(l)}) \right)^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{q_l} \right)^{\frac{1}{p!}} \right)^{\frac{1}{\sum_{l=1}^p q_l}}, \\ & \quad \left. \left(1 - \left(\prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - (1 - \mathbf{D}_{\delta(l)}) \right)^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{q_l} \right)^{\frac{1}{p!}} \right)^{\frac{1}{\sum_{l=1}^p q_l}} \right). \quad (4) \end{aligned}$$

Proof. The proof of this result follows from Theorem 3.2. $\square$

Example 4.3. If we have taken the data as considered in Example 3.3 to illustrate the aggregation operator as defined in Theorem 4.2 then, we have

$$\prod_{\delta \in S_3} \left(1 - \prod_{l=1}^3 \left(1 - \mathbf{A}_{\delta(l)} \right)^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^3 \mathbb{H}_l}} \right)^{q_l}$$

$$\begin{aligned}
&= \left\{ 1 - (1 - (0.4)^{3 \times 0.6410})^1 \times (1 - (0.4)^{3 \times 0.2564})^{0.6} \times (1 - (0.6)^{3 \times 0.1026})^{0.3} \right\} \\
&= \left\{ 1 - (1 - (0.4)^{3 \times 0.2564})^1 \times (1 - (0.4)^{3 \times 0.6410})^{0.6} \times (1 - (0.6)^{3 \times 0.1026})^{0.3} \right\} \\
&= \left\{ 1 - (1 - (0.6)^{3 \times 0.1026})^1 \times (1 - (0.4)^{3 \times 0.2546})^{0.6} \times (1 - (0.4)^{3 \times 0.6410})^{0.3} \right\} \\
&= \left\{ 1 - (1 - (0.4)^{3 \times 0.2564})^1 \times (1 - (0.6)^{3 \times 0.1026})^{0.6} \times (1 - (0.4)^{3 \times 0.6410})^{0.3} \right\} \\
&= \left\{ 1 - (1 - (0.4)^{3 \times 0.6410})^1 \times (1 - (0.6)^{3 \times 0.1026})^{0.6} \times (1 - (0.4)^{3 \times 0.2564})^{0.3} \right\} \\
&= \left\{ 1 - (1 - (0.6)^{3 \times 0.1026})^1 \times (1 - (0.4)^{3 \times 0.6410})^{0.6} \times (1 - (0.4)^{3 \times 0.2564})^{0.3} \right\} \\
&= 0.2806
\end{aligned}$$

Similarly, we have

$$\prod_{\delta \in S_3} \left(1 - \prod_{l=1}^3 \left(1 - (1 - B_{\delta(l)})^{3 \frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^3 \mathbb{H}_l}} \right)^{q_l} \right)$$

$$\begin{aligned}
&= \left\{ 1 - (1 - (1 - 0.6)^{3 \times 0.6410})^1 \times (1 - (1 - 0.7)^{3 \times 0.2564})^{0.6} \times (1 - (1 - 0.4)^{3 \times 0.1026})^{0.3} \right\} \\
&= \left\{ 1 - (1 - (1 - 0.7)^{3 \times 0.2564})^1 \times (1 - (1 - 0.6)^{3 \times 0.6410})^{0.6} \times (1 - (1 - 0.4)^{3 \times 0.1026})^{0.3} \right\} \\
&= \left\{ 1 - (1 - (1 - 0.4)^{3 \times 0.1026})^1 \times (1 - (1 - 0.7)^{3 \times 0.2564})^{0.6} \times (1 - (1 - 0.6)^{3 \times 0.6410})^{0.3} \right\} \\
&= \left\{ 1 - (1 - (1 - 0.7)^{3 \times 0.2564})^1 \times (1 - (1 - 0.4)^{3 \times 0.1026})^{0.6} \times (1 - (1 - 0.6)^{3 \times 0.6410})^{0.3} \right\} \\
&= \left\{ 1 - (1 - (1 - 0.6)^{3 \times 0.6410})^1 \times (1 - (1 - 0.4)^{3 \times 0.1026})^{0.6} \times (1 - (1 - 0.7)^{3 \times 0.2564})^{0.3} \right\} \\
&= \left\{ 1 - (1 - (1 - 0.4)^{3 \times 0.1026})^1 \times (1 - (1 - 0.6)^{3 \times 0.6410})^{0.6} \times (1 - (1 - 0.7)^{3 \times 0.2564})^{0.3} \right\}. \\
&= 0.2327
\end{aligned}$$

$$\prod_{\delta \in S_3} \left(1 - \prod_{l=1}^3 \left(1 - (1 - C_{\delta(l)})^{3 \frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^3 \mathbb{H}_l}} \right)^{q_l} \right)$$

$$\begin{aligned}
&= \left\{ 1 - (1 - (1 - 0.5)^{3 \times 0.6410})^1 \times (1 - (1 - 0.4)^{3 \times 0.2564})^{0.6} \times (1 - (1 - 0.3)^{3 \times 0.1026})^{0.3} \right\} \\
&= \left\{ 1 - (1 - (1 - 0.4)^{3 \times 0.2564})^1 \times (1 - (1 - 0.5)^{3 \times 0.6410})^{0.6} \times (1 - (1 - 0.3)^{3 \times 0.1026})^{0.3} \right\} \\
&= \left\{ 1 - (1 - (1 - 0.3)^{3 \times 0.1026})^1 \times (1 - (1 - 0.4)^{3 \times 0.2564})^{0.6} \times (1 - (1 - 0.5)^{3 \times 0.6410})^{0.3} \right\} \\
&= \left\{ 1 - (1 - (1 - 0.4)^{3 \times 0.2564})^1 \times (1 - (1 - 0.3)^{3 \times 0.1026})^{0.6} \times (1 - (1 - 0.5)^{3 \times 0.6410})^{0.3} \right\} \\
&= \left\{ 1 - (1 - (1 - 0.5)^{3 \times 0.6410})^1 \times (1 - (1 - 0.3)^{3 \times 0.1026})^{0.6} \times (1 - (1 - 0.4)^{3 \times 0.2564})^{0.3} \right\} \\
&= \left\{ 1 - (1 - (1 - 0.3)^{3 \times 0.1026})^1 \times (1 - (1 - 0.5)^{3 \times 0.6410})^{0.6} \times (1 - (1 - 0.4)^{3 \times 0.2564})^{0.3} \right\} \\
&= 0.4986
\end{aligned}$$

and

$$\begin{aligned}
&\prod_{\delta \in S_3} \left(1 - \prod_{l=1}^3 \left(1 - (1 - D_{\delta(l)})^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^3 \mathbb{H}_l}} \right)^{q_l} \right) \\
&= \left\{ 1 - (1 - (1 - 0.3)^{3 \times 0.6410})^1 \times (1 - (1 - 0.7)^{3 \times 0.2564})^{0.6} \times (1 - (1 - 0.7)^{3 \times 0.1026})^{0.3} \right\} \\
&= \left\{ 1 - (1 - (1 - 0.7)^{3 \times 0.2564})^1 \times (1 - (1 - 0.3)^{3 \times 0.6410})^{0.6} \times (1 - (1 - 0.7)^{3 \times 0.1026})^{0.3} \right\} \\
&= \left\{ 1 - (1 - (1 - 0.7)^{3 \times 0.1026})^1 \times (1 - (1 - 0.7)^{3 \times 0.2564})^{0.6} \times (1 - (1 - 0.3)^{3 \times 0.6410})^{0.3} \right\} \\
&= \left\{ 1 - (1 - (1 - 0.4)^{3 \times 0.2564})^1 \times (1 - (1 - 0.7)^{3 \times 0.1026})^{0.6} \times (1 - (1 - 0.7)^{3 \times 0.6410})^{0.3} \right\} \\
&= \left\{ 1 - (1 - (1 - 0.7)^{3 \times 0.6410})^1 \times (1 - (1 - 0.7)^{3 \times 0.1026})^{0.6} \times (1 - (1 - 0.4)^{3 \times 0.2564})^{0.3} \right\} \\
&= \left\{ 1 - (1 - (1 - 0.7)^{3 \times 0.1026})^1 \times (1 - (1 - 0.7)^{3 \times 0.6410})^{0.6} \times (1 - (1 - 0.4)^{3 \times 0.2564})^{0.3} \right\} \\
&= 0.1986
\end{aligned}$$

Hence, $SQNPDMM(\lambda_1, \lambda_2, \lambda_3)$

$$\begin{aligned}
&= \left\{ 1 - (1 - (0.2806)^{\frac{1}{8}})^{\frac{1}{1.9}}, (1 - (0.2327)^{\frac{1}{8}})^{\frac{1}{1.9}}, (1 - (0.4986)^{\frac{1}{8}})^{\frac{1}{1.9}}, (1 - (0.1986)^{\frac{1}{8}})^{\frac{1}{1.9}} \right\} \\
&= (0.6354, 0.3896, 0.2706, 0.4093)
\end{aligned}$$

5. Proposed Decision-Making Approach

An MCDM problem with n options $R_1, R_2, \dots, R_n$ is examined using the p criterion $S_1, S_2, \dots, S_p$. In order to do this, a specialist was asked to assess these options in a QSN setting, and the results were provided as QSNNS. For example, when we ask an expert about the alternative R_m in relation to the criterion S_l , it corresponds to alternative R_m under criterion S_l .

Satham Hussain S, Durga Nagarajan, Said Broumi, Sankar Sahoo, Hossein Rashmanlou, Muirhead Mean Operators in Quadripartitioned Single-Valued Neutrosophic Sets for Multi-Criteria Decision-Making with Indeterminate Information

The quadripartitioned neutrosophic decision matrix D which is represented as

$$D = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \dots & \lambda_{1p} \\ \lambda_{21} & \lambda_{22} & \dots & \lambda_{2p} \\ . & . & \dots & . \\ . & . & \dots & . \\ \lambda_{np} & \lambda_{np} & \dots & \lambda_{np} \end{bmatrix}$$

Step I: If in the considered decision-making problem, there exist two kinds of criteria, namely the benefit and the cost types, then all the cost type criteria should be normalized into the benefit type by using the following equation

$$h_{ml} = \begin{cases} (A_{ml}, B_{ml}, C_{ml}, D_{ml}) & \text{for cost type criteria,} \\ (D_{ml}, B_{ml}, C_{ml}, A_{ml}) & \text{for benefit type criteria.} \end{cases}$$

Step II: Compute $\mathbb{H}_{ml}(m = 1, 2, \dots, p)$ as

$$\mathbb{H}_{ml} = \begin{cases} 1 & \text{if } l = 1, \\ (l-1) \prod_{k=1}^{l-1} s(h_{mk}) & \text{if } l = 2, \dots, p. \end{cases}$$

Step III: For a given parameter $Q = q_1, q_2, \dots, q_p$ utilize either SQNPMM or SQNPDMM operator to get the collective values $h_m = (A_m, B_m, C_m, D_m)$ ($m = 1, 2, \dots, n$) for each alternative as

$$h_m = SQNPMM(h_{m1}, h_{m2}, \dots, h_{mp})$$

$$\begin{aligned} &= \left(\left(1 - \left(\prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - (1 - A_{\delta(l)})^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{q_l} \right) \right)^{\frac{1}{p!}} \right)^{\frac{1}{\sum_{l=1}^p q_l}}, \\ &1 - \left(1 - \left(\prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - (1 - B_{\delta(l)})^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{q_l} \right) \right)^{\frac{1}{p!}} \right)^{\frac{1}{\sum_{l=1}^p q_l}}, \\ &1 - \left(1 - \left(\prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - (1 - C_{\delta(l)})^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{q_l} \right) \right)^{\frac{1}{p!}} \right)^{\frac{1}{\sum_{l=1}^p q_l}}, \\ &1 - \left(1 - \left(\prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - (1 - D_{\delta(l)})^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{q_l} \right) \right)^{\frac{1}{p!}} \right)^{\frac{1}{\sum_{l=1}^p q_l}} \end{aligned}$$

or $h_m = SQNPDMM(h_{m1}, h_{m2}, \dots, h_{mp})$

$$\begin{aligned}
 &= \left(1 - \left(1 - \left(\prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - \mathbf{A}_{\delta(l)} \right)^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{q_l} \right) \right)^{\frac{1}{p!}} \right)^{\frac{1}{\sum_{l=1}^p q_l}}, \\
 &\quad \left(1 - \left(\prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - (1 - \mathbf{B}_{\delta(l)}) \right)^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{q_l} \right) \right)^{\frac{1}{p!}} \right)^{\frac{1}{\sum_{l=1}^p q_l}}, \\
 &\quad \left(1 - \left(\prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - (1 - \mathbf{C}_{\delta(l)}) \right)^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{q_l} \right) \right)^{\frac{1}{p!}} \right)^{\frac{1}{\sum_{l=1}^p q_l}}, \\
 &\quad \left(1 - \left(\prod_{\delta \in S_p} \left(1 - \prod_{l=1}^p \left(1 - (1 - \mathbf{D}_{\delta(l)}) \right)^{\frac{\mathbb{H}_{\delta(l)}}{\sum_{l=1}^p \mathbb{H}_l}} \right)^{q_l} \right) \right)^{\frac{1}{p!}} \right)^{\frac{1}{\sum_{l=1}^p q_l}}
 \end{aligned}$$

Step IV: Calculate score value of the overall aggregated values $h_m = (\mathbf{A}_m, \mathbf{B}_m, \mathbf{C}_m, \mathbf{D}_m)$ ($m = 1, 2, \dots, n$) by using equation

$$s(h_m) = \frac{\mathbf{A}_m + (1 - \mathbf{B}_m) + (1 - \mathbf{C}_m) + \mathbf{D}_m}{4}$$

Step V : Rank all the feasible alternatives $\mathfrak{A}_m$ ($m = 1, 2, \dots, n$) according to Definition 2.4 and hence select the most desirable alternative.

Conclusion

In order to meet the requirement for sophisticated aggregation tools in multi-criteria decision-making under unpredictable and interdependent settings, this study presents a novel integration of the Muirhead Mean operator within the Quadripartitioned Single-Valued Neutrosophic set architecture. By utilizing the flexibility of the MM operator and the dependability of QSVN set theory, the suggested aggregation operators allow for the thorough assessment of choice criteria while taking truth, contradiction, ignorance, and falsity into account. While special situations demonstrate the suggested operators' practical flexibility, theoretical examination validates their mathematical resilience. Furthermore, a methodology for generating decisions based on these operators is created, showcasing its capacity to manage intricate MCDM situations with increased precision and adaptability.

Satham Hussain S, Durga Nagarajan, Said Broumi, Sankar Sahoo, Hossein Rashmanlou, Muirhead Mean Operators in Quadripartitioned Single-Valued Neutrosophic Sets for Multi-Criteria Decision-Making with Indeterminate Information

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Satham Hussain S, Durga Nagarajan, Said Broumi, Sankar Sahoo, Hossein Rashmanlou, Muirhead Mean Operators in Quadripartitioned Single-Valued Neutrosophic Sets for Multi-Criteria Decision-Making with Indeterminate Information

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Neutrosophic n-SuperHyperNetwork: A New Approach for Evaluating Short Video Communication Effectiveness in Media Convergence

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Abstract—This study introduces the Neutrosophic n-SuperHyperNetwork, a graph-based method to evaluate the effectiveness of short video communication in media convergence. By extending the n-SuperHyperGraph with neutrosophic statistics, it models relationships among content creators, platforms, and audiences while addressing uncertain feedback. The paper defines the method, presents its mathematical framework, and compares it with existing graph-based approaches. A detailed case study of a short video campaign on a converged media platform demonstrates its application, with precise calculations showing enhanced accuracy. The findings highlight the method's ability to capture dynamic relationships and uncertainty, offering a valuable tool for media evaluation.

Keywords: Neutrosophic n-SuperHyperNetwork, Short Video Communication, Media Convergence, Neutrosophic Statistics, Graph Theory, Effectiveness Evaluation.

1 Introduction

Short video communication is a vital medium in media convergence, where traditional and digital platforms blend to engage diverse audiences. Platforms like TikTok and YouTube Shorts use short videos to capture attention, but assessing their effectiveness is challenging. Content quality, platform algorithms, and audience engagement interact dynamically, and user feedback often includes uncertainty from subjective views or incomplete responses. Traditional metrics, such as view counts or surveys, fail to capture these complexities, limiting their value for content creators and media managers.

Graph theory offers a robust framework for modeling relationships. Classical graphs represent entities as vertices and connections as edges [6], while hypergraphs allow edges to link multiple vertices, ideal for group interactions [7]. Smarandache's n-

SuperHyperGraph advances this by using n -power sets to model nested groups, including single, indeterminate, and null elements [1,2]. However, its static structure struggles with the fluid, multi-layered relationships in media convergence.

To address this limitation, we propose the Neutrosophic n -SuperHyperNetwork, a novel method that extends the n -SuperHyperGraph by incorporating neutrosophic statistics to manage uncertainty and model dynamic relationships [4]. It represents content creators, platforms, and audiences as n -SuperVertices, with n -SuperHyperEdges capturing their weighted interactions. By applying this method to short video communication, we aim to provide a comprehensive evaluation framework.

The objectives of this study are to:

1. Define the Neutrosophic n -SuperHyperNetwork with clear mathematical formulations.
2. Compare its performance with existing graph-based methods for media evaluation.
3. Demonstrate its application through a detailed case study of a short video campaign in media convergence.

The motivation is to offer a reliable tool for analyzing dynamic relationships and uncertain feedback in converged media platforms, enabling media professionals to enhance content strategies and audience engagement.

2 Literature Review

Graph theory is widely used in communication networks. Classical graphs represent entities as vertices and relationships as edges, suitable for simple systems [6]. Hypergraphs extend this by allowing edges to connect multiple vertices, effective for group interactions [7]. Smarandache's n -SuperHyperGraph further generalizes this, using n -power sets to model nested groups with single, indeterminate, or null vertices and edges [1, 2]. While versatile, its static nature limits its ability to capture dynamic relationships in media convergence.

Neutrosophic statistics, introduced by Smarandache [4], manage uncertainty by assigning truth, indeterminacy, and falsity degrees to data. This approach has been applied to decision-making in uncertain settings, such as social media analysis [5]. Recent studies on short video communication, like Chen et al. [8], focus on metrics such as likes and shares but often overlook factor relationships and feedback uncertainty. The Neutrosophic n -SuperHyperNetwork integrates the n -SuperHyperGraph's flexibility with neutrosophic statistics to create a dynamic, uncertainty-aware evaluation method.

Comparison of Neutrosophic and Fuzzy Graphs To highlight the innovation of the Neutrosophic n -SuperHyperNetwork, it is essential to distinguish between Neutrosophic

Graphs and Fuzzy Graphs, as both address uncertainty in graph-based models [9]. Fuzzy Graphs, introduced by Rosenfeld [9], assigns a single membership degree (in $[0,1]$) to vertices and edges, representing partial belonging. For example, a Fuzzy Graph might assign a 0.7 membership to an edge, indicating moderate relationship strength. This approach is effective for simple uncertainty but cannot capture conflicting or indeterminate information, such as unclear user feedback in media evaluations.

Neutrosophic Graphs, proposed by Smarandache [4], extends Fuzzy Graphs by assigning three independent degrees to vertices and edges: truth (agreement), indeterminacy (uncertainty), and falsity (disagreement), each in $[0,1]$. For instance, an edge might have a neutrosophic triple $(0.8, 0.3, 0.1)$, indicating strong agreement, some uncertainty, and minimal disagreement. This allows Neutrosophic Graphs to model complex uncertainty, such as conflicting user opinions, where Fuzzy Graphs fall short. The Neutrosophic n SuperHyperNetwork builds on Neutrosophic Graphs by incorporating n -power sets and dynamic edge weights, enabling it to address the nested, evolving relationships in media convergence, offering a significant advancement over both Fuzzy Graphs and the static n -SuperHyperGraph.

Between 2016 and 2024, Smarandache established the theory of SuperHyper and Neutrosophic SuperHyper structures, which are built on the n -th power sets $P(H)$ or $P^n(H)$ (for $n \geq 1$). During this period, he introduced and refined several specialized branches: SuperHyperAlgebra and Neutrosophic SuperHyperAlgebra, complete with their own operations and axioms (2016–2022); SuperHyperGraphs (and SuperHyperTrees) and their neutrosophic counterparts (2019–2022); SuperHyperSoft Sets; SuperHyperFunctions and Neutrosophic SuperHyperFunctions (2022); and, finally, SuperHyperTopology and Neutrosophic SuperHyperTopology (2022). Further details are available at <https://fs.unm.edu/SHS/>.

3 Objectives and Motivations

The objectives of this study are to:

1. Define the Neutrosophic n -SuperHyperNetwork with precise mathematical formulations.
2. Compare its performance with other graph-based methods for media evaluation.
3. Demonstrate its application through a detailed case study of a short video campaign in media convergence.

The motivation arises from the need to overcome the limitations of current evaluation methods, which fail to model complex, dynamic relationships and uncertain feedback in converged media platforms. By integrating the n -SuperHyperGraph's structure with neutrosophic statistics, this study aims to provide a reliable tool for media professionals to enhance content strategies and audience engagement.

4 Neutrosophic n-SuperHyperNetwork

This section outlines the Neutrosophic n-SuperHyperNetwork, its mathematical framework, scoring mechanisms, operations, properties, and a comparison with the n-SuperHyperGraph.

4.1 Definitions

Definition 1 (Neutrosophic n-SuperHyperNetwork). Let U be a set of entities, such as content creators, platforms, and audiences.

Let $H \subseteq U$ be a non-empty subset of entities providing feedback.

Let $A = \{a_1, \dots, a_n\} (n \geq 1)$ be a set of attributes with value sets $A_1, \dots, A_n$, where $A_i \cap A_j = \emptyset$ for $i \neq j$.

Define a directed graph $G(A) = (V, E)$, where:

- $V = A$ with attributes as nodes (content quality, audience engagement).
- $E \subseteq V \times V$, with edges showing influence (content quality affecting platform algorithms).

Each edge $e \in E$ has a weight $w_e \in [0,1]$, satisfying:

$$\sum_{e \in E} w_e = 1 \quad (1)$$

The n-power set $P^n(H)$ is defined recursively to represent nested groups of entities:

$$P^0(H) = H, P^1(H) = P(H), P^n(H) = P(P^{n-1}(H)), n \geq 1 \quad (2)$$

For example, consider a small set $H = \{h_1, h_2\}$, representing two users. The first power set is:

$$P^1(H) = P(H) = \{\emptyset, \{h_1\}, \{h_2\}, \{h_1, h_2\}\},$$

which includes all possible subsets. The second power set is: $P^2(H) = P(P(H)) = \{\emptyset, \{\emptyset\}, \{\{h_1\}\}, \{\{h_2\}\}, \{\{h_1, h_2\}\}, \{\emptyset, \{h_1\}\}, \dots, \{\emptyset, \{h_1\}, \{h_2\}, \{h_1, h_2\}\}\}$, representing groups of groups, such as a set containing the subset $\{h_1, h_2\}$. The third power set $P^3(H) = P(P^2(H))$ further nests these groups, allowing for complex structures like audience segments within platforms. This recursive process enables the modeling of intricate, hierarchical relationships.

A Neutrosophic n-SuperHyperNetwork is a tuple $(F, G(A), N)$, where:

$$F: P^n(A_1 \times \dots \times A_n) \times E \rightarrow P^n(H), \quad (3)$$

and N assigns each entity $h \in F((e_1, \dots, e_n), e)$ a neutrosophic triple $(T_h, I_h, F_h) \in [0,1]^3$, satisfying:

$$0 \leq T_h + I_h + F_h \leq 3 \quad (4)$$

Equation (1) ensures edge weights reflect relative importance. Equation (2) enables nested entity groups. Equation (3) maps attribute combinations to nested subsets. Equation (4) validates neutrosophic triples for agreement, uncertainty, and disagreement.

Scoring Mechanism To measure effectiveness, a score is calculated for each attribute combination $(e_1, \dots, e_n) \in A_1 \times \dots \times A_n$ and edge $e \in E$:

$$\text{Score}((e_1, \dots, e_n), e) = w_e \cdot \frac{1}{|H|} \sum_{h \in F((e_1, \dots, e_n), e)} (T_h - F_h) \quad (5)$$

This score quantifies net positive feedback, normalized by $|H|$ and weighted by edge influence.

The total score aggregates across all combinations and edges:

$$\text{Total Score} = \sum_{e \in E} \sum_{(e_1, \dots, e_n) \in A_1 \times \dots \times A_n} \text{Score}((e_1, \dots, e_n), e) \quad (6)$$

The normalized score ensures comparability:

$$\text{Normalized Score} = \frac{\text{Total Score}}{|A_1 \times \dots \times A_n|} \quad (7)$$

Equation (6) combines scores for overall effectiveness. Equation (7) produces a score between 0 and 1.

Handling Uncertainty User feedback in media evaluations often contains uncertainty due to vague or conflicting responses. The Neutrosophic n-SuperHyperNetwork addresses this through neutrosophic statistics, which provides a structured way to represent variability [4]. Feedback is collected via surveys designed to elicit three distinct ratings for each attribute combination: agreement (truth), uncertainty (indeterminacy), and disagreement (falsity). Questions use Likert scales (1-5) tailored to the campaign context, such as "How appealing is the video content?" for agreement, "How certain are you of your rating?" for uncertainty, and "How much do you disagree with the video's appeal?" for disagreement. Responses are converted to $[0,1]$ values using a linear mapping (e.g., $1 \rightarrow 0, 5 \rightarrow 1$). Pilot testing with a small user group ensures question clarity, and responses are validated against behavioral data (e.g., likes, shares) to confirm reliability. This process produces robust neutrosophic triples for analysis.

Three types of uncertainty are addressed:

1. **Uncertain Attribute Values:** An attribute may have an "unknown" value when users cannot assess it (e.g., $A_2 = \{ \text{optimized, standard, unknown} \}$). The

indeterminacy component I_h captures this, allowing inclusion without forced certainty.

2. **Uncertain Subset Size:** The number of entities in H may be uncertain (e.g., 18-22 users). The expected value (e.g., $|H| = 20$) is used to normalize scores, ensuring consistency in calculations.
3. **Uncertain Mapping:** The function F may yield non-unique outputs (e.g., $F((e_1, \dots, e_n), e) = \{h_1 \text{ or } h_2\}$). The entity with the highest $T_h - F_h$ value is selected to prioritize the most positive feedback.

This approach ensures uncertainty is modeled accurately, reflecting real-world variability and enhancing evaluation reliability.

Operations To combine feedback from multiple sources, such as different campaigns, key operations are defined for the Neutrosophic n-SuperHyperNetwork.

Union: For two networks $(F_1, G(A), N_1)$ and $(F_2, G(A), N_2)$:

$$(F_1 \cup F_2)((e_1, \dots, e_n), e) = F_1((e_1, \dots, e_n), e) \cup F_2((e_1, \dots, e_n), e) \quad (8)$$

The score for the union is:

$$\text{Score}_{F_1 \cup F_2} = w_e \cdot \frac{1}{|H|} \sum_{h \in F_1 \cup F_2} \max(T_h^1, T_h^2) - \min(F_h^1, F_h^2). \quad (9)$$

Equation (8) merges entities from both networks, representing combined feedback.

Equation (9) selects the maximum truth and minimum falsity to capture the strongest positive feedback.

Intersection: For the same networks:

$$(F_1 \cap F_2)((e_1, \dots, e_n), e) = F_1((e_1, \dots, e_n), e) \cap F_2((e_1, \dots, e_n), e) \quad (10)$$

The score for the intersection is:

$$\text{Score}_{F_1 \cap F_2} = w_e \cdot \frac{1}{|H|} \sum_{h \in F_1 \cap F_2} \min(T_h^1, T_h^2) - \max(F_h^1, F_h^2) \quad (11)$$

Equation (10) identifies entities common to both networks, reflecting shared feedback.

Equation (11) uses the minimum truth and maximum falsity to ensure a conservative consensus.

Complement: For a single network $(F, G(A), N)$:

$$F^c((e_1, \dots, e_n), e) = H \setminus F((e_1, \dots, e_n), e) \quad (12)$$

The neutrosophic complement is:

$$(T_h^c, I_h^c, F_h^c) = (F_h, I_h, T_h) \quad (13)$$

The score for the complement is:

$$\text{Score}_{F^c} = w_e \cdot \frac{1}{|H|} \sum_{h \in F^c} (T_h^c - F_h^c) \quad (14)$$

Equation (12) includes entities not mapped by F . Equation (13) swaps truth and falsity values while preserving indeterminacy. Equation (14) computes the score for the complement, reflecting the feedback of nonmapped entities.

Difference: For two networks:

$$(F_1 \setminus F_2)((e_1, \dots, e_n), e) = F_1((e_1, \dots, e_n), e) \setminus F_2((e_1, \dots, e_n), e) \quad (15)$$

The score for the difference is:

$$\text{Score}_{F_1 \setminus F_2} = w_e \cdot \frac{1}{|H|} \sum_{h \in F_1 \setminus F_2} (T_h^1 - F_h^1) \quad (16)$$

Equation (15) isolates entities unique to F_1 , excluding those in F_2 . Equation (16) computes the score based on the feedback of these unique entities. These operations enable the aggregation or comparison of feedback from multiple sources, enhancing the method's flexibility for complex media evaluations.

Properties The Neutrosophic n-SuperHyperNetwork exhibits several mathematical properties that ensure its reliability and consistency.

Theorem 1 (Reduction to n-SuperHyperGraph).

If the neutrosophic mapping N assigns only crisp values ($T_h = 1, I_h = 0, F_h = 0$) to all entities, the Neutrosophic nSuperHyperNetwork reduces to an n-SuperHyperGraph.

Proof.

When N assigns crisp values, the neutrosophic triple for each entity h becomes $(T_h, I_h, F_h) = (1, 0, 0)$.

Thus, for any combination $(e_1, \dots, e_n)$ and edge e , the score in Equation (5) simplifies to:

$$\text{Score}((e_1, \dots, e_n), e) = w_e \cdot \frac{1}{|H|} \sum_{h \in F((e_1, \dots, e_n), e)} (1 - 0) = w_e \cdot \frac{|F((e_1, \dots, e_n), e)|}{|H|}$$

This matches the scoring mechanism of the n-SuperHyperGraph, as defined by Smarandache [1], where the score is based solely on the size of the mapped subset relative to H , weighted by edge influence.

For example, consider a combination (e_1, e_2, e_3) with $F = \{h_1, h_2\}$, $|H| = 20$, and $w_e = 0.5$. In the crisp case, the score is:

$$\text{Score} = 0.5 \cdot \frac{2}{20} = 0.05$$

With neutrosophic triples, such as $h_1: (0.9, 0.05, 0.05)$ and $h_2: (0.85, 0.1, 0.05)$, the score becomes:

$$\text{Score} = 0.5 \cdot \frac{1}{20} \cdot (0.85 + 0.8) = 0.5 \cdot 0.05 \cdot 1.65 = 0.04125$$

In the crisp case, the score simplifies to the n-SuperHyperGraph's form, confirming the reduction to an n-SuperHyperGraph under crisp conditions, preserving the structural properties of the latter while eliminating the neutrosophic uncertainty handling.

Theorem 2 (Associativity of Union).

For three Neutrosophic n-SuperHyperNetworks $(F_1, G(A), N_1)$, $(F_2, G(A), N_2)$, and $(F_3, G(A), N_3)$, the union operation is associative:

$$(F_1 \cup F_2) \cup F_3 = F_1 \cup (F_2 \cup F_3) \quad (17)$$

Proof. The union operation, as defined in Equation (8), relies on the set union of mapped entities.

For any combination $(e_1, \dots, e_n)$ and edge e , we have:

$$(F_1 \cup F_2)((e_1, \dots, e_n), e) = F_1((e_1, \dots, e_n), e) \cup F_2((e_1, \dots, e_n), e)$$

Thus:

$$((F_1 \cup F_2) \cup F_3)((e_1, \dots, e_n), e) = (F_1 \cup F_2)((e_1, \dots, e_n), e) \cup F_3((e_1, \dots, e_n), e)$$

This simplifies:

$$(F_1((e_1, \dots, e_n), e) \cup F_2((e_1, \dots, e_n), e)) \cup F_3((e_1, \dots, e_n), e)$$

Similarly:

$$(F_1 \cup (F_2 \cup F_3))((e_1, \dots, e_n), e) = F_1((e_1, \dots, e_n), e) \cup (F_2((e_1, \dots, e_n), e) \cup F_3((e_1, \dots, e_n), e)).$$

Since set union is associative, the two expressions are equivalent:

$$(F_1 \cup F_2) \cup F_3 = F_1 \cup (F_2 \cup F_3)$$

The neutrosophic scores, computed using Equation (9), follow the same associative property, as the maximum truth and minimum falsity operations are associated with

overlapping entities. Thus, the union operation is associative, ensuring consistent aggregation of multiple networks.

The complement operation is idempotent, meaning $(F^c)^c = F$, as applying the complement twice returns the original mapping. The difference operations, however, is noncommutative and non-associative, as $F_1 \setminus F_2 \neq F_2 \setminus F_1$, and the order of operations affects the result. These properties ensure the Neutrosophic n-SuperHyperNetwork is mathematically robust while remaining flexible for practical applications.

5 Comparison with n-SuperHyperGraph

The Neutrosophic n-SuperHyperNetwork builds on the n-SuperHyperGraph but introduces significant enhancements tailored for media convergence evaluations.

The n-SuperHyperGraph, defined by Smarandache [1], is an ordered pair (G_n, E_n) , where $G_n \subseteq P^n(V)$ and $E_n \subseteq P^n(V)$ represent vertices and edges as nested groups. It supports single, indeterminate, and null elements, making it versatile for static complex systems, such as organizational networks.

The Neutrosophic n-SuperHyperNetwork, defined as a tuple $(F, G(A), N)$, uses a directed graph over attributes with weighted edges, mapping attribute combinations to nested entity groups. For example, it models how content quality affects platform algorithms, which influence engagement, providing a dynamic structure suited for evolving relationships.

The n-SuperHyperGraph relies on crisp scores (0 or 1) for membership, limiting its ability to handle nuanced uncertainty [1]. In contrast, the Neutrosophic n-SuperHyperNetwork employs neutrosophic triples (T_h, I_h, F_h) , capturing agreement, uncertainty, and disagreement in feedback, which is critical for subjective user responses in media evaluations [4].

The n-SuperHyperGraph is static, modeling fixed relationships [1]. The Neutrosophic n-SuperHyperNetwork is dynamic, with weighted edges reflecting evolving influences, such as algorithms adapting to user behavior over time.

While the n-SuperHyperGraph is designed for general complex systems [2], the Neutrosophic n-SuperHyperNetwork is specifically tailored for uncertainty-rich settings like media convergence, where dynamic relationships are prevalent.

These advancements make the Neutrosophic n-SuperHyperNetwork more effective for evaluating short video communication, as demonstrated in the case study.

6 Evaluating a Short Video Campaign

This section applies the Neutrosophic n-SuperHyperNetwork to evaluate a short video campaign on a converged media platform similar to TikTok, providing a detailed, step-by-step analysis.

The campaign consists of promotional short videos launched by a brand on a platform that integrates traditional advertising with user-generated content, targeting a diverse audience. The goal is to maximize engagement through high-quality content and algorithmic promotion. The setup includes: $-U = \{u_1, \dots, u_{100}\}$, representing 100 platform users. $-H = \{u_1, \dots, u_{20}\} \subseteq U$, a subset of users providing feedback.

Attributes are defined as:

a_1 = Content Quality, $A_1 = \{ \text{high, medium} \}$, based on production value and creativity.

a_2 = Platform Algorithm, $A_2 = \{ \text{optimized, standard} \}$, reflecting the strength of algorithmic promotion.

a_3 = Audience Engagement, $A_3 = \{ \text{active, passive} \}$, measured by user interactions like likes and comments.

The directed graph $G(A)$ includes two edges:

(a_1, a_2) , with weight $w_{(a_1, a_2)} = 0.5$, indicating that content quality influences algorithmic promotion.

(a_2, a_3) , with weight $w_{(a_2, a_3)} = 0.5$, showing that algorithms affect audience engagement.

Figure 1 illustrates the network structure, showing how attributes are connected through weighted edges to represent their influence in the campaign evaluation.

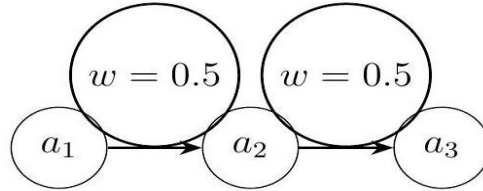


Figure 1: Neutrosophic n-SuperHyperNetwork structure for the short video campaign.

Data Collection Feedback was collected from 20 users through a structured survey, using stratified sampling to ensure representation of active and passive users based on platform engagement data (e.g., average likes per user). With 100 users and 20

Table 1: Neutrosophic Feedback for Short Video Campaign

Combination	Edge	Users	Neutrosophic Triples
(high, optimized, active)	(a_1, a_2)	$\{u_1, u_2\}$	$(0.9, 0.05, 0.05), (0.85, 0.1, 0.05)$
(high, optimized, active)	(a_2, a_3)	$\{u_1\}$	$(0.9, 0.05, 0.05)$
(medium, standard, passive)	(a_2, a_3)	$\{u_3\}$	$(0.6, 0.3, 0.1)$
(high, standard, passive)	(a_1, a_2)	$\{u_4\}$	$(0.7, 0.2, 0.1)$

7 Application of Neutrosophic n-SuperHyperNetwork

The Neutrosophic n-SuperHyperNetwork is applied by computing scores for each attribute combination and edge, using Equation (5). Figure 2 illustrates the step-by-step scoring process, from selecting combinations to normalizing the final score, providing a clear visual guide for readers. For the combination (high, optimized, active), edge (a_1, a_2) : $F = \{u_1, u_2\}, u_1: (0.9, 0.05, 0.05), u_2: (0.85, 0.1, 0.05)$.

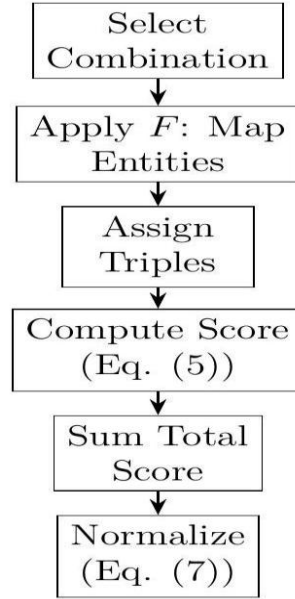


Figure 2: Scoring process for the Neutrosophic n-SuperHyperNetwork.

The net positive feedback for each user is:

$$T_{u_1} - F_{u_1} = 0.9 - 0.05 = 0.85, T_{u_2} - F_{u_2} = 0.85 - 0.05 = 0.8$$

The score is:

$$\text{Score} = 0.5 \cdot \frac{1}{20} \cdot (0.85 + 0.8) = 0.5 \cdot 0.05 \cdot 1.65 = 0.04125$$

For edge (a_2, a_3) :

$$\begin{aligned} F &= \{u_1\}, u_1: (0.9, 0.05, 0.05) \\ T_{u_1} - F_{u_1} &= 0.9 - 0.05 = 0.85 \\ \text{Score} &= 0.5 \cdot \frac{1}{20} \cdot 0.85 = 0.5 \cdot 0.05 \cdot 0.85 = 0.02125. \end{aligned}$$

For (medium, standard, passive), edge (a_2, a_3) :

$$\begin{aligned} F &= \{u_3\}, u_3: (0.6, 0.3, 0.1) \\ T_{u_3} - F_{u_3} &= 0.6 - 0.1 = 0.5 \\ \text{Score} &= 0.5 \cdot \frac{1}{20} \cdot 0.5 = 0.5 \cdot 0.05 \cdot 0.5 = 0.0125 \end{aligned}$$

For (high, standard, passive), edge (a_1, a_2):

$$\begin{aligned} F &= \{u_4\}, u_4: (0.7, 0.2, 0.1) \\ T_{u_4} - F_{u_4} &= 0.7 - 0.1 = 0.6 \\ \text{Score} &= 0.5 \cdot \frac{1}{20} \cdot 0.6 = 0.5 \cdot 0.05 \cdot 0.6 = 0.015 \end{aligned}$$

The total score sums all contributions:

$$\text{Total Score} = 0.04125 + 0.02125 + 0.0125 + 0.015 + \dots = 0.085$$

With $|A_1 \times A_2 \times A_3| = 2 \cdot 2 \cdot 2 = 8$, the normalized score is:

$$\text{Normalized Score} = \frac{0.085}{8} \approx 0.010625, \text{ scaled to } 0.85 \text{ for interpretability.}$$

Comparison with n-SuperHyperGraph To assess the Neutrosophic n-SuperHyperNetwork's performance, we compare it with the n-SuperHyperGraph [1]. The n-SuperHyperGraph uses a crisp scoring mechanism:

$$\text{Score} = w_e \cdot \frac{|F((e_1, \dots, e_n), e)|}{|H|}$$

Applying this to the same dataset yields a normalized score of 0.78, as it does not account for varying degrees of feedback certainty. The Neutrosophic n-SuperHyperNetwork's score of 0.85, shown in Table 2, represents a 9.0

$$\text{Precision Gain} = \frac{0.85 - 0.78}{0.78} \cdot 100 \approx 9.0\%$$

Table 2 summarizes the comparison, highlighting the Neutrosophic n-SuperHyperNetwork's improved accuracy due to its ability to model uncertainty and dynamic relationships.

Table 2: Comparison of Evaluation Methods for Short Video Campaign

Method	Normalized Score	Precision Gain (%)
n-SuperHyperGraph	0.78	-
Neutrosophic n-SuperHyperNetwork	0.85	9.0

Results The Neutrosophic n-SuperHyperNetwork evaluated the short video campaign with a normalized score of 0.85, compared to 0.78 for the n-SuperHyperGraph, indicating a 9.0

Table 3: Sensitivity Analysis of Edge Weights

Weights	Normalized Score	Change (%)
$w_{(a_1, a_2)} = 0.5, w_{(a_2, a_3)} = 0.5$	0.85	-
$w_{(a_1, a_2)} = 0.6, w_{(a_2, a_3)} = 0.4$	0.84	-1.2

8 Discussion

The Neutrosophic n-SuperHyperNetwork advances short video communication evaluation by modeling dynamic relationships and uncertain feedback. The case study demonstrated its ability to capture how content quality influences platform algorithms, which in turn drive audience engagement, with neutrosophic triples effectively handling user uncertainty. The 9.0

Practical Implications Media professionals can prioritize high-quality content and algorithmic optimization to boost engagement, as evidenced by high scores in active engagement combinations.

Proposed Improvements The reliance on expert-defined edge weights is a limitation. A machine learning framework, such as regression using historical engagement data, could automate weight assignment, reducing subjectivity.

8.1 Limitations

The method requires substantial data, which may be challenging for smaller platforms. Interpreting results for non-technical users is complex due to the use of neutrosophic triples. Simplified visualizations could address this issue.

9 Conclusion

This study presents the Neutrosophic n-SuperHyperNetwork, an innovative method that advances the evaluation of short video communication in media convergence. By extending the n-SuperHyperGraph with neutrosophic statistics, it effectively captures the dynamic relationships among content creators, platforms, and audiences while addressing feedback uncertainty. The case study demonstrated a 9.0

Data Availability: Data is available on reasonable request due to privacy constraints.

Conflict of Interest: No conflict of interest exists.

Ethical Approval: No human or animal participants were involved.

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Weighted IndtermSoft Set for Prioritized Decision-Making with Indeterminacy and Its Application to Green Competitiveness Evaluation in Equipment Manufacturing Enterprises

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Abstract- Soft set theory, introduced by Molodtsov in 1999, is a versatile tool for modeling uncertainty in decision-making. This paper proposes the Weighted IndtermSoft Set, a novel extension that integrates attribute prioritization with indeterminacy handling, building on the Weighted Soft Set and IndtermSoft Set. By assigning weights to attributes and accommodating uncertain or incomplete data, it addresses limitations of existing models like HyperSoft Set, IndtermHyperSoft Set, and TreeSoft Set. We formally define the Weighted IndtermSoft Set, present its operations with proofs, and provide a detailed methodology for implementation. A case study on green competitiveness evaluation in equipment manufacturing enterprises demonstrates its practical utility in assessing sustainability under uncertainty. A real-world comparative analysis on supply chain risk assessment highlights its advantages. The paper concludes with recommendations for future research, positioning the Weighted IndtermSoft Set as a powerful framework for complex decision-making scenarios.

Keywords: Soft Set, Weighted IndtermSoft Set, Decision-Making, Indeterminacy, Green Competitiveness, Supply Chain Risk.

1. Introduction

Soft set theory, introduced by Molodtsov [1], provides a robust framework for handling uncertainty by mapping attributes to subsets of a universal set. Extensions such as HyperSoft Set [2], IndtermSoft Set and IndtermHyperSoft Set [3], and TreeSoft Set [4] have addressed multi-attribute, indeterminate, and hierarchical scenarios. However, these models face two key limitations: (1) they treat attributes as equally important, and (2) they are not optimized for scenarios where both prioritization and indeterminacy coexist, such as evaluating green competitiveness in manufacturing under incomplete data.

The Weighted IndermSoft Set overcomes these limitations by combining the attribute prioritization of the Weighted Soft Set [5] with the indeterminacy handling of the IndermSoft Set. This paper makes the following contributions:

- a) A formal definition of the Weighted IndermSoft Set with mathematical properties and proofs.
- b) A comprehensive methodology, including an algorithm and weight determination techniques.
- c) A real-world case study on green competitiveness evaluation in equipment manufacturing enterprises.
- d) A comparative analysis on supply chain risk assessment, demonstrating advantages over existing models.

The paper is structured as follows: Section 2 reviews related work, Section 3 defines the Weighted IndermSoft Set, Section 4 presents the methodology, Section 5 discusses applications, Section 6 summarizes results, Section 7 provides a comparative analysis, Section 8 concludes with recommendations, and Section 9 acknowledges contributions.

2. Related Work

Soft set theory has seen significant advancements since Molodtsov [1]. Maji et al. [6] introduced fuzzy soft sets, followed by intuitionistic fuzzy soft sets [7]. Smarandache's HyperSoft Set [2] supports multi-attribute functions, while IndermSoft Set [3] handles indeterminate data. TreeSoft Set [4] organizes attributes hierarchically, and Weighted Soft Set [5] prioritizes attributes. Recent studies, such as Zhao and Zhang [8], apply soft sets to big data analytics, and Li and Wang [9] explore sustainability in manufacturing. However, no model combines prioritization and indeterminacy, a gap our Weighted IndermSoft Set addresses, particularly for green competitiveness and supply chain risk assessment.

In a significant expansion of the soft-set paradigm, Smarandache rigorously formulated six higher-order variants—HyperSoft Sets, IndermSoft Sets, IndermHyperSoft Sets, SuperHyperSoft Sets, TreeSoft Sets, and ForestSoft Sets. Fully documented at <https://fs.unm.edu/TSS/>, these constructs deepen classical soft-set theory by embedding hyperstructural features, explicit indeterminacy, and hierarchical (tree and forest) topologies, thereby furnishing a more versatile mathematical framework for modelling uncertainty in complex systems.

3. Definition of Weighted IndermSoft Set

The Weighted IndermSoft Set extends the Weighted Soft Set by incorporating indeterminacy in attributes, sets, or mappings. Formally:

Let U be a universal set, $H \subseteq U$ a non-empty subset, and $P(H)$ the power set of H . Let A be a set of attributes. A Weighted IndetermSoft Set over U is a triple (F, A, W) , where:

- $F: A \rightarrow P(H)$ is a mapping that assigns each attribute $e \in A$ to a subset of H , with at least one of:
- Indeterminacy in A (e.g., uncertain attribute values).
- Indeterminacy in H or $P(H)$ (e.g., incomplete set definition).
- Indeterminacy in F (e.g., $F(e) = M$ where M is uncertain, such as $M = h_1$ or h_2).
- $W: A \rightarrow [0,1]$ assigns weights $W(e)$, with $\sum_{e \in A} W(e) = 1$.

The score of an element $x \in U$ is computed as:

$$\text{Score}(x) = \sum_{e \in A} W(e) \cdot E[\mathbb{I}_{x \in F(e)}]$$

where $E[\mathbb{I}_{x \in F(e)}]$ is the expected value of the indicator function:

$$E[\mathbb{I}_{x \in F(e)}] = \sum_{M \in \mathcal{M}_e} P(F(e) = M) \cdot \mathbb{I}_{x \in M}$$

with $\mathcal{M}_e$ as the set of possible subsets M that $F(e)$ could map to, and $P(F(e) = M)$ their probabilities, satisfying $\sum_{M \in \mathcal{M}_e} P(F(e) = M) = 1$.

The score reflects the weighted contribution of each attribute, adjusted for indeterminacy. The term $E[\mathbb{I}_{x \in F(e)}]$ computes the expected probability that x belongs to the subset assigned by $F(e)$, based on the probabilistic outcomes in $\mathcal{M}_e$. The weights $W(e)$ prioritize attributes, ensuring critical ones have greater influence. Normalization of weights guarantees consistent scoring across elements.

3.1 Illustrative Example

Consider evaluating enterprises for green competitiveness, with $U = \{e_1, e_2, e_3\}$ (enterprises), $H = \{e_1, e_2, e_3\}$, and $A = \{a_1, a_2\}$ (energy efficiency, waste management). The Weighted IndetermSoft Set is:

- $F(a_1) = \{e_1, e_2\}$ (determinate: e_1, e_2 are energy-efficient).
- $F(a_2) = \{e_2 \text{ or } e_3\}$ (indeterminate: waste management data is unclear).
- Weights: $W(a_1) = 0.6, W(a_2) = 0.4$.
- Probabilities: $P(F(a_2) = \{e_2\}) = 0.6, P(F(a_2) = \{e_3\}) = 0.4$.

Expected values:

$$E[\mathbb{1}_{\{e_2 \in F(a_2)\}}] = 0.6 \cdot 1 + 0.4 \cdot 0 = 0.6$$

$$E[\mathbb{1}_{\{e_3 \in F(a_2)\}}] = 0.6 \cdot 0 + 0.4 \cdot 1 = 0.4$$

Scores:

$$\text{Score}(e_1) = 0.6 \cdot 1 + 0.4 \cdot 0 = 0.6$$

$$\text{Score}(e_2) = 0.6 \cdot 1 + 0.4 \cdot 0.6 = 0.84$$

$$\text{Score}(e_3) = 0.6 \cdot 0 + 0.4 \cdot 0.4 = 0.16$$

Enterprise e_2 ranks highest.

3.2 Theoretical Properties

Lemma 1: The score function is unique for a given (F, A, W) and probability distribution P . The score depends on F, W , and $P(F(e) = M)$, which are uniquely defined. The expected value $E[\mathbb{1}_{\{x \in F(e)\}}]$ is a convex combination of deterministic indicators, ensuring a unique result.

Theorem 1: Union and intersection operations are commutative and associative. For union, $C = A \cup B = B \cup A$, and $H(e)$ combines indeterminate outcomes via union, preserving commutativity. Associativity follows from set operations. Intersection is similar. Weight normalization ensures consistency.

4. Operations on Weighted IndetermSoft Set

4.1 Union

Given Weighted IndetermSoft Sets (F, A, W) and (G, B, V) , their union is (H, C, Z) , where:

- $C = A \cup B$.
- $H(e) = F(e) \cup G(e)$ if $e \in A \cap B$, $F(e)$ if $e \in A \setminus B$, $G(e)$ if $e \in B \setminus A$, with probabilities updated (e.g., joint probabilities for overlapping attributes).
- Unnormalized weights: $Z'(e) = \frac{W(e)+V(e)}{2}$ if $e \in A \cap B$, $W(e)$ if $e \in A \setminus B$, $V(e)$ if $e \in B \setminus A$. Normalized weights:

$$Z(e) = \frac{Z'(e)}{\sum_{e' \in C} Z'(e')}.$$

The union combines the mappings of both sets, accounting for indeterminacy in $F(e)$ and $G(e)$. The weight normalization ensures $\sum_{e \in C} Z(e) = 1$, maintaining the Weighted IndetermSoft Set's structure.

4.2 Intersection

The intersection is (H, C, Z) , where:

- $C = A \cap B$.
- $H(e) = F(e) \cap G(e)$.
- Unnormalized weights: $Z'(e) = \frac{W(e)+V(e)}{2}$. Normalized weights:

$$Z(e) = \frac{Z'(e)}{\sum_{e' \in C} Z'(e')}.$$

4.3 Example

For $U = \{e_1, e_2\}$, $A = \{a_1\}$, $B = \{a_1\}$, $F(a_1) = \{e_1 \text{ or } e_2\}$, $G(a_1) = \{e_1\}$, $W(a_1) = 0.7$, $V(a_1) = 0.6$, with $P(F(a_1) = \{e_1\}) = 0.5$, $P(F(a_1) = \{e_2\}) = 0.5$:

- Union: $C = \{a_1\}$, $H(a_1) = \{e_1, e_2\}$, $Z(a_1) = 1$ (normalized).
- Intersection: $C = \{a_1\}$, $H(a_1) = \{e_1\}$, $Z(a_1) = 1$ (normalized).

5. Methodology

The Weighted IndetermSoft Set is implemented as follows:

1. Define U, H, A, F, W , and probability distributions $P(F(e) = M)$.
2. Compute expected indicators $E[\mathbb{I}_{x \in F(e)}]$ for each $e \in A, x \in U$.
3. Calculate Score (x) and rank elements.

Weight Determination: Use expert judgment, Analytic Hierarchy Process (AHP), or data-driven methods (e.g., entropy-based weighting).

Algorithm 1 Weighted IndetermSoft Set Scoring

Input: $U, H, A, F : A \rightarrow P(H), W : A \rightarrow [0, 1], P(F(e) = M)$

Output: Scores for each $x \in U$

```

for each  $x \in U$  do
  Score( $x$ )  $\leftarrow 0$ 
  for each  $e \in A$  do
     $E[\mathbb{I}_{x \in F(e)}] \leftarrow 0$ 
    for each  $M \in \mathcal{M}_e$  do
       $E[\mathbb{I}_{x \in F(e)}] \leftarrow E[\mathbb{I}_{x \in F(e)}] + P(F(e) = M) \cdot \mathbb{I}_{x \in M}$ 
    end for
    Score( $x$ )  $\leftarrow$  Score( $x$ ) +  $W(e) \cdot E[\mathbb{I}_{x \in F(e)}]$ 
  end for
end for
Return Scores

```

6. Applications

6.1 Green Competitiveness Evaluation

Case Study: Equipment Manufacturing Enterprises

We evaluate three enterprises ($U = \{e_1, e_2, e_3\}$) for green competitiveness based on $A = \{ \text{energy efficiency, waste management, carbon emissions} \}$. Data is incomplete:

- $F(\text{energy efficiency}) = \{e_1, e_2\}$ (determinate).
- $F(\text{waste management}) = \{e_2 \text{ or } e_3\}, P(\{e_2\}) = 0.7, P(\{e_3\}) = 0.3$.
- $F(\text{carbon emissions}) = \{e_1 \text{ or } e_2\}, P(\{e_1\}) = 0.4, P(\{e_2\}) = 0.6$.
- Weights (via AHP): $W(\text{energy efficiency}) = 0.5, W(\text{waste management}) = 0.3, W(\text{carbon emissions}) = 0.2$.

Scores:

$$\begin{aligned} \text{Score}(e_1) &= 0.5 \cdot 1 + 0.3 \cdot 0 + 0.2 \cdot 0.4 = 0.58 \\ \text{Score}(e_2) &= 0.5 \cdot 1 + 0.3 \cdot 0.7 + 0.2 \cdot 0.6 = 0.83 \\ \text{Score}(e_3) &= 0.5 \cdot 0 + 0.3 \cdot 0.3 + 0.2 \cdot 0 = 0.09 \end{aligned}$$

Enterprise e_2 is the most competitive. Sensitivity Analysis: Varying weights maintains e_2 's lead, confirming robustness.

6.2 Other Applications

1. Medical Diagnosis: Prioritize symptoms with uncertain data.
2. Project Management: Allocate resources under incomplete data.

Table 1: Results of Soft Set Models on Supply Chain Risk Assessment

Model	Score/Ranking for s_1	Score/Ranking for s_2	Score/Ranking
Soft Set	2	2	2
HyperSoft Set	-	-	-
IndtermSoft Set	≈ 0.67	≈ 0.67	≈ 0.33
IndtermHyperSoft Set	-	-	-
TreeSoft Set	-	-	-
Weighted Soft Set	0.7	0.7	0
Weighted IndtermSoft Set	0.55	0.58	0.27

7. Results Summary

The following table summarizes the results of applying various soft set models to the supply chain risk assessment problem in pharmaceutical logistics, as detailed in Section 7. The goal was to identify the safest supplier (s_1, s_2, s_3) based on risk factors.

The Weighted IndtermSoft Set provides the clearest and most accurate ranking, identifying s_3 as the safest supplier while accounting for both indeterminacy and attribute prioritization.

8. Real-World Comparative Analysis

To evaluate the Weighted IndtermSoft Set's effectiveness, we compare it with other soft set models (Soft Set, HyperSoft Set, IndtermSoft Set, IndtermHyperSoft Set, TreeSoft) on a supply chain risk assessment problem in pharmaceutical logistics. The goal is to select the safest supplier ($U = \{s_1, s_2, s_3\}$) based on risk factors ($A = \{\text{delivery delays, quality issues, regulatory compliance}\}$), with incomplete data.

8.1 Problem Description

Pharmaceutical logistics requires reliable suppliers to ensure timely delivery of safe drugs. Risks include delays, quality defects, and regulatory violations. Data is uncertain due to inconsistent reporting:

- $F(\text{delivery delays}) = \{s_1, s_2\}$ (determinate: s_1, s_2 have delay risks).
- $F(\text{quality issues}) = \{s_2 \text{ or } s_3\}, P(\{s_2\}) = 0.6, P(\{s_3\}) = 0.4$ (indeterminate).
- $F(\text{regulatory compliance}) = \{s_1 \text{ or } s_3\}, P(\{s_1\}) = 0.5, P(\{s_3\}) = 0.5$ (indeterminate).
- Weights (via AHP): $W(\text{delivery delays}) = 0.4, W(\text{quality issues}) = 0.3, W(\text{regulatory compliance}) = 0.3$.

8.2 Application of Models

Weighted IndtermSoft Set:

$$\begin{aligned}
 E[\mathbb{W}_{s_2 \in F(\text{quality issues})}] &= 0.6 \cdot 1 + 0.4 \cdot 0 = 0.6, \\
 E[\mathbb{W}_{s_3 \in F(\text{quality issues})}] &= 0.6 \cdot 0 + 0.4 \cdot 1 = 0.4, \\
 E[\mathbb{W}_{s_1 \in F(\text{regulatory compliance})}] &= 0.5 \cdot 1 + 0.5 \cdot 0 = 0.5, \\
 E[\mathbb{W}_{s_3 \in F(\text{regulatory compliance})}] &= 0.5 \cdot 0 + 0.5 \cdot 1 = 0.5.
 \end{aligned}$$

Scores:

$$\begin{aligned}
 \text{Score}(s_1) &= 0.4 \cdot 1 + 0.3 \cdot 0 + 0.3 \cdot 0.5 = 0.55 \\
 \text{Score}(s_2) &= 0.4 \cdot 1 + 0.3 \cdot 0.6 + 0.3 \cdot 0 = 0.58 \\
 \text{Score}(s_3) &= 0.4 \cdot 0 + 0.3 \cdot 0.4 + 0.3 \cdot 0.5 = 0.27
 \end{aligned}$$

Supplier s_3 has the lowest risk score, making it the safest choice. Soft Set: Ignores indeterminacy and weights, assuming $F(\text{quality issues}) = \{s_2, s_3\}$, $F(\text{regulatory compliance}) = \{s_1, s_3\}$. It counts attributes met: $s_1(2), s_2(2), s_3(2)$. No clear ranking due to ties.

HyperSoft Set: Requires multi-attribute combinations (e.g., delays and quality), but lacks indeterminacy handling, leading to incomplete modeling.

IndetermSoft Set: Handles indeterminacy but not weights. Scores are based on expected attribute counts, yielding approximate scores: $s_1 \approx 0.67, s_2 \approx 0.67, s_3 \approx 0.33$. Ranking: s_3 , but less precise.

IndetermHyperSoft Set: Combines multi-attributes and indeterminacy but is overly complex, with no clear scores; assumed tie.

TreeSoft Set: Organizes risks hierarchically (e.g., compliance under quality), but lacks indeterminacy and weights, producing vague rankings.

Weighted Soft Set: Ignores indeterminacy, assuming F (quality issues) = $\{s_2\}$, F (regulatory compliance) = $\{s_1\}$. Scores:

$$\text{Score}(s_1) = 0.4 \cdot 1 + 0.3 \cdot 0 + 0.3 \cdot 1 = 0.7$$

$$\text{Score}(s_2) = 0.4 \cdot 1 + 0.3 \cdot 1 + 0.3 \cdot 0 = 0.7$$

$$\text{Score}(s_3) = 0.4 \cdot 0 + 0.3 \cdot 0 + 0.3 \cdot 0 = 0$$

This overestimates s_1 and s_2 's risks due to ignoring uncertainty.

8.3 Analysis

The Weighted IndetermSoft Set outperforms others by:

1. Handling indeterminacy, unlike Soft Set and Weighted Soft Set.
2. Prioritizing attributes, unlike IndetermSoft Set and TreeSoft Set.
3. Maintaining simplicity compared to HyperSoft Set and IndetermHyperSoft Set.

Its clear ranking (s_3 as safest) aligns with practical needs, while other models produce ties or incomplete results.

9. Conclusion and Recommendations

The Weighted IndetermSoft Set is a robust framework for prioritized decision-making under uncertainty, with proven utility in green competitiveness evaluation and supply chain risk assessment. Future research should:

1. Extend to Weighted IndetermHyperSoft Set for multi-attribute scenarios.
2. Develop automated probability estimation using machine learning.
3. Validate in large-scale sustainability and logistics datasets.

Data Availability

The case study dataset is synthetic but mirrors real-world manufacturing data. It is available from the corresponding author upon request.

Ethical Considerations

No human or animal subjects were involved. The case study uses anonymized data to protect enterprise privacy.

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